Relational Calculus

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Formal Definition

The general form of the relational calculus is:

$$\{t|P(t)\}$$

t is a free variable. P is the formula. A tuple variable is a *free variable* unless it is quantified by $\exists, \forall$. If quantified, it is *bound variable*. Free variable are

- not in scope of any quantifier
- free variables are the parameters of the formula

$$t \in Purchases \wedge \exists c \in Customers[t(custid) = c(custid)]$$

Atoms

- $t \in R$, where t is a tuple variable, R is a relation
- $t(x)\Theta u(y)$, where t, u are tuple variables, x, y are the attributes. Θ is comparison operator $(<, \leq, =, \neq, \geq, >)$.
- $t(x)\Theta c$, where c is a constant.

Formulae

- An atom P is a formula
- $\neg P, P_1 \wedge P_2, P_1 \vee P_2$
- $P_1 \longrightarrow P_2$
- $P(t)$ contains a free tuple t, and.

$$\exists t \in R[P(t)]$$
$$\forall t \in R[P(t)]$$

Equivalence

- $P_1 \wedge P_2 \equiv \neg(\neg P_1 \vee \neg P_2)$
- $\forall t \in R[P(t)] \equiv \neg \exists t \in R[\neg P(t)]$
- $P_1 \rightarrow P_2 \equiv \neg P_1 \vee P_2$

Each unicorn has a horn.

$$\forall a \in Animals, a \text{ is a unicorn} \rightarrow a \text{ has a horn}$$
$$\forall a \in Animals, a \text{ is a unicorn} \wedge a \text{ has a horn}$$

Some horses have horns.

$$\exists a \in Animals, a \text{ is a horse} \wedge a \text{ has a horn}$$

Attributes binding

Schema: Books (bookid, title, author, year)
When we need all attributes:

$$\{t | t \in Books\}$$

When we need to subset of attributes:

$$\{t : title | \exists b \in Books[b(title) = t(title)]\}$$

Note that the tuple relational calculus does not have any equivalent of the aggregate operation, but it can be extended to support aggregation. Extending the tuple relational calculus to handle arithmetic expressions is straightforward.

Consider the following database schema for a BOOKSTORE database:

- Books (bookid, title, author, year)
- Customers (customerid, name, email)
- Purchases (customerid, bookid, year)
- Reviews (customerid, bookid, rating)
- Pricing (bookid, format, price)

Find books (show their titles) written by ’EDMUND MORGAN’ since year 1990.

SELECT title
FROM Books
WHERE author = ‘EDMUND MORGAN’
AND year $>=$ 1990;

SELECT title
FROM Books
WHERE author = ’EDMUND MORGAN’
AND year >= 1990;

$\{r : \text{title} \mid \ldots\}$

SELECT title
FROM Books
WHERE author = 'EDMUND MORGAN'
AND year >= 1990;

$\{r : \text{title} \mid \exists b \in \text{Books}[...]\}$

SELECT title
FROM Books
WHERE author = ’EDMUND MORGAN’
AND year $>=$ 1990;

$\{r : \text{title} \mid \exists b \in$
$\text{Books}[b(\text{author}) = \text{‘EDMUND MORGAN’} \wedge b(\text{year}) \geq 1990]\}$
DONE?

SELECT title
FROM Books
WHERE author = 'EDMUND MORGAN'
AND year >= 1990;

$$\{r : \text{title} \mid \exists b \in \text{Books}[b(\text{author}) = \text{‘EDMUND MORGAN’} \wedge b(\text{year}) \geq 1990 \wedge r(\text{title}) = b(\text{title})]\}$$

What are the titles of the newest books?

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$$\{r : \text{title} \mid \exists b \in \text{Books}[\ldots]\}$$

What are the titles of the newest books?

$\{r : \text{title} \mid \exists b \in \text{Books}[\forall o \in \text{Books}[\ldots]]\}$

What are the titles of the newest books?

$\{r : \text{title} \mid \exists b \in \text{Books}[\forall o \in \text{Books}[b(\text{year}) \geq o(\text{year})]]\}$
DONE?

What are the titles of the newest books?

$$\{r : \text{title} \mid \exists b \in \text{Books}[\forall o \in \text{Books}[b(\text{year}) \geq o(\text{year})]$$
$$\land b(\text{title}) = r(\text{title})]\}$$

What are the titles of the newest books?

$$\{r : \text{title} \mid \exists b \in \text{Books}[\neg\exists o \in \text{Books}[b(\text{year}) < o(\text{year})] \\ \wedge b(\text{title}) = r(\text{title})]\}$$

What are the titles of the newest books?

$$\{r : \text{title} \mid \exists b \in \text{Books}[\neg \exists o \in \text{Books}[b(\text{year}) < o(\text{year})]$$
$$\wedge b(\text{title}) = r(\text{title})]\}$$

```
SELECT b.title
FROM Books b
WHERE ...
```

What are the titles of the newest books?

$$\{r : \text{title} \mid \exists b \in \text{Books}[\neg\exists o \in \text{Books}[b(\text{year}) < o(\text{year})]$$
$$\wedge b(\text{title}) = r(\text{title})]\}$$

```
SELECT b.title
FROM Books b
WHERE NOT EXISTS (SELECT *
  FROM Books o
  WHERE b.year < o.year
);
```

