A GENERIC EVENT-B THEORY FOR THE FORMALISATION OF THE INTERNATIONAL SYSTEM OF UNITS

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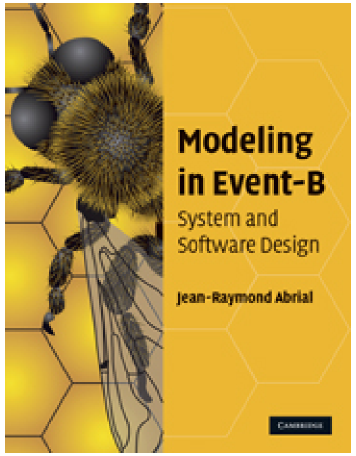
THE EVENT-B METHOD

- The **Event-B method** is an evolution of the classical **B method**.
  - modeling a system by a set of events instead of operations.

- The **Event-B method** is a formal method based on first-order logic and set theory.

- The **Event-B method** is based on :
  - the notions of pre-conditions and post-conditions (**Hoare**),
  - the weakest pre-condition (**Dijkstra**),
  - and the calculus of substitution (**Abrial**).



USING EVENT-B METHOD

- The **Rodin** platform (an Eclipse-based IDE) is intended to support the construction and verification of **Event-B models**.

- The use of the **Event-B method** has continued to increase.
  - applied to various applications and domains.
  - railway, automotive, aeronautics, cybersecurity, nuclear-energy, ...

- The **Event-B method** is adapted to analyse discrete systems.
  - offers the possibility of modelling **discrete behaviors**.

THE EVENT-B METHOD

```
CONTEXT ctx₁
EXTENDS ctx₂

SETS s
CONSTANTS c
AXIOMS
   A(s, c)
THEOREMS
   T(s, c)
END
```

```
MACHINE mch₁
REFINES mch₂
SEES ctxᵢ

VARIABLES v
INVARIANTS
   I(s, c, v)
THEOREMS
   T(s, c, v)
EVENTS
   [events_list]
END
```

```
event ≙
   any x
   where
      G(s, c, v, x)
   then
      BA(s, c, v, x, v′)
   end
```

$$A(s,c) \quad \vdash \quad T(s,c)$$
$$A(s,c) \wedge I(s,c,v) \quad \vdash \quad T(s,c,v)$$
$$A(s,c) \wedge I(s,c,v) \wedge G(s,c,v,x) \quad \vdash \quad \exists v'.\, BA(s,c,v,x,v')$$
$$A(s,c) \wedge I(s,c,v) \wedge G(s,c,v,x) \wedge BA(s,c,v,x,v') \quad \vdash \quad I(s,c,v')$$
$$\ldots$$

THE EVENT-B METHOD

STATIC TYPE CHECKING

- **Event-B** support static type checking using tools such as **Rodin** or **AtelierB**.
- These tools generate proof obligations (**POs**) for arithmetic operations - known as **Well-Defined** (**WD**) **POs**.
- **WD POs** ensure that expressions (axioms, theorems, invariants, guards, actions, etc.) are mathematically well-defined.
- **Example** $\longrightarrow$ For the expression $E \div F$, a **WD PO** ensures that $F \neq 0$.

THE EVENT-B METHOD

THE THEORY PLUGIN

- **Theory Plug-in** provides capabilities to extend **the Event-B mathematical language** and **the Rodin proving infrastructure**.

- An **Event-B theory** can contain :
  - new datatype definitions,
  - new polymorphic operator definitions,
  - axiomatic definitions,
  - theorems,
  - associated rewrite and inference rules.

- Michael J. Butler and Issam Maamria.
  Practical theory extension in Event-B. Theories of Programming and Formal Methods. 2013.
- Thai Son Hoang, Laurent Voisin, Asieh Salehi, Michael J. Butler, Toby Wilkinson, and Nicolas Beauger.
  Theory plug-in for Rodin 3.x. CoRR, abs/1701.08625, 2017.

