目次

1 Lesson 1: Orientation 2

1.1 Process . . . 2

1.2 Knowledge requirement . . . 2

1.3 Intro . . . 2

2 Lesson 2: Preference, Efficiency, and Games in Normal Form(1) 3

2.1 Concepts and symbols on logic . . . 3

2.2 Concepts and symbols on set theory . . . 3

2.3 Preference of agents on outcomes . . . 3

2.4 Efficiency . . . 4

2.5 Games in normal form . . . 4

3 Lesson 3: Rationality Games in Normal Form(2) 5

3.1 Definition 1: Best replies(BR) . . . 5

3.2 Definition 2: Domimamt strategies(DS) . . . 5

3.3 Definition 3: Dominant strategy equilibria(DSE) . . . 6

3.4 Definition 4: Nash equilibria(NE) . . . 6

4 Lesson 4: Games in Extensive Form with perfect imformation 6

4.1 Definition: Games in Extensive Form . . . 6

4.2 Backward induction . . . 7

5 Lesson 5: How to change an extensive form to a normal form 7

5.1 Examples . . . 7

5.2 Subgame perfect equilibria . . . 9

6 Lesson 6: Option forms for confilict analysis 9

6.1 Defination . . . 9

6.2 Example . . . 10

6.3 Transformation of games in normal form into option form for conflict analysis . . . 11

6.4 Example of transformation . . . 11

7 Lesson 7: Graph model for confilict resolution(GMCR) 12

7.1 Defination . . . 12

7.2 Example . . . 12

8 Lesson 8: GMCR(2) and stability 12

8.1 Reachable list of agents . . . 12

8.2 Reachable lists of coalitions . . . 13

8.3 Unilateral improvement list of agents . . . 13

8.4 Unilateral improvement list of coalitions . . . 13

8.5 Equally or less preferred outcomes . . . 13
8.6 Stability concepts . . . 13
8.7 Definition 1 (Nash stability (Nash)) . . . 13
8.8 Definition 2 (General metarationality (GMR)) . . . 14
8.9 Definition 3 (Symmetric metarationality (SMR)) . . . 14
8.10 Definition 4 (Sequential stability (SEQ)) . . . 14

9 Lesson 9: Simple games, weighted majority games, and committees 14
9.1 Simple game . . . 14
9.2 Weighted majority game . . . 15
9.3 Group . . . 15
9.4 Committee . . . 15
9.5 Meeting . . . 15

10 Lesson 12: Compare power of coalitions 16
10.1 Desirability relation . . . 16
10.2 Other relation for comparing coalitions . . . 16
10.3 Power index . . . 16
10.4 Core of commitees . . . 18

11 19

1 Lesson 1: Orientation

1.1 Process

1. Form groups
2. Summary reports
3. 3 reports(background, modal, analysis)
4. Group presentation

1.2 Knowledge requirement

1. Game theory
2. Discrete mathematics
3. Set theory, logic, graph theory

1.3 Intro

1. Differences of coorporation and non-coorporation model
   coorporation model:
   non-coorporation model:
2. An example of how methematic models work

2 Lesson 2: Preference, Efficiency, and Games in Normal Form(1)

2.1 Concepts and symbols on logic

1. $p \vee q$: p or q
2. $p \wedge q$: p and q
3. $\neg p$: not p
4. $p \rightarrow q$: if p, than q.
5. $p \leftrightarrow q$: $p \rightarrow q$ and $q \rightarrow p$. p if and only if q(p iff q)
6. $(\forall x)(P(x))$: for all x, $P(x)$
7. $(\exists x)(P(x))$: there exists (at least one) x such that $P(x)$

2.2 Concepts and symbols on set theory

1. $X, Y, \ldots, A, B, \ldots$: sets
2. $\emptyset$: empty set
3. $x, y, \ldots, a, b, \ldots$: elements
4. $x \in X$: x is an exlement of X
5. $A \subseteq B$: A is a subset of B, defined by $(\forall x)(x \in A \rightarrow x \in B)$
6. $A = B$: A is equal to B, Also defined by $A \subseteq B$ and $B \subseteq A$
7. $A \cup B$: $\{x \mid x \in A \vee x \in B\}$(The union of A and B)
8. $A \cap B$: $\{x \mid x \in A \wedge x \in B\}$(The intersection of A and B)
9. $A - B$(or $A \setminus B$): $\{x \mid x \in A \wedge x \notin B\}$(The difference of A from B)

2.3 Preference of agents on outcomes

$\succeq$ is a relation on a set T(that is, $\succeq \ \subseteq T \times T$), which denotes the preference of an agent on the set T of all possible outcomes of the focal decison making situation.

The set T of all possible outcomes depends on the model of the focal decison making situation. For outcomes t and $t' \in T$, (t, t') is denoted by $t \succeq t'$.

- $t \succeq t'$: t is more or equally preferred to t'
- $t \sim t'$: t is equally preferred to t', defined by $[t \succeq t'$ and $t' \succeq t]$
- $t \succ t'$: t is strictly more preferred to t', defined by $[t \succeq t'$ and $\neg(t' \succeq t)]$

2.3.1 Characteristics of Preference: $\succeq$ is said to be($i \in N$):

1. reflexive on $T \leftrightarrow \forall t \in T, t \succeq_i t$
2. anti-symmetric on $T \leftrightarrow \forall t \in T$, if $[t \succeq_i t'$ and $t' \succeq_i t]$ then $t = t'$
3. transitive on $T \leftrightarrow \forall t, t', t'' \in T$, if $[t \succeq_i t'$ and $t' \succeq_i t'']$ then $t \succeq t''$
4. comeplete on $T \leftrightarrow \forall t, t' \in T, [t \succeq_i t'$ or $t' \succeq_i t]$
   (every pair of preference can be compared)

5. acyclic on $T \leftrightarrow \neg\left[\exists t_1, t_2, \ldots, t_m \in T \text{ such that } t_1 \succ t_2 \succ \cdots \succ t_m \succ t_1\right]$

2.3.2 **Definition 1**: Orderings

1. $\succeq$ on T is a linear ordering $\leftrightarrow$ $\succeq$ is reflexive, anti-symmetric, transive, comeplete
2. $\succeq$ on T is a weak ordering $\leftrightarrow$ $\succeq$ is reflexive, transive, comeplete
3. $\succeq$ on T is a partial ordering $\leftrightarrow$ $\succeq$ is reflexive, anti-symmetric, transive

2.4 Efficiency

in $\left(N, T, (\succeq_i)_{i\in N}\right)$, N is the set of all agents, T is the set of all possible outcomes, and $(\succeq_i)_{i\in N}$ is a list of the preference $\succeq_i$ of agent i for each $i \in N$.

