Parallel Programming

Data-Parallel Primitives:

Reduction

`

Overview

- The Reduction Operation
- Sequential Implementation
- Baseline Reduction Kernel
- Improved Reduction Kernel

Reduce (Reduction)

- A commonly used strategy for processing large input data sets
- There is no required order of processing elements in a data set (associative and commutative)
  - Partition the data set into smaller chunks
  - Have each thread to process a chunk
  - Use a reduction tree to summarize the results from each chunk into the final answer
- Google and Hadoop MapReduce frameworks support this strategy

Reduction in Other Parallel Operations

- Reduction is also needed to clean up after some commonly used transformations
- Privatization
  - Multiple threads write into an output location
  - Replicate the output location so that each thread has a private output location
  - Use a reduction tree to combine the values of private locations into the original output location

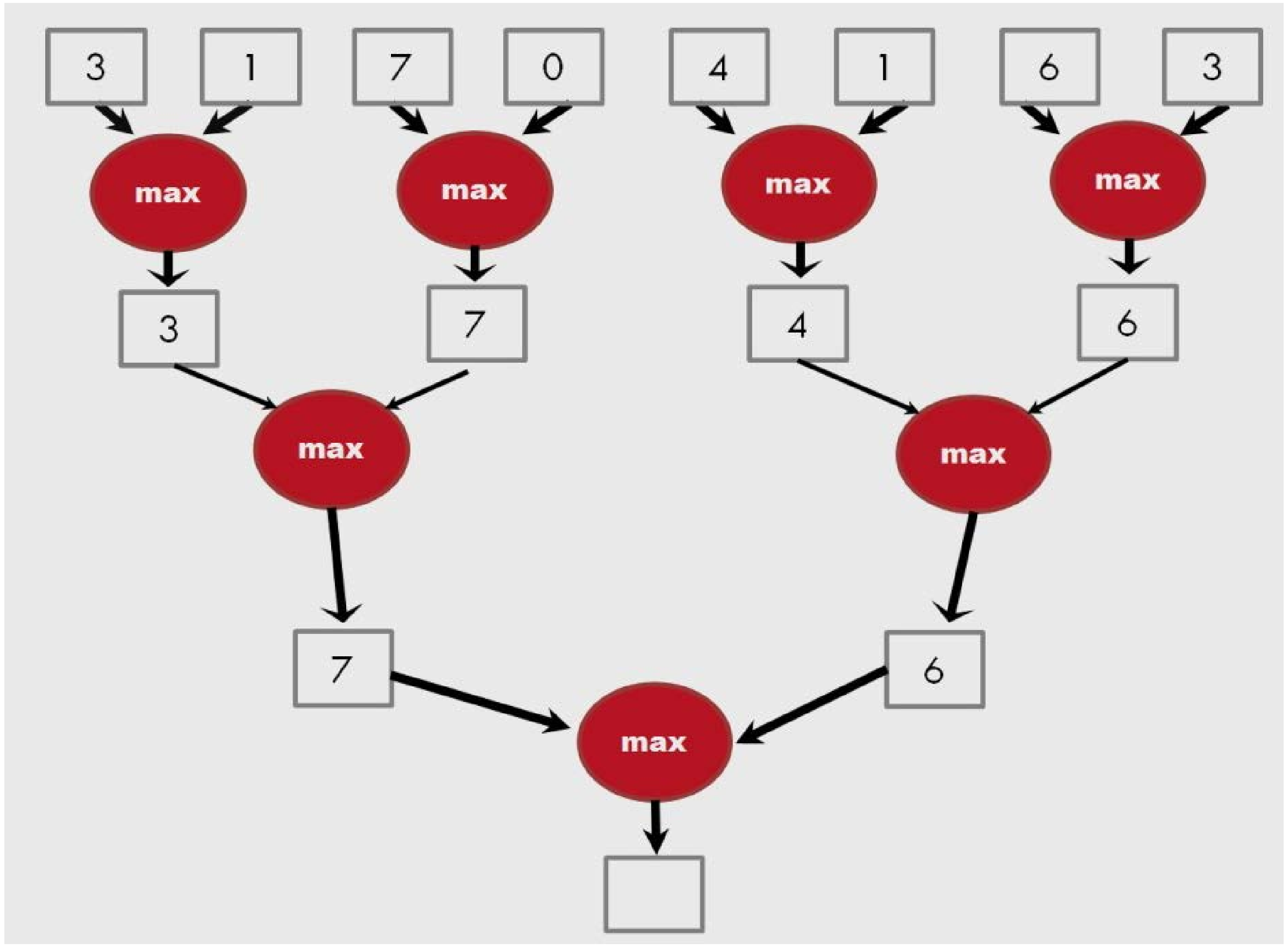
Computation used in Reduction

- Summarize a set of input values into one value using a "reduction operation"
  - Max
  - Min
  - Sum
  - Product
  - User defined reduction operation function as long as the operation
    - Is associative and commutative
    - Has a well-defined identity value (e.g., 0 for sum)

