CONCEPTION ET VÉRIFICATION DE SYSTÈMES CRITIQUES

LA SPÉCIFICATION DES PROPRIÉTÉS AVEC LA LOGIQUE CTL

2A Cursus Ingénieurs - ST5 : Modélisation fonctionnelle et régulation
CentraleSupelec - Université Paris-Saclay - 2024/2025

Idir AIT SADOUNE
idir.aitsadoune@centralesupelec.fr

OUTLINE

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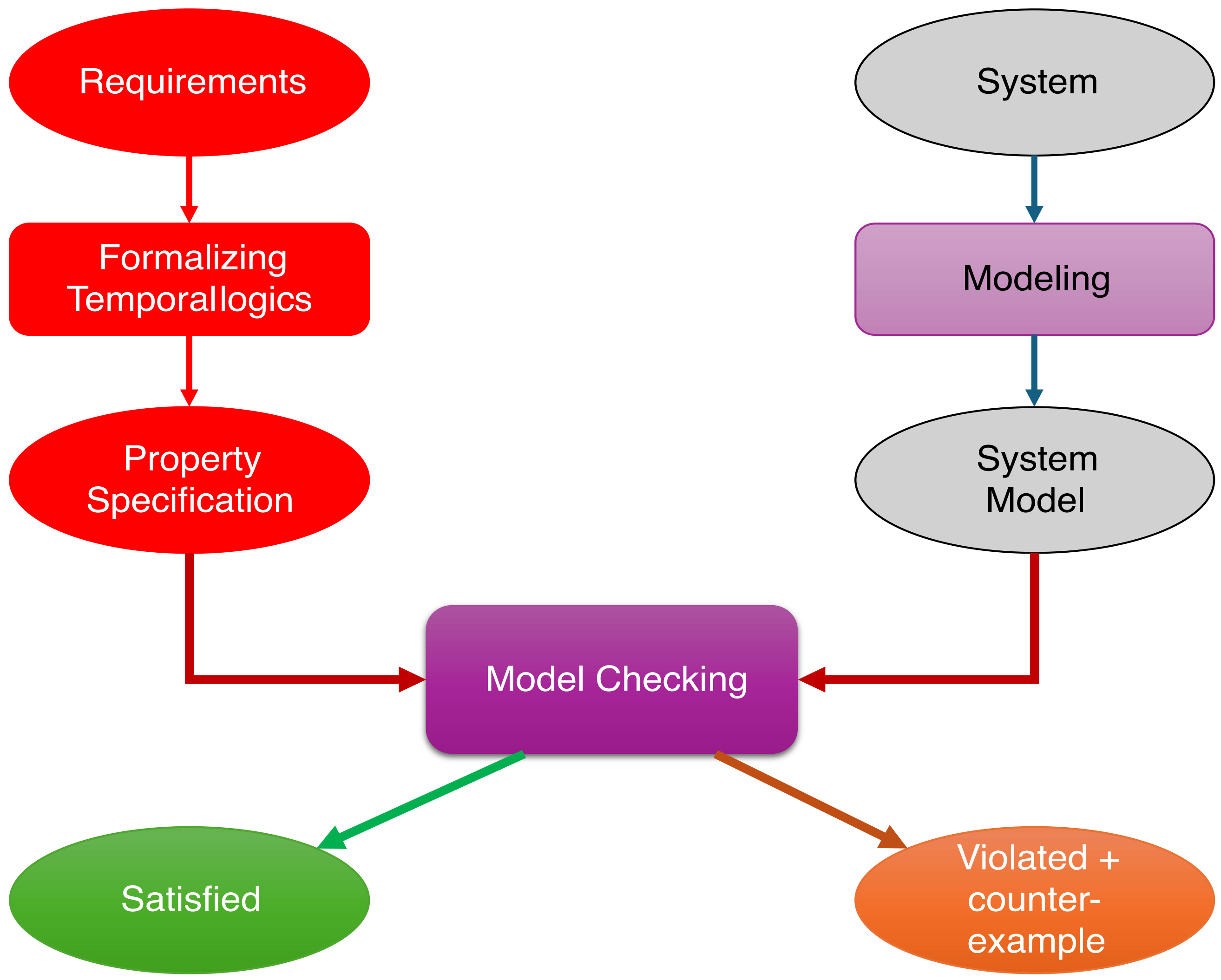
PRINCIPLE OF MODEL-CHECKING



OUTLINE

- Introducing CTL
- CTL Logics
- CTL running example
- Example : Dining Philosophers

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LTL AND CTL

- **LTL - (linear-time logic)**
  - Describes properties of individual executions.
  - Semantics defined as a set of executions.
- **CTL - (computation tree logic)**
  - Describes properties of a computation tree: formulas can reason about many executions at once. (CTL belongs to the family of branching-time logics.)
  - Semantics defined in terms of states.

COMPUTATION TREE

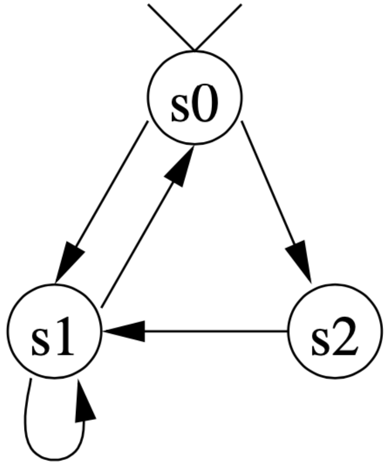
- Let $\mathcal{T} = (S, \rightarrow, s^0)$ be a transition system.
  Intuitively, the **computation tree** of $\mathcal{T}$ is the acyclic unfolding of $\mathcal{T}$.

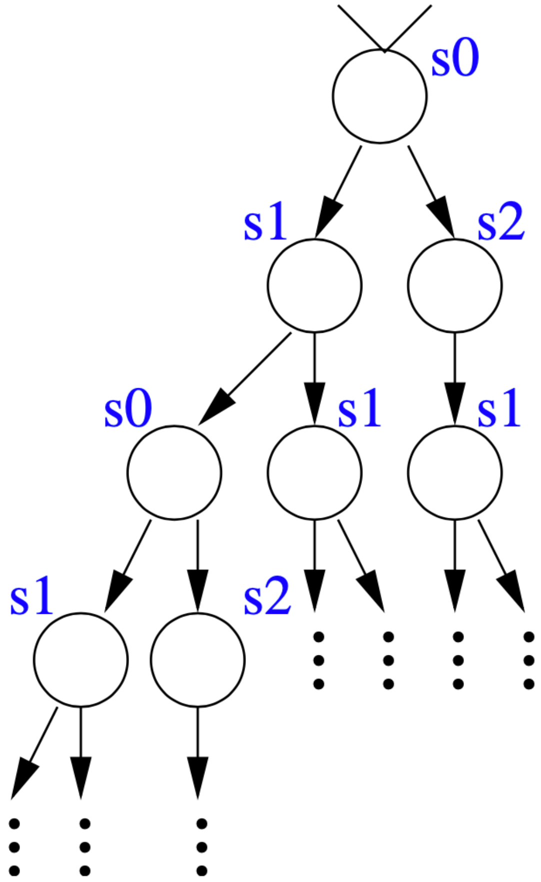
- Formally, we can define the unfolding as the least (possibly infinite) transition system $(U, \rightarrow', u^0)$ with a labelling $l : U \rightarrow S$ such that :
  - $u^0 \in U$ and $l(u^0) = s^0$
  - if $u \in U, l(u) = s$, and $s \rightarrow s'$ for some u, s, s'
    then there is $u' \in U$ with $u \rightarrow' u'$ and $l(u') = s'$
  - u^0 does not have a direct predecessor, and all other states in U have exactly one direct predecessor

Note: For model checking CTL, the construction of the computation tree will not be necessary. However, this definition serves to clarify the concepts behind CTL.

