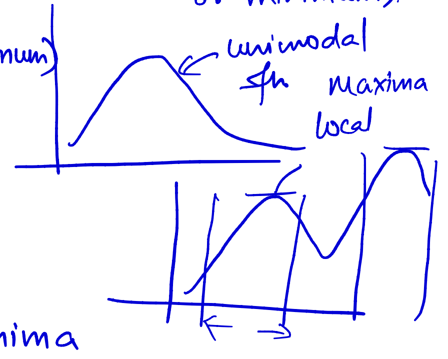
Golden Section search

→ find the minimum & maximum of a unimodal fn.

→ don't required derivative.

Single peak (maximum or minimum).

Single Extermum (one extermum (either in maximum or minimum) with in a specific interval

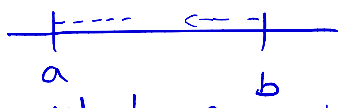


Monotonicity ← either non-decreasing or non-increasing

No other local Extrema.

no ~~others~~ local maxima or minima in the interval other than the single peak or trough.

→ fix the bound is computing $[a, b]$



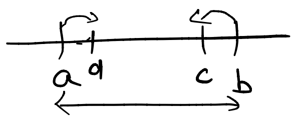
bracketing Method.

→ it reduces the search interval by successively narrowing it down using the golden ratio. (τ)

$$\tau = \frac{\sqrt{5}-1}{2} \approx 0.6180339887 \text{ (golden ratio conjugate)}$$

Algorithm

I/p unimodal f, bracket $[a, b]$, tolerance $\varepsilon > 0$



$c = b - \tau(b-a)$

$d = a + \tau(b-a)$.

Evaluate $f(c)$ and $f(d)$

if $[f(c) \leq f(d)]$: the minimizer lies in $[a, d]$

Set $b \leftarrow d$, $d \leftarrow c$ and comute new $d = b - \tau(b-a)$

old $f(d)$ as the old $f(c)$; $f(c)$.

else minimizer lies in $[c, b]$

Set $a \leftarrow c$, $c \leftarrow d$ and compute new $d = a + \tau(b-a)$

$(b-a) < \varepsilon$ → stop

Number of iteration

$L_0 = b_0 - a_0$

$L_n = \tau^n L_0$

$L_n \leq \varepsilon$

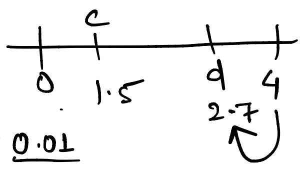
$$\log(L_n) = n\log(\tau) + \log(L_0)$$

$$\log(\varepsilon) = n\log(\tau) + \log(L_0)$$

$$\frac{\log(\varepsilon) - \log(L_0)}{\log(\tau)} \Rightarrow \left\lceil \frac{\log(\varepsilon/L_0)}{\log \tau} \right\rceil$$

$$n \geq \left\lceil \frac{\log(\varepsilon/L_0)}{\log \tau} \right\rceil$$

Minimize $f(x) = (x-1.5)^2 + 1$.



$[a, b] = [0, 4]$ $\varepsilon = 0.01$

$c = b - 0.618 \times 4$

$= 4 - 0.618 \times 4 = 1.5278$

$d = a + \tau(b-a) = 0 + 4 \times 0.618$

$= 2.472$

$f(c) = 1.000$

$f(d) = 1.945$ $f(c) < f(d)$

Fibonacci Search optimization

→ derivative free

→ unimodal fn.

→ closed interval.

fixed number of iteration

N $\boxed{F_N \geq \frac{b_0 - a_0}{e}}$

F_K is the K^{th} fibonacci number $(1, 1, 2, 3, 5 \dots)$

$$|b-a| \leq \frac{b_0 - a_0}{F_N} \leq \varepsilon$$

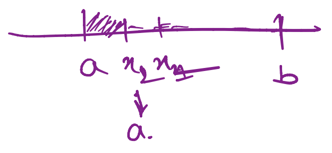
$F_1 = F_2 = 1$

choose the smallest N with $F_N \geq (b-a)/e$.

minimization

$$x_1 = a + \frac{F_{N-2}}{F_N}(b-a), \quad x_2 = a + \frac{F_{N-1}}{F_N}(b-a)$$

$f_1 = f(x_1)$ $f_2 = f(x_2)$



$K = 1, 2, \dots N-2$

if $f_1 > f_2$

$a = x_1$

$x_1 = x_2$ $f_1 = f_2$

$$x_2 = a + \frac{F_{N-K-1}}{F_{N-K}}(b-a)$$

else $f_2 \geq f_1$

Set $b = x_2$ (discard right piece)

Reuse x_1 Set $x_2 = x_1$ $f_2 = f_1$

$$x_1 = a + \frac{F_{N-K-2}}{F_{N-K}}(b-a).$$

then compute f_1