Assume that $\succeq_i$ is **comeplete** on T for each $i \in N$:

$t \in T$ is said to be **Pareto efficient** in T iff there is no other outcome in which all agents are improved from the outcome t at the same time(It is unable to improving all agents' t at the same time?).

2.4.1 **Definition 2**: Strong Pareto Efficiency(sPE)

Consider a 3-tuple $\left(N, T, (\succeq_i)_{i\in N}\right)$,

$t \in T$ is said to be **sPE** $\leftrightarrow \neg\left[\exists t' \in T, \left[\left[\forall i \in N, t' \succeq_i t\right] \text{ and } \left[\exists j \in N, t' \succ_j t\right]\right]\right]$

or:

$\forall t' \in T, \left[\left[\exists i \in N, t' \succ_i t\right] \rightarrow \left[\exists j \in N, t \succ_j t'\right]\right]$.

2.4.2 **Definition 3**: Weak Pareto efficiency(wPE)

Consider a 3-tuple $\left(N, T, (\succeq_i)_{i\in N}\right)$,

$t \in T$ is said to be **wPE** $\leftrightarrow \neg\left[\exists t' \in T, \left[\forall i \in N, t' \succ_i t\right]\right]$

or:

$\forall t' \in T, \left[\exists i \in N, t' \succeq_i t\right]$.

2.4.3 Propositions

1. Consider a **complete** $\succeq$ on T, we have:
   - $\forall t, t', \left[\neg\left(t \succeq t'\right) \leftrightarrow t' \succ t\right]$.
   - $\forall t, t', \left[\neg\left(t' \succeq t\right) \leftrightarrow t \succeq t'\right]$.
2. If a $\succeq$ on T is **transive**, it is **acyclic**.

2.5 Games in normal form

A game in normal form is a 3-truple $\left(N, (T_i)_{i\in N}, (\succeq_i)_{i\in N}\right)$, where N is the set of all agents, T_i is the set of all strategies of agent $i \in N$, and $\succeq_i$ is the preference of agent $i \in N$.

It is assumed that $\forall i \in N, |T_i| \geq 2$.

The set T of all possible outcomes is defined as follows:

$$T = \prod_{i \in N} T_i = \{(t_i)_{i \in N} \mid \forall i \in N, t_i \in T_i\}$$

.

Thus, $\succeq_i$ is the preference of agent i on T.

An outcome $t = (t_i)_{i \in N} \in T$ is often expressed by (t_i, t_{-i}) for a particular $i \in N$, where $i_{-i} = (t_j)_{j \neq i}$. The set $\prod_{j \neq i} T_j$ of all t_{-i} is denoted by T_{-i}. Thus, we have that $T = T_i \times T_{-i}$.

A game $\left(N, (T_i)_{i \in N}, (\succeq_i)_{i \in N}\right)$ in normal form is said to be a 2×2 game, if$|N| = 2$ and $|T_i| = 2$ for all $i \in N$.

表 1: Matrix representation of a 2×2 game: Left number is agent 1's preference of outcome and the right is 2's. Outcomes with bigger numbers are more preferred.

agent		2	
	strategy	a_2	b_2
1	a_1	x_1,x_2	y_1,y_2
	b_1	z_1,z_2	w_1,w_2

Examples (A.Rapoport and M.Guyer 1966):

1. Prisoner's Dilemma(PD) game: Strongly stable deficient equilibrium.
2. Chicken(Ch) game: A two-equilibria game with non-equilibrium outcome.
3. Battle of the sexes(BS) game: A two-equilibria game with non-equilibrium outcome.
4. *The Gift of the Magi* (GM) game: A game with strongly stable equilibria.

3 Lesson 3: Rationality Games in Normal Form(2)

Dominant strategies, Dominant strategy equilibria, and Nash equilibria are representative concepts for rationality analysis of competitive decison making situations.

3.1 Definition 1: Best replies(BR)

For $i \in N, t_i^\star \in T_i, t = (t_i, t_{-i}) \in T$, $T_i^\star$ is said to be a **best reply(BR)** of agent i at t, iff

$$\forall t_i' \in T_i, (t_i^\star, t_{-i}) \succeq_i (t_i', t_{-i}).$$

$BR_i(t)$ denotes the set of all BRs of i at t.

Choosing a best reply $t_i^\star \in BR_i(t)$ when others select t_{-i} is an individual rational behavior of agent i.

3.2 Definition 2: Domimamt strategies(DS)

For $i \in N$ and $t_i^\star \in T_i$, $t_i^\star$ is said to be a **dominant strategy(DS)** iff

$$\forall t \in T, t_i^\star \in BR_i(t)$$

The set of all DSs of agent i is deboted by DS_i.

A DS $t_i^\star$ is abest reply for all $t \in T$. $t_i^\star$ does not always exist. If it exists, choosing it is an individually rational behavior of agent i.

3.3 Definition 3: Dominant strategy equilibria(DSE)

For $t^\star \in T$, $t^\star$ is said to be a **dominant strategy equilibrium(DSE)** iff

$$\forall i \in N, t_i^\star \in DS_i$$

The set of all $DSEs$ is denoted by DSE.

A DSE is an outcome which will be achieved as a result of all agents' individually rational behaviors. DSE does not always exist.