COMPUTATION TREE EXAMPLE

A transition system and its computation tree (labelling l given in blue)





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CTL - OVERVIEW

- Combines temporal operators with quantification over runs
- Operators have the following form: $Q\ T$
  - Q
    - E : there exists an execution
    - A : for all executions
  - T
    - $X \equiv \bigcirc$: next
    - $F \equiv \Diamond$: finally
    - $G \equiv \Box$: globally
    - $U \equiv \bigcup$: until
    - (and possibly others)

CTL - SYNTAX

- We define a minimal syntax first. Later we define additional operators with the help of the minimal syntax.

- Let AP be a set of atomic propositions: The set of CTL formulas over AP is as follows:

  if $a \in AP$, then a is a CTL formula;

  if $\phi_1, \ \phi_2$ are CTL formulas, then so are

  $$\neg\phi_1, \qquad \phi_1 \vee \phi_2, \qquad EX\ \phi_1, \qquad EG\ \phi_1, \qquad E\ (\phi_1\ U\ \phi_2)$$

CTL - SEMANTICS

- let $\mathcal{K} = (S, \to, s^0, AP, v)$ be a **Kripke** structure.
- We define the semantic of every CTL formula ϕ over AP with respect to $\mathcal{K}$ as a set of states $[\![\phi]\!]_\mathcal{K}$, as follows :

$$
\begin{aligned}
[\![a]\!]_\mathcal{K} &= v(a) \qquad a \in AP \\
[\![\neg\phi_1]\!]_\mathcal{K} &= S \backslash [\![\phi_1]\!]_\mathcal{K} \\
[\![\phi_1 \vee \phi_2]\!]_\mathcal{K} &= [\![\phi_1]\!] \cup [\![\phi_2]\!]_\mathcal{K} \\
[\![EX\ \phi_1]\!]_\mathcal{K} &= \{s \mid \text{there is a state } t \text{ with } s \to t \text{ and } t \in [\![\phi_1]\!]_\mathcal{K}\} \\
[\![EG\ \phi_1]\!]_\mathcal{K} &= \{s \mid \text{there is a run } \sigma \text{ with } \sigma(0) = s \text{ and } \sigma(i) \in [\![\phi_1]\!]_\mathcal{K}\ \forall i \geq 0\} \\
[\![E\ (\phi_1\ U\ \phi_2)]\!]_\mathcal{K} &= \{s \mid \text{there is a run } \sigma \text{ with } \sigma(0) = s \text{ and } k \geq 0,\ \sigma(i) \in [\![\phi_1]\!]_\mathcal{K}\ \forall i < k,\ \sigma(k) \in [\![\phi_2]\!]_\mathcal{K}\}
\end{aligned}
$$

CTL - EXTENDED SYNTAX

$$
\begin{aligned}
false &\equiv \neg true \\
\phi_1 \vee \phi_2 &\equiv \neg(\neg\phi_1 \wedge \neg\phi_2) \\
EF\ \phi &\equiv E\ (true\ U\ \phi) \\
AX\ \phi &\equiv \neg EX\ \neg\phi \\
AG\ \phi &\equiv \neg EF\ \neg\phi \\
AF\ \phi &\equiv \neg EG\ \neg\phi \\
A\ (\phi_1\ U\ \phi_2) &\equiv \neg E\ \neg(\phi_1\ U\ \phi_2)
\end{aligned}
$$

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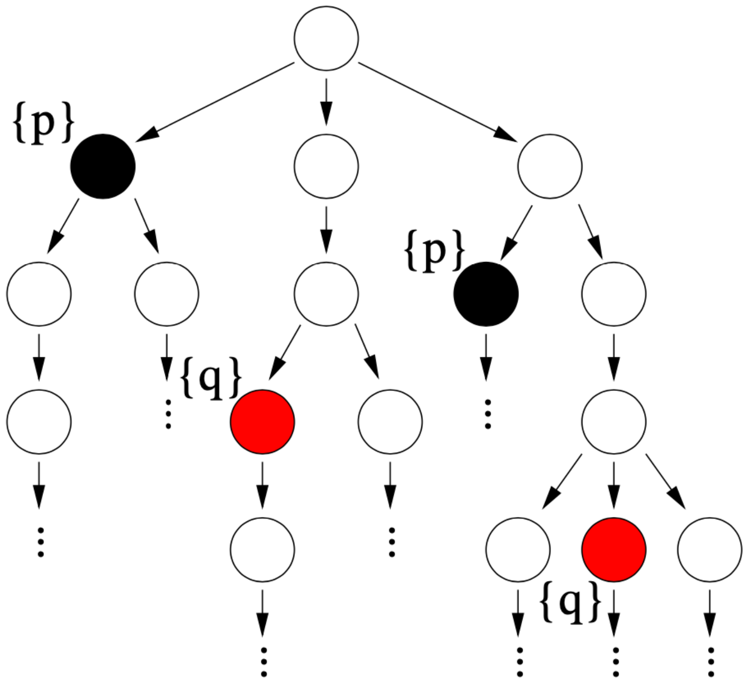
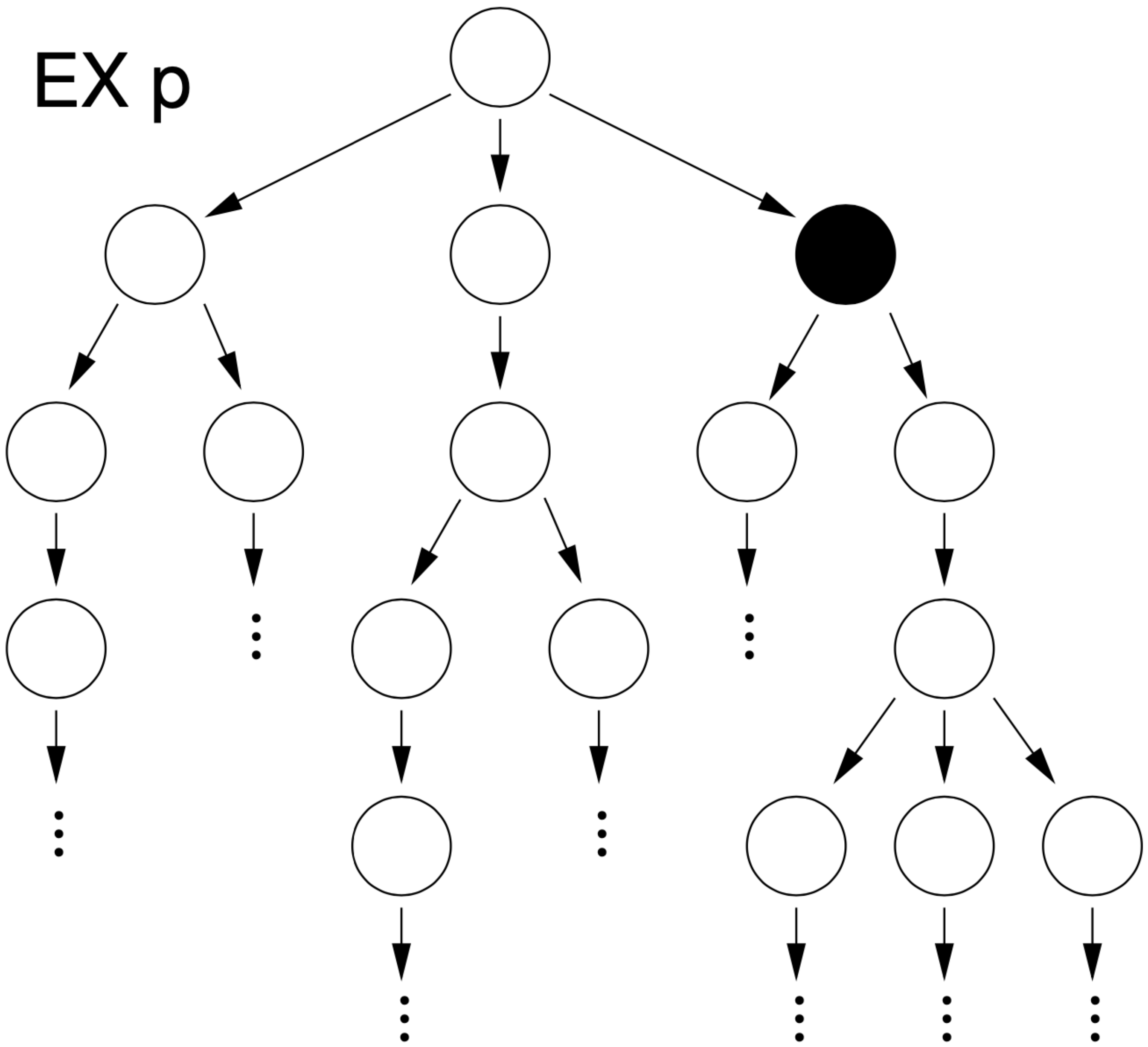
CTL EXAMPLES

