CONCEPTION ET VÉRIFICATION DE SYSTÈMES CRITIQUES

INTRODUCTION AUX MÉTHODES FORMELLES

2A Cursus Ingénieurs - ST5 : Modélisation fonctionnelle et régulation
CentraleSupelec - Université Paris-Saclay - 2024/2025

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OUTLINE

- On the need of Verification
- On the need of Formal Methods
- Program Proof
- First Order Logic
- Principle of Model-Checking
- System Modeling

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DEPENDABILITY OF CONTROL SYSTEMS

- A Control System is composed of 3 parts:
  1. Sensors
  2. Actuators
  3. Control **Software** that is **critical** in the Nuclear context!

Critical Software

For which a failure can be catastrophic:

- fatal or/and extremely costly

- Some spectacular failures of critical softwares :
  - Crash of Ariane 5
  - LASCAD : Crash of London Ambulance CAD service
  - Therac-25 : 7 deaths of cancer patients due to overdoses of radiation

CONTROL SOFTWARE VERIFICATION

1. Take the software
2. Determine what the software is supposed to do
3. Prove that the software does what it is supposed to do

Software verification

Software verification checks/proves whether a system fulfills the qualitative requirements that have been identified in its specification

- Imposed by Certification Organisations
  - Several famous examples of abandoned projects, caused by impossibility of the verification step
  - Ex : P20 portion of the french nuclear reactor protection

VERIFICATION vs TESTING

- Testing is a common dynamic technique where the system is executed

- **Testing procedure**:
  - take an implementation
  - stimulate it with certain inputs, i.e., the tests
  - observe reaction and check whether this is "desired"

- **Testing drawbacks**:
  - number of possible behaviors is very large (or even infinite)
  - unexplored behaviors may contain the fatal bug
  - testing is biased towards the most probable scenarios

- Testing may prove the presence of errors, **not their absence!**
- **Verification proves the absence of errors (or finds them)**

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ON THE NEED OF FORMAL METHODS

Verification has to be **provable**!

Definition of Formal Methods:
Formal Methods are the applied mathematics
for **modeling**, **analyzing** and **verifying** systems

- The formal form of the verification problem is $M \models^{?} \varphi$ where:
  - M is the formal representation of the system under observation
  - φ is the formal representation of the property to be verified

IAEA SAFETY STANDARDS SERIES

Safety Guide of Nuclear Power Plants - IAEA

- Requirements and descriptions of designs should be stated **formally** . . .
- When formal languages are used to specify requirements or designs, **theorem provers** and **model checkers** may also aid in verifiability . . .
- When software requirements have been formally specified, it is possible to undertake **formal code verification**. However, formal verification generally requires considerable expertise, and therefore consulting competent analysts should be considered . . .

6 MYTHS ON FORMAL METHODS

1. The use of formal methods guarantees perfect software
   - Nonsense, a formal specification is a model of the real world. Modeling may bring mistakes, omissions and ambiguities
2. The use of formal methods is restricted to proving software
   - Before program proving, formal specification of a system forces a detailed analysis, early in the development
3. The use of formal methods is restricted to critical systems
   - Industrial developments show that using formal methods reduce costs for all types of systems (of mass production)

6 MYTHS ON FORMAL METHODS

4. Only mathematicians can use formal methods
    - Nonsense, the mathematics that are used are elementary
5. Formal methods increase development costs
    - Unproved, costs are shifted to the beginning of the cycle
6. Formal methods are used only for small systems
    - Several very large projects have used formal methods

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FORMAL VERIFICATION OF SEQUENTIAL SOFTWARE

Definition of Sequential Software

A sequence of instructions that terminates and the **result** is computed from **initial data**

- **Pre-Condition** : property satisfied by the program initial data before the execution of the instructions
- **Post-Condition** : property satisfied by the program result and variables after the execution of the instructions

Verification of Sequential Software (Program Proof)

- Prove that if pre-condition is satisfied then post-condition is satisfied
- Find the **most general** pre-condition

EXAMPLE

- Software : Array Sort
  - Initial data : Array T of size N
  - Result : Sorted Array T of size N

- Post-Condition :

$$\forall n, m \in [1..N],\ n < m \Longrightarrow T[n] < T[m]$$

- Most general Pre-Condition :

$$\forall n, m \in [1..N],\ n \neq m \Longrightarrow T[n] \neq T[m]$$

THE HOARE PROOF SYSTEM

The Hoare Proof System provides for each type of instructions an **Axiom**/Rule to find the most general pre-condition φ (general form : $\{\varphi\}\ P\ \{\psi\}$)

- Assignment axiom : $\{\varphi[expr/x]\}\ x = expr\ \{\varphi\}$
  - ex : $\{y == 5\}\ x = y + 5\ \{x == 10\}$
  - ex : $\{x^2 < 4\}\ x = x * x\ \{x < 4\}$
- Loop axiom : $\{\varphi\}$ while(C) $P\ \{\varphi \wedge \neg C\}$
  - if φ is a loop **invariant** : $\{\varphi \wedge C\}\ P\ \{\varphi\}$
- Choice axiom : $\{\varphi\}$ if(C) $P1$ else $P2\ \{\psi\}$
  - if : $\{\varphi \wedge C\}\ P1\ \{\psi\}$
  - else : $\{\varphi \wedge \neg C\}\ P2\ \{\psi\}$

