Parallel Programming

CUDA Example: Matrix Multiplication

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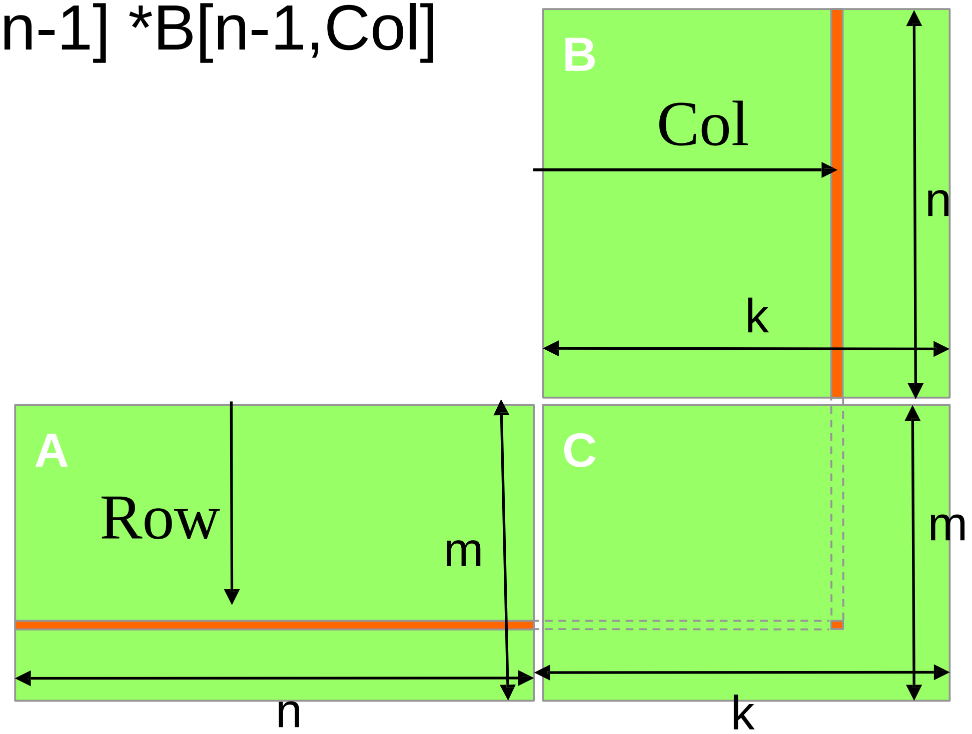
Overview

- Matrix multiplication as an example in CUDA
  - Math operation review
  - Baseline implementation
  - Tiling for shared memory/blocking

Math Review: Matrix Multiplication

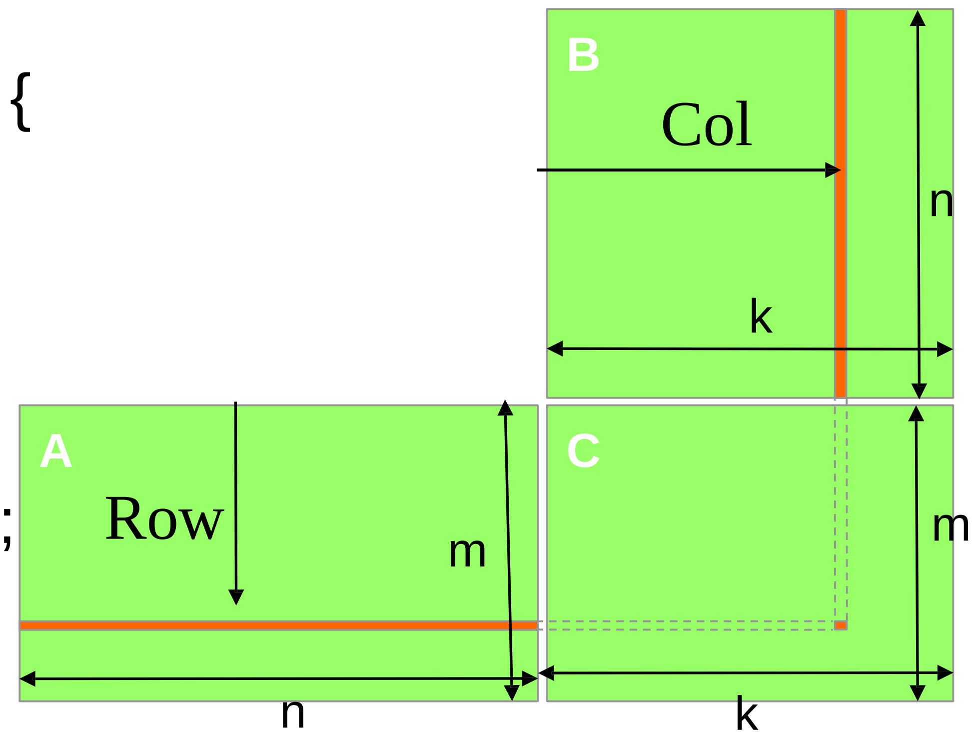
A_{mxn} X B_{nxk} = C_{mxk}

C[Row,Col] = A's row at Row· B's column at Col
= A[Row,0] * B[0, Col] + A[Row,1]*B[1,Col} + …
+A[Row,n-1] *B[n-1,Col]



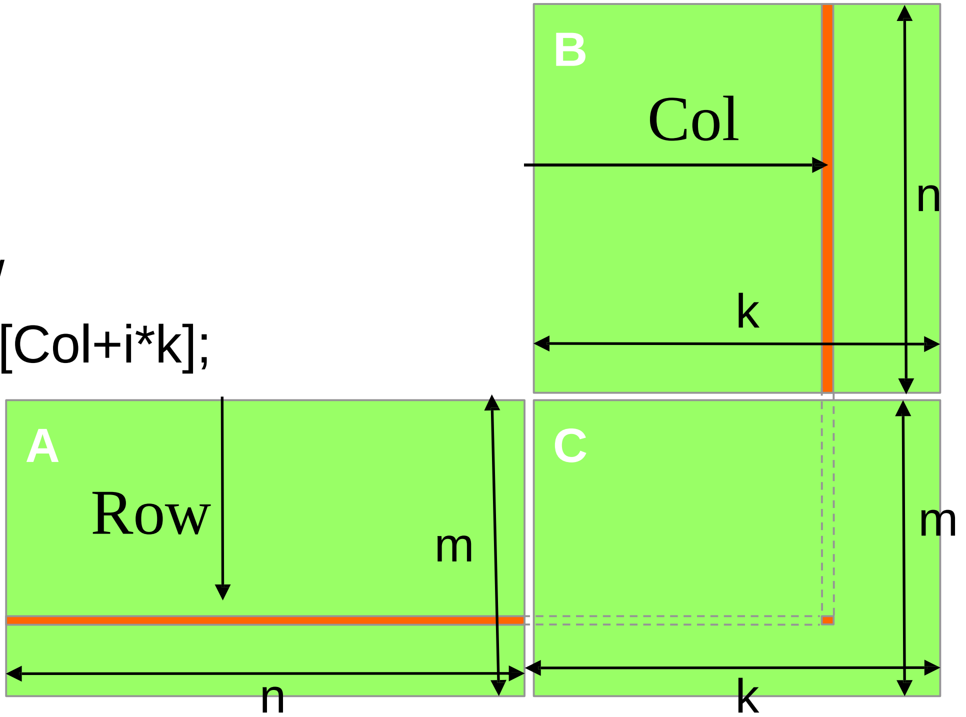
Sequential C code

```
void MatrixMulOnHost(int m, int n, int k, float* A, float* B, float* C)
{
 for (int Row = 0; Row < m; ++Row)
  for (int Col = 0; Col < k; ++Col) {
    float sum = 0;
    for (int i = 0; i < n; ++i) {
    float a = A[Row*n + i];
    float b = B[Col + i*k];
    sum += a *b;
    }
    C[Row*k + Col] = sum;
}
```

