

Notes for Graduate Lecture in Cognition, Mathematics and Information S1A

CHEM, Xudong

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1 Lesson 7: Notes for Quiz

1. 意思決定論: 多分野 (数学, 心理学, 統計学, 経済学)
2. 規範的研究と記述的研究
 - 相違
 - いつから記述的研究がはじまったか
3. 意思決定論を学ぶ理由・メリット：合理・不合理を知り，合理的判断をするため
4. 選択・判断における合理とはなにか：ロジック的に矛盾がない。基準が統一。
5. 類似性による判断への影響
 - Gamblers' fallacy: ランダムに対する無意識 (自我矯正があると思い込む)
 - Tom W. Study: 人物像情報 (ステレオタイプ) による判断が確率情報の上に行く
 - 既存経験・知識による判断：過去あったことから目下の状況を考えてしまう。
 - Representativeness Heuristic and Bayes' Theorem(事前確率の考慮)..
6. 類似性による判断の誤差を避けるために
 - 類似点と相違点をリストアップし，表層的な類似か深層的な類似かが判断できる。
 - 過去の成功と現在の状況の類似と相違を考え，高い類似度と低い相違の選択肢を選ぶ。
7. Heuristic
 - Availability heuristic : 心理的にアクセスしやすい要素を判断の基準にする。
最近おきたこと，メディアに取り上げられた事件，自分の経験したこと， **Hindsight bias**
 - Numerosity heuristic : 量的判断の不確かさ，直感と現実の差が大きい，直感の正確さへの過信
 - Planning Fallacy : 計画・計画が順調に進むことに着目し，可能な失敗に目をそらす。過信による有用情報の検出の失敗。
8. Heuristic の罠
 - Salient な情報が Heuristic を左右する
 - Here and Now による感覚の過度な影響が判断の誤差につながる
9. 効用理論とその発展
 - (a) St. Peterburg Paradox と Bernoulli's EU
 - St. Peterburg Paradox :

コインのギャンブルの収益が無限大，人が無限大の金を投入し参加するはずだがそうしない。なぜか？
 - Expected Utility における金額増の効果が減っていく (対数関数に近い)
人間の金額に対する敏感がその増加につれ失われる。

初期所持金も EU に影響する。お金持ちにとっての 1w が貧乏人にとっての 1w の

Utility が異なる。

- 人が Maximum Expected Utility を選択するのが合理的
- $eu(x) = p \times u(x) = p \times \log(W + x)$

(b) von Neumann & morgenstern's EU

- $EU(A) > EU(B)$ iff A is preferred than B

Compared with $EU(A) > EU(B)$ then prefer A than B (Bernoulli's EU)

- 7 Choice Axioms : 人間の判断はこの基準から逸脱する

(Tversky, 1967: Intransitivity)

(Allais' paradox: contradiction with independence)

(c) Allais' paradox と Savage's SEU

- Allais' Paradox: EU の理論では矛盾が生じる
- SEU: 客観的な確率ではなく主観的確率

(d) SEU では確率的要素のない Ellsberg paradox の矛盾を解決できない

10. Prospect Theory(Kahneman and Tversky 1979)

(a) Ellesberg paradox : 曖昧な選択肢を回避する

(b) Allais' and Ellesburg paradox から, 規範的研究から記述的研究 :

人間の意思決定の不条理の記述の研究パラダイムがここからはじまる

(c) 早期の理論背景 (記述的研究)

- リスク回避 (in gain) とリスク追求 (in loss)(Markowitz)
- 選択の重要度, 確率の重要度が各選択で異なる (確率にもウェイトがある)

Ellesberg と矛盾しない (曖昧でないほうの確率の重要度が高い?)

(質的変化には敏感, 質的な変化が起こる選択肢の決定が重要)(Edwards, 1954)

(d) Decision Theory の 3 の重要な側面

- Value function(価値のウェイト)
- Probability function(起こる確率のウェイト)
- Combination

(e) Prospect Theory

- Probability Weighting function(Gonzale and Wu 1999) : Allais を解決
- Reference Point(参照点) : gain でのリスク回避と loss でのリスク追求現象の説明。

(f) Reflection effect(参照点による判断の矛盾) と Framing effect(言い方・着目点による判断の矛盾)

11. Dual Process(Kahneman 2002)

- System 1: 直感的, 経済的, 自動的, 速い処理。
- System 2: 論理的思考, 熟慮
- 1/10 と 9/100 の確率であったるくじ引きでは, 結果が得か損かにかかわらず, 人は 9/100 のほうを好む。