PROOF EXAMPLE

pre-condition: $n \geq 0$ // initial data : n
$0 == 0 \wedge 0 \leq n$
$\sum_{k=0}^{0} k == 0 \wedge 0 \leq n$
```
i = 0;
```
$\sum_{k=0}^{i} k == 0 \wedge i \leq n$
```
res = 0;
```
$\sum_{k=0}^{i} k == res \wedge i \leq n$ // φ
```
while (i < n) { // C
```
$(\sum_{k=0}^{i} k == res \wedge i \leq n) \wedge i < n$ // $\varphi \wedge C$
$\sum_{k=0}^{i} k == res \wedge i < n$
$\sum_{k=0}^{i+1} k == res + i + 1 \wedge (i+1) \leq n$
```
    i = i + 1;
```
$\sum_{k=0}^{i} k == res + i \wedge i \leq n$
```
    res = res + i;
```
$\sum_{k=0}^{i} k == res \wedge i \leq n$ // φ
```
}
```
$(\sum_{k=0}^{i} k == res \wedge i \leq n) \wedge i \geq n$ // $\varphi \wedge \neg C$
$\sum_{k=0}^{i} k == res \wedge i == n$
post-condition: $\sum_{k=0}^{n} k == res$ // result : res

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FIRST ORDER LOGIC

- Program Proof is based on the Formal System of the First Order Logic (FOL)
  - pre-conditions, post-conditions, invariants, assertions . . .
- FOL is the logic you are used to use in mathematics
- The syntax :

$$
\begin{array}{lcl}
t & ::= & c \mid x \mid f(t, \ldots, t) \\
\phi & ::= & true \mid a \mid t = t \mid P(t, \ldots, t) \mid \phi \wedge \phi \mid \neg \phi \mid \exists x.\ \phi
\end{array}
$$

- The semantics are as usual in mathematics

FOL FORMAL SYSTEM

Definition

- A Formal System consists of a set of **axioms** and a set of **inference rules** (reasoning) that are combined to **derive well formed formulas**
- A derivation that leads to a wff $\mathcal{F}$ is called a **proof** of $\mathcal{F}$

Axioms

$(ax1)\ A_x(t) \ \Rightarrow\ \exists x.\ A$
$(ax2)\ x = x$
$(ax3)\ x = y \Rightarrow (A \Rightarrow A_x(y))$
$(ax4)\ A \Rightarrow (B \Rightarrow A)$
$(ax5)\ \neg\neg A \Rightarrow A$
$(ax6)\ (A \Rightarrow (B \Rightarrow C)) \Rightarrow ((A \Rightarrow B) \Rightarrow (A \Rightarrow C))$

Rules

$(r1)\ A,\ A \Rightarrow B \qquad \vdash B$
$(r2)\ A \Rightarrow B \qquad \vdash \exists x.\ A \Rightarrow B$

FOL FORMAL SYSTEM

Soundness
A formal system is sound if each derived formula is valid i.e. semantically true.
A valid formula is called a **theorem**

Completeness
A formal system is complete if each valid formula could be derived,
i.e. it exists a proof leading to the theorem

- First Order Logic is sound and **complete**!
- An automatic program prover tries many possible derivations (infinite) and after a time limit :
  - option 1/2 : it reaches the formula to prove $\longrightarrow$ **YES!**
  - option 2/2 : it doesn't (it needs some help) $\longrightarrow$ Inconclusive
- First Order Logic is **semi-decidable**!

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HISTORY OF FORMAL VERIFICATION METHODS

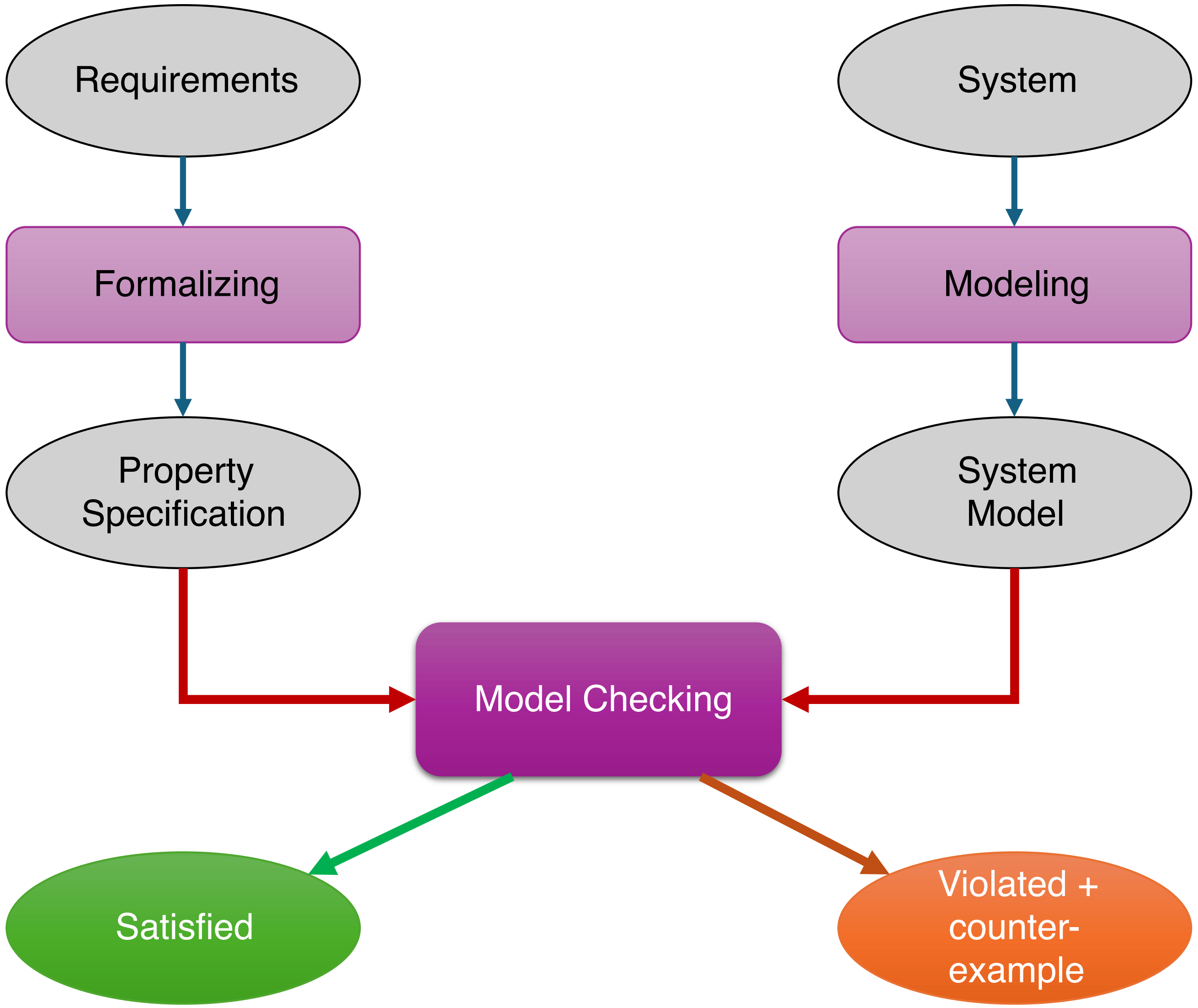
Before . . .