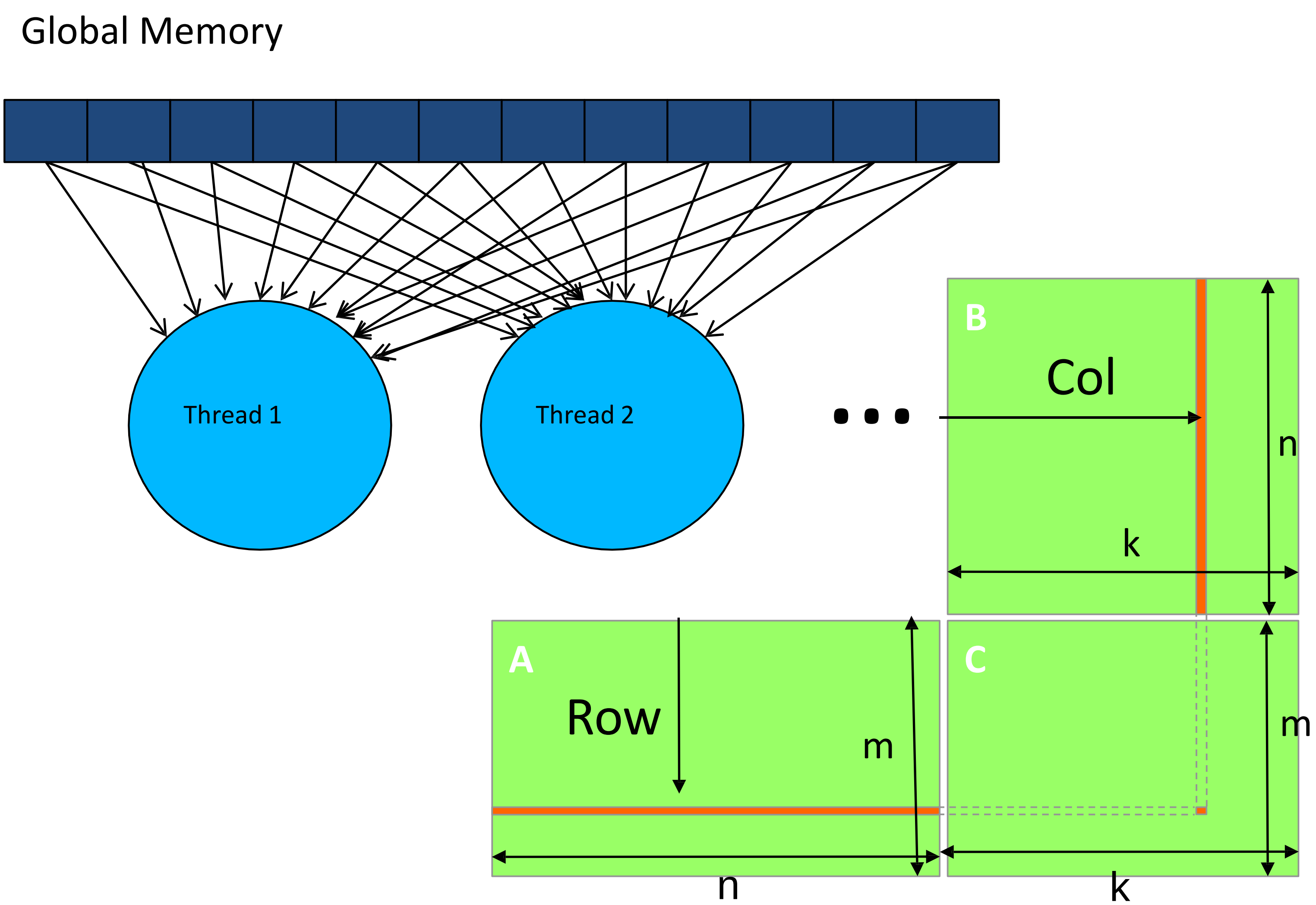


Baseline Kernel

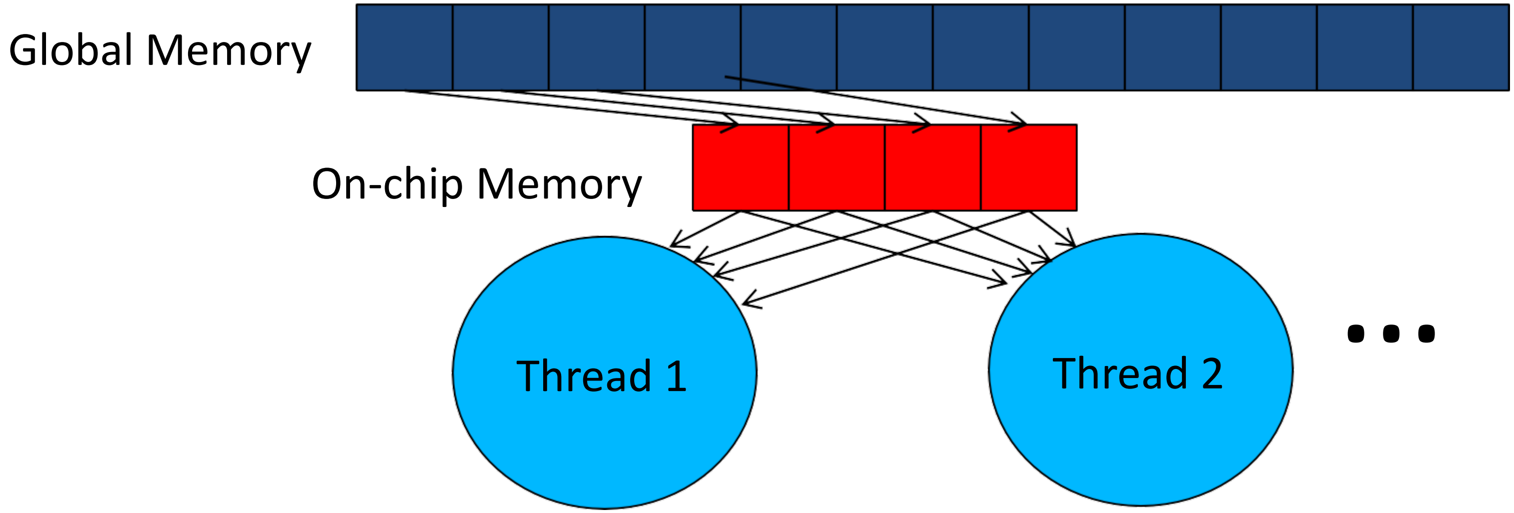
```
_global___void MatrixMulKernel(int m,int n,int k,float* A,float* B, float* C)
{
    int Row = blockIdx.y*blockDim.y+threadIdx.y;
    int Col   = blockIdx.x*blockDim.x+threadIdx.x;

    if ((Row < m)   && (Col < k)) {
     float Cvalue = 0.0;
     for (int i = 0; i < n; ++i)
         /* A[Row, i] and B[i, Col] */
         Cvalue += A[Row*n+i] * B[Col+i*k];
         C[Row*k+Col] = Cvalue;
     }
    }
```