Sequential Reduction

- Initialize the result as an identity value for the reduction operation
  - Smallest possible value for max reduction
  - Largest possible value for min reduction
  - 0 for sum reduction
  - 1 for product reduction
- Iterate through the input and perform the reduction operation between the result value and the current input value
  - N reduction operations performed for N input values

A Reduction Tree



Analysis of Reduction Tree

- For N input values, the reduction tree performs

(1/2)N + (1/4)N + (1/8)N + … 1 = (1- (1/N))N = N-1 operations

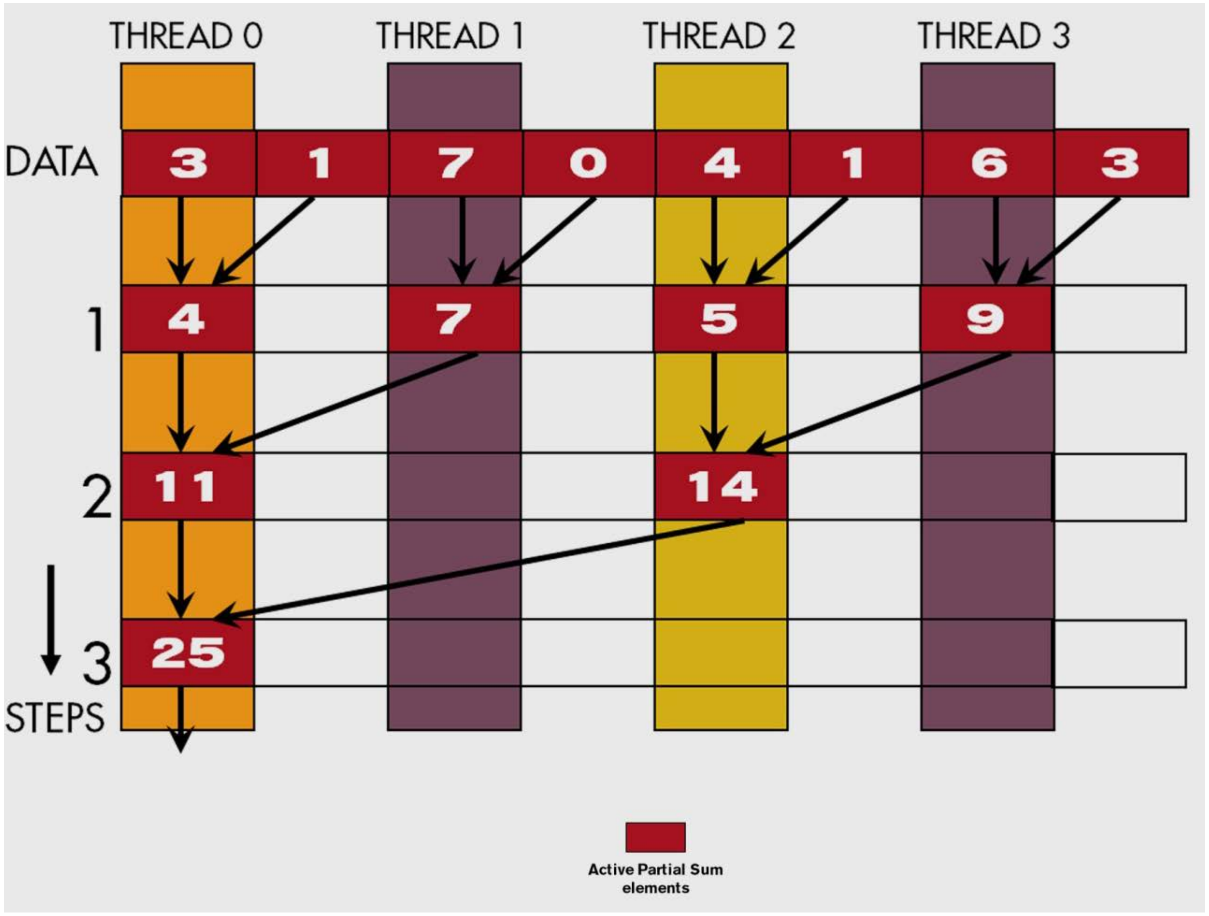
- In Log (N) steps – 1,000,000 input values take 20 steps
  - Assuming that we have enough execution resources
- Average Parallelism (N-1)/Log(N))
  - For N = 1,000,000, average parallelism is 50,000
  - However, peak resource requirement is 500,000!
  - This is not resource efficient.
- This is a work-efficient parallel algorithm
  - The amount of work done is comparable to sequential
  - Many parallel algorithms are not work efficient

Parallel Implementation

- Parallel execution of reduction tree
  - Add two values per thread in each step
  - Halve # of threads for next step
  - Takes log(n) steps for n elements
  - Requires n/2 threads at most in a step
- In-place reduction using shared memory
  - The original vector is in device global memory
  - The shared memory is used to hold a partial sum vector
  - Each step brings the partial sum vector closer to the sum
  - The final sum will be in element 0
  - Reduces global memory traffic due to partial sum values

n<=2048 for current GPU due to limit of number of threads per SM

Example of Parallel Reduction



Baseline Thread-to-Data Mapping

- Each thread is responsible for an even-index location of the partial sum vector
  - In each step, one of the input is always from the location of responsibility
  - The other input comes from an increasing distance away
- After each step, half of the threads are no longer needed