- Software code was sequential
- Properties were expressed in **First-Order Predicate Logic**
- **Theorem provers** : partial/total correctness
- e.g. B Method
- Hardly automated : **semi-decidable**

After 80's

- Software is **concurrent** and reactive
- Properties are expressed in **Temporal Logic**
- Solving accurate properties like safety, liveness, fairness . . .
- e.g. Model Checking
- Push-Button : **decidable**

PRINCIPLE OF MODEL-CHECKING



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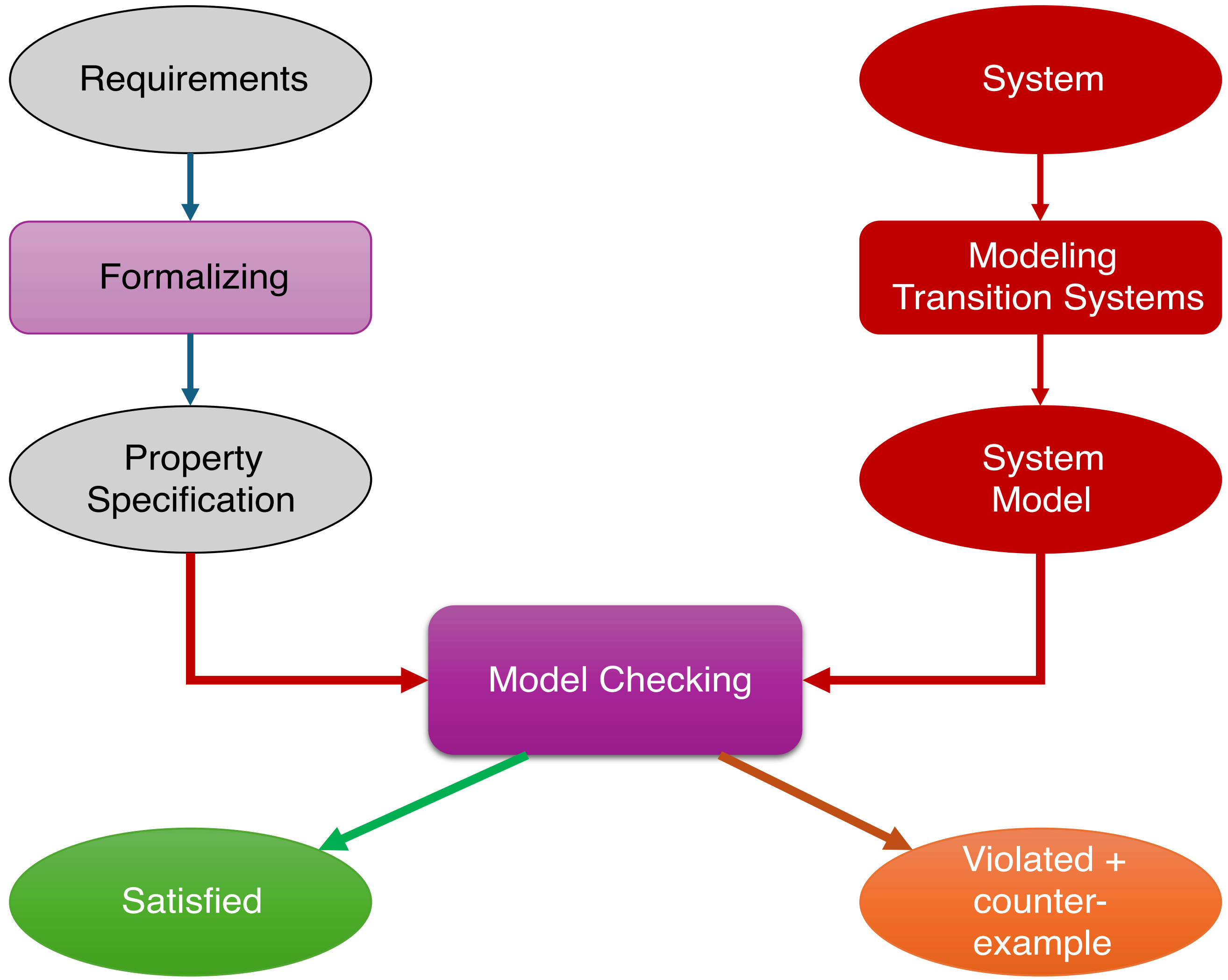
PRINCIPLE OF MODEL-CHECKING