We use the following computation tree as a running example
(with varying distributions of **red** and **black** states)

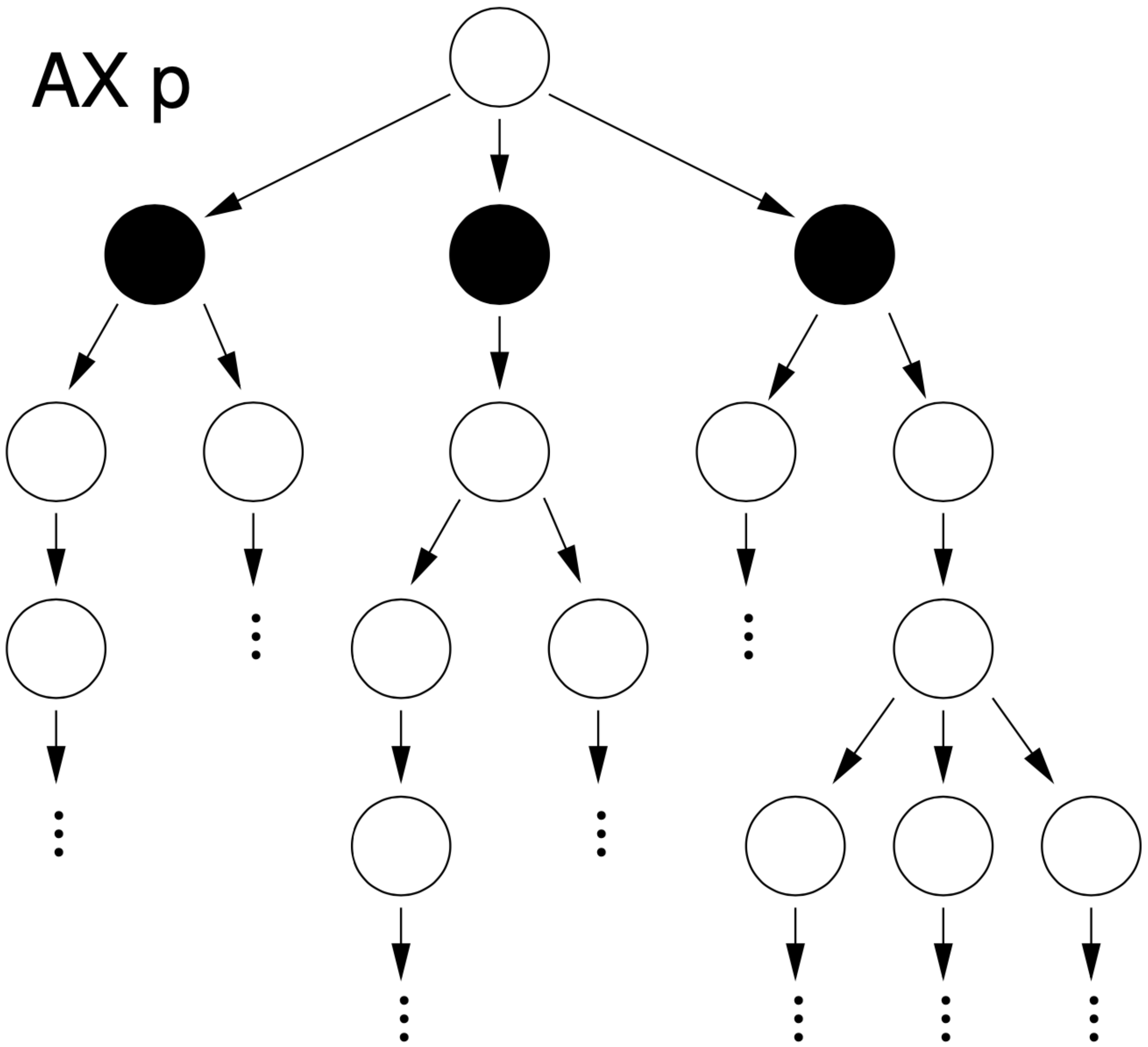


In the following slides, the topmost state satisfies the given formula
if the black states satisfy p and the red states satisfy q.