3.4 Definition 4: Nash equilibria(NE)

For $t^\star \in T$, $t^\star$ is said to be a **Nash equilibrium(NE)** iff

$$\forall i \in N, t_i^\star \in BR_i(t^\star)$$

.

The set of all NEs is denoted by NE.

At a NE $t^\star = (t_i^\star, t_{-i}^\star)$, each agent i chooese a BR $t_i^\star$ to the others' choices $t_{-i}^\star$.

All $DSEs$ are NEs, while NEs are not always $DSEs$.

4 Lesson 4: Games in Extensive Form with perfect imformation

- With perfect imformation
- With imperfect imformation
- Nomal Form: without a sequential order of each decision making
- Extensive Form: with a sequential order of each decision making

4.1 Definition: Games in Extensive Form

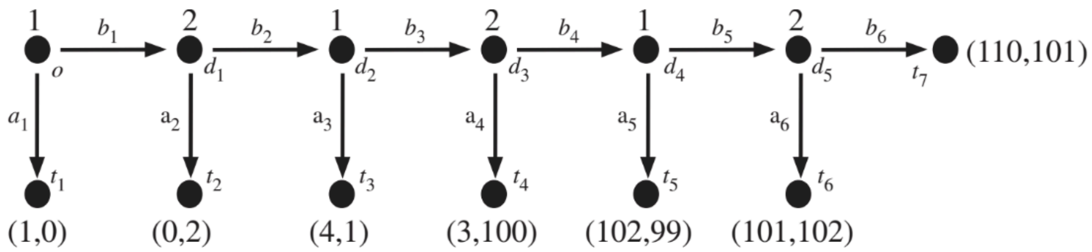


図 **1:** A simple example: A centipete game

A game in extensive form is a 9-tuple:

$$(N, V, \sigma, K, \kappa, (D_i)_{i\in N}, (\succeq_i)_{i\in N}, U, (K(u))_{u\in U})$$

N is the set of DMs. $N = \{1, 2\}$.

V is the set of all nodes(Vertices), where V has an element o called *origin* in it. $V = \{o, d_1, \cdots, d_5, t_1, \cdots, t_7\}$.

V is partitioned into the set D of all *decision nodes* and the set J of all *terminal nodes*.

$\{o\} \in V$ is partitioned into $d_1 \in D$ and $t_1 \in J$. $D = \{o, d_1, \cdots, d_5\}, J = \{t_1, \cdots, t_7\}$.

σ is the **function** that indicate the way of connection among the nodes. $\forall v \in V\backslash\{o\}$, $\sigma(\mathrm{v})$ is the preceding node of v. $\sigma(\mathrm{o})=\sigma(\mathrm{d_1})=\sigma(\mathrm{t_1})=\mathrm{o}$, $\sigma(\mathrm{d_2})=\sigma(\mathrm{t_2})=\mathrm{d_2}$, $\cdots$, $\sigma(\mathrm{t_6})=\sigma(\mathrm{t_7})=\mathrm{d_5}$.

σ satisfies that $\sigma(o) = o$ and $\forall v \in V, \exists k > 0, [\sigma^k(v) = o]$, where $\sigma^k(v)$ is the value obtained by k times operarion of σ toward v.

K is the set of all actions $(a_i)_{i\in N}$ and $(b_i)_{i\in N}$. $K = \{a_1, \cdots, a_6, b_1, \cdots, b_6\}$.

κ is the **function** that indicate the relation between the nodes and the actions. $\forall v \in V\backslash\{o\}$, $\kappa(v)$ is the action that is needed to be selected by a DM in order to arrive at v from its preceding node. $\kappa(t_1) = a_1, \kappa(t_2) = a_2, \cdots, \kappa(t_7) = b_6$.

$(D_i)_{i\in N}$ is the list of all decision nodes for each DM. $(D_i)_{i\in N}$ forms a partition of D. $D_i = \{o, d_2, d_4\}, D_2 = \{d_1, d_3, d_5\}$.

DM1's preference: $t_7 \succeq_1 t_5 \succeq_1 t_6 \succeq_1 t_3 \succeq_1 t_4 \succeq_1 t_1 \succeq_1 t_2$,

DM2's preference: $t_6 \succeq_2 t_7 \succeq_2 t_4 \succeq_2 t_5 \succeq_2 t_2 \succeq_2 t_3 \succeq_2 t_1$,

U is the set of all imformation sets. U is a partition of D, the partition which is finer than the partition $\{D_i\}_{i\in N}$ of D. $U = \{\{o\}, \{d_i\}, \cdots, \{d_5\}\}$;

$\forall i \in N, u \in U$, $u \subseteq D_i$ means that DM i cannot distinguish all of the nodes in u.

The list $(K(u))_{u\in U}$ indicates the relation between the imformation sets and the actions. $\forall u \in U$, $K(u)$ indicates the set of all actions available at each node $d \in u$.

In this form, the set Ω of all possible outcomes of the focal decision making situations is expressed by J. Thus, $\succeq_i$ is the preference of agent $i \in N$ on J.

4.2 Backward induction

The procedure of backward induction start with the decision nodes such that all nodes that follows are terminal nodes, that is, the decision nodes in the set $\{d \in D \mid V(d) \subseteq T\}$.

5 Lesson 5: How to change an extensive form to a normal form

5.1 Examples

In fig.1, we can find the strategies of DM1 shown in table 2 and the strategies of DM2 shown in table 3. Then we can simply transfer the possible strategies of the 2 DMs into a game in normal form shown in table 4.

Other examples are shown in fig.2 and fig.3.