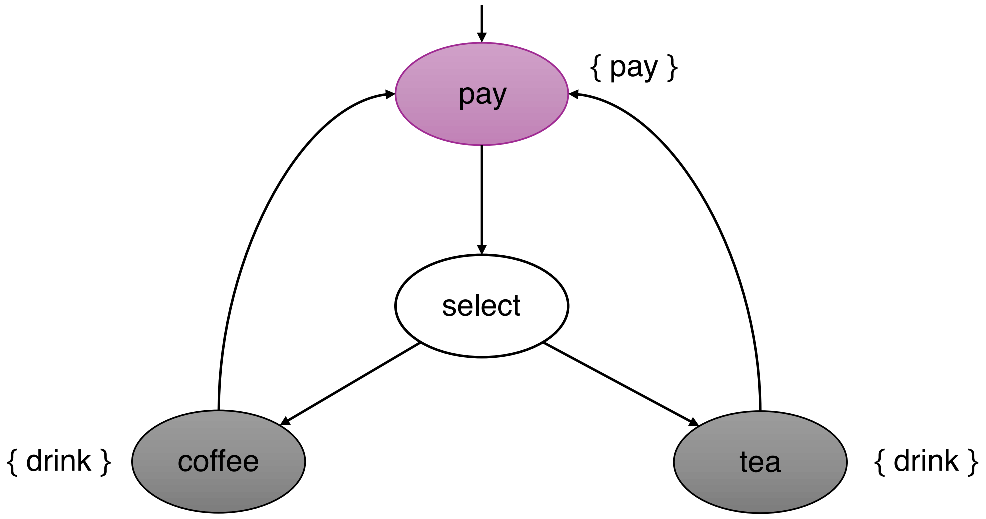
TRANSITION SYSTEMS

- **model** to describe the behaviour of systems
- **digraphs** where **nodes** represent states, and **edges** represent transitions
- **states** :
  - the current colour of a traffic light : red, green, orange.
  - **software :** the current values of all program variables + the program counter
  - **hardware :** the current value of the registers together with the values of the input bits
- **transitions** : ("state change")
  - a switch from one colour to another
  - **software :** the execution of a program statement
  - **hardware :** the change of the registers and output bits for a new input

FORMAL DEFINITION

- A **transition system** TS is a tuple $(S, \delta, I, AP, \mathcal{L})$ where
  - S is a set of states
  - $\delta \subseteq S \times S$ is a transition relation
    Notation: $s \rightarrow s'$ instead of $(s, s') \in \delta$
  - $I \subseteq S$ is a set of initial states
  - AP is a set of Atomic Propositions
  - $\mathcal{L} : S \longrightarrow 2^{AP}$ is a Labeling function

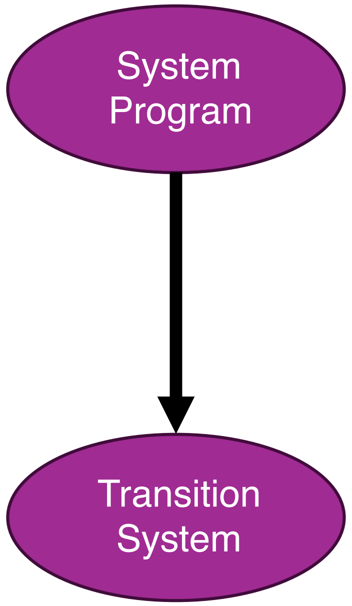
EXAMPLE



- **States** : $S = \{pay, select, tea, coffee\}$
- **Initial states** : $I = \{pay\}$
- **Atomic Propositions, Labeling function** :
  - suppose $AP = S$, $\mathcal{L}(s) = \{s\}$
  - suppose $AP = \{pay, drink\}$, $\mathcal{L}(tea) = \mathcal{L}(coffee) = \{drink\}$
    $\mathcal{L}(pay) = \{pay\}, \quad \mathcal{L}(select) = \emptyset$

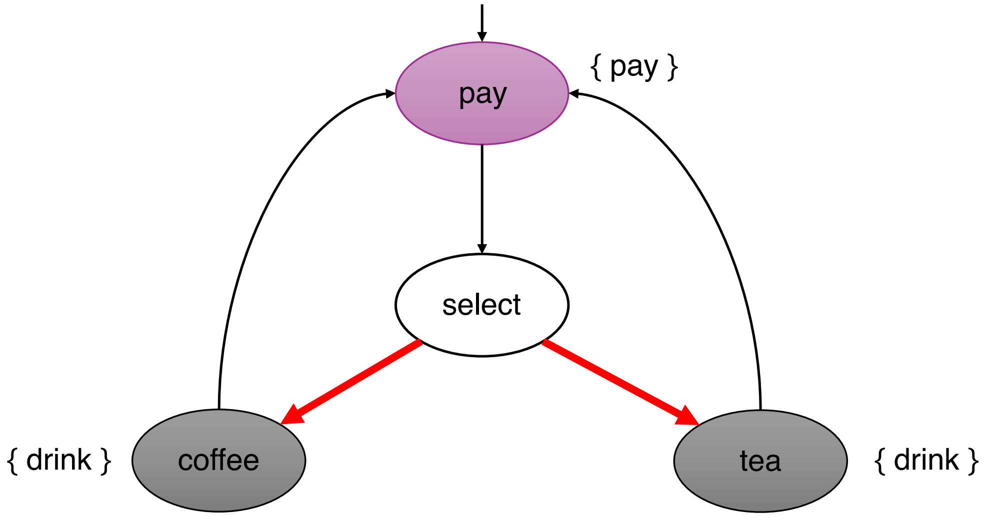
FROM PROGRAMMING LANGUAGES TO TRANSITION SYSTEMS

- Transition systems are an elementary modeling language
  - describe all the states that the system may reach
  - describe the behavior of the system (transitions)
- Even a basic system may have thousands of states!
  - `int i=0; while(i<1000) i++;`
  - modeling could be tedious !
- What if the transition system is automatically generated from the system's program ?
  - modeling would be automatic !
  - many tools exist from **C**, **Java** . . . to TS