この傾向の原因は人の System 1 が 9/100 を benefit と結びつけて考えるからである (Implicit Association Test でわかった)

- Affect Heuristic: 単一個体への同情がグループへの同情が働きやすい (Slovic らの一連の研究)

- Somatic Marker Hypothesis

人は経験を生理的反応・状態と結びつけ, 二度目にその経験に遭遇するとき生理的に自動的に

反応する

Iowa gamble task で人は損失を食らった経験により生理的反応を更新し、戦略を変える。

- Frontal Lobe patients:

Somatic Marker が作用しない

2 Lesson 1: Trailer

2.1 Introduction to decision sciences

- An interdisciplinary field:

mathematics, psychology, economy, statistics involved.

- Decision science research:

Normative research - what one should decide?

Descriptive research - how people decide?

- Choice, prose & walking:

Everybody does but not actually in a correct way.

- Why study decision making:

1. A rational decision maker acts like weight-ordering system: shows no contradiction.

2. Human choices systematically deviate from rational prescriptions.

3. So decision science is to let you know when s/he is likely to make irrational choices(to make decision wisely.)

- *An intro from net:*

Decision Science is the collection of quantitative techniques used to inform decision-making at the individual and population levels. It include decision analysis, risk analysis, cost-benefit and cost-effectiveness analysis, constrained optimization, simulation modeling, and behavioral decision theory, as well as parts of operations research, microeconomics, statistical inference, management control, cognitive and social psychology, and computer science. By focusing on decisions as the unit of analysis, decision science provides a unique framework for understanding public health problems, and for improving policies to address those problems.

2.2 3 examples

- Given a)mean IQ = 100, b)IQ of 1 person out of 50 = 150. Guess the mean of the 50. The core answer is 100(should be 101 under a rational consideration)
- Guess how many animals of each kind did Moses take on the Ark. The core answer is 2.
- Stimulus list of names: more male names or female?

What we can see: Incompatibility between logic and human choice. humans are not like:

1. video cameras. don't record physical changes.
2. search engines. recalled contents shows distortions.
3. weather forecasts. make mistakes in probability assessment.

2.3 Rationality in choice

- A rational choice agent shows no contradiction over infinite opportunities to choose.
- Rational Decision making shows no effect among different ways to make logically equivalent choices.
- **No contradiction** in the sense of weight measurement systems.

2.4 Tom W. Study (Kahneman and Tversky 1972): a psychology assessment

- Tom W.'s personality described:
 1. High intelligence
 2. Lacking creativity
 3. etc.(see wiki)
- Nine majors ranked by the estimated proportion of freshmen
- Answer what's his major should be. CS is the most probably chosen major, while education is the least.
 1. Similarity group: based on personality = high positive correlation
 2. Base-rate group: based on rank = middle negative correlation
 3. Conclusion: people make predictions based on similarity, rather than base rate information.

2.5 Ellsberg paradox

The bucket contains 90 marbles. Among them, 30 are Red(others are Yellow / Blue). Given 4 choices:

1. F: Win 1w if you drew a R.
2. G: Win 1w if you drew a Y.
3. H: Win 1w if you drew R/B.
4. I: Win 1w if you drew B/Y.

In FG, F is preferred; in HI, I is preferred.

- A preference for F based on the **expected value** implies:

$$p(R) > p(Y)$$

- A preference for I implies:

$$p(Y) + p(B) > p(R) + p(B)$$

- Contradiction!(Irrationality; People tend to avoid ambiguity)

2.6 St. Petersburg paradox

Bernoulli wondered why people were obsessed with gambling even if every gamble offers a negative expected value and in the long run a gambler must lose money.

- (A coin game with) An infinite prospect vs. low probability. people seem willing to pay money (= expected value) to enter the game but actually they don't.

3 Lesson 2: Similarity and judgement

3.1 Examples

1. Is Linda(majoring philosophy) a Feminist or a bank teller or a feminist?

bank teller: feminist bank teller is often chosen though a lower probability.

2. Tom W. Study(cf. 2.4)

3. A family Survey: FFFMMM or FFMFMF:

FFMFMF is more easily chosen, FFFMMM is too systematic even if FFFMMM is more probable.

4. Rocket hit a random target:

Citizens thought that a secret military is more probably hit.

5. Ipod shuffle:

People feel that the less random is more random.

6. **Gamblers' fallacy**: “random sequence corrects it self.” But self-correction don't occur.