表 **2:** Possible Strategies of DM1

No.	$\{o\}$	$\{d_2\}$	$\{d_4\}$
x_1	a_1	a_3	a_5
x_2	a_1	b_3	a_5
x_3	a_1	a_3	b_5
x_4	a_1	b_3	b_5
x_5	b_1	a_3	a_5
x_6	b_1	b_3	a_5
x_7	b_1	a_3	b_5
x_8	b_1	b_3	b_5

表 **3:** Posssible strategies of DM2

No.	$\{d_1\}$	$\{d_3\}$	$\{d_5\}$
y_1	a_2	a_4	a_6
y_2	a_2	b_4	a_6
y_3	a_2	a_4	b_6
y_4	a_2	b_4	b_6
y_5	b_2	a_4	a_6
y_6	b_2	b_4	a_6
y_7	b_2	a_4	b_6
y_8	b_2	b_4	b_6

表 **4:** A transferred game from fig.1. The underlined part is **NE**.

DMs		DM2							
	strategies	y_1	y_2	y_3	y_4	y_5	y_6	y_7	y_8
DM1	x_1	$\underline{(1,0)}$	$\underline{(1,0)}$	$\underline{(1,0)}$	$\underline{(1,0)}$	(1,0)	(1,0)	(1,0)	(1,0)
	x_2	$\underline{(1,0)}$	$\underline{(1,0)}$	$\underline{(1,0)}$	$\underline{(1,0)}$	(1,0)	(1,0)	(1,0)	(1,0)
	x_3	$\underline{(1,0)}$	$\underline{(1,0)}$	$\underline{(1,0)}$	$\underline{(1,0)}$	(1,0)	(1,0)	(1,0)	(1,0)
	x_4	$\underline{(1,0)}$	$\underline{(1,0)}$	$\underline{(1,0)}$	$\underline{(1,0)}$	(1,0)	(1,0)	(1,0)	(1,0)
	x_5	(0,2)	(0,2)	(0,2)	(0,2)	(4,1)	(4,1)	(4,1)	(4,1)
	x_6	(0,2)	(0,2)	(0,2)	(0,2)	(3,100)	(102,99)	(3,100)	(102,99)
	x_7	(0,2)	(0,2)	(0,2)	(0,2)	(4,1)	(4,1)	(4,1)	(4,1)
	x_8	(0,2)	(0,2)	(0,2)	(0,2)	(3,100)	(101,102)	(3,100)	(110,101)

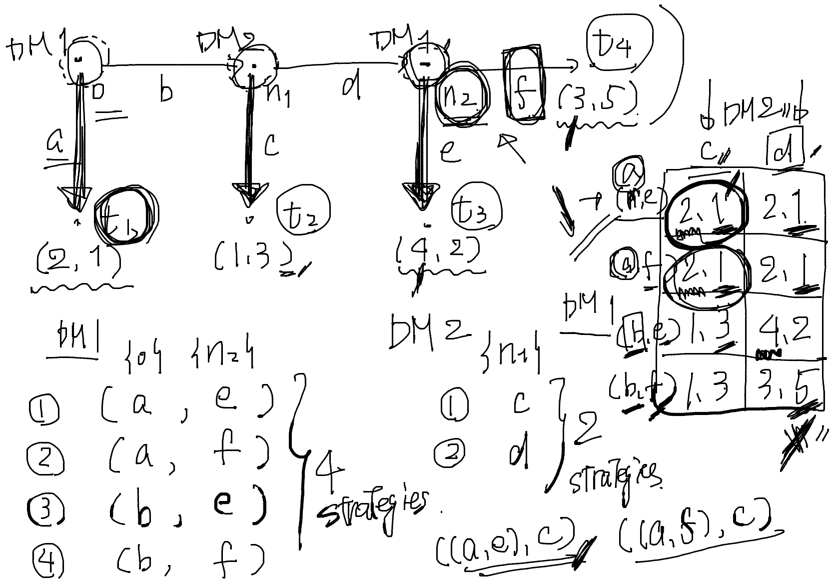
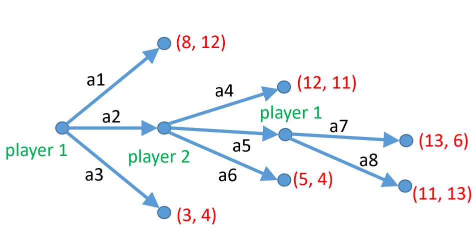


図 **2:** Other examples for the transformation 1

Answer



We can get the game in normal form, as in the right table, by transforming the origial game in extensive form.

Player 1 has 6 strategies:
(a1, a7), (a1, a8), (a2, a7), (a2, a8), (a3, a7), (a3, a8).

player 2 has 3 strategies: a4, a5, a6.

Throught the analysis of the game in normal form, we can see that there are 3 Nash equilibria indicated in the right table.

	a4	a5	a6
a1, a7	8,12	8,12	8,12
a1, a8	8,12	8,12	8,12
a2, a7	12,11	13,6	5,4
a2, a8	12,11	11,13	5,4
a3, a7	3,4	3,4	3,4
a3, a8	3,4	3,4	3,4

図 **3:** Other examples for the transformation 2

5.2 Subgame perfect equilibria

6 Lesson 6: Option forms for confilict analysis

6.1 Defination

An option form of a conflict is a 5-tuple

$$(N, (O_i)_{i\in N}, (\mathbb{Y}_i)_{i\in N}, \mathbb{U}, (\succeq_i)_{i\in N})$$

.

O_i the non-empty set of all options of agent $i \in N$. An option is a smallest unit of selection. For each option, an agent needsto decide if she/he selects it or not.

$\mathbb{Y}_i$ a family of subsets of O_i, where $|Y_i| \geq 2$. Each element of $\mathbb{Y}_i$ called a strategy of agent $i \in N$. That is, a strategy of anagent is a feasible combination of options selected by the agent.

In this form the set ω of all possible outcomes of the focal decision making situation is the set $\mathbb{U}$, where $|\mathbb{U}| \geq 2$.Uis a subset of $\mathbb{Y} = \prod_{i \in N} \mathbb{Y}_i$. That is, an outcome of a situation is a feasible combination of strategies selected by the agents.

6.2 Example

An option form of a conflict can be described by two tables.