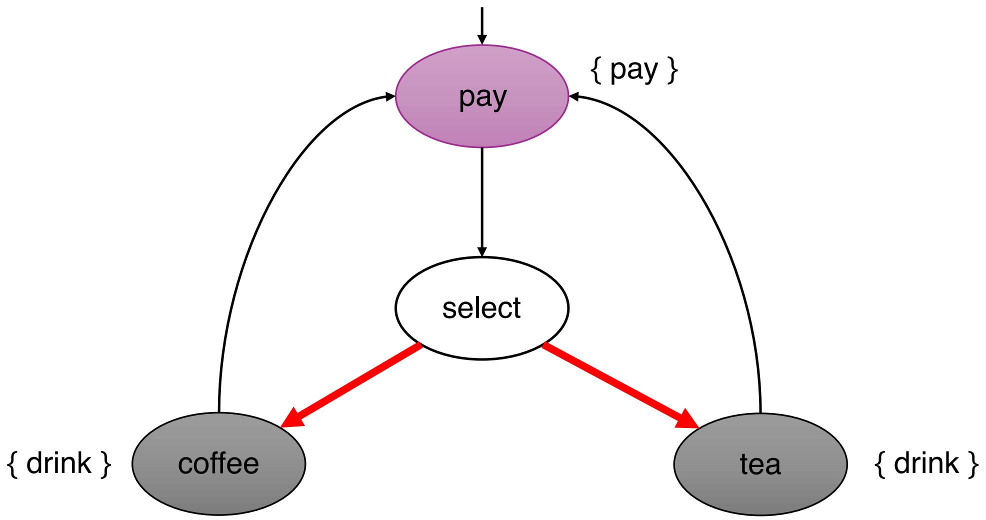


DETERMINISM AND NONDETERMINISM

- Let $TS = (S, \delta, I, AP, \mathcal{L})$ be a transition system, TS is **deterministic**
  - iff $\forall\, s, s_1', s_2' \in S,\quad s \to s_1', s \to s_2' \in \delta \Rightarrow s_1' = s_2'$
  - iff $\forall\, s \in S,\quad \#(\delta(s)) \leq 1$



SOURCES OF NONDETERMINISM



- Incomplete information on the system environment
  - User selection
  - Triggered events

INTERLEAVING OF CONCURRENT SYSTEMS

- the system is composed by many concurrent components
- **one** transition system for modeling **one** component behavior
- e.g. threading, distributed algorithms and communication protocols

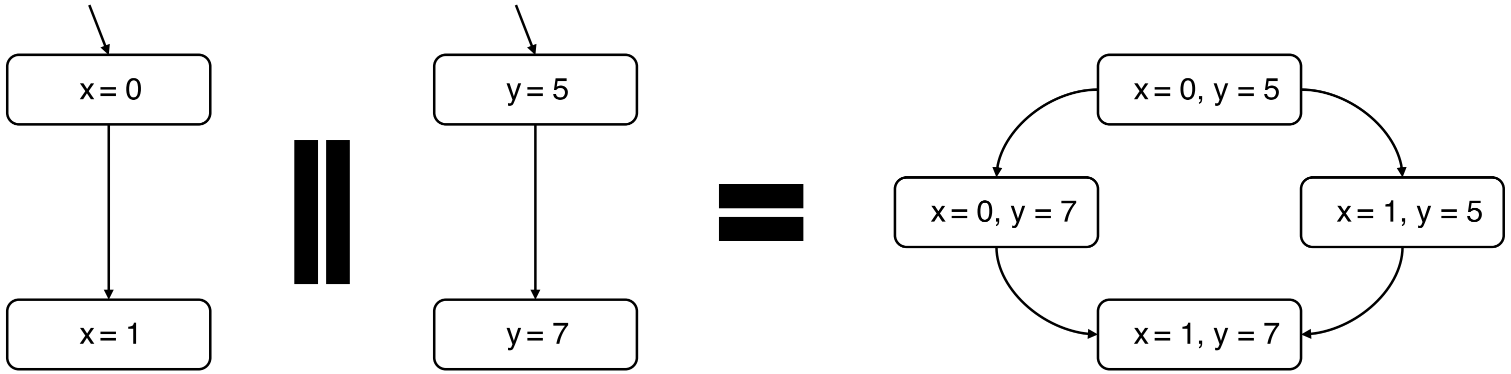
INTERLEAVING PRINCIPLE

- Actions of independent components are interleaved
  - a single processor is available
  - on which each component executes for a quantum of time
- No assumptions are made on the order of executions
  - possible orders for non-terminating independent components $Loop(P) \parallel Loop(Q)$:

$$\begin{array}{l} P\ Q\ P\ Q\ P\ Q\ P\ Q\ P\ Q \ldots \\ P\ P\ Q\ P\ P\ Q\ P\ P\ Q\ P \ldots \\ P\ Q\ P\ P\ Q\ P\ P\ P\ Q\ P \ldots \end{array}$$

main source of **nondeterminism** that can be avoided by adding a **scheduler** with a particular strategy

INTERLEAVING EXAMPLE



- Justification for interleaving:
  - the effect of **concurrently** executed independent actions equals the effect when they are **successively** executed in **arbitrary** order

INTERLEAVING $TS_1 \parallel TS_2$ FORMAL DEFINITION

Let $TS_i = (S_i, \delta_i, I_i, AP_i, \mathcal{L}_i), i = 1, 2$ be two transition systems.