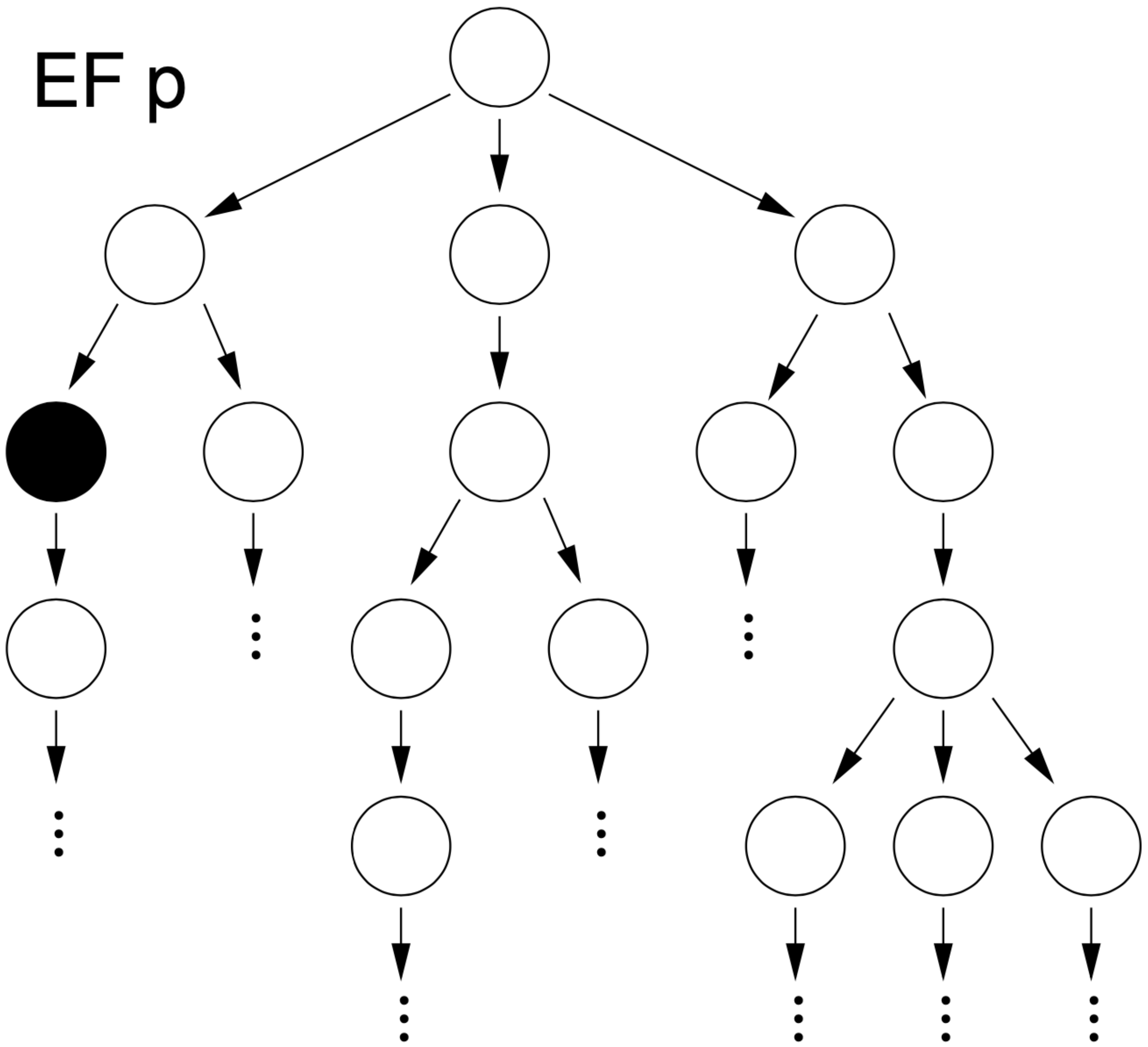
EX p



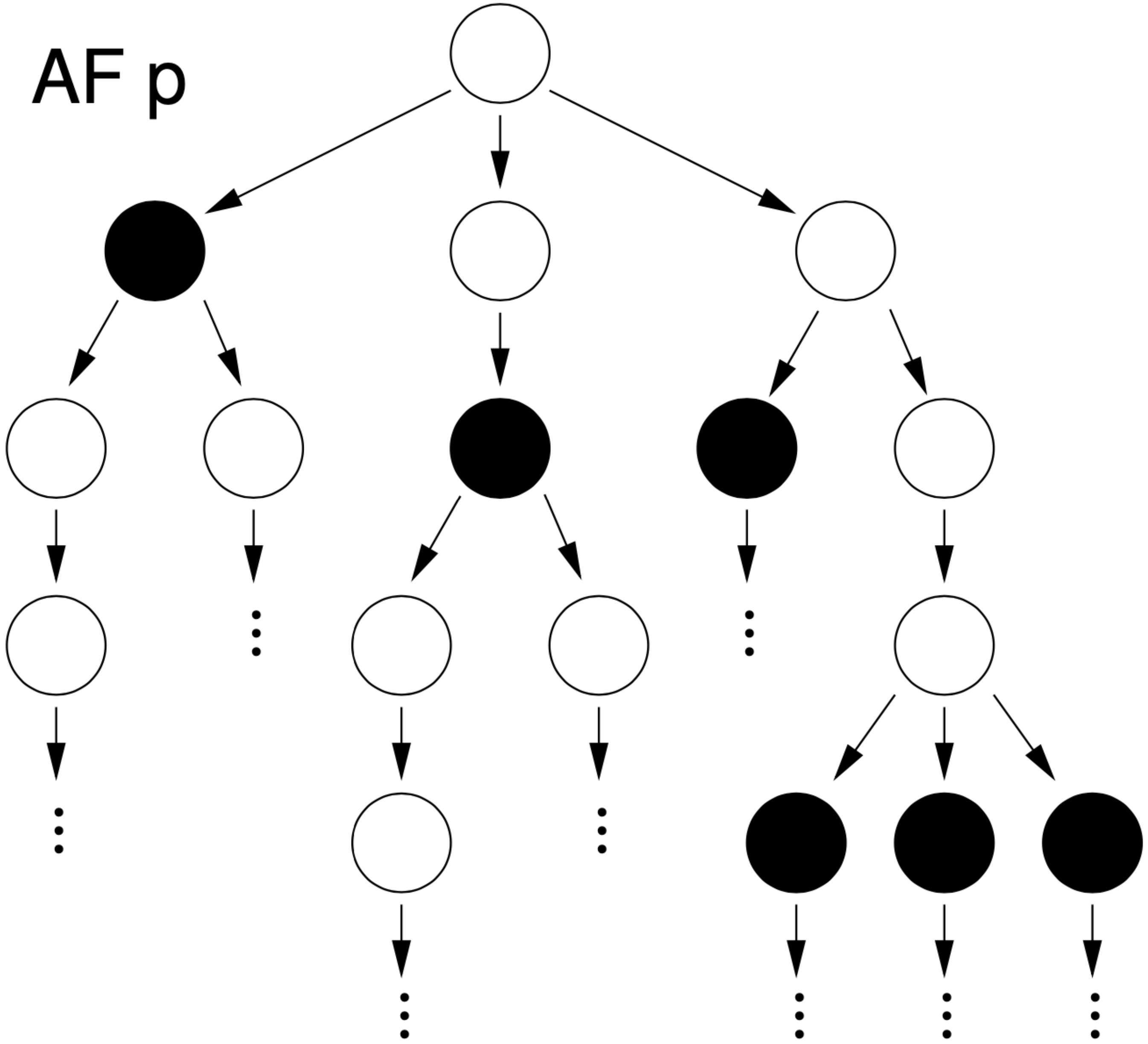
AX p



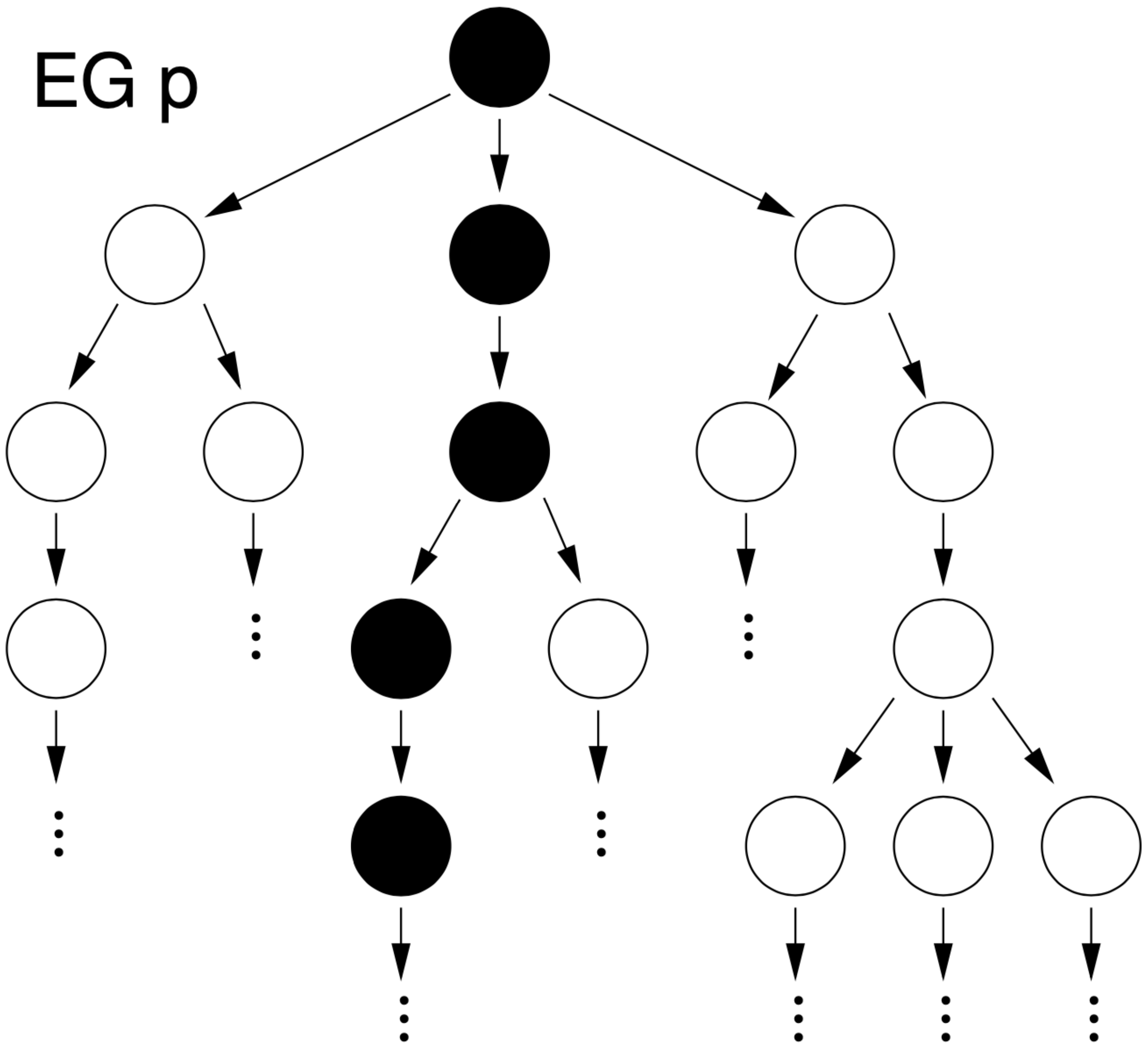
EF p



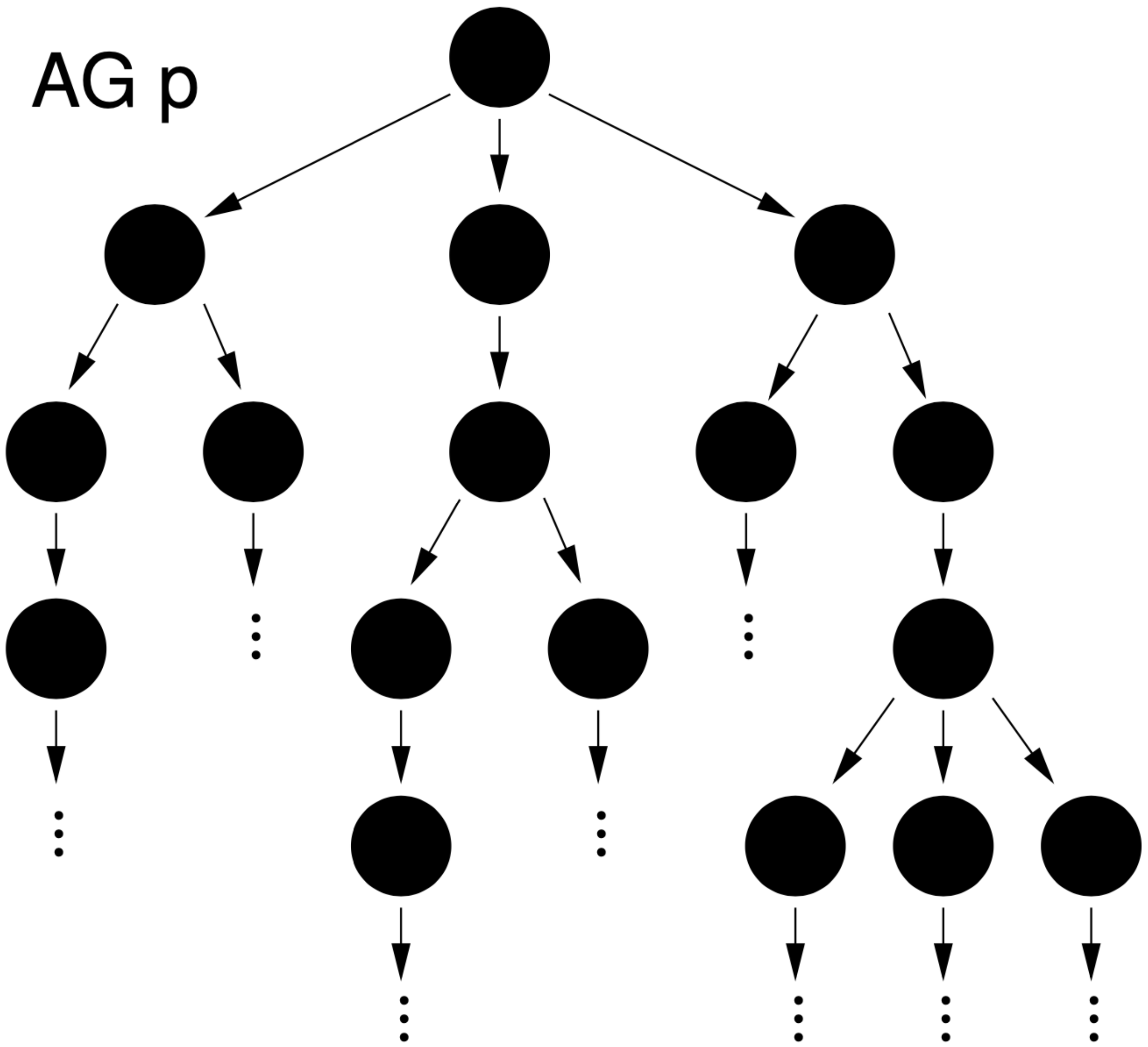
AF p

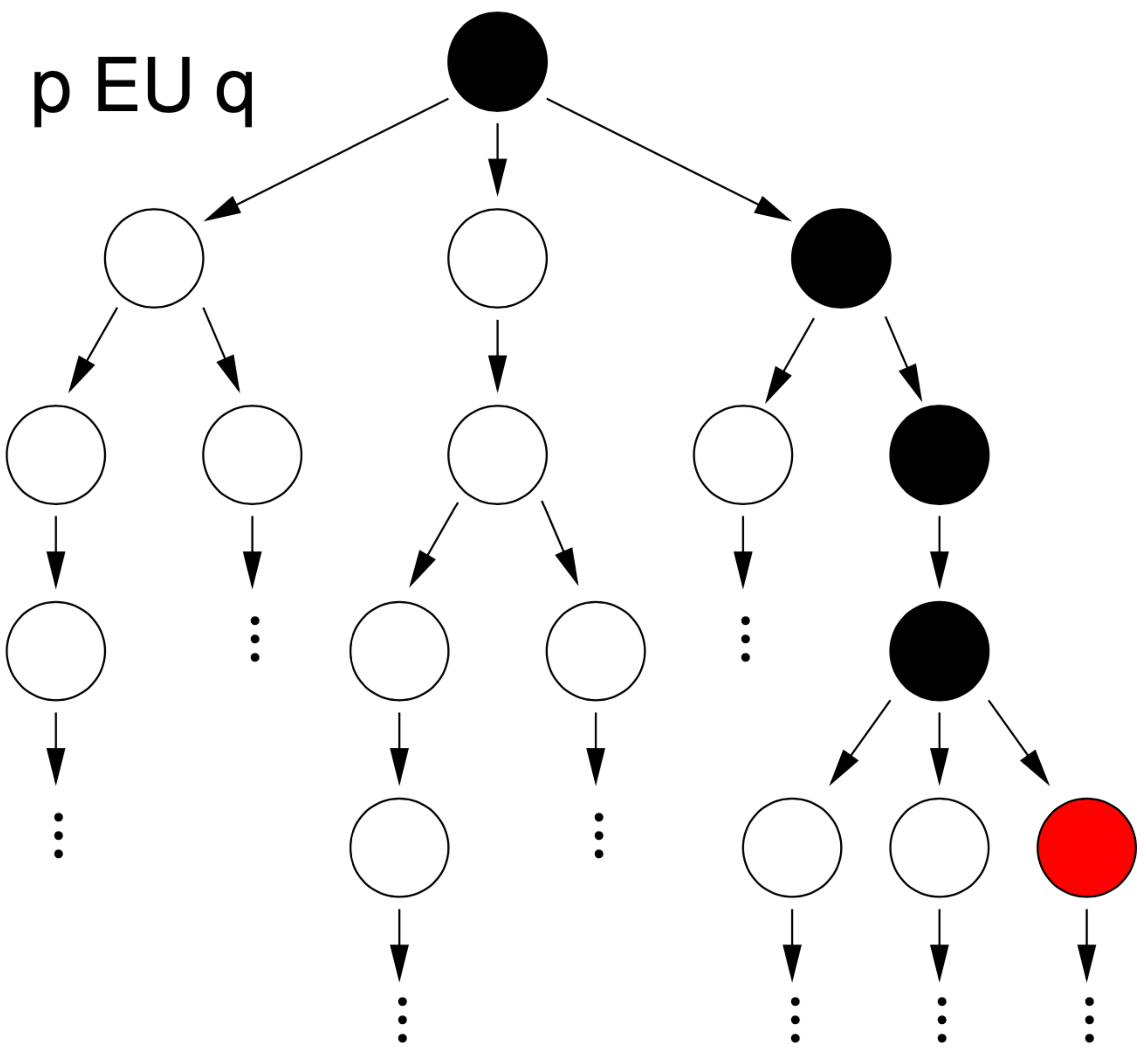


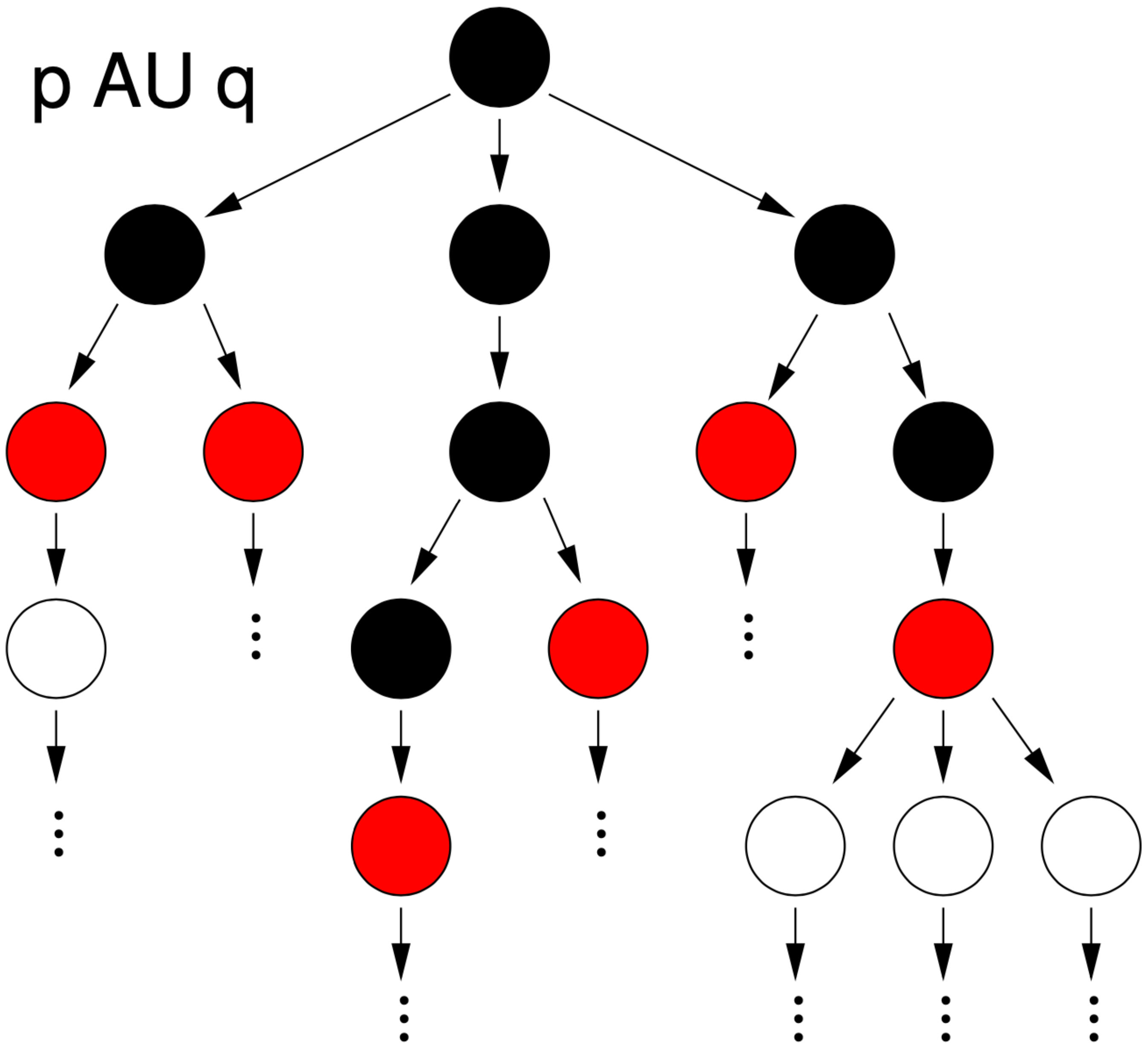
EG p



AG p

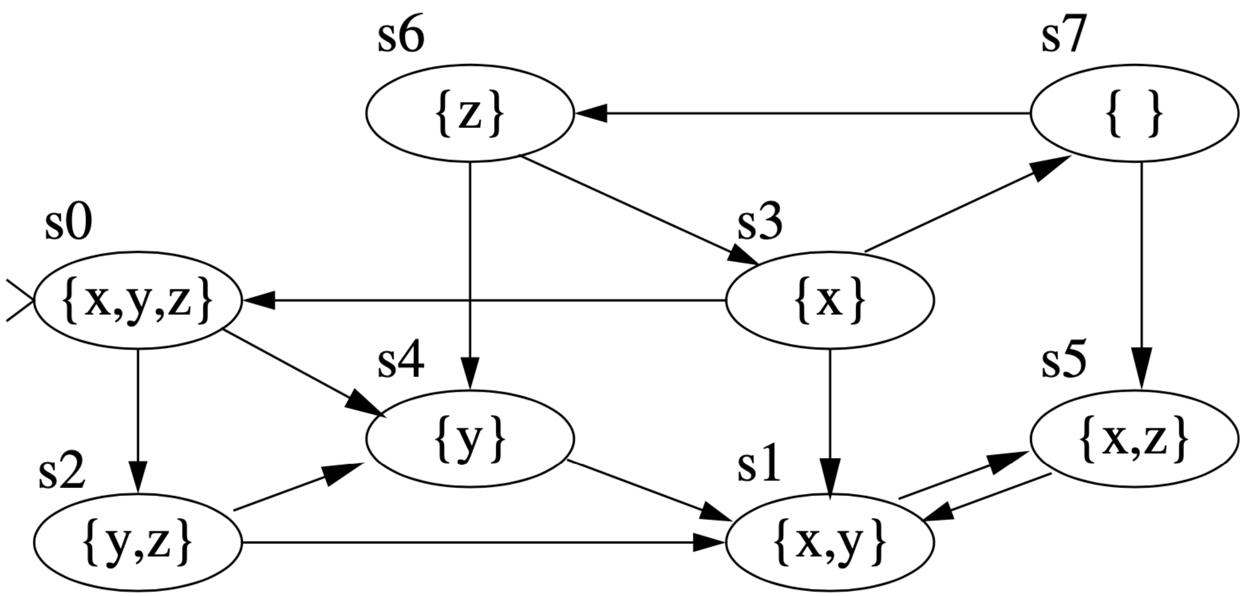



p EU q


p AU q