Memory Access Pattern



Shared Memory Tiling/Blocking

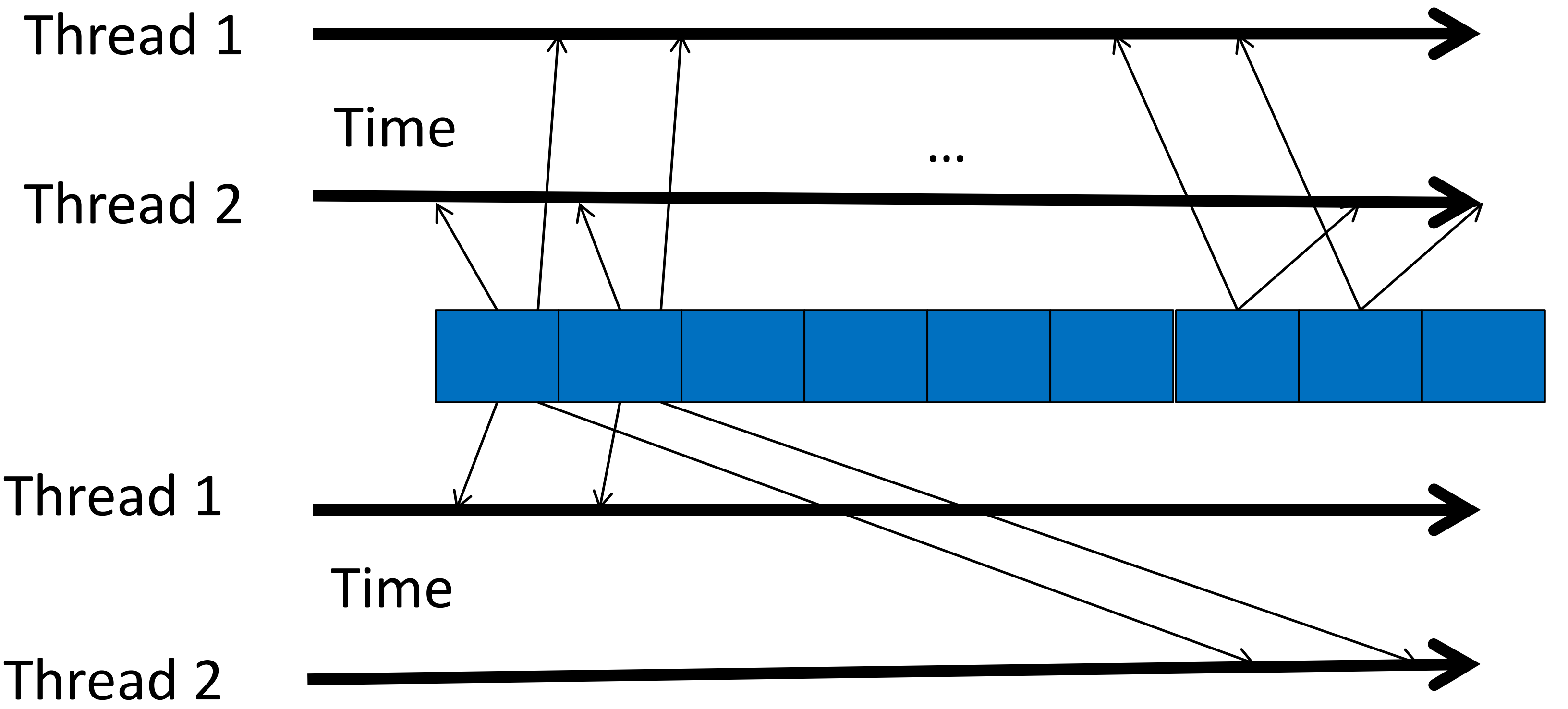


Divide the global memory content into tiles

Focus the computation of small number of tiles in multiple threads at each point in time