The **Interleaving Product** (**Asynchronous product**) is the transition system:

$$TS_1 \parallel TS_2 = (S_1 \times S_2, \delta, I_1 \times I_2, AP_1 \cup AP_2, \mathcal{L})$$

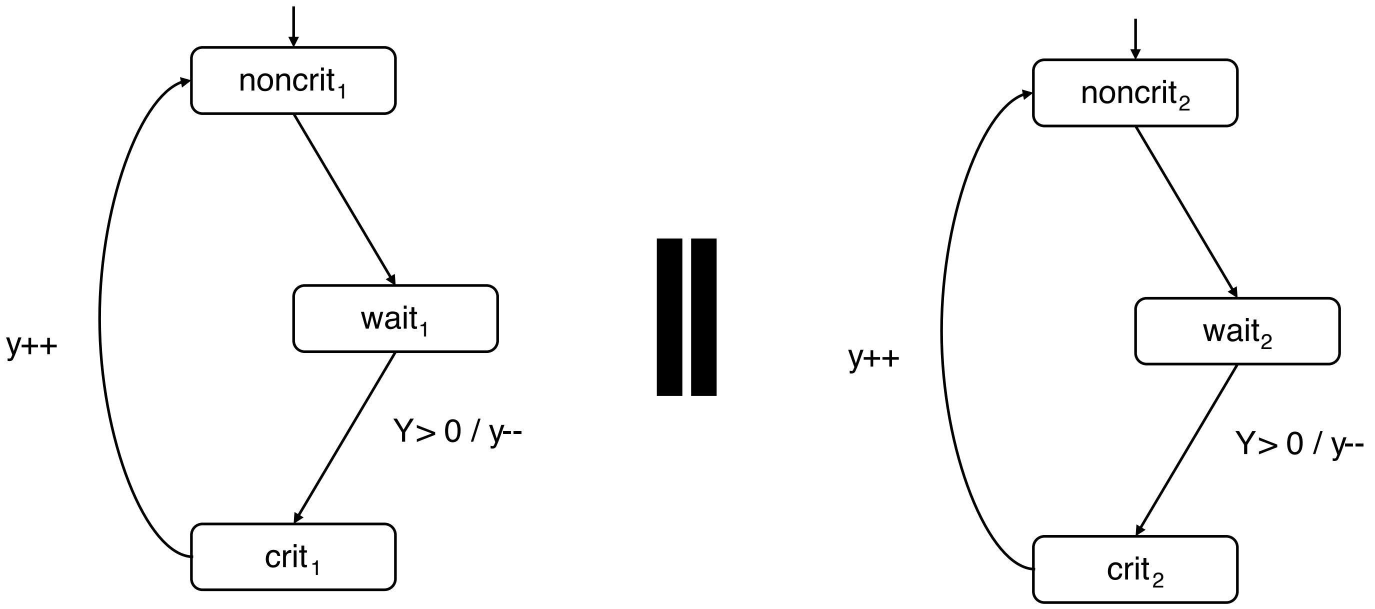
where δ verifies:

$$\frac{s_1 \longrightarrow s_1'}{\langle s_1, s_2 \rangle \longrightarrow \langle s_1', s_2 \rangle} \quad and \quad \frac{s_2 \longrightarrow s_2'}{\langle s_1, s_2 \rangle \longrightarrow \langle s_1, s_2' \rangle}$$

and $\mathcal{L}$ verifies :

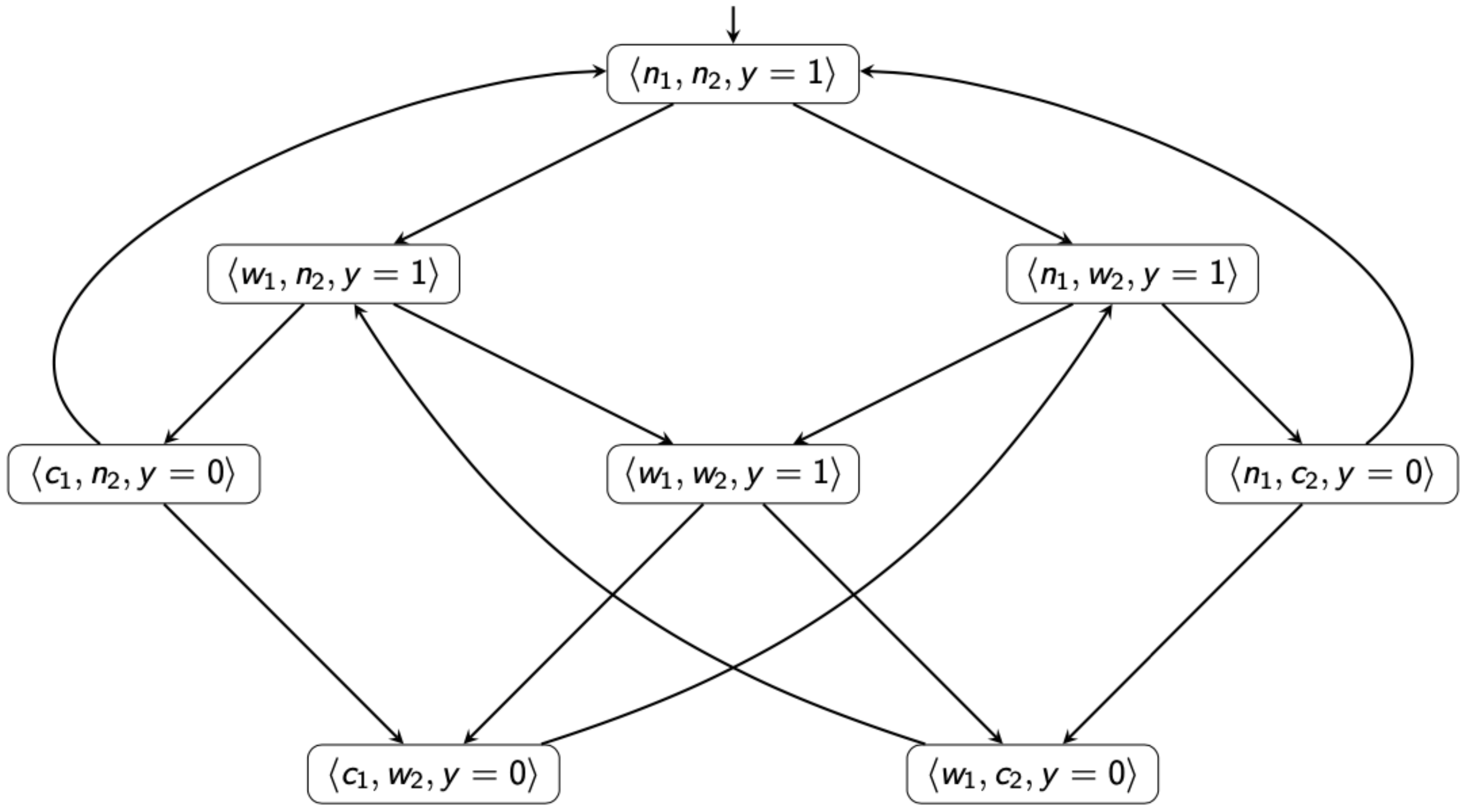
$$\mathcal{L}(\langle s_1, s_2 \rangle) = \mathcal{L}_1(s_1) \cup \mathcal{L}_2(s_2)$$