SOLVING NESTED FORMULAS

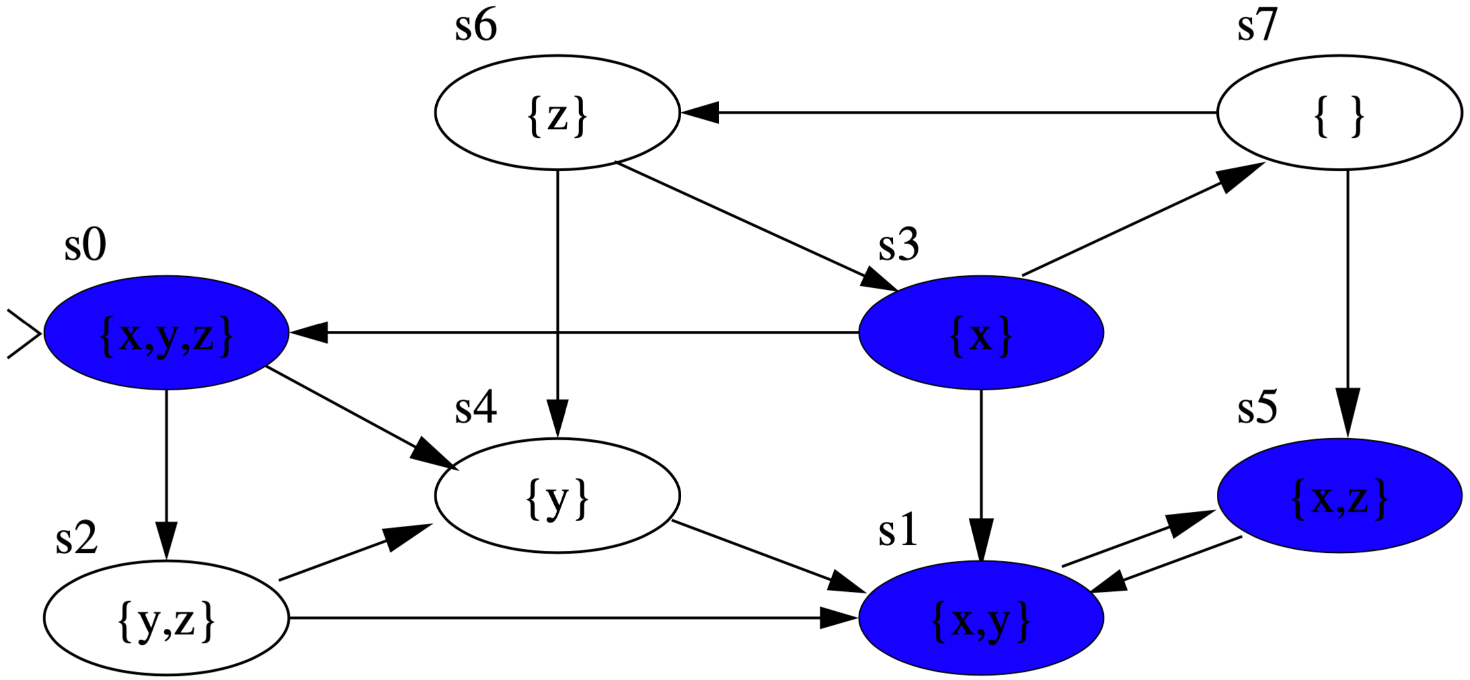
$$s^0 \in [\![AFAG\ x]\!]$$



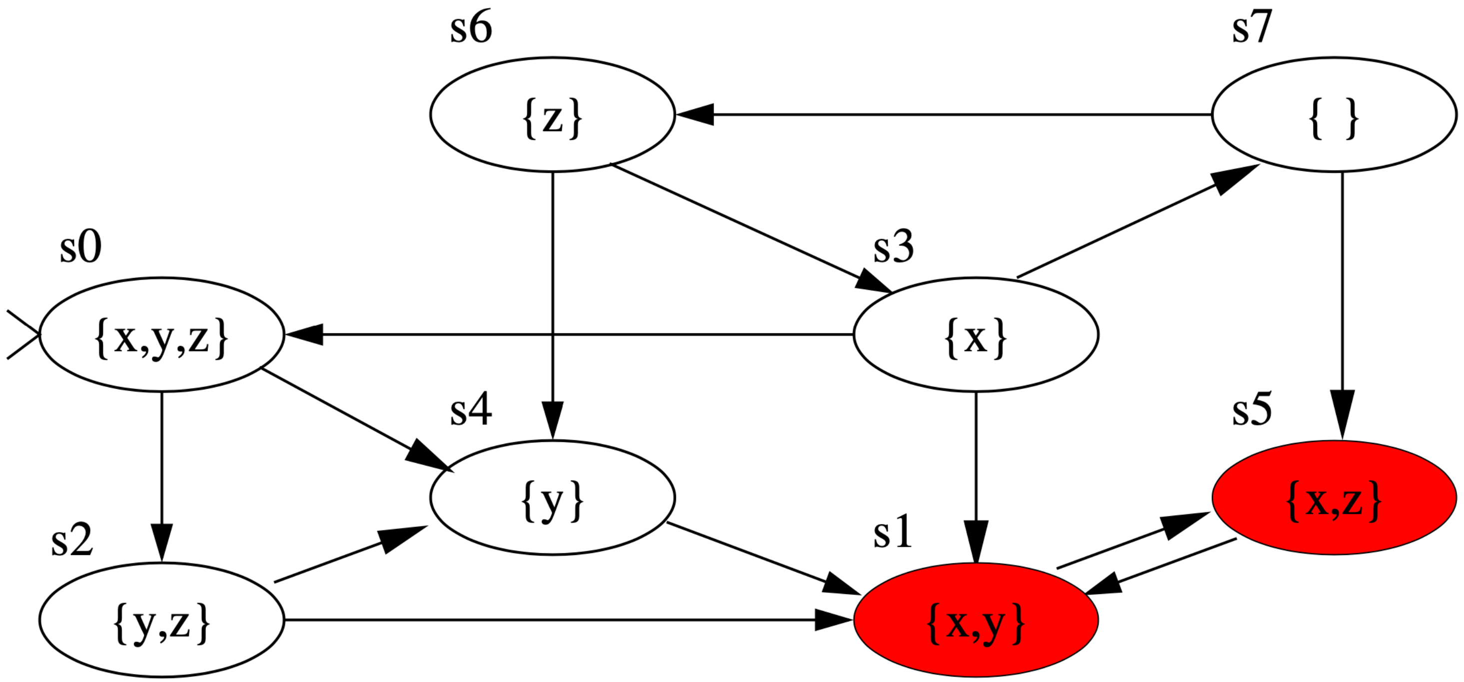
- To compute the semantics of formulas with nested operators,
  - we first compute the states satisfying the innermost formulas;
  - then we use those results to solve progressively more complex formulas.