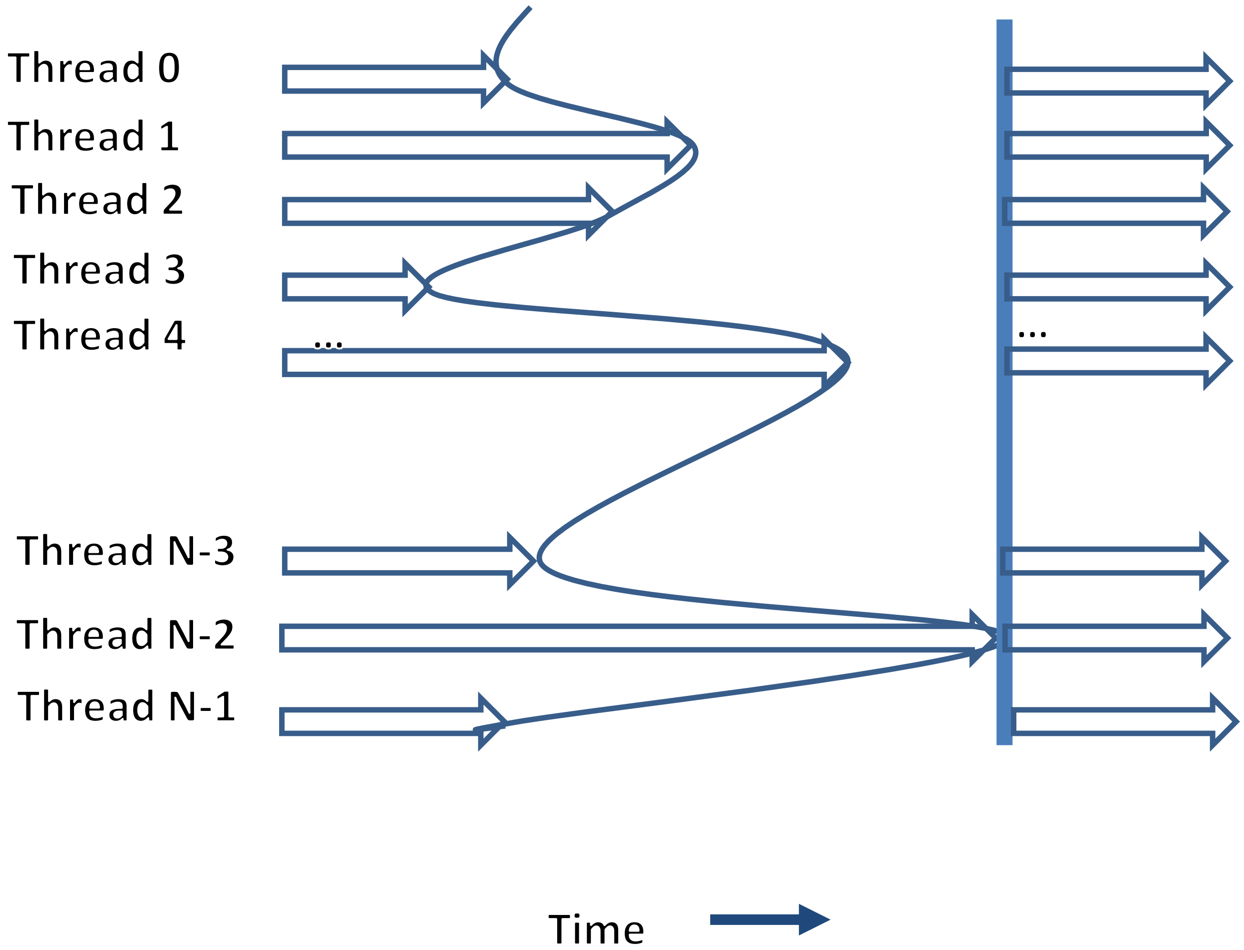
Timing with Tiling

- Good: when threads have similar access timing



- Bad: when threads have very different timing

Barrier Synchronization for Tiling



Barrier Synchronization

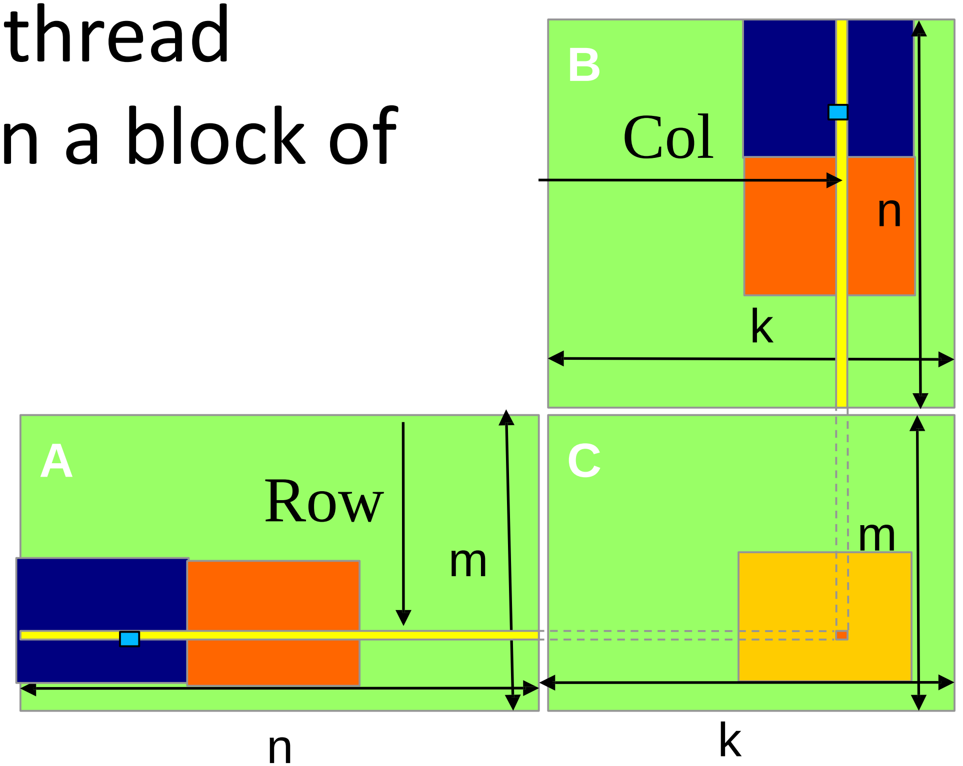
- Synchronize all threads in a thread block: __syncthreads()
- All threads in the same block must reach the __syncthreads() before any of them can move on
- Best used to coordinate tiled algorithms
  - To ensure that all elements of a tile are loaded at the beginning of a phase
  - To ensure that all elements of a tile are consumed at the end of a phase

Outline of Tiling

- Identify a tile of global memory contents that are accessed by multiple threads
- Load the tile from global memory into on-chip memory
- Use barrier synchronization to make sure that all threads are ready to start the phase
- Have the multiple threads to access their data from the on-chip memory
- Use barrier synchronization to make sure that all threads have completed the current phase
- Move on to the next tile

Matrix Multiplication Tiled

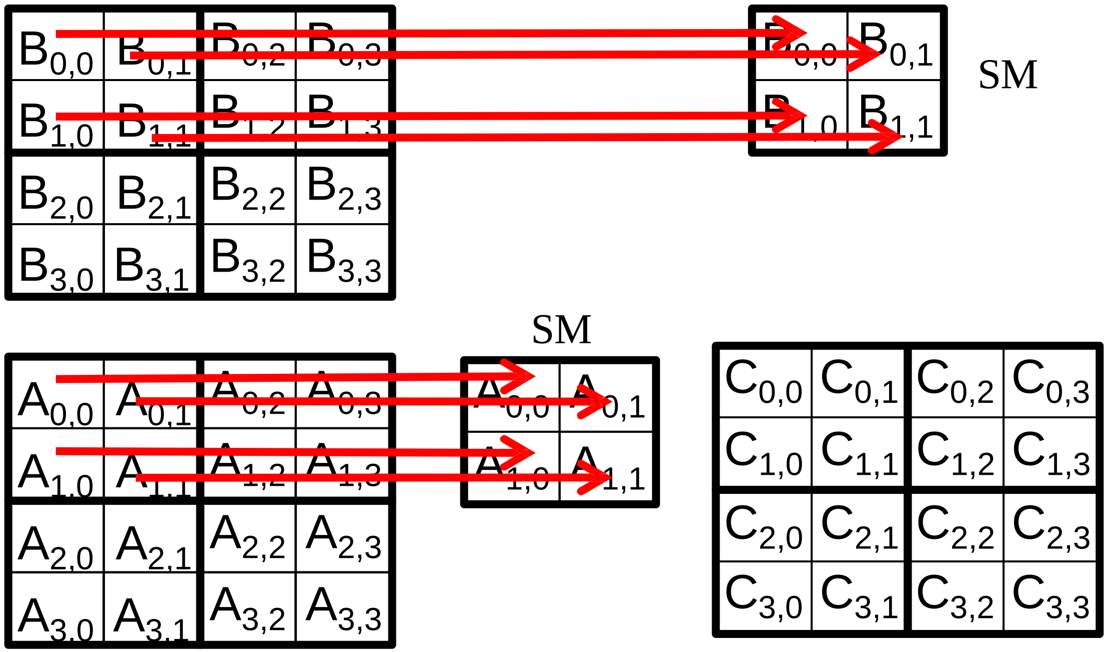
- Break up the execution of each thread into phases so that the data accessed by a thread block is contained in a block of A and a block of B.