- Pirre-Simon Laplace(1796): The boys already born would increase the chance of the births of girls next time.
- Mihajlo Petrobić: win next game after a lost
- Italian tragedy in 2005: for 2 years, number lotteries did not result 53. So people thought 53 would come. Consequently, in February, Italian lottery sales tallied 671 million euro. A man shot his wife and sons and suicide., etc..

3.2 Numbness with randomness

1. Human information processing mechanism is selectively sensitve to environmental patterns and orderings.

2. Consequently we don't have natural sense of “randomness”

3.3 Heuristic

1. Heristic refers to thinking strategies by simplifying the problem and reaches an **approximate** solution
2. Contrasted to a **algorithm**, Heristic is not totally correct.

3. Heristic - estimate.
4. Representativeness heuristic:
 - We make judgements of probability based on **similarity** to our expectations rather than other rules.
 - (Bayes' Theorem)
5. Representativeness and similarity:
 - When social categories appear in judgement, representativeness heuristics take the form of similarity judgements to stereotypes.
 - **People replace probability judgements with similarity judgements.**

3.4 Out of Psychology: seeing the past in the present

Do Superficial Features Affect Availability of Analogies?

Gilovich, T. (1981). Seeing the past in the present: The effect of associations to familiar events on judgments and decisions. *Journal of Personality and Social Psychology*, 40, 797-808.

FYI: This Gilovich study was not discussed in the Goldstein textbook.

- Basic structure of the scenarios:
Country A is threatening to invade its peaceful neighbor, Country B.
Neither country is strategically important. Country B has asked the U.S. for help.
- Question: Should the U.S. intervene to help Country B?

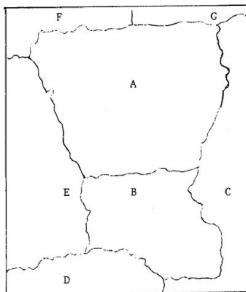


Figure 1. Map of crisis area.

Psych 355, Miyamoto, Spr '18

Two Prototypes of American Military Intervention 31

図 1: [Gilovich \(1981\)](#): p.803): Should the U.S. intervene? - Scenario 1 -

1. Pictures from the past shape our understanding of the present.

2. Should the U.S. intervene?([Gilovich 1981](#))

(Scenario 1):

- Nation A invaded B by Bilzkrieg
- Minorities in A fled to G via trains. A and D had a non-aggression pact
- U.S. troops via troop transports; Winston Churchill Hall

(Scenario 2):

- Nation A invaded B by quickstrike
- Minorities in A fled to G via small boats. A and D had a pact
- U.S. troops via Chinook helicopters

(some explanation):

- In early WW2, The U.S. did not intervene in international conflicts.
- They actively participated in Vietnam.
- Resemblance to either incident guide policy choice involving a new conflict.

3. Seeing the past in the present(cf. fig.1)

- The first scenario is similar with Nazi Germany's invasion to Poland

- The second scenario was similar to Vietnam
- Poli Sci majors who read 1 supported U.S. intervention, whereas those who read 2 did not.

3.5 Similarity affect our judgements: how to prevent

- **Similar vs. Dissimilar**
- One can calculate similarities between apples and oranges, Yamaha, BMW luxury cars, etc.
- By listing up common and distinct features, one can distinguish between profound and superficial similarity.

3.6 Conclusion: Similarity and rational choice

1. probability judgements via heuristic obey rules of similarity
2. When wishing to decide wisely, assess the present and dissimilarity between the present and success in the past
3. Take the option with high similarity and low dissimilarity.

4 Lesson 3: Memory and judgement(Heuristic)

4.1 Availability heuristic(Heuristic: cf. 3.3)

1. People make decisions based on how easily examples come to mind. (More famous ones come easily) The ease sometimes misguides judgements.
2. Examples
 - airline accidents are more common than motor vehicle accidents.
 - Tokyo undergraduates risk judgements:
Nagoya airline accidents(**a temporal factors**) food poisoning(**mass media coverage**) influenced the risk judgements.
 - Rating danger of risks such as traffic accidents, terrorism, nuclear powerplants and railway accidents:
Rating strengths were more consistent with **media coverage** (Soviet residents gave safest rating to nuclear power risks)
 - 9.11 attacks ([Gigerenzer 2006](#)):
people chose driving then airline after 9.11 though driving is more dangerous.
 - Wife and husband's self-reports of *proportion of household chores I'm doing*. ([Ross and Sicoly 1979](#))
Report more than s/he is actually doing.
 - **Hindsight bias** ([Fischhoff and Beyth 1975](#)): *I knew it all along.*
What will happen: Fight?
Before Nixon's China visit(**pre-estimates**) and after the visit(**post-recalling**) differs:
I predicted correctly

(other e.g., UK-EU case predictions and post-recalling)

4.2 Numerosity heuristic

- How many block units fill up a pizza.