THE EVENT-B METHOD

THE THEORY PLUGIN

```
THEORY thy₁
IMPORT thy₂

DATATYPES
  DT₁, ..., DTₙ
OPERATORS
  OP₁₁, ..., OP₁ₙ
AXIOMATIC DEFINITIONS
  operators
    OP₂₁, ..., OP₂ₙ
  axioms
    A
THEOREMS
  T
PROOF RULES
  PR
END
```

```
CONTEXT ctx₁
EXTENDS ctx₂

SETS s
CONSTANTS c
AXIOMS
  A(s, c)
THEOREMS
  T(s, c)
END
```

```
MACHINE mch₁
REFINES mch₂
SEES ctxᵢ

VARIABLES v
INVARIANTS
  I(s, c, v)
THEOREMS
  T(s, c, v)
EVENTS
  [events_list]
END
```

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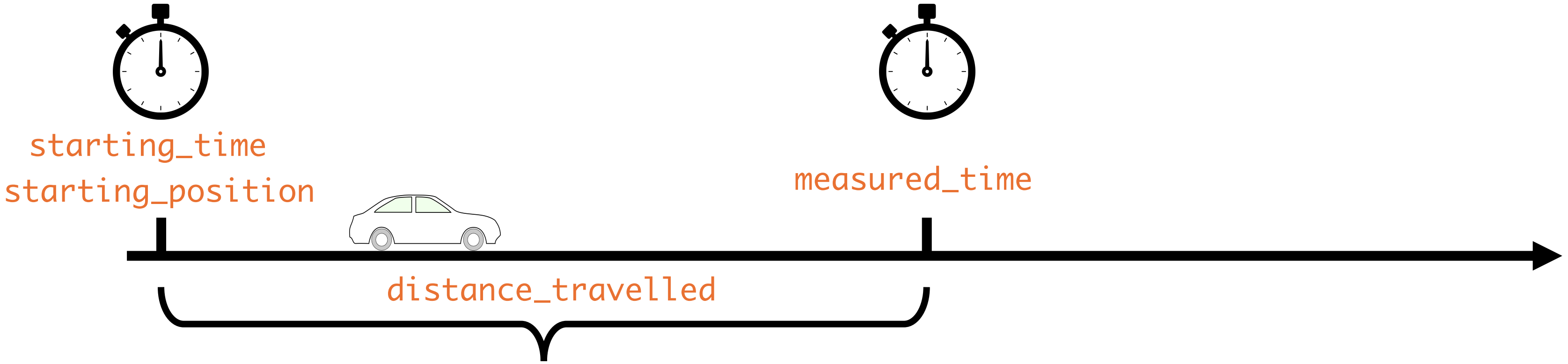
A SIMPLE EXAMPLE

System that continuously calculates **a moving object's speed**



- Analysing **two functional properties**:
  - **PROP-1** : **the speed of the moving object** is equal to the $distance\_travelled$ divided by the $measured\_time$ ($v = d/t$).
  - **PROP-2** : when the $distance\_travelled$ is strictly positive, the $speed$ of the moving object must also be strictly positive.
    - **the object moves** when its $speed$ is different from zero.

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A SIMPLE EXAMPLE

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    - **the object moves** when its $speed$ is different from zero.

Objectives $\longrightarrow$ showing some **modelling and verification problems** :

- using **integer** variables to handle **small values.**
- analysing **physical phenomena.**
  - expressions that come from the physics laws.

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CHALLENGES IN MODELLING CPS SYSTEMS

- Cyber-Physical Systems (CPS) models require numerical variables representing physical measurements.
- Event-B does not support physical unit annotations for such variables.
- Formal verification of CPS systems requires a physical measurement model, e.g. the International System of Units (SI).
- Using explicit units improves rigour by ensuring unit compatibility in computations.

PROPOSED APPROACH

- **Objective**
  - Formally annotate numerical variables with measurement units in Event-B.
  - Provide automatic checking of correct unit usage in arithmetic expressions.
  - Define Well-Defined Proof Obligations (WD POs) to ensure unit consistency.
  - Example: $b = v/2a_{max,brake}$
    $\rightarrow$ must ensure that the unit of b matches that of $v/2a_{max,brake}$.
- **Proposal**
  - Develop a measurement units theory using the Theory plugin.
  - Extend the Event-B type-checking system to handle reasoning about measurement units.
  - Introduce a formal method for annotating Event-B variables with their associated units of measurement.

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THANK YOU

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