表 **5:** $N, (O_i)_{i \in N}, (\mathbb{Y}_i)_{i \in N}, \mathbb{U}$

anagent	options	outcome 1	2	3	4	5	6	7	8
1	o_{11}		✓		✓		✓		✓
2	o_{21}	✓	✓			✓	✓		
	o_{22}			✓	✓			✓	
	o_{23}								✓
3	o_{31}					✓	✓	✓	✓

表 **6:** Preference $\succeq_i$ of Outcomes for the Agents ordering

Agents	more $\leftrightarrow$ less preferred
1	7 3 4 8 6 1 2 6
2	1 4 8 5 3 7 2 6
3	7 3 5 1 8 6 4 2

In this example, $N = \{1, 2, 3\}$.

Agent 1 has one option, so that $O_1 = \{o_{11}\}$.

Agent 2 has three opitons, so that $O_2 = \{o_{21}, o_{22}, o_{23}\}$.

Agent 3 has one option, so that $O_3 = \{o_{31}\}$.

A strategy Y_i of agent $i \in N$ is expressed by a subset of the setOiof all options of the agent.

Agent 1's strategies $\mathbb{Y}_1$ consists of and $\{o_{11}\}$, denoted by Y_{11} and Y_{12}, respectively.

For agent 2, the set of all strategies, $\mathbb{Y}_2$, is $\{\{o_{21}\}, \{o_{22}\}, \{o_{23}\}\}$. Let $Y_{21} = \{o_{21}\}$, $Y_{22} = \{o_{22}\}$ and $Y_{23} = \{o_{23}\}$.

The set $\mathbb{Y}_3$ of all strategies of agent 3 is $\{, \{o_{31}\}\}$. and $\{o_{31}\}$ are denoted by Y_{31} and Y_{32}, respectively.

A strategy may consist of two or more options.

One can distinguish feasible combinations of options from infeasible ones in this model.

An outcome Y is a list $(Y_i)_{i \in N}$ of strategies Y_i of agent i for each $i \in N$.

One can distinguish feasible combinations of strategies from infeasible ones in this model, too.

In this example, there are eight (feasible) oucomes, although there are $12 (= 2 \times 3 \times 2)$ (logically) possible listsof strategis of the agents.

Specifically, the setUof all outcomes consists of:

$$\begin{array}{ll} 1{:}(Y_{12}, Y_{21}, Y_{32}) & 2{:}(Y_{11}, Y_{21}, Y_{32}) \\ 3{:}(Y_{12}, Y_{22}, Y_{32}) & 4{:}(Y_{11}, Y_{21}, Y_{32}) \\ 4{:}(Y_{12}, Y_{21}, Y_{31}) & 6{:}(Y_{12}, Y_{21}, Y_{31}) \\ 7{:}(Y_{11}, Y_{21}, Y_{31}) & 8{:}(Y_{11}, Y_{21}, Y_{31}) \end{array}$$

The preference $\succeq_i$ of agent $i \in N$ over the outcomes can beexpressed by a *preference vector* as in Table 6.2.

An outcome which is more preferred by an agent is placed moreto the left the agent' s preference vector.

For instance, in Table 6.2, it is indicated that agent 1 prefersoutcome 7 most, and outcome 6 least.

6.3 Transformation of games in normal form into option form for conflict analysis

A game $(N, (T_i)_{i\in N}, (\succeq_i)_{i\ \in\ N})$ in normal form can be canonically transforemd into an option form of a conflict.

$$(N, (O_i)_{i\in N}, (\mathbb{Y}_i)_{i\in N}, \mathbb{U}, (\succeq'_i)_{\text{ß}\in N})$$

. We use T_i as O_i for each $i \in N$.

$\mathbb{Y}_i$ is obtained as the set of all singleton subsets of $T_i(= O_i)$ for each $i \in N$, that is, $Y_i = \{\{t_i\} \mid t_i \in T_i\}$.

$\mathbb{Y} = \prod_{i\in N} \mathbb{Y}_i$ is set as $\mathbb{U}$.

For $i \in N$ and $Y, Y' \in \mathbb{Y}$, where $Y = (Y_i)_{i\in N} = (\{t_i\})$ and $Y' = (Y'_i)_{i\in N} = (\{t'_i\})$, $Y \succeq'_i Y'$ iif $(t_i)_{i\in N} \succeq_i (t'_i)_{i\in N}$.

6.4 Example of transformation

Consider the Prisoners' Dilemma(PD) game.

Agent		2	
	strategy	a_2	b_2
1	a_1	3,3	1,4
	b_1	4,1	2,2

In this game, $N = \{1, 2\}$ and $T_i = \{a_i, b_i\}$ for each $i \in N$.

The preference $\succeq_i$ of agent $i \in N$ is expressed by the ordinal numbers, where a larger number indicates a more preferred outcome.

As shown in Table 6.4, in this case,

- $O_1 = \{a_1, b_1\} = T_1$ and $O_2 = \{a_2, b_2\} = T_2$;
- $\mathbb{Y}_1 = \{\{a_1\}, \{b_1\}\}$ and $Y_2 = \{\{a_2\}, \{b_2\}\}$;
- $\mathbb{U} = \{(\{a_1\}, \{a_2\}), (\{a_1\}, \{b_2\}), (\{b_1\}, \{a_2\}), (\{b_1\}, \{b_2\})\}$;
- $\succeq_1$ and $\succeq_2$ is described in Table 6.4.

表 **7:** An Option Form of a Conflict for the PD game

Agent	Option	Outcome 1	2	3	4
1	a_1	✓	✓		
	b_1			✓	✓
2	a_2	✓		✓	
	b_2		✓		✓

表 **8:** Preference of the Agents over Outcomes Ordering of Outcomes

Agent	more $\leftrightarrow$ less preferred
1	3 1 4 2
2	2 1 4 3

7 Lesson 7: Graph model for confilict resolution(GMCR)

7.1 Defination

A graph model of a conflict is a 4-tuple: $(N, C, (A_i)_{i\in N}, (\succeq_i)_{i\in N})$.