When devided into more slices people estimate that more blocks needed.

Judgements via the numerosity heuristic.

- Which route is safer?

- A: 5 mines, 20% chance of explosion
- B: 10 mines 10% chance of explosion
- B is more safer but people tend to choose A.

- Confidence intervals: What was the height of Socrates?

An interval of 0-1000 cm contains the answer with 100% certainty, but is a useless estimates.

Try to estimate the following:

- The hottest temperature created by man
- The distance covered by the worlds' longest highway
- The ground temperature on Pluto
- etc.

Usually subjective 90% confidence intervals are correct for 0~3 out of 10

People show overconfidence for their correctness

4.3 Planning Fallacy: focus on successes

- Time for finishing honors ([Buehler et al. 1994](#)):

estimated time is 33.9 days. The actual time is 55.5.

- Bay of Pigs Invasion ([Wyden 1979](#)):

U.S. launched a civilian battle against Cuba, expecting that Cubans would willingly join to throw Socialism away, but failed.

Planners often focus exclusively on successes and overlook possible failures.

- Predicting college football ([Heath and Gonzalez 1995](#)):

Document reading group and *discussion* group both expressed their confidence to win.

- Both groups were overconfident
- Document* group examined reliable information and showed mild overconfidence
- Discussion* group was severely overconfident.

Overconfidence reflects difficulty to distinguish useful from useless pieces of knowledge.

4.4 Conclusion: Traps in Numerical Judgements

- Heuristics such as **Availability** or **Numerosity** indicate human susceptibility to salient information.
- Judgemental bias occur by over-generalization of the feeling of "Here and Now"

5 Lesson 4: Utility theory and its development

5.1 Bernoulli's solution for St. Petersburg paradox and Gamble metaphor

1. An infinite prospect in coin gamble.

2. St. Petersburg Paradox(cf. 2.6)

3. Bernoulli's solution, proposal and contribution

- Solution and proposal (expected utility function): see fig.2 and fig.3:

People turn insensitive to increasing value and infinite prospect is beyond human imagination.

A rational decision maker chooses the maximum expected utility.

- Contribution:

Bernoulli first proposed maximization as a rational choice rule.

This proposal was mathematician's educated guess.

4. Gamble Metaphor Gamble Metaphor is the standard analogy in decision theories.

Gambles consisting of unambiguous payoffs and probabilities serve like fruit files in genetics research.

- A rational decision maker chooses gambles offering the maximum expected utility.
- The calculation for each gamble with payoff x and the probability p is $p \times u(x)$.

Daniel Bernoulli's Solution

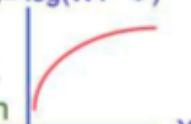
- ※ X ducats worth X ducats.
- ※ Humans' subjective worth is the Utility of X , or $u(X)$.
- ※ Utility reflects each person's asset.
- ※ The utility of 100 ducats differ between a pauper and a millionaire.

Bernoulli's proposal

- ※ $u(X) = \log(W + X)$, where W is the current asset.
- ※ $[X, p]$ denotes a gamble with probability p and payoff X . The expected utility is $p \cdot u(X)$
- ※ Expected utilities of gambles A and B are $EU(A)$ and $EU(B)$.
- ※ A rational decision maker chooses the gamble with maximum expected utility.

図 2: Bernoulli's solution

Utility function and human mind
 $u(Y) = \log(W + Y)$



- ※ For positive payoffs, human utility function is concave downward
- ※ The feeling of infinite wealth goes beyond human imagination.

$$U(Y) = \log(W + Y)$$

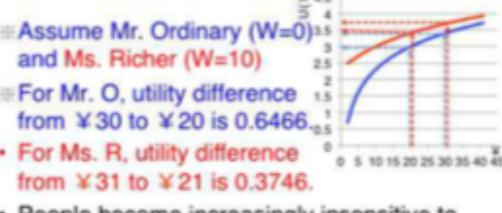
- ※ Assume Mr. Ordinary ($W=0$) and Ms. Richer ($W=10$)
 - ※ For Mr. O, utility difference from ¥30 to ¥20 is 0.6466
 - For Ms. R, utility difference from ¥31 to ¥21 is 0.3746.
 - People become increasingly insensitive to value increments/decrements as the values go further away from the origin.
 - Diminishing Marginal Sensitivity.
- 

図 3: Bernoulli's proposal

5.2 20C: von Neumann & Morgenstern's EU Theory and Choice axioms

1. Bernoulli suggested:

$$EU(A) > EU(B) \rightarrow A \succ B.$$

2. von Neumann and Morgenstern (1945) prove that, if choices adhere to certain requirements,
 $EU(A) > EU(B) \leftrightarrow A \succ B.$

Such decision makers are rational and produce no contradiction.