Example 3

What are the titles of the books which have been purchased by every Customer?

$$\{r\ {:}title \mid \exists b \in Books, \forall c \in Customers, \exists p \in \text{Purchases}[$$
$$c(customerid) = p(customerid) \wedge p(bookid) = b(bookid)$$
$$\wedge\, b(title) = r(title)$$
$$]\}$$

Example 3

$$\{r:title \mid \exists b \in Books, \neg\exists c \in Customers, \neg\exists p \in \text{Purchases}[$$
$$c(customerid) = p(customerid) \wedge p(bookid) = b(bookid)$$
$$\wedge\, b(title) = r(title)]\}$$

```
SELECT b.title
FROM Books b
WHERE NOT EXISTS (SELECT *
  FROM Customers c
  WHERE NOT EXISTS (SELECT *
   FROM Purchases p
   WHERE c.customerid = p.customerid AND
     p.bookid = b.bookid));
```

Exercise 1

Find books (show their titles, authors and prices) that are on ‘CIVIL WAR’ (i.e., the title field contains ‘CIVIL WAR’), available in ‘AUDIO’ format [1].

$$
\begin{aligned}
&\{t : title, author, price | \exists b \in Books, \exists p \in Pricing \\
&\quad [(b(title) = t(title) \wedge b(author) = t(title) \wedge p(price) = t(price)) \wedge \\
&\quad b(title) =' \%CIVILWAR\%' \wedge \\
&\quad p(bookid) = b(bookid) \wedge p(format) =' AUDIO' \\
&\ ]\}
\end{aligned}
$$

[1] We don’t need to get the of columns of Books t.

Exercise 2

Find the set of books purchased by 'JOHN CHAMBERS'

$$
\begin{aligned}
&\{t|t \in Books \wedge \exists p \in Purchases, \exists c \in Customers \\
&\quad [t(bookid) = p(bookid) \wedge c(customerid) = p(customerid) \wedge \\
&\quad c(name) = \text{'JOHN CHAMERS'} \\
&]\}
\end{aligned}
$$

Exercise 3

Find customers (show their names and email addresses) who purchased more than one book in year 2003.(Do it without aggregation)

$$
\begin{aligned}
&\{t : name, email \mid \exists c \in Customers \\
&\quad [(t(name) = c(name) \wedge t(email) = c(email)) \wedge \\
&\quad \exists p_1, p_2 \in Purchase[p_1 \neq p_2 \wedge p_1(customerid) = c(customerid) \wedge \\
&\qquad p_2(customerid) = c(customerid) \wedge \\
&\qquad p_1(year) = 2003 \wedge p_2(year) = 2003] \\
&\ ]\}
\end{aligned}
$$

Exercise onward

The schema about battleships and the battles they fought in:

- Ships(name, yearLaunched, country, numGuns, gunSize, displacement)
- Battles(ship, battleName, result)

Exercise 4

Which battleships launched before 1930 had 16-inch guns? List their names, their country, and the number of guns they carried.

$$\{t : names, country, numGuns | \exists s \in Ships[$$
$$t(name) = s(name) \wedge t(country) = s(country)$$
$$\wedge\, t(numGuns) = s(numGuns) \wedge$$
$$s(yearLaunched) < 1930 \wedge s(gunSize) = 16$$
$$]\}$$

Exercise 5

Which battleship(s) had the guns with the largest gun size?

$$
\begin{aligned}
&\{t \mid t \in \mathit{Ships} \wedge \\
&\quad \forall s \in \mathit{Ships}[t(\mathit{gunSize}) \geq s(\mathit{gunSize}) \\
&]\}
\end{aligned}
$$

Exercise 6

Which battleships had the guns with the second largest gun size. More precisely, find the ships whose gun size was exceeded by only one gun size, no matter how many other ships had that larger gun size.

$$
\begin{aligned}
&\{t|t \in Ships \wedge \\
&\quad \exists m \in Ships \Big[\forall s \in Ships[m(gunSize) \geq s(gunSize)] \wedge \\
&\qquad t(gunSize) < m(gunSize) \wedge \forall s' \in Ships[s'(gunSize) < m(gunSize) \\
&\qquad \rightarrow t(gunSize) \geq s'(gunSize)] \\
&\Big]\}
\end{aligned}
$$

Exercise 7

List all the pairs of countries that fought each other in battles. List them with the country that comes first in alphabetical order first[2].

$$
\begin{aligned}
\{t :& c_1, c_2 | t(c_1) < t(c_2) \wedge \\
& \exists s, s' \in Ships, \exists b, b' \in Battles \Big[\\
& s(country) = t(c_1) \wedge b(ship) = s(name) \wedge \\
& s'(country) = t(c_2) \wedge b'(ship) = s'(name) \wedge \\
& b(battleName) = b'(battleName) \\
\Big]\}&
\end{aligned}
$$

[2] may contains ted entries

Reference

1. "*Database Systems Concepts*" by Silberschatz, Korth and Sudarshan, 6th edition, McGraw-Hill.