C is the set of all states of the focal decision making situation, where $|C| \geq 2$.

For $i \in N, (C, A_i)$ constitutes agent i's graph, denoted by G_i, for which C is the set of all vertices and $A_i \subseteq C \times C$ is the set of all arcs.

For c, c' in C, $(c, c') \in A_i$ means that agent i can shift the state of the conflict from state c to state c'.

In this form, the set ω of all possible outcomes of the focal decision making situations is the set C of all states.

Thus $\succeq_i$ is the preference of agention C.

7.2 Example

8 Lesson 8: GMCR(2) and stability

8.1 Reachable list of agents

For each $i \in N$, a Functuin S_i from C to the power set $\mathbb{P}(C)$ of C, $S_i : C \to \mathbb{P}(C)$, such that $S_i(c)$ is defined as the set $\{c' \in C \mid (c, c') \in A_i\}$.

Note that it is satisfied that for each $c \in C, c \notin S_i(c) \in \mathbb{P}(C)$.

S_i is called the irreflexive reachable list function of agent $i \in N$.

$S_i(c)$ the irreflexive reachable list of agent $i \in N$ at $c \in C$.

8.2 Reachable lists of coalitions

For coalition $H \subset N$, the irreflexive reachable list function of coalitionHis a function S_H from C to the power set $\mathbb{P}(C)$ of C, $S_H(c)$.

such that for all $c \in C$, $S_H(c)$ is inductively defined as the setthat satisfies the next two conditions:

1. if $i \in H$ and $c' \in S_i(c)$, then $c' \in S_H(c)$, and
2. if $i \in H$ and $c' \in S_H(c)$ and $c'' \in S_i(c')$, then $c'' \in S_H(c)$.

$S_H(c)$ is called the irreflexive reachable list of coalition H at c.

8.3 Unilateral improvement list of agents

S_i^+ is a function from C to the power set $\mathbb{P}(C)$ of C, $S_i^+ : C \to \mathbb{P}(C)$, such that $S_i^+(c) = \{c' \in S_i(c) \mid c' \succeq_i c\} \subset S_i(c) \subset C$ for $c \in C$.

Note that it is satisfied that for each $c \notin S_i^+(c) \in \mathbb{P}(C)$ for all $c \in C$.

S_i^+ is called the irreflexive reachable improvement list function of agent $i \in N$.

$S_i^+(c)$ the irreflexive reachable reachable improvemen list of agent $i \in N$ at $c \in C$.

8.4 Unilateral improvement list of coalitions

For coalition $H \subset N$, the irreflexive reachable list function of coalitionHis a function S_H^+ from C to the power set $\mathbb{P}(C)$ of C, $S_H^+(c)$.

such that for all $c \in C$, $S_H^+(c)$ is inductively defined as the setthat satisfies the next two conditions:

1. if $i \in H$ and $c' \in S_i^+(c)$, then $c' \in S_H^+(c)$, and
2. if $i \in H$ and $c' \in S_H^+(c)$ and $c'' \in S_i^+(c')$, then $c'' \in S_H^+(c)$.

$S_H^+(c)$ is called the irreflexive reachable list of coalition H at c.

8.5 Equally or less preferred outcomes

$\phi_i^{\approxeq}(c)$ denote the set of all outcomes that are equally or less preferred to outcome c by agent i, that is, $\{c' \in C \mid c \succeq_i c'\}$.

8.6 Stability concepts

Nash stability, general metarationality, symmetric metarationality and sequential stability are repersentative stability concepts forthe stability analysis of states in an graph form of a conflict.

A state is said to be an equilibrium with respect to a stability concept if and only if the state is stable with respect to the stability concept for all agents.

8.7 Definition 1 (Nash stability (Nash))

For $i \in N$,state $c \in C$ is said to be Nash stable(Nash) for agent i, iff $S_i^+(c) = \emptyset$.

The set of all state c which is Nash stable for agent i is denoted by C_i^{Nash}.

State c is stable for agent i, because there is no unilateral improvement of agent i from c.

8.8 Definition 2 (General metarationality (GMR))

For $i \in N$, state $c \in C$ is said to be generally metarational(GMR) for agent i, iff for all $c' \in S_i^+(c), S_{N\setminus\{i\}}(c') \cap \Phi\phi_i^{\approx}(c) \neq \emptyset$.

The set of all state c which is generally metarational for agent i is denoted by C_i^{GMR}.

State c is stable for agent i, because for each unilateral improvement of agent i from c, there exists at least onethe others' countermove which causes an equally or less preferred outcome for agent i to c.

8.9 Definition 3 (Symmetric metarationality (SMR))

For $i \in N$, state $c \in C$ is said to be symmetrically metarational(SMR) for agent i, iff for all $c' \in S_i^+(c)$, there exists $c'' \in S_{N\setminus\{i\}}(c') \cap \phi_i^{\approx}(c)$ such that $c''' \in \phi_i^{\approx}(c)$ for all $c''' \in S_i(c'')$.

The set of all state c which is symmetrically metarational for agent i is denoted by C_i^{SMR}.

Statecis stable for agent i, because for each unilateral improvement of agent i from c, there exists at least one the others' countermove which causes an equally or less preferred outcome for agent i to c, and agent i cannot achieve more preferred outcomes for him/herself to c by responding to the others' countermove.

8.10 Definition 4 (Sequential stability (SEQ))

For $i \in N$, state $c \in C$ is said to be sequentially stable(SEQ) for agent i, iff for all $c' \in S_i^+(c), S_{N\setminus\{i\}}^+(c*\prime) \cap \phi_i^{\approx}(c) \neq \emptyset$.

The set of all state c which is symmetrically metarational for agent i is denoted by C_i^{SEQ}.

State c is stable for agent i, because for each unilateral improvement of agent i from c, there exists at least one the others' countermove which is rational for the others and causes an equally or less preferred outcome for agent i to c.