3. Choice Axioms

- Comparability
- **Transitivity:** $A \succeq B, B \succeq C \rightarrow A \succeq C.$
 - Behold the fashion. Human preferences violate transitivity(cf. 5.3).
 - A person prefers hotdogs to apples; grapes to hotdogs; but apples to grapes(violates transitivity).
- Closure
- Distribution of probability
- **Independence:** $A \succeq B \leftrightarrow A + C \succeq B + C.$
- Consistency
- Solvability

4. Axioms and Rationality:

- The choice axioms are identical to those guaranteeing the consistency of weight-measurement.
- Human psyche systematically deviates from requirements of the axioms.

5.3 Intransititivity of preference (Tversky 1967)

1. Preferences may violate transitivity if multiple criteria apply sequentially.
2. An example: admission in state univ's in the US.

2 criteria: test score and ethnical diversity

- If more than 1 point differences in test, than choose by test.
- If difference of score is within 1 point, then choose by diversity.

5.4 Allais' paradox and L.J. Savage's analysis

- X : Win 1w with probability 0.11

Y : Win 2w with probability 0.10

- K : Win 1w for sure

L : Win 2w with probability 0.10, win 1w with 0.89, otherwise nothing.

- Allais paradox is observed whenever $K \succ L$ while $Y \succ X$:

$K \succ L \leftrightarrow 1.0 \times u(1w) > 0.1 \times u(2w) + 0.89 \times u(1w)$ according to 3 and 4, with which $0.11 \times u(1w) > 0.1 \times u(2w)$ can be found.

However, $X \succ Y \leftrightarrow 0.11 \times u(2w) > 0.10 \times u(1w)$.

- L.J. Savage's analysis([Subjective EU Theory](#), see [5.5](#)):
 1. KL and XY differ only in the payoffs for 89%.
 2. Allais' paradox is a violation of [3](#)(see table.[1](#))

表 1: L.J. Savage's analysis

| | .89 | .01 | .1 |
|-----|-----|-----|----|
| K | 1w | 1w | 1w |
| L | 1w | 0 | 2w |
| X | 0 | 1w | 1w |
| Y | 0 | 0 | 2w |

5.5 Subjective expected utility theory and Savage's novelty

1. von Neumann & Morgenstern formulated EU Theory.
2. Savage extended EU by incorporating subjective probability.
3. Savage's Subjective EU Theory does not contradict with Allais' paradox
4. Savage's novelty:
 - In EU, the probability is the **objective** probability.
 - In SEU, the decision maker relies on Subjective Probability $S(p)$.
 - In general, $S(p) \neq p$.
 - SEU reduces to EU when $S(p) = p$.

5.6 Ellsberg paradox refutes Subjective EU Theory

Ellsberg paradox see [2.5](#). People tend to avoid ambiguity(**Ambiguity Aversion**):

1. The Ellsberg paradox indicate people's aversion of uncertainty without known probability.
2. Subjective probability fails to account for ambiguity aversion.
3. The Ellsberg paradox refutes Subjective EU Theory

6 Lesson 5: Prospect theory

6.1 Norms and descriptions

1. Before Allais and Ellsberg, decision analysis assumed rational humans.
2. The paradoxes uncovered human irrationality.
3. Modern decision science consists of Normative theories in search of **rationality** and Descriptive theories to account for human **choice behaviors**.

6.2 Early descriptive work: Markowitz, Edwards

6.2.1 **Markowitz**

The amount of money that an agent would have to receive to be indifferent between that payoff and a given gamble is called that gamble's **CERTAINTY EQUIVALENT(CE)**.

- $[+200; 0.5] \approx 73$
- $[-200; 0.5] \approx -86$
- The pain of losing X yen is larger than the pleasure of earning X yen.

6.2.2 Risk Seeking and Risk Averse(Markowitz)

- A sure gain of 1000 yen(1)
- A 50% chance to win 2000 yen(2)

1 tends to be chosen. In the domain of gains, people are RISK AVERSE.

- A sure loss of 1000 yen(3)
- A 50% chance to lose 2000 yen(4)

4 tends to be chosen. In the domain of losses, people are RISK SEEKING.