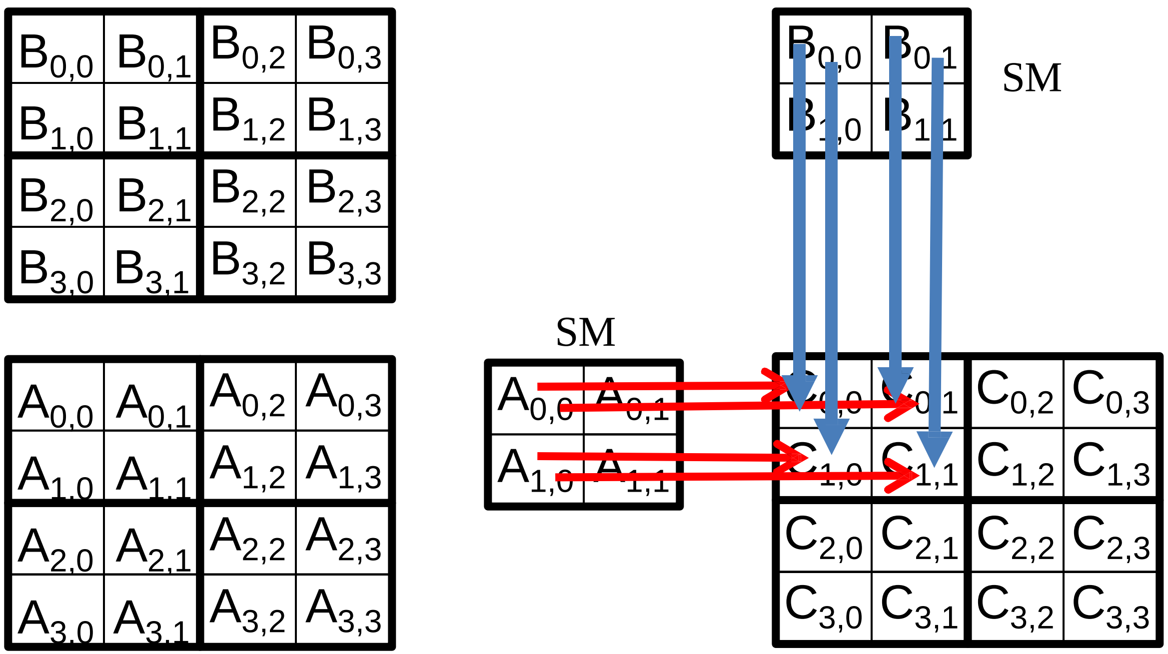
Loading a Tile

- All threads in a block participate
  - Each thread loads one A element and one B element in the tiled code
- Assign the loaded element to each thread such that the accesses within each warp are coalesced

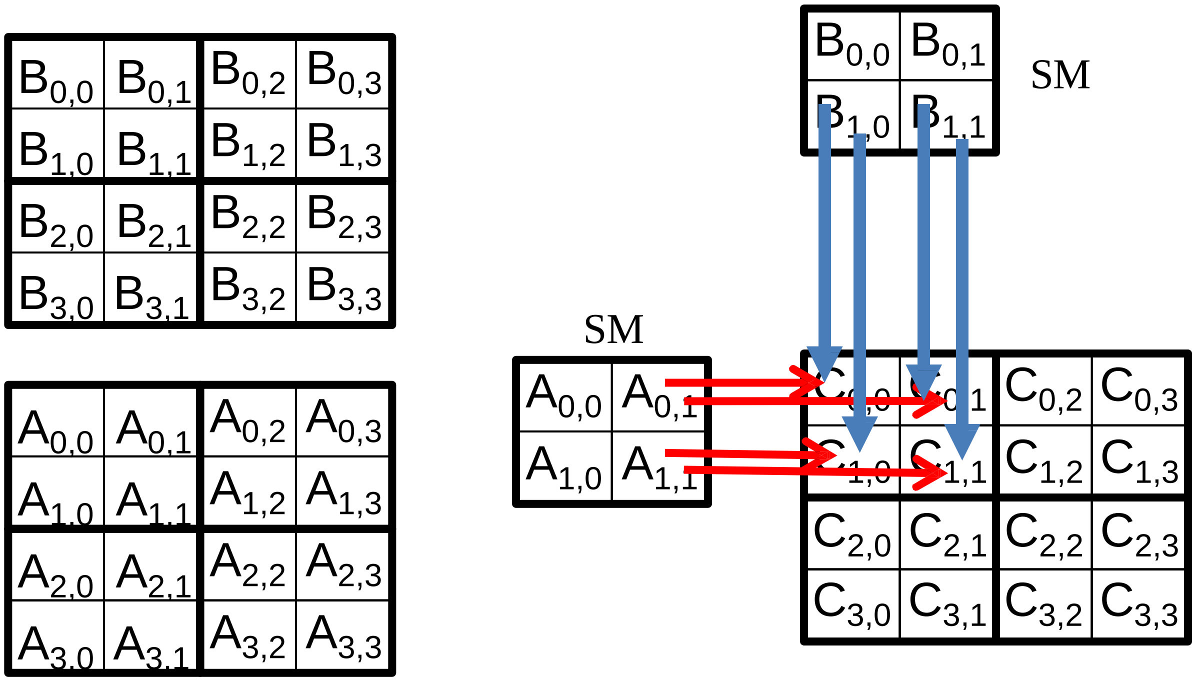
Phase 0: Load for Block (0,0) of C



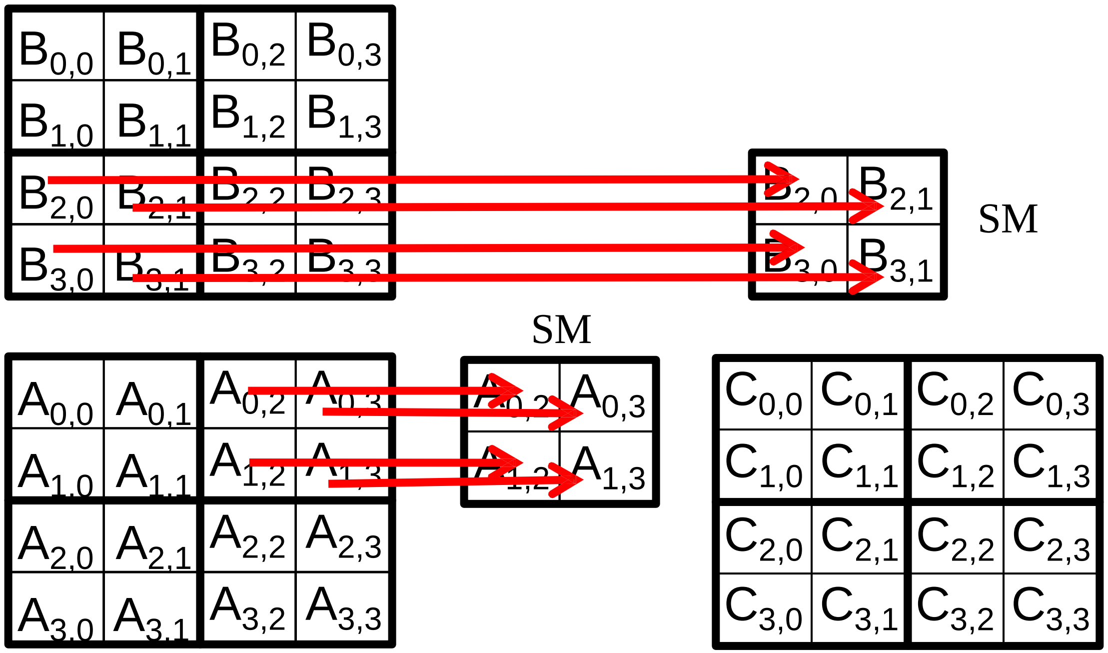
Phase 0: Compute Block (0,0) Iteration 0