SEMAPHORE-BASED MUTUAL EXCLUSION



$y = 0$ means “lock is currently possessed”; $y = 1$ means “lock is free”

INTERLEAVING PRODUCT



Typical source of state explosion

suppose there were 3 concurrent components

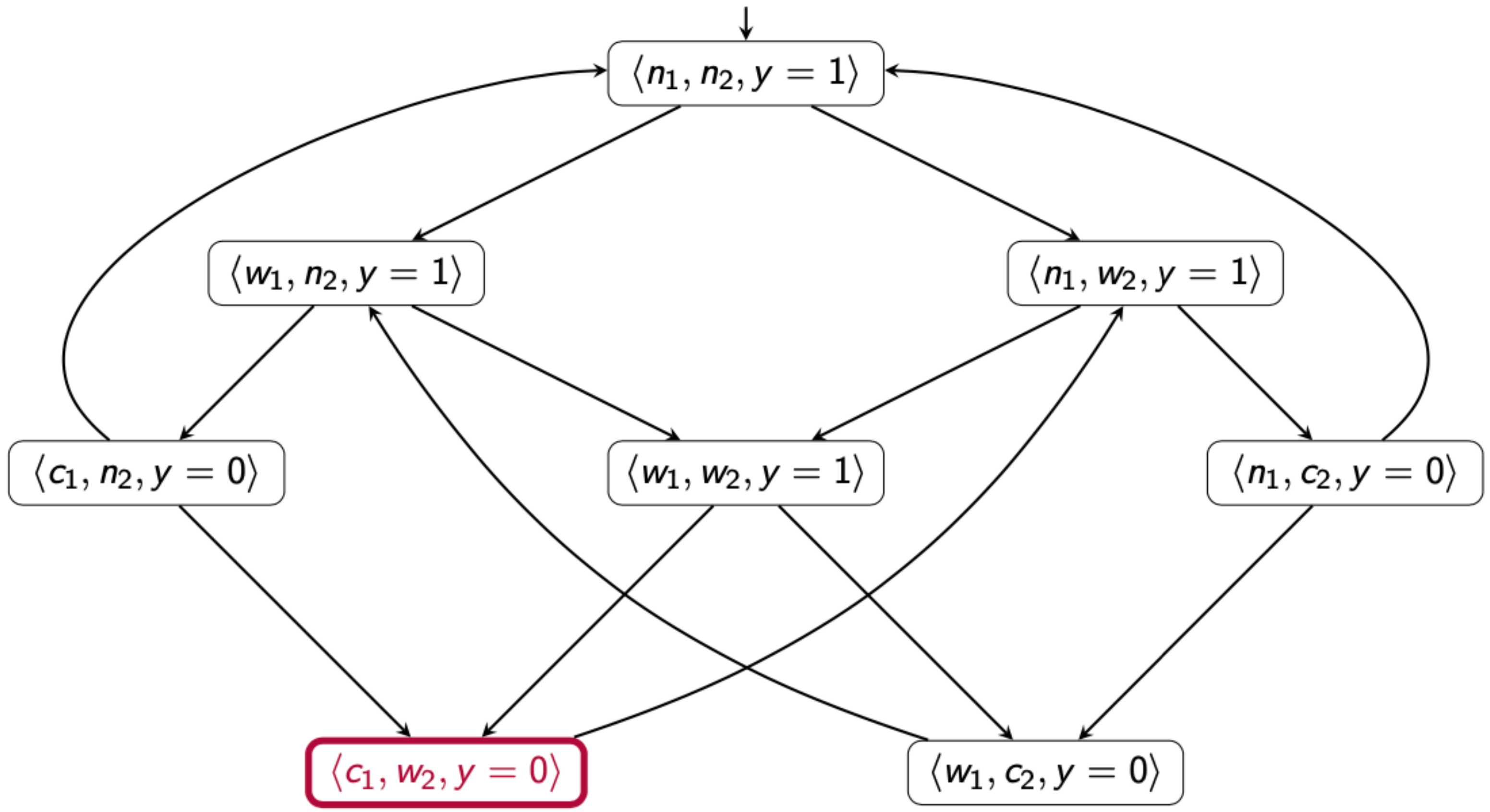
PATHS AND REACHABLE STATES

- An **infinite path fragment** π is an infinite state sequence:
  $\pi = s_0 s_1 s_2 \ldots$ such that $\forall i > 0, s_i \longrightarrow s_{i+1} \in \delta$
- $Paths(s)$ is the set of infinite path fragments π with $first(\pi) = s$
- $Paths(TS) = \bigcup\limits_{s \in I} Paths(s)$ is the set of initial path fragments
- A state $s \in S$ is called **reachable** in TS if there exists an initial path π fragment such that