6.2.3 **Probability Weighting** and decision importance(Edwards, 1954)

1. Decisions are **not** based upon the strength of **Subjective Probability**.
2. $S(p)$ proposed by Savage denotes belief strength, Edwards' $w(p)$ denotes decision importance.
 - Decision importance(Belief and Weight). An example:
Imagine you are forced to play a Russian Roulette, how much you will pay to buy **1** bullet when **2** are loaded? Or when **last 1** is loaded?
Subjective probability decreases by $1/6$ in both situations, yet the **decision importance** differs between $2/6 \rightarrow 1/6$ and $1/6 \rightarrow 0$ (The latter means that you are safe)
3. $w(p)$ does **not contradict** with ambiguity aversion (Ellesberg paradox).

6.3 Three Aspects of Decision Theories and Modern Decision Theories

1. We have covered **Expected value**, **Expected Utility(EU)**, and **Subjective Expected Utility(SEU)** as normative theories.
2. For the descriptive side. **Markowitz** demonstrated **loss aversion**, and **Edwards** proposed **probability weighting**.
3. Aspects of Decision Theories:
 - Value function
 - Probability function
 - Combination
4. The development of Decision Theories is shown in Table.2.

Prospect theory combined Edwards' and Markowitz's theory.

表 2: Modern Decision Theories

| | 1.VALUE | 2.PROBABILITY | 3.COMBO | |
|------------------------|----------------|----------------------|-------------|-------------|
| Cardano(1663): EV(5.1) | $X = X$ | $p = p$ | Multiply | Normative |
| vN & M: EU(5.2) | Utility $u(X)$ | $p = p$ | Multiply | Normative |
| Savage(1954):SEU(5.5) | Utility $u(X)$ | $p = S(p)$ | Multiply | Normative |
| Markowitz(6.2.2) | Loss aversion | ? | ? | Descriptive |
| Edwards(6.2.3) | ? | Prob Weight $w(p)$ | ? | Descriptive |
| Prospect Theory(6.4) | Loss aversion | Prob Weight $\pi(p)$ | complicated | Descriptive |

6.4 Prospect Theory ([Kahneman and Tversky 1979](#))

Accounting for human decisions requires **value function**, **probability weighting**, and **Reference point**.

Among many theories, Prospect Theory is widely recognized by explaining the Allais' Paradox.

6.4.1 Probability Weighting Function ([Gonzalez and Wu 1999](#))

$$\pi(p) = \frac{\delta p^\gamma}{\delta p^\gamma + (1-p)^\gamma}$$

1. Function $\pi(p)$ overreacts to certainty, overvalues low probability and undervalues mid-high probability.
 - Parameter δ controls the intersection with $y = x$.
 - Parameter γ controls the curvature extremity of the inverse S .
2. Psychology of $\pi(p)$
 - Infants' understanding of uncertainty is Yes, No, and Maybe.
 - As they mature, they start understanding degrees of uncertainty.
 - Even grownups are sensitive to changes from $0 \rightarrow 0.1$ and $0.9 \rightarrow 1$, but not too sensitive to $0.35 \rightarrow 0.45$.
3. Explanations for Allais' Paradox(in 6.4.3)
 - Objective probability requires $0.89 + 0.11 = 1$,
 - The choices in Allais' Paradox imply $\pi(0.89) + \pi(0.11) < 1$,
 - $\pi(p) + \pi(1-p) \leq 1$ “subadditivity” is a characteristic of $\pi(p)$ according to 6.4.2.

6.4.2 Choice rules in Prospect Theory

- Premises:
 1. $\pi(0) = 0, \pi(1) = 1$.
 2. $\pi(p) + \pi(1-p) \leq 1$
 3. $V(x, 1.0) = V(x)$

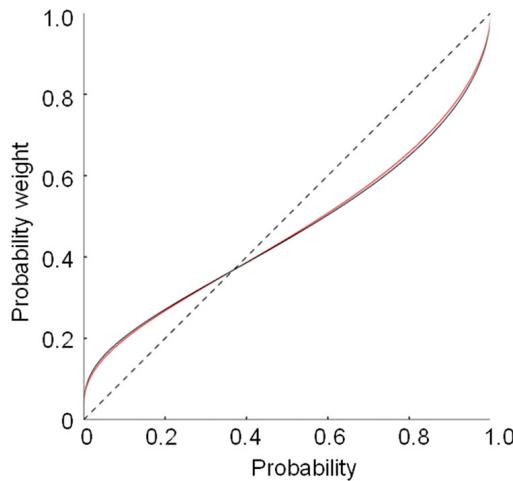


図 4: Probability Weighting Function

- If $p + q < 1$, or $X \geq 0 \geq Y$, or $X \leq 0 \leq Y$, then
 $V(X, p; Y, q) = \pi(p)V(X) + \pi(q)V(Y).$
- If $p + q = 1$, and $X > Y > 0$, or $0 > X > Y$, then
 $V(X, p; Y, q) = V(Y)\pi(p)(V(X) - V(Y)).$

