Fol Excercse problems for practise

1 message

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FOL-Based Equivalence Formulas

1. Show that $\forall x(P(x) \lor Q(x)) \equiv (\forall x P(x)) \lor (\forall x Q(x))$.
2. Prove $\neg(\exists x P(x)) \equiv \forall x \neg P(x)$.
3. Simplify $\forall x(\neg P(x) \lor Q(x)) \equiv \forall x(P(x) \Rightarrow Q(x))$.
4. Prove $(\forall x P(x)) \lor Q \equiv \forall x(P(x) \lor Q)$.

Modus Ponens in FOL

5. Given $\forall x(P(x) \Rightarrow Q(x))$ and $P(a)$, derive $Q(a)$.
6. Prove $(\forall x(R(x) \Rightarrow S(x))) \land R(c) \Rightarrow S(c)$ using Modus Ponens.
7. If $\forall x(P(x) \Rightarrow Q(x)) \land \exists y P(y)$, prove $\exists y Q(y)$.
8. Given $\forall x(A(x) \land B(x) \Rightarrow C(x))$ and $A(a) \land B(a)$, derive $C(a)$.
9. Prove $(\forall x(L(x) \lor M(x))) \land \neg M(c) \Rightarrow L(c)$.

And Eliminations in FOL

10. From $\forall x(P(x) \land Q(x))$, derive $\forall x P(x)$.
11. Prove $(\forall x(R(x) \land S(x))) \Rightarrow (\forall x R(x))$.
12. Show $(\exists x(A(x) \land B(x))) \Rightarrow (\exists x A(x)) \land (\exists x B(x))$.

Forward Chaining in FOL

13. In a knowledge base $\forall x(P(x) \Rightarrow Q(x))$, $Q(c) \Rightarrow R(c)$, and $P(c)$, derive $R(c)$.
14. Given $\forall x(A(x) \Rightarrow B(x))$, $B(c) \Rightarrow C(c)$, and $A(c)$, prove $C(c)$.
15. Use forward chaining to derive $Q(c)$ from $P(c) \Rightarrow Q(c)$ and $P(c)$.
16. In a rule-based system with $\forall x(P(x) \Rightarrow (Q(x) \land R(x)))$ and $P(a)$, derive $Q(a)$ and $R(a)$.
17. Prove $\forall x(A(x) \Rightarrow B(x))$, $A(c) \Rightarrow B(c)$ using forward chaining.

Backward Chaining in FOL

18. Prove $\forall x(P(x) \Rightarrow Q(x)), Q(c)$ using backward chaining from $P(c)$.
19. Verify $S(c)$ from $\forall x(R(x) \Rightarrow S(x))$, $R(c)$.
20. Use backward chaining to derive $Q(a)$ from $Q(x) \Rightarrow R(x)$ and $R(a)$.
21. Prove $(\forall x(A(x) \Rightarrow B(x))) \land B(c) \Rightarrow A(c)$.
22. Verify $C(c)$ using backward chaining in $(\forall x(A(x) \Rightarrow C(x))) \land A(c)$.

Resolution in FOL

23. Resolve $P(a) \lor Q(a)$ and $\neg Q(a) \lor R(a)$ to derive $P(a) \lor R(a)$.
24. Prove $Q(c)$ from $\forall x(P(x) \lor Q(x)), \neg P(c)$.
25. Derive $R(x)$ from $\forall x(\neg Q(x) \lor R(x)), Q(c)$.
26. Resolve $(\forall x(\neg A(x) \lor B(x)))$ and $\neg B(a)$ to derive $\neg A(a)$.
27. Prove $Q(b) \lor R(b)$ from $P(a) \lor Q(a), \neg P(a) \lor R(a), Q(a)$.

Mixed and Advanced Problems

28. Convert $\forall x(P(x) \land Q(x)) \lor R(x)$ to Skolemized form.
29. Simplify $\exists x(A(x) \lor \neg A(x))$ using equivalence rules.
30. Use forward chaining to derive conclusions from $\forall x(P(x) \Rightarrow Q(x)), Q(x) \Rightarrow R(x), P(a)$.
31. Use backward chaining to verify $T(c)$T(c) in a system $T(x) \Rightarrow (A(x) \lor B(x)), \neg A(c)$.
32. Resolve$(\neg P(x) \lor Q(x)) \land (\neg Q(a) \lor R(a))$ to derive $R(a) \lor \neg P(a)$.

33. Show $\forall x(\neg P(x) \lor \neg Q(x)) \equiv \neg \exists x(P(x) \land Q(x))$.
34. Prove $\forall x(P(x) \Rightarrow Q(x))$, $P(c) \Rightarrow Q(c)$.
35. Simplify $\neg(\forall x A(x)) \lor (\exists y B(y))$.
36. Use forward chaining to derive $C(a)$ from $A(a) \Rightarrow B(a)$, $B(a) \Rightarrow C(a)$.
37. Prove $R(a)$ using resolution with $\forall x(P(x) \lor R(x)), \neg P(a)$.