9 Lesson 9: Simple games, weighted majority games, and committees

Games in extensive form, option forms for confilict analysis, graph model for resolution, These models are focus on individual rationality and are non-coorperate models. GMs in non-coorperate models compete with each others.

While in coorperational models, GMs can coorperate with each others.

9.1 Simple game

A simple gameis a pair $(N, \mathbb{W})$.

$\mathbb{W}$ is non-empty family of subsets of N.

Each element of $\mathbb{W}$ is called a winning coalition.

It is assumed that the following two conditions are satisfied:

1. effectiveness: $\emptyset \notin \mathbb{W}, N \in \mathbb{W}$
2. monotonicity: for $H, H' \subseteq N$, if $H \in \mathbb{W}$ and $H \subseteq H'$, then $H' \in \mathbb{W}$.

9.2 Weighted majority game

Weighted majority games constitute a special class of simple games.

A weighted majority gameis a tuple $[r; w_1, w_2, \cdots, w_n]$, where

1. $\{1, 2, \cdots, n\}$ constitutes the set N of all agents;
2. $r, w1, w2, ..., w_n \in \Re^+$($\Re^+$: the set of all positive real numbers) which satisfies that $w_N \geq r$, where
   (a) w_i is the weight of agent $i \in N$;
   (b) $\Sigma_{i \in H} w_i$ is the weight of the coalition $H \subset N$, denoted by w_H(in partiqular, $w_N = \Sigma_{i \in H} w_i$).
   (c) r is the threshold of the weight for a coalition to be winning, that is, a coalition H is said to be winning, if and only if $w_H \geq r$.

Taking a family $\mathbb{W}$ of subsets of N by $\{H \subset N \mid w_H \geq r\}$, we have a simple game $(N, \mathbb{W})$.

9.3 Group

A group V is a pair (N, A), where A is a finite set of all alternatives.

A group V with $\succeq = (\succeq_i)_{i \in N}$ constitutes a meeting, which is denoted by $V_{\succeq}$. That is, $V_{succeq} = (N, A, (\succeq_i)_{i \in N})$.

9.4 Committee

A committee is a 4-tuple $(N, \mathbb{W}, A, (\succeq_i)_{i \in N})$, where

1. $(N, \mathbb{W})$ is a simple game;
2. $V = (N, A)$ is a group;
3. $V_{\succeq} = (N, Am(\succeq_i)_{i \in N})$ is a meeting, where $\succeq = (\succeq_i)_{i \in N}$.

A committee $(N, \mathbb{W}, A, (\succeq_i)_{i \in N})$ can be denoted by $V_{\succeq}(\mathbb{W})$, where $V = (N, A)$ and $\succeq = (\succeq_i)_{i \in N}$.

9.5 Meeting

In a meeting V_{succeq} and a committee $(N, \mathbb{W}, A, (\succeq_i)_{i \in N})$, where

1. $V = (N, A)$;
2. $\succeq = (\succeq_i)_{i \in N}$

the set Ω of all possible outcomes of the focal decision making situations is the set A of all alternatives.

Thus, $\succeq_i$ is the preference of agent $i \in N$ on A.

10 Lesson 12: Compare power of coalitions

10.1 Desirability relation

According to Einy (1985), The desirability relation on a simple game $(N, \mathbb{W})$ on N is defined as follows:

Consider a simple game $(N, \mathbb{W})$ on N. For $H, H' \subseteq N, H \succeq^d H'$ is defined as:
for all $B \subseteq N$ such that $B \cap (H \cup H') = \emptyset$, if $B \cup H' \in \mathbb{W}$ then $B \cup H \in \mathbb{W}$.

10.1.1 Exercise

Compare the coalitions H and H' in the simple game $(N, \mathbb{W})$, where $\mathbb{W} = \{12, 123, 124, 1234, 134\}$ through desirability relations.

When $H = \{2\}, H' = \{3\}$, We can get $B = \{\emptyset, 1, 4, 14\}$ such that $B \cap (H \cup H') = \emptyset$:

	$\emptyset$	$\{1\}$	$\{4\}$	$\{1,4\}$		No. in $\mathbb{W}$
2	2	$\underline{12}$	24	$\underline{124}$	$B \cup H$	2
3	3	13	34	$\underline{134}$	$B \cup H'$	1

So, $H \succeq^d H'$.

10.2 Other relation for comparing coalitions

- Blockability relations
- Viability relations

10.3 Power index

A power index is a function from the setNof all agents to the set of all real numbers.

10.3.1 Shapley-Shubik power index(Shapley and Shubik 1954)

Consider a permutation $i_1 i_2 \cdots i_n$ of all agents.

Putting agents together one by one depending on the permutation, we will have a unique agent i_k who satisfies the following properties:

The agents before him/her, that is, $i_1 i_2 \cdots ß_{k-1}$ do not constitutea winning coalition, and once he/she get together to the agents before him/her, they, that is, $i_1 i_2 \cdots i_{k-1} i_k$ constitute a winning coalition.

The agent i_k is called the **pivot** in the permutation.

For $i \in N$, the number of all permutations in which agent i is the pivot is denoted by λ_i.

Then, the <u>Shapley-Shubik power index</u> ϕ_i of agent i is defined as:

$$\phi_i = \frac{\lambda_i}{n!}$$

, where $n = |N|$.

We see that

$$\phi_i = \Sigma_{H \in \mathbb{W}, H \setminus \{i\} \notin \mathbb{W}} \frac{(h-1)!(n-h)!}{n!}$$

, where $h = |H|$.

10.3.2 Exercise

For the weighted majority game $[7; 4, 3, 2, 1]$, we have the simple game $(N, \mathbb{W})$, where $\mathbb{W} = \{12, 123, 124, 1234, 134\}$, through the canonical transformation of a weighted majority game into a simple game(Use the Shapley-Shubik indices $(\phi_i)_{i \in N}$).