6.4.3 Prospect Theory and Allais' Paradox(cf. 5.4)

$$\begin{aligned} \therefore K \succ L &\leftrightarrow V(1w) > \pi(.89)V(1w) + \pi(.1)V(2w), \\ \therefore K \succ L &\leftrightarrow \pi(.1)V(2w) > \pi(.11)V(1w), \\ \therefore V(1w) &> \pi(.89)V(1w) + \pi(.11)V(1w), \\ \therefore 1 &> \pi(.89) + \pi(.11). \end{aligned}$$

Shows no contradiction with the Premises in 6.4.2.

6.4.4 The equivalence of AB and CD and Reflection Effect

- Assume yourself richer by 3w yen.

Earn 1w for sure(A). $A[3+1, 1]$

Win 2w with the probability 0.5(B). $B[3+2, 0.5; 3, 0.5]$

- Assume yourself richer by 5w yen.

Lose 1w for sure(C). $C[5-1, 1]$

Lose 2w with $p = 0.5$ (D). $D[5, 0.5; 5-2, 0.5]$

- $\therefore A = C, B = D$. A rational decision maker selects between satisfying final asset positions.

If s/he chooses A, s/he will choose C. However, people tend to choose without such a balance(**Reflection Effect**).

- This Reflection Effect may caused by different **Reference Points** of DMs(see 6.4.5).

1. $A \succ B$: risk averse in gains.

2. $D \succ C$: risk seeking in losses.

3. Initial gains of 3w or 5w do NOT count, but **reference point shifts** (ref fig.5, red curve is steeper: Loss aversion).

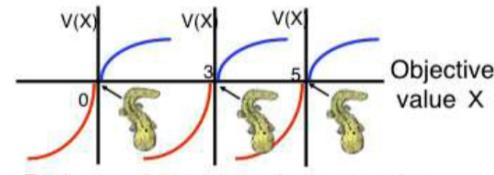


図 5: Reference point shift

6.4.5 Reference Point(Value Function)

- Human decision agents form reference point, such as desired goals, or status quo.
- They evaluate **gains** and **losses** relative to the reference point.
- Gains and losses prior to the reference point do NOT count. Therefore they do not maximize the final asset position.
- Goals as Reference Point(An example)

Bronze medalists happier than silver medalists in the Olympic ([Medvec et al. 1995](#)): Goals as reference point are different.

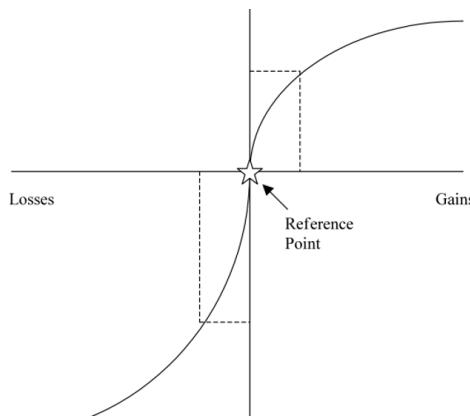


図 6: Reference Point

6.4.6 Framing Effect

1. Asian disease problem ([Kahneman and Tversky 1984](#)):

$A = C, B = D$, yet choice of expressions *will die* or *will be saved* triggers a **framing effect**.

See fig. 7.

- A : 200 people will be saved(72%) or
- B : 1/3 probability that 600 will be saved and 2/3 that nobody will be saved(28%)
- C : 400 people will die(22%) or
- D : 1/3 probability nobody die and 2/3 probability that 600 people will die(78%)

2. Framing Effect in Action ([Levin and Gaeth 1988](#))

- 75% lean ground meat(A) and 25% fat meat(B). $A = B$.

- A was evaluated as healthier and better quality.

In tasting, A was preferred.



図 7: Asian disease problem

7 Lesson 6: Dual Processes

7.1 Dual-Process Theory

- Kahneman(2002) proposed a dichotomy of cognitive functioning.
- System 1:
Quick, automatic, associative.
Often emotional, difficult to control consciously
- System 2:
Slow, serial, effortful.
Easy to control consciously. Rule-governed.