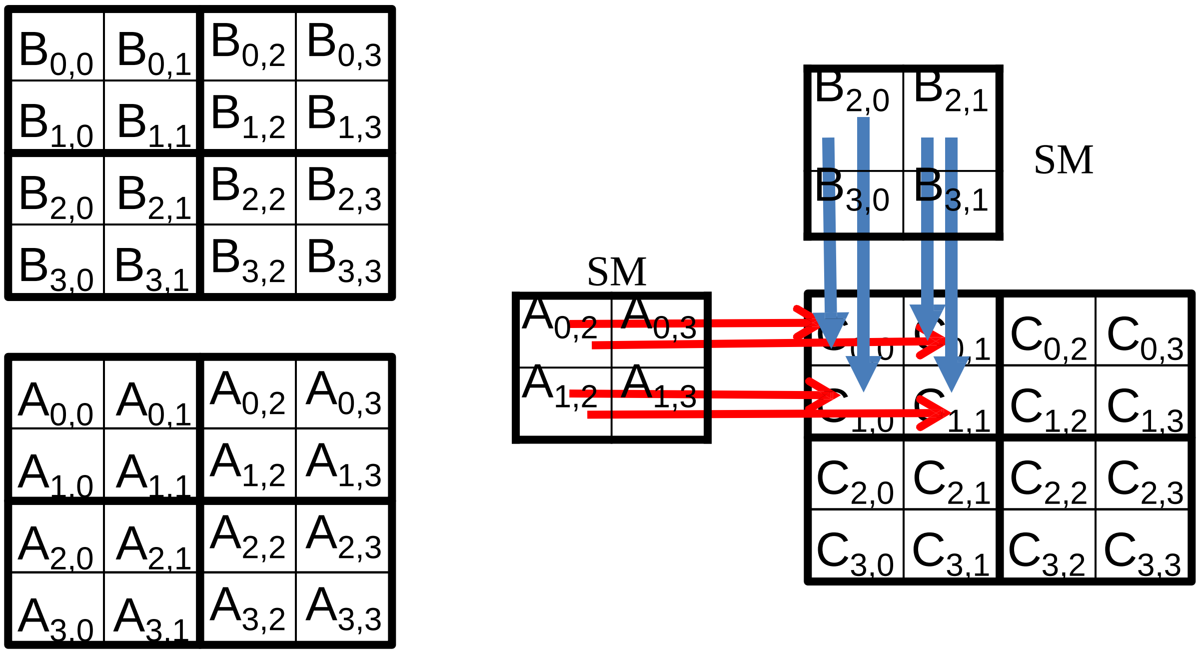
Phase 0: Compute Block (0,0) Iteration 1



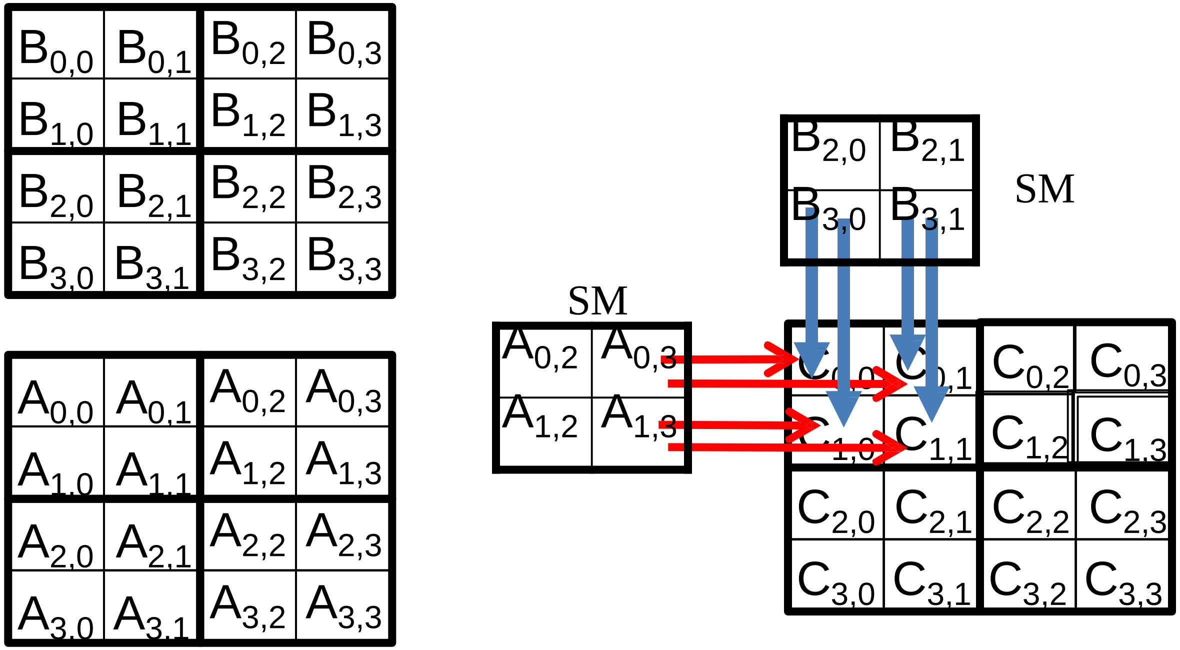
Phase 1: Load for Block (0,0) of C



Phase 1: Compute Block (0,0) Iteration 0



Phase 1: Compute Block (0,0) Iteration 1



Loading a Tile: 2D Element Index

Have each thread to load an A element and a B element at the same relative position as its C element.

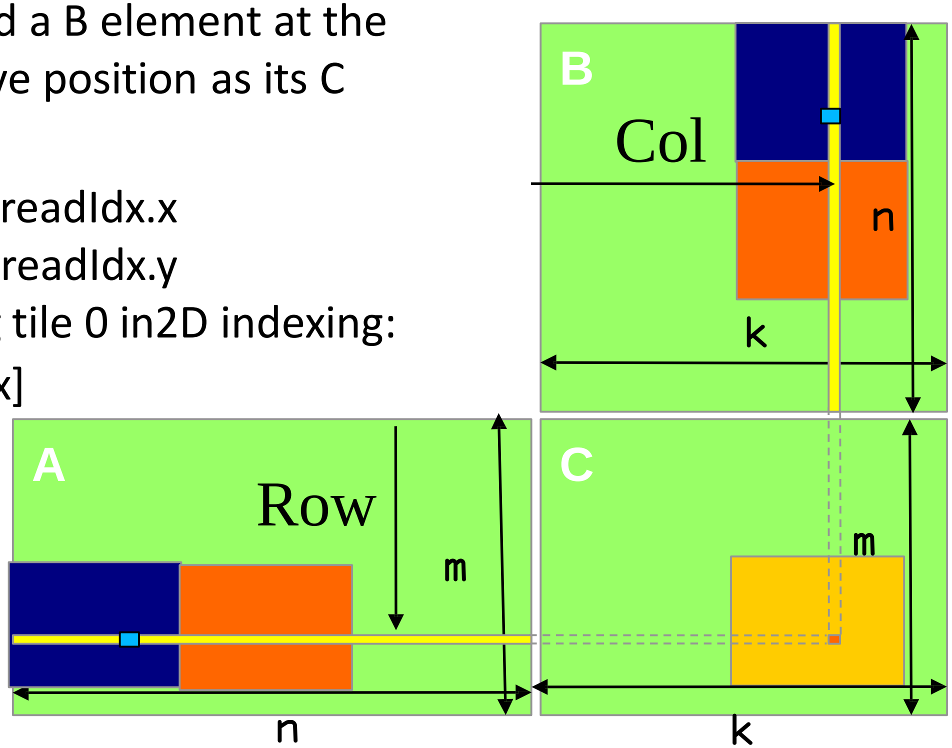
int tx = threadIdx.x

int ty = threadIdx.y

Accessing tile 0 in2D indexing:

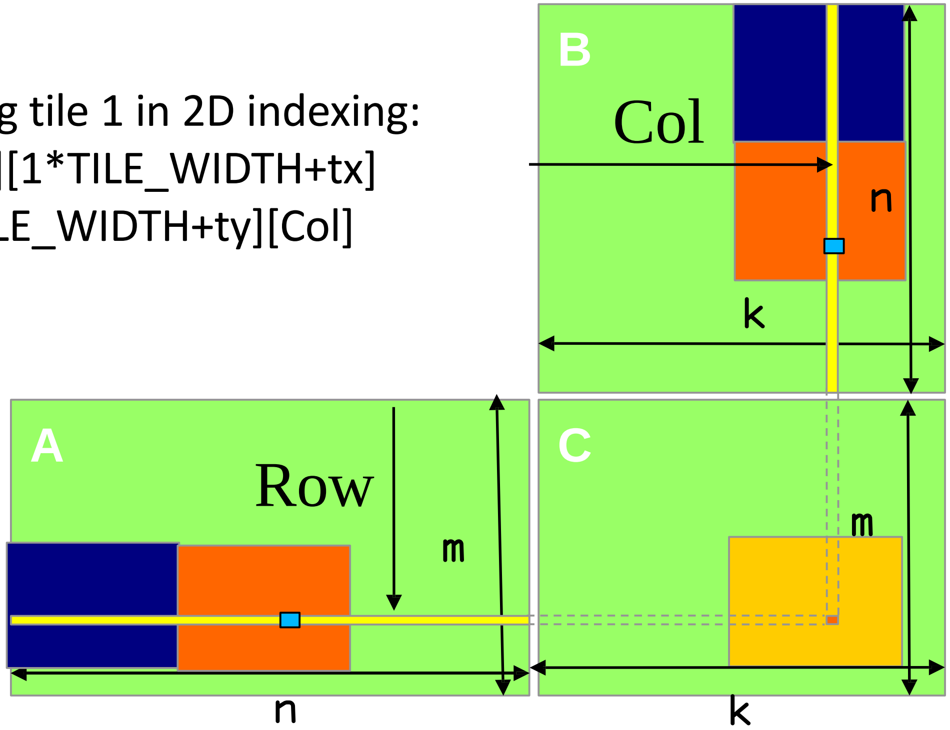
A[Row][tx]

B[ty][Col]



Loading a Tile: 2D Element Index (cont.)

Accessing tile 1 in 2D indexing:
A[Row][1*TILE_WIDTH+tx]
B[1*TILE_WIDTH+ty][Col]

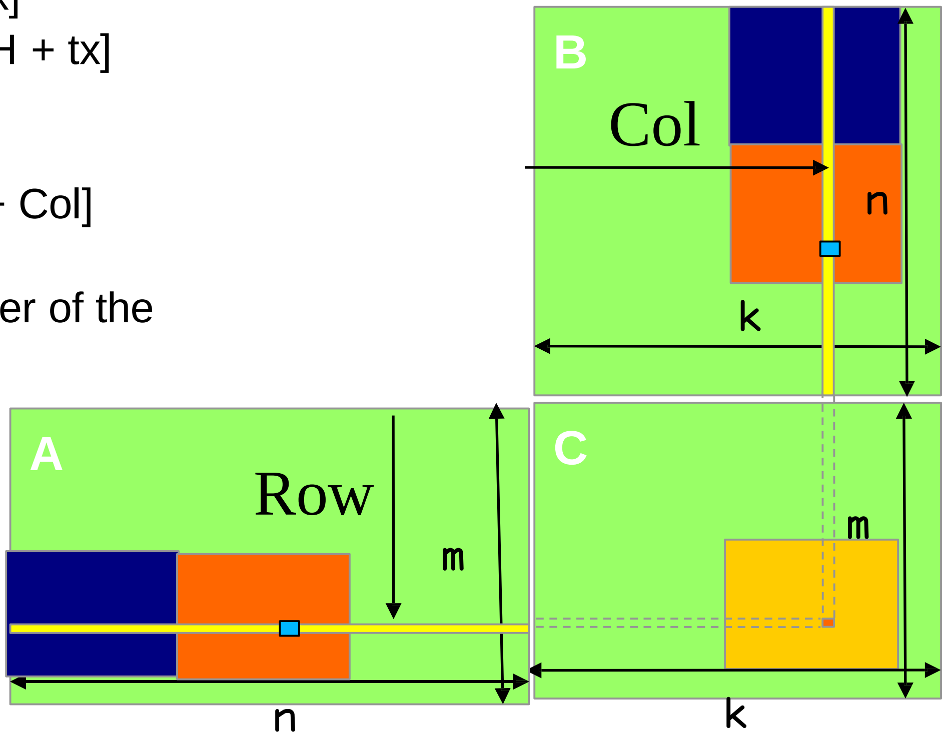


Loading a Tile: Element in 1D Index

A[Row][t*TILE_WIDTH+tx]
➡ A[Row*n + t*TILE_WIDTH + tx]

B[t*TILE_WIDTH+ty][Col]
➡ B[(t*TILE_WIDTH+ty)*k + Col]

where t is the tile sequence number of the current phase



Tiled Matrix Multiplication Kernel

```
__global__ void MatrixMulKernel(int m, int n, int k, float* A, float* B, float* C)
{

_shared__ float ds_A[TILE_WIDTH][TILE_WIDTH];
_shared__ float ds_B[TILE_WIDTH][TILE_WIDTH];

int   bx = blockIdx.x;   int      by = blockIdx.y;
int   tx = threadIdx.x; int      ty = threadIdx.y;

int   Row = by    * blockDim.y    + ty;
int   Col = bx    * blockDim.x    + tx;
float Cvalue = 0;
```

Tiled Matrix Multiplication Kernel (cont.)

```
//Loop over the A and B tiles as required to compute the C
    for (int      t = 0; t      < n/TILE_WIDTH; ++t) {
 // Collaborative loading of A and B tiles into memory

        ds_A[ty][tx] = A[Row*n + t*TILE_WIDTH+tx];
        ds_B[ty][tx] = B[(t*TILE_WIDTH+ty)*k + Col];
        _syncthreads();

        for (int i = 0; i < TILE_WIDTH; ++i)
            Cvalue += ds_A[ty][i] * ds_B[i][tx];
      _synchthreads();

    }
      C[Row*k+Col] = Cvalue;
}
```

Block Size Consideration

- Each thread block should have many threads
  - TILE_WIDTH of 16 gives 16*16 = 256 threads
  - TILE_WIDTH of 32 gives 32*32 = 1024 threads
- For 16, each block performs 2*256 = 512 float loads from global memory for 256 * (2*16) = 8,192 mul/add operations. (memory traffic reduced by a factor of 16)
- For 32, each block performs 2*1024 = 2048 float loads from global memory for 1024 * (2*32) = 65,536 mul/add operations. (memory traffic reduced by a factor of 32)
- However, the thread count limitation of threads per SM in current generation GPUs will reduce the number of blocks per SM (e.g., with a limit of 1536 threads per SM, we have 1536/256 = 6 16*16blocks, 1536/1024 = 1 block).