- In this example, we compute $[\![x]\!]$, $[\![AG\ x]\!]$, and $[\![AFAG\ x]\!]$, in that order

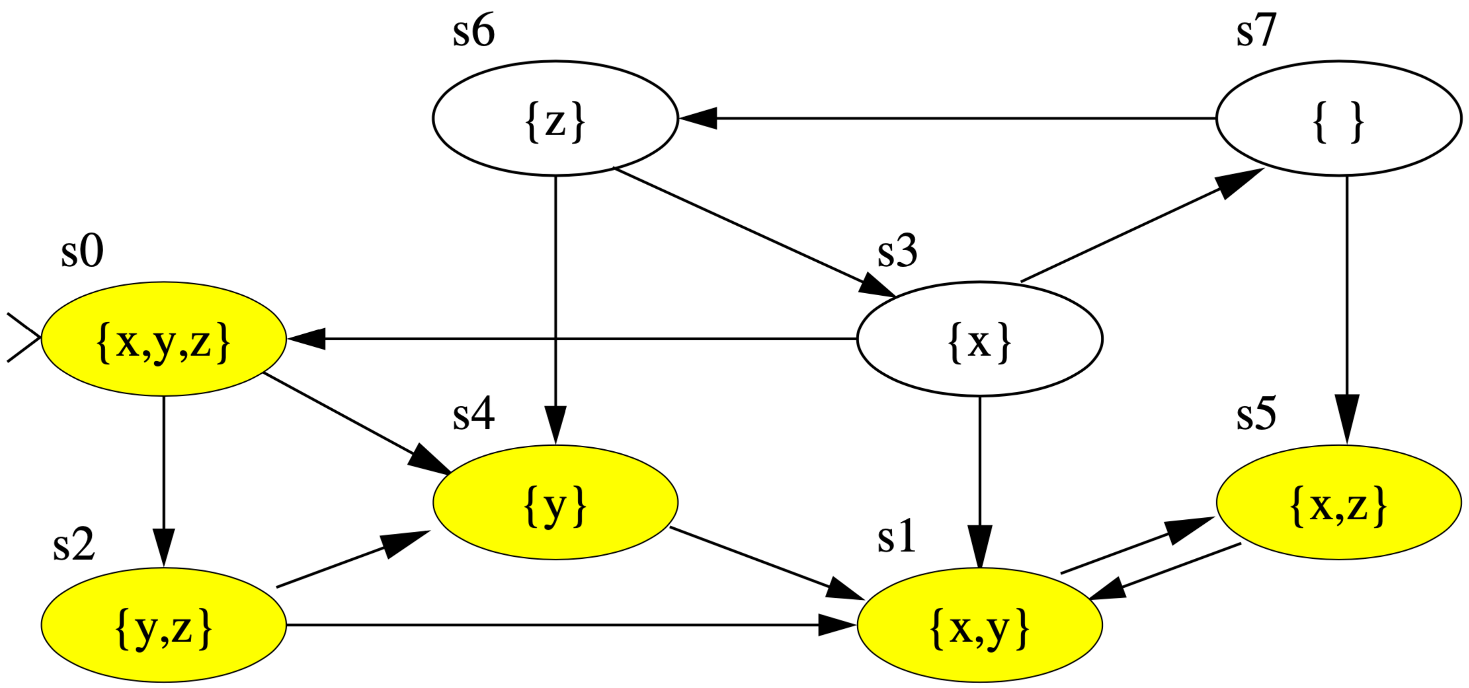
Compute $[\![x]\!]$



Compute $[\![AG\ x]\!]$



Compute $[\![AFAG\ x]\!]$



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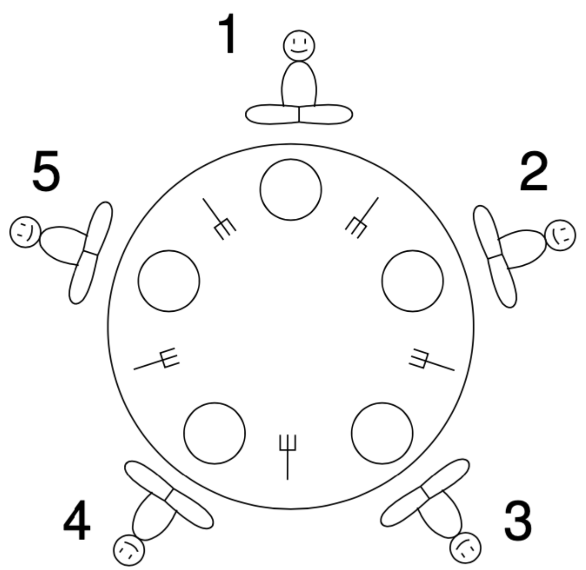
EXAMPLE : DINING PHILOSOPHERS



- Five philosophers are sitting around a table, taking turns at thinking and eating.
- We shall express a couple of properties in CTL. Let us assume the following atomic propositions:
  - $e_i \rightarrow$ philosopher i is currently eating
  - $f_i \rightarrow$ philosopher i has just finished eating

EXAMPLE : DINING PHILOSOPHERS

- Philosophers 1 and 4 will never eat at the same time.

$$AG\ \neg(e_1 \wedge e_4)$$

- Whenever philosopher 4 has finished eating, he cannot eat again until philosopher 3 has eaten.

$$AG\ (f_4 \Rightarrow A\ (\neg e_4\ W\ e_3))$$

- Philosopher 2 will be the first to eat.

$$A(\neg(e_1 \vee e_3 \vee e_4 \vee e_5)\ U\ e_2)$$

THANK YOU

PDF version of the slides

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