INDR 220: Introduction to Computing for Operations Research
Homework 5: The Distance Between Ellipses
Deadline: January 9, 2022, 11:59 PM

In this homework, you will implement a Python script that finds the distance between two ellipses using CPLEX. In analytic geometry, an ellipse is defined as the set of points (x, y) of the Cartesian plane that, in non-degenerate cases, satisfy the following equality

$$ax^2 + bxy + cy^2 + dx + ey + f = 0,$$

where $b^2 - 4ac < 0$. The Euclidean distance between two ellipses is the minimum distance between two points (i.e., (x_1, y_1) and (x_2, y_2)) within these two ellipses (i.e., $(a_1, b_1, c_1, d_1, e_1, f_1)$ and $(a_2, b_2, c_2, d_2, e_2, f_2)$), respectively.

The quadratically constrained quadratic programming formulation of this problem becomes

$$\begin{aligned} \text{minimize} \quad & z = (x_1 - x_2)^2 + (y_1 - y_2)^2 \\ \text{subject to:} \quad & a_1x_1^2 + b_1x_1y_1 + c_1y_1^2 + d_1x_1 + e_1y_1 + f_1 \leq 0 \\ & a_2x_2^2 + b_2x_2y_2 + c_2y_2^2 + d_2x_2 + e_2y_2 + f_2 \leq 0. \end{aligned}$$

After solving the optimization problem, $\sqrt{z}$ will give the Euclidean distance between the two ellipses.

This problem will be represented using a `.txt` file, namely, `ellipses.txt`. This file contains the parameters of two ellipses in two rows (i.e., a_1, b_1, c_1, d_1, e_1, and f_1 in the first row and a_2, b_2, c_2, d_2, e_2, and f_2 in the second row), and it is composed of the following two lines for an example problem:

```
ellipses.txt
--------------------
9 0 25 -18 100 -116
16 0 9 160 -72 400
```

The quadratically constrained quadratic programming model for the example problem can be written as

$$\begin{aligned} \text{minimize} \quad & z = x_1^2 - x_1x_2 - x_2x_1 + x_2^2 + y_1^2 - y_1y_2 - y_2y_1 + y_2^2 \\ \text{subject to:} \quad & -18x_1 + 100y_1 + 9x_1^2 + 0x_1y_1 + 25y_1^2 \leq 116 \\ & +160x_2 - 72y_2 + 16x_2^2 + 0x_2y_2 + 9y_2^2 \leq -400. \end{aligned}$$

The optimum solution of the example problem is as follows:

$$\begin{aligned} x_1^\star &= -2.9511316241907335, \\ y_1^\star &= -0.16155469550487994, \\ x_2^\star &= -3.4943180761546437, \\ y_2^\star &= 0.5402833705123231, \\ \text{distance}^\star &= 0.8874841928214324. \end{aligned}$$

Implement your algorithm to find the distance between two ellipses in a single interactive Python notebook using Azure Lab Services. Your notebook should include at least the following function definition that takes the file path of the input file as parameter and returns the solution found.

```
def distance_between_ellipses(ellipses_file):
    #implement your algorithm here
    return(x1_star, y1_star, x2_star, y2_star, distance_star)
```

What to submit: You need to submit your source code in a single file (`.py` file that you will download from Azure Lab Services by following "File" / "Download as" / "Python (.py)" menu items) named as `STUDENTID.py`, where `STUDENTID` should be replaced with your 7-digit student number.

How to submit: Submit the file you created to Blackboard. Please follow the exact style mentioned and do not send a file named as `STUDENTID.py`. Submissions that do not follow these guidelines will not be graded.

Late submission policy: Late submissions will not be graded.

Cheating policy: Very similar submissions will not be graded.