Shared Memory Size Consideration

- For an SM with 16KB shared memory
  - For TILE_WIDTH = 16, each thread block uses 2*256*4B = 2KB of shared memory. We can have up to 8 thread blocks. This allows up to 8*512 = 4,096 pending loads. (2 per thread, 256 threads per block)
  - The next TILE_WIDTH 32 would lead to 2*32*32*4 Byte= 8K Byte shared memory usage per thread block, allowing 2 thread blocks active at the same time.
- Each __syncthread() can reduce the number of active threads for a block
  - More thread blocks can be advantageous

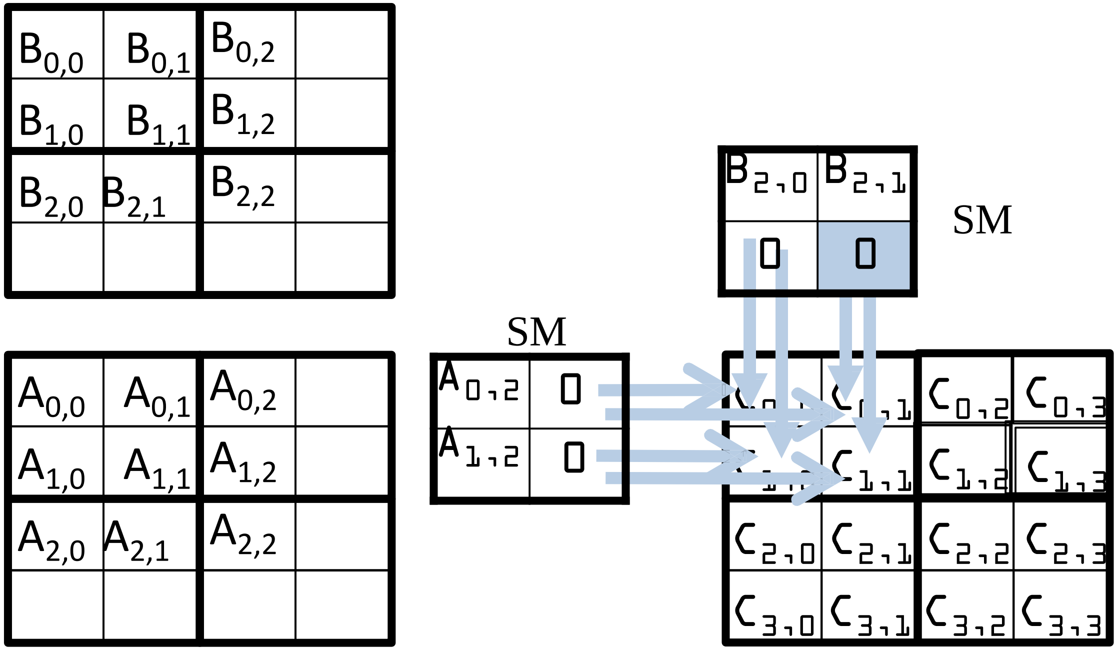
What If Tiles Exceed Matrix Boundaries

- When a thread is to load any input element, test if it is in the valid index range
  - If valid, proceed to load
  - Else, do not load, just write a 0
- Rationale: a 0 value will ensure that the multiply-add step does not affect the final value of the output element

Compute Elements Exceeding Boundaries

- If a thread does not calculate a valid output element, it can still perform multiply-add into its register as long as it is not allowed to write to the global memory at the end of the kernel
- This way, the thread does not need to be turned off by an if-statement as in the baseline kernel; it can participate in the tile loading process

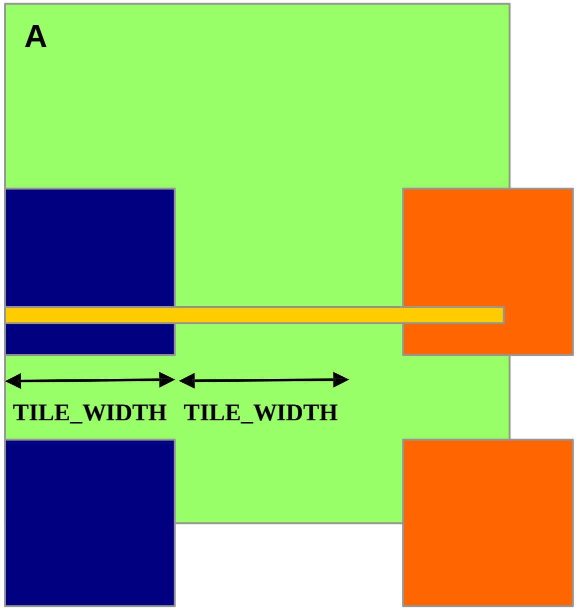
Illustration



The multiply-add will not affect the output due to 0's.

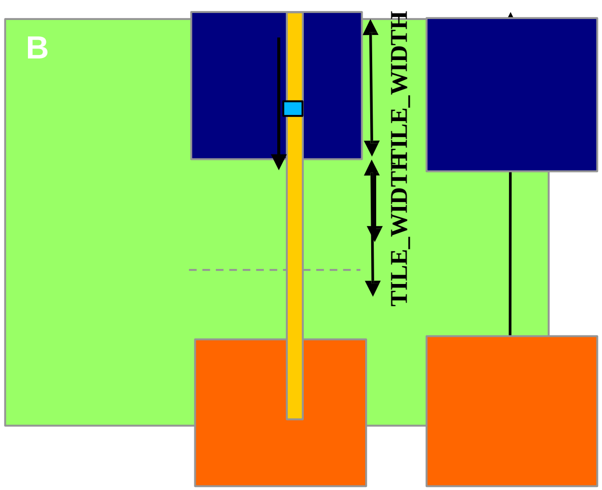
Testing Boundary Condition on A

- Each thread loads
  - A[Row][t*TILE_WIDTH+tx]
  - A[Row*Width + t*TILE_WIDTH+tx]
- Need to test
  - (Row < m) && (t*TILE_WIDTH+tx < n)
  - If true, load A element
  - Else , load 0



Testing Boundary Condition on B

- Each thread loads
  - B[t*TILE_WIDTH+ty][Col]
  - B[(t*TILE_WIDTH+ty)*k+ Col]
- Need to test
  - (t*TILE_WIDTH+ty < n) && (Col< k)
  - If true, load B element
  - Else , load 0



Code: Loading A and B Tiles with Boundary Checks

```
for (int t = 0; t < (n-1)/TILE_WIDTH + 1; ++t) {
        if(Row      < m && t*TILE_WIDTH+tx < n) {
                    ds_A[ty][tx] = A[Row*n + t*TILE_WIDTH+ tx];
        } else {
                   ds_A[ty][tx] = 0.0;
        }
        if (t*TILE_WIDTH+ty < n && Col < k) {
                   ds_B[ty][tx] = B[(t*TILE_WIDTH + ty)*k+col];
        } else {
                   ds_B[ty][tx] = 0.0;
        }
       __syncthreads();
```

Code: Calculate C Values and Store

```
        for (int i = 0; i < TILE_WIDTH; ++i) {
                Cvalue += ds_A[ty][i] * ds_B[i][tx];
        }
        _syncthreads();
    } /* end of outer for loop */
    if (Row < m && Col < k)
            P[Row*k + Col] = Cvalue;
} /* end of kernel */
```

Summary

- Matrix multiplication is a common computation task in many applications.
- Its parallelization in CUDA can be optimized by tiling and use of shared memory.
- When tiles exceed matrix boundaries, loading the input and storing the result needs to check the boundary conditions.