Advanced Mixed Problems

38. Resolve $(\forall x(A(x) \lor B(x))) \land (\neg A(c))$ to derive $B(c)$.
39. Use backward chaining to verify $S(x)$ from $A(x) \Rightarrow S(x)$, $A(c)$.
40. Prove $\neg \exists x(\neg P(x)) \equiv \forall x(P(x))$.
41. Derive $Q(c) \lor R(c)$ from $\neg P(c) \lor Q(c), P(c) \lor R(c)$.
42. Simplify $\forall x(A(x) \lor \neg B(x)) \land (\forall y B(y))$.
43. Resolve $(\forall x(P(x) \lor Q(x)))$ and $\neg Q(a)$ to derive $P(a)$.
44. Prove $(\forall x(R(x) \Rightarrow S(x))) \land R(c) \Rightarrow S(c)$.
45. Use forward chaining to derive $C(a)$ from $A(a) \Rightarrow B(a), B(a) \Rightarrow C(a)$.
46. Prove $\forall x(A(x) \land B(x)) \Rightarrow \forall x A(x) \land \forall x B(x)$.
47. Convert to CNF: $\forall x(P(x) \lor Q(x)) \Rightarrow R(x)$. Hint: Rewrite the implication, eliminate universal quantifier, and distribute disjunction.
48. Convert to CNF: $\neg(\exists x(P(x) \land Q(x))) \lor R(a)$. : Use De Morgan's laws and convert the existential quantifier to universal.
49. Convert to CNF: $\forall x(\neg P(x) \lor (Q(x) \Rightarrow R(x)))$. Hint: Replace the implication and distribute disjunction over conjunction.
50. Convert to CNF: $(\forall x P(x)) \land (\exists y Q(y) \lor R(c))$. Hint: Skolemize the existential quantifier and standardize variables.
51. Convert to CNF: $\forall x(\neg Q(x) \lor \exists y R(x,y))$. Hint: Skolemize the existential quantifier and distribute logical operators.
52. Convert to CNF: $(\exists x P(x)) \Rightarrow \forall y(Q(y) \lor R(y))$. Hint: Replace implication, Skolemize, and standardize variables.
53. Convert to CNF: $\neg(\forall x(P(x) \Rightarrow Q(x)))$. Hint: Eliminate negation and implication, then apply standard CNF conversion steps.
54. Convert to CNF: $\forall x \forall y((P(x) \lor Q(y)) \Rightarrow R(x,y))$. Hint: Replace implication, Skolemize if necessary, and distribute.
55. Convert to CNF: $\exists x(\forall y(P(x,y) \lor Q(x)) \Rightarrow R(x))$. Hint: Carefully Skolemize and distribute disjunctions.
56. Convert to CNF: $\forall x \exists y(\neg P(x) \lor Q(y))$. Hint: Skolemize the existential quantifier and simplify using logical equivalences.

Unification Problems

57. Unify $P(A,x)$ and $P(y,B)$.Hint: Find substitutions for x and y.
58. Determine if $Q(f(A),B,z)$ and $Q(x,B,g(y))$ can be unified. If yes, find the unifier. Hint: Match function symbols and variables.
59. Unify $P(A,f(x))$ and $P(A,f(B))$. Hint: Focus on the arguments of the function.
60. Unify $R(x,g(A))$ and $R(f(y),g(z))$. Hint: Consider nested terms carefully.
61. Determine if $S(x,A,y)$S(x,A,y) and $S(g(B),A,g(z))$S(g(B),A,g(z)) can be unified. Hint: Compare xx and $g(B)$g(B), yy and $g(z)$g(z).
62. Unify $Q(x,y,z)$ and $Q(A,f(B),g(C))$.Hint: Derive substitutions for x,y, and z.
63. Determine if $P(f(A,B),x)$ and $P(f(y,z),g(A))$ can be unified. Hint: Focus on matching arguments of the function f.
64. Unify $R(A,g(x))$ and $R(y,g(B))$. Hint: Identify substitutions for x and y.
65. Can $P(f(x), g(A))$ and $P(f(A),g(y))$ be unified? Find the substitution if possible. Hint: Ensure consistent substitutions.
66. Determine if $S(f(x,A),B)$ and $S(f(y,z),w)$ can be unified. Hint: Resolve nested terms $f(x,A)$ and $f(y,z)$.

Lifting Problems

67. Apply the lifting principle to prove $\exists x P(x) \Rightarrow P(A)$. Hint: Instantiate x with a constant.

68. Demonstrate lifting by generalizing $P(A) \vee P(B)$ to $\forall x P(x)$. Hint: Show the relationship between instances and universal quantification.
69. Use lifting to prove $\forall x(P(x) \vee Q(x)) \Longrightarrow P(A) \vee Q(A)$. Hint: Substitute x with a constant and derive the conclusion.
70. Show that $\forall x(P(x) \wedge Q(x)) \Longrightarrow P(y) \wedge Q(y)$ using lifting. Hint: Replace x with y.
71. Apply lifting to derive $\exists y(P(y) \vee Q(y))$ from $P(A) \vee Q(A)$. Hint: Introduce an existential quantifier over instances.
72. Prove $\exists x(P(x) \wedge Q(x)) \Longrightarrow P(A) \wedge Q(A)$ using lifting. Hint: Instantiate x with A.
73. Use lifting to transform $P(A) \wedge P(B)$ into $\forall x P(x)$. Hint: Generalize from specific instances.
74. Apply lifting to validate $\forall x(P(x) \vee Q(x)) \Longrightarrow \exists y(P(y) \vee Q(y))$. Hint: Use a general-to-specific reasoning.
75. Show that $\forall x(P(x) \wedge R(x)) \Longrightarrow R(y)$ using lifting. Hint: Replace x with y.
76. Demonstrate lifting for $\exists x P(x) \vee \exists y Q(y) \Longrightarrow \exists z(P(z) \vee Q(z))$. Hint: Use existential generalization.

Thanks & Regards

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