$$\text{New value} = \text{old value} - \frac{\text{corr. key col. val.} \times \text{corr. key row value}}{\text{pivot element}}$$

for optimality condition

for max $\forall\ c_j - z_j \leq 0$

for min $\forall\ c_j - z_j \geq 0$

Our objective is maximize $(c_j - z_j)$

↦ select maximum value.

z_j

Simplex Method

Maximize $Z = 12x_1 + 16x_2$

Subject to

$$10x_1 + 20x_2 \leq 120$$
$$8x_1 + 8x_2 \leq 80$$

Slack variable S_1 and S_2.

$$10x_1 + 20x_2 + S_1 = 120$$
$$8x_1 + 8x_2 + S_2 = 80$$
$$Z = 12x_1 + 16x_2 + 0S_1 + 0S_2$$

Initial Simplex Table.

C_j		12	16	0	0		
CB_i	Basic variable	x_1	x_2	S_1	S_2	RHS	Ratio
0	S_1	10	20	1	0	120	$120/20 = 6$
0	S_2	8	8	0	1	80	$80/8 = 10$
	Z_j	0	0	0	0	0	
	$C_j - Z_j$	12	16	0	0		

$$Z_j = \sum_{i=1}^{2} (CB_i) a_{ij}$$

$$(0 \times 10) + (0 \times 8)$$

Select the maximum value $(C_j - Z_j)$

Select the minimum value (least value)

Pivot element $\boxed{x_2 = 20}$

For optimality

- for max: $\forall\ C_j - Z_j \leq 0$.
- for min: $\forall\ C_j - Z_j \geq 0$.

Iteration 1

	c_j	12	16	0	0		
C_{Bi}	Basic variable	x_1	x_2	S_1	S_2	RHS	Ratio
16	x_2	1/2	1	1/20	0	6	12
0	S_1	4	0	−2/5	1	32	8
	Z_j	8	16	4/5	0	96	
	$c_j - z_j$	4	0	−4/5	0		

$\sum_{i=2} C_{Bi} * a_{ij}$

New val = old val − [Corres key col. × Corresp key row / Pivot elemen]

Optim

$8 - \frac{8 \times 10}{20} = 4$

$8 - \frac{8 \times 20}{20} = 0$

$0 - \frac{2 \times 1}{5} = -2/5$

$1 - \frac{8 \times 0}{20}$

1

$80 - \frac{8 \times 120}{20}$

$80 - 48 = 32$

$x_1 =$

Pivot element is 4

$-4/5 \times 4$ 2

Iteration 2

	c_j	12	16	0	0		
C_{Bi}	Basic variable	x_1	x_2	S_1	S_2	RHS	Ratio
16	x_2	0	1	1/10	−1/8	2	
12	x_1	1	0	−1/10	1/4	8	
	Z_j	12	16	2/5	1	128	
	$c_j - z_j$	0	0	−2/5	−1		

$\sum_{i=2} C_{Bi} * a_{ij}$

$\frac{16}{10} - \frac{12}{10}$

$-\frac{16}{8} + \frac{12}{4}$ $\frac{4}{10}$ $\frac{2}{5}$

32 + 96

128

$x_2 = 2$

$x_1 = 8$

$Z = 128$

≤ 0

$$\text{new val} = \text{old val} - \left[\frac{\text{corre Key col.} * \text{Key row}}{\text{pivot}}\right]$$

$$\frac{1}{2} - \frac{4 \times \frac{1}{2}}{4} = 0.$$

$$\frac{1}{20} - \frac{-\frac{2}{5} \times \frac{1}{2}}{4}$$

$$\frac{1}{20} + \frac{1}{20} = \frac{1}{10}$$

$$0 - \frac{1 \times \frac{1}{2}}{4} = -\frac{1}{8}$$

$$6 - \frac{\frac{16}{32} \times \frac{1}{2}}{4} \quad (2)$$

	x_1	x_2	S_1	S_2	RHS	
S_1	10	20	1	0	120	6
S_2	8	8	0	1	80	10
$-Z$	-12	-16	0	0	0	

my

Maximize $Z = 12x_1 + 16x_2$

Subject to

$10x_1 + 20x_2 \leq 120$

$8x_1 + 8x_2 \leq 80$

Select the most negative value in $-Z$ row.

Pivot is 20

$S_1 = S_1/\text{pivot}$ $\quad [\frac{1}{2}, 1, \frac{1}{20}, 0 \mid 6]$

$S_2 \leftarrow S_2 - 8S_1$

$\leftarrow S_2 - (S_1 \times 8)$ (pivot) $\quad [8, 8, 0, 1 \mid 80] - [4, 8, 2/5, 0 \mid 48]$

$[4, 0, -2/5, 1 \mid 32]$

		x_1	x_2	S_1	S_2	RHS	Ration
16	x_2	1/2	1	1/20	0	6	12
	S_2	4	0	-2/5	1	32	8
	$-Z_j$	-4	0	4/5	0	96	

$-Z \leftarrow -Z + 16 * S_1$

$-12 + \cancel{8}\,16 \times \frac{1}{2}$ $\quad | \quad -16 + 16 \times 1$ $\quad 0 + 16 \times 0$

$0 + 16 \times \frac{1}{20}$ $\quad 0 + 16 \times 6$

pivot 4 x_1

$S_2 \leftarrow S_2/\text{pivot}$

Basis	x_1	x_2	S_1	S_2	RHS.
x_2	0	1	$1/10$	$-1/8$	2
x_1	1	0	$-1/10$	$1/4$	8
$-Z_j$	0	0	$2/5$	1	128

positive.

$x_2 \leftarrow x_2 - \frac{1}{2} S_2$

$\left[\frac{1}{2} \quad 1 \quad \frac{1}{20} \quad 0 \quad \middle| 6\right] - \frac{1}{2}\left[1, 0, -\frac{1}{10}, \frac{1}{4}, 8\right]$

$-Z_j \leftarrow -Z_j + 4S_2$

$0 + 4x$

$-4 + 4x1$

$0 + 4x0$ $\quad \frac{4x2}{5x2} + 4x\frac{-1}{10}$

$96 + 4x8$

$96 + 32 = 128$