$$\pi = s_0 s_1 \ldots s_{n-1}(s_n = s)s_{n+1} \ldots \in Paths(TS)$$

- $Reach(TS)$ denotes the set of all reachable states in TS

BACK TO OUR EXAMPLE



$$Paths(\langle c_1, w_2, y = 0\rangle)\ ?, Paths(TS)\ ?, Reach(TS)\ ?$$

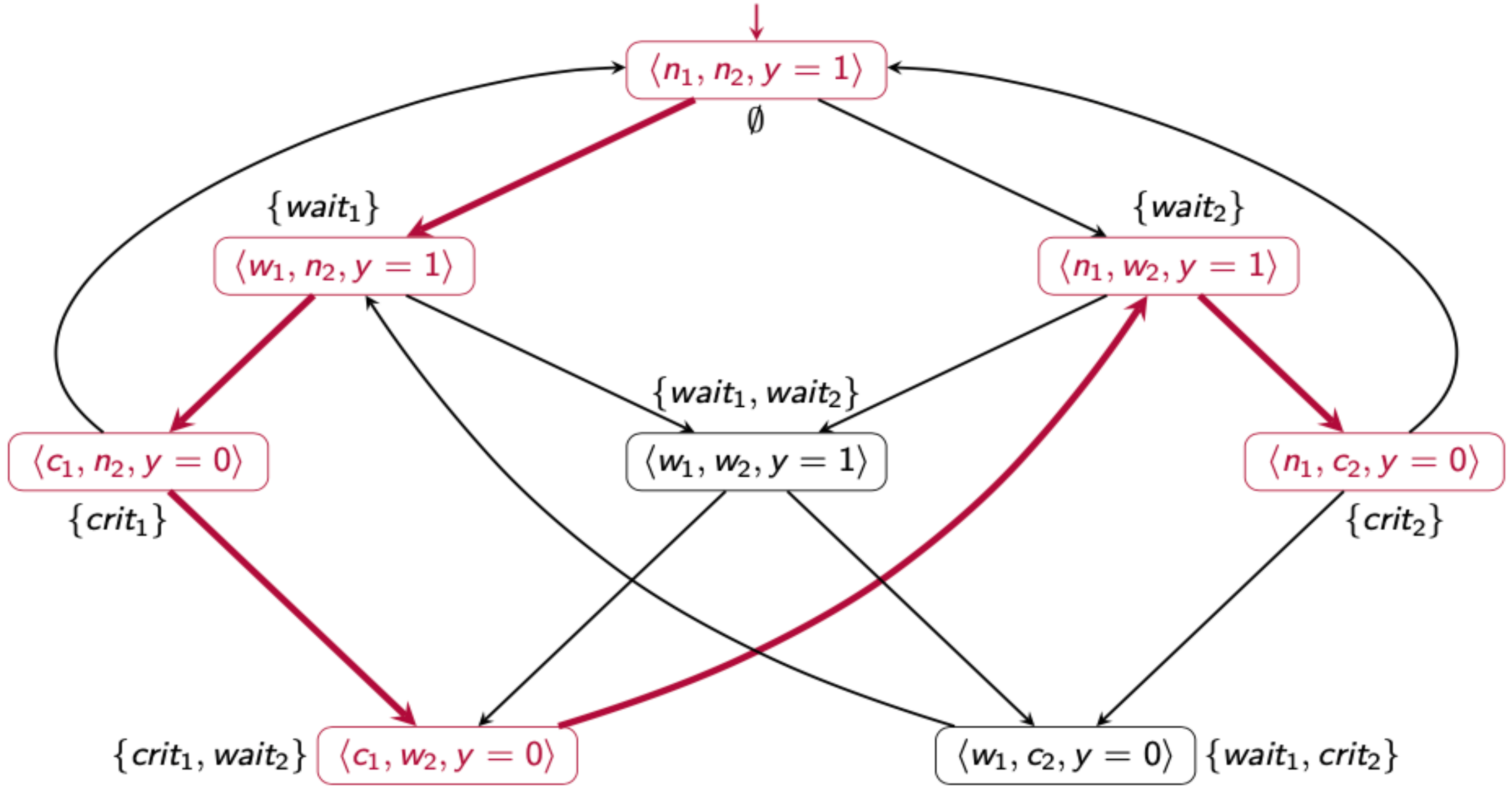
TRACES

- States are observable through their atomic propositions
- **Traces** only focus on the (set of) atomic propositions that are valid along the execution (path)
- The trace of the path $\pi = s_0 s_1 s_2 \ldots \in S^\omega$ with $\mathcal{L} : S \longrightarrow 2^{AP}$
  - $trace(\pi) = \mathcal{L}(s_0)\mathcal{L}(s_1)\mathcal{L}(s_2) \ldots \in (2^{AP})^\omega$
- Traces are **infinite** words over the alphabet 2^{AP}
- $trace(\Pi) = \{trace(\pi) | \pi \in \Pi\}, Traces(s) = trace(Paths(s))$

and $Traces(TS) = \bigcup_{s \in I} Traces(s)$

BACK TO OUR EXAMPLE

Let $AP = \{wait_1, crit_1, wait_2, crit_2\}$



$$Trace(\pi \ldots) = \emptyset\{wait_1\}\{crit_1\}\{crit_1, wait_2\}\{wait_2\}\{crit_2\}\ldots$$

THANK YOU

PDF version of the slides

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