Assume that $N = \{1, 2, 3, 4\}$

1. A permutation is 1234: 1, $\underline{12}$, $\underline{123}$, $\underline{1234}$, where the order is following the order of the permutation, the underlined parts are winning coalitions $\mathbb{W}$. We can kmow that the first coalition in $\mathbb{W}$ is 12 when agent 2 joins in. So agent 2 is the pivot.
2. A permutation is 1342: 1, 13, $\underline{134}$, 1342. Agent 4 is the pivot.

Like this, we can calculate the number of permutations where 1/2/3/4 is a pivot $(\lambda_i, i = 1/2/3/4)$ and the Shapley-Shubik indices $\phi_i = \lambda_i / (1 \times 2 \times 3 \times 4)$.

10.3.3 Banzhaf index(Banzhaf 1965)

An agent, who is a member of a winning coalition, is said to becritical (or a swing) in the winning coalition, **iff**

the withdrawal of agent from the winning coalition makes the winning coalition non-winning.

That is, an agent $i \in N$ is said to be a swing in a winning coalition $H\ in \mathbb{W}$, **iff**

$H \setminus \{i\} \notin \mathbb{W}$.

For $i \notin N$, η_i denotes the number of the winning coalitions in which agent i is a swing.

Then, the <u>Banzhaf index</u> β_i of agent i is defined as

$$\beta_i = \frac{\eta_i}{\Sigma_{i=1}^{n} \eta_i}$$

.

10.3.4 Exercise

For the weighted majority game $[7; 4, 3, 2, 1]$, we have the simple game $(N, \mathbb{W})$, where $\mathbb{W} = \{12, 123, 124, 1234, 134\}$, through the canonical transformation of a weighted majority game into a simple game(Use the Banzhaf indices $(\beta_i)_{i \in N}$).

Assume that $N = \{1, 2, 3, 4\}$, we can get all $H \setminus \{i\} \notin \mathbb{W}$, the underlined parts are not in $\mathbb{W}$:

$H \in \mathbb{W}$	1	2	3	4
12	$\underline{2}$	$\underline{1}$	12	12
123	$\underline{23}$	$\underline{13}$	12	123
124	$\underline{24}$	$\underline{14}$	124	12
134	$\underline{34}$	134	$\underline{14}$	$\underline{13}$
1234	$\underline{234}$	134	124	123
No. of H where i is swing	5	3	1	1
Banzhaf index β_i	5/10	3/10	1/10	1/10

10.4 Core of commitees

Core of committees is a useful concept to analyze committees.

Core of committees is defined using the concept of dominancerelation on alternatives.

10.4.1 Committee(review)

A committee is a 4-tuple $(N, \mathbb{W}, A, (\succeq_i)_{i\in N})$, where

1. $(N, \mathbb{W})$ is a simple game;
2. $V = (N, A)$ is a group;
3. $V_{\succeq} = (N, A, (\succeq_i)_{i\in N})$ is a meeting, where $\succeq = (\succeq_i)_{i\in N}$.

A committee $(N, \mathbb{W}, A, (\succeq_i)_{i\in N})$ can be denoted by $V_{\succeq}(\mathbb{W})$, where $V = (N, A)$ and $\succeq = (\succeq_i)_{i\in N}$.

10.4.2 Dominance relation on alternatives

Consider a committee $V_{\succeq}(\mathbb{W})$, where $V = (N, A)$ and $\succeq = (\succeq_i)_{i\in N}$.

For $x, y \in A$ such that $x \neq y$, alternative x is said to **dominate** alternative y, iff

there exists $H \in \mathbb{W}$ such that for all $i \in H, x \succeq_i y$.

x **Dom** y and $\neg(x$ **Dom** $y)$ denote that alternative x dominates alternative y, and that alternative x does not dominate alternative y, respectively.

10.4.3 The core of committee

The core of a committee is the set of all alternatives that arenot dominated by any other alternatives.

Consider a committee $V_{\succeq}(\mathbb{W})$, where $V = (N, A)$ and $\succeq = (\succeq_i)_{i\in N}$. The core of a committee $V_{\succeq}(\mathbb{W})$, denoted by $Core(V_{\succeq}(\mathbb{W}))$, is the set of all alternatives that are not dominated by any other alternatives, that is,

$$Core(V_{\succeq}(\mathbb{W})) = \{x \in A \mid \forall y \neq x, \neg[y \textbf{ Dom } x]\}$$

.

10.4.4 Example

Consider two meetings $V\succeq = (N, A, \succeq)$ and $V\succeq' = (N, A, \succeq')$, such that

$$N = \{1, 2, 3\};$$

$$A = \{a, b, c\};$$

$$\succeq_1 = [a,b,c]; \succeq_2 = [c,a,b]; \succeq_3 = [c,b,a];$$

We have a preference matrix:

1	2	3
a	c	c
b	a	b
c	b	a

When $\mathbb{W} = \{12, 23, 13, 123\}$,

	$H = 12$	$H = 23$	$H = 13$
a, b	a Dom b	c Dom a	-
b, c	-	c Dom b	-
a, c	-	-	-

$Core(V_{\succeq}(\mathbb{W})) = \{c\}$.

11

参考文献

A.Rapoport and M.Guyer (1966) “A taxonomy of 2×2 games,” in Bertalanffy, L.von and A.Rapoport eds. *General system: Yearbook of the society for general systems research*, Vol. 11, Michigan: Ann Arbor, pp. 203–214.

Banzhaf, J. F. (1965) “Weighted voting doesn't work : A mathematical analysis,” *Rutgers Law Review*, Vol. 19, pp. 317-343.

Einy, Ezra (1985) “The Desirability Relation of Simple Games,” *Mathematical Social Sciences*, Vol. 10, No. 2, pp. 155-168, DOI: 10.1016/0165-4896(85)90032-0.

Shapley, L. S. and Martin Shubik (1954) “A Method for Evaluating the Distribution of Power in a Committee System,” *American Political Science Review*, Vol. 48, No. 3, pp. 787-792, DOI: 10.2307/1951053.