Laboratoire Méthodes Formelles

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DÉVELOPPEMENT DE SYSTÈMES CRITIQUES AVEC LA MÉTHODE EVENT-B

LE CONCEPT DE LA MODÉLISATION PAR RAFFINEMENT

🎓 3A cursus ingénieurs - Mention Sciences du Logiciel
🏛 CentraleSupelec - Université Paris-Saclay - 2025/2026

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PLAN

Retour au plan - Retour à l'accueil

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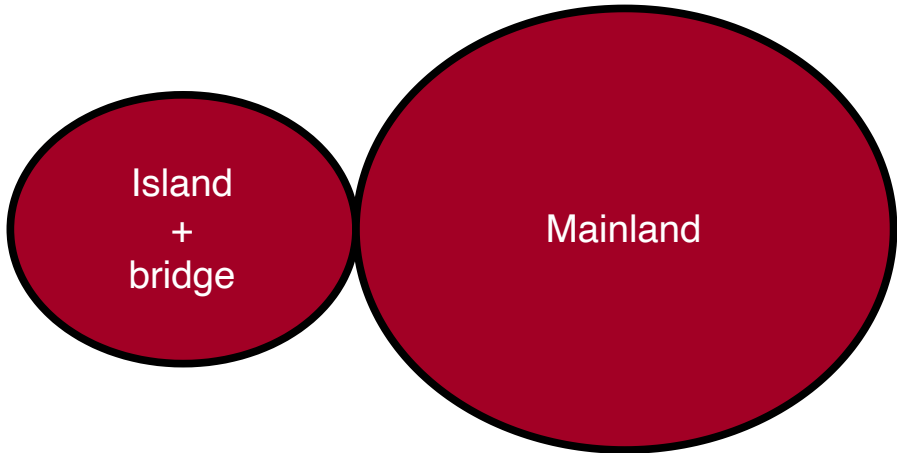
A SITUATION AS SEEN FROM THE SKY



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