Simple Thread Block Design

- Each thread block takes 2* BlockDim.x input elements
- Each thread loads 2 elements into shared memory

```
__shared__ float partialSum[2*BLOCK_SIZE];
unsigned int t = threadIdx.x;
unsigned int start = 2*blockIdx.x*blockDim.x;
partialSum[t] = input[start + t];
partialSum[blockDim.x+t] = input[start + blockDim.x+t];
```

Reduction

```
for (unsigned int stride = 1; stride <= blockDim.x;
stride *= 2)
{
    __syncthreads();
    if (t % stride == 0)
        partialSum[2*t]+=partialSum[2*t+stride];
}
```

Synchronization Barrier

- __syncthreads() is needed to ensure that all elements of each version of partial sums have been generated before we proceed to the next step

Finishing Up Reduction

- At the end of the kernel, Thread 0 in each thread block writes the sum of the thread block in partialSum[0] into a vector indexed by the blockIdx.x
- There can be a large number of such sums if the original input array for reduction is very large
  - The host code may iterate and launch another kernel
- If there are only a small number of sums, the host can simply transfer the data back and add them together.

Problems in the Simple Reduction Kernel

- In each iteration, two control flow paths will be sequentially traversed for each warp
  - Threads that perform addition and threads that do not
  - Threads that do not perform addition still consume execution resources

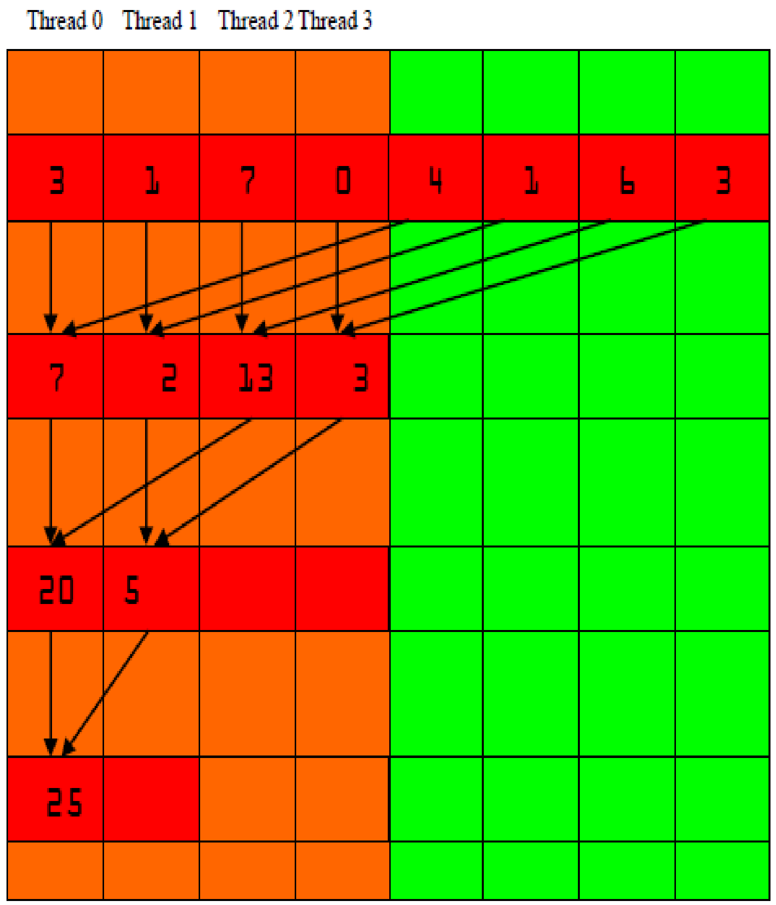
Problems in the Simple Reduction Kernel

- Half or fewer of threads will be executing after the first step
  - All odd-index threads are disabled after first step
  - After the 5th step, entire warps in each block will fail the if test, poor resource utilization but no divergence.
  - This can go on for a while, up to 6 more steps (stride = 32, 64, 128, 256, 512, 1024), where each active warp only has one productive thread until all warps in a block retire

Thread Index Usage Matters

- In some algorithms, one can shift the index usage to improve the divergence behavior
  - Commutative and associative operators
- Always compact the partial sums into the front locations in the partialSum[] array
- Keep the active threads consecutive

An Example of Four Threads



A Better Reduction Kernel

```
for (unsigned int stride = blockDim.x; stride > 0; stride /= 2)
{
   __syncthreads();
   if (t < stride)
      partialSum[t] += partialSum[t+stride];
}
```

Analysis on the Better Kernel

- For a 1024 thread block
  - No divergence in the first 5 steps
    - 1024, 512, 256, 128, 64, 32 consecutive threads are active in each step
    - All threads in each warp either all active or all inactive
  - The final 5 steps will still have divergence

Summary

- Reduction or reduce is also a data-parallel primitive
- Sequential implementation is of O(n) time complexity
- Parallel reduction tree algorithm is work efficient
- Thread index mapping improves reduction kernel performance