表 3: Dual System

| | Intuition System 1 | Reasoning System 2 |
|---------|--------------------|---------------------|
| Process | Quick | Slow |
| | Parallel | Serial |
| | Effortless | Effortful |
| | Associative | Rule-Governed |
| Content | Perception | Memory |
| | Sentation | Past-present-future |
| | | Recallable |

7.2 IAT accesses associative System 1

7.2.1 Denes-Raj and Epstein (1994): Prefer 9/100 than 1/10 regardless of ir/rationality
2 Gambles:

1. 1/10 probability to get a jackpot
2. 9/100 probability to get a jackpot

With positive payoffs. 2 is irrationaly choce.

With Negative payoffs. 2 is rationalily choce

7.2.2 Nishimura and Yamagishi (2006): Irrational choice in Gains and Rational choice in loss

Hypothesis: People showing Irrational choice in Gains and Rational choice in loss (the preference for 9/10) would associate the 9/100 with **benifits**.

- Use **IAT** (Implicit Association test): Respondents who readily associate 2 things together would choose the combination more quickly.
- People choose combination of the word *benifit* and the fixture of 9/100 more quickly than that they choose *benifit* and 1/10.
- The implicit association was stronger between 9/100 and *benifit* than 1/10 and *benifit*.
- **System 1 constantly prefers 9/100.**

7.3 The Affect Heuristic driven by System 1: Indifference with mass but act for one

1. **The affect heuristic** ([Slovic et al. 2002](#)):

- Under emotional stimulation, people start acting, driven by **System 1**.
- Imagery & Attention → Affect → Action.

2. People willingly help after natural disasters but ignore genocides.

- The U.S. citizens donated 738 Million dollars after Hurricane Katrina.
- After the 2004 Indian Ocean earthquake, U.S. citizens donated 239 million dollars.

3. Lack of interests in People, Affect do not extend too far ([Slovic et al. 2002](#))

- Affect has its limitations. One does not sympathize with the masses.

4. Rokia and the Mass in Malawi:

- Small et al.(2005) called for donation from their participants.
- Food shortage in Malawi is affecting **3 million** children. More than **11 million** in Ethiopia are starving.
- Some description of **Rokia**.
- People donated dollars to Rokia as 2 times as they donated to Malawi.

5. Västfjäll et al. (2008)

- People donated more to Rokia and Moussa respectively than to Rokia and Moussa as a pair.

6. A lot of Media used such kind of Human psychological characteristics.

- Texas refugees' poster of a single girl on Facebook.
- Single dead boy than mass: a news photo of Indian Ocean earthquake.
- Terror of War, Phan Thi Kim Phuc.
- Death of Aylan Kurdi.
- CNN shooting in Aleppo, Syria, 2018.8.27

7.4 Frontal Lobe patients, Somatic Marker Hypothesis(Damasio, 1996), Iowa gamble taskn

1. in 1848, Iron rod penetrated Phineas Gage's frontal lobe.
Kind and responsible Gage became short-tempered, impulsive of restraint.
Gage is no longer gage.
2. **Frontal lobe** is the most developed in Humans. It grows up in the last in life.
3. Frontal-Lobe patients
 - **Maintains** the **IQ** level prior to injury
 - **Memory disturbance**: what s/he has done, when, how
 - **Difficulties with planning**, behaving as planned, changing plans.
 - **Little self-control**: Acting like an infant.(If interested, keeps playing around with others' possessions)
4. **Somatic Marker Hypothesis**
 - Arousal and emotion usually accompany bodily **somatic responses**.
 - Somatic markers are **associations** between **stimuli** and **physiological affective state**.
 - Existing associations induce somatic responses.
 - **Somatic signals** are **twofold**: *Good* and *bad*.
 - Somatic markers are physiological **System 1** to encourage quick and functional decisions.
 - Example of somatic marker: Experience of nausea when eating a half-rot mackerel will **associate** pain and mackerel and **produce** a somatic marker.
In the next time eating a mackerel, somatic marker emit *danger!!!* signal.
 - Somatic markers may be regarded as experience-based System 1 choice heuristic.

5. Iowa gambling task(LGT), Frontal Coortex and Somatic Marker

- LGT ([Bechara et al. 1994](#)): 4card desks:
AB: Win more with a high probability and lose big number sometimes(loss in the long run)
CD: Win less with a high probability and lose small number sometimes(Win in the long run)
- Decision of **helthy participants**: Stick to the **AB** to win big number but after 10 turns change to choose **CD** because they found they would lose more if they choose **AB**
- Decision of **Patients with orbitofrontal cortex dysfunction**: They would keep shooting **AB**
- The patients never develop this physiological reaction to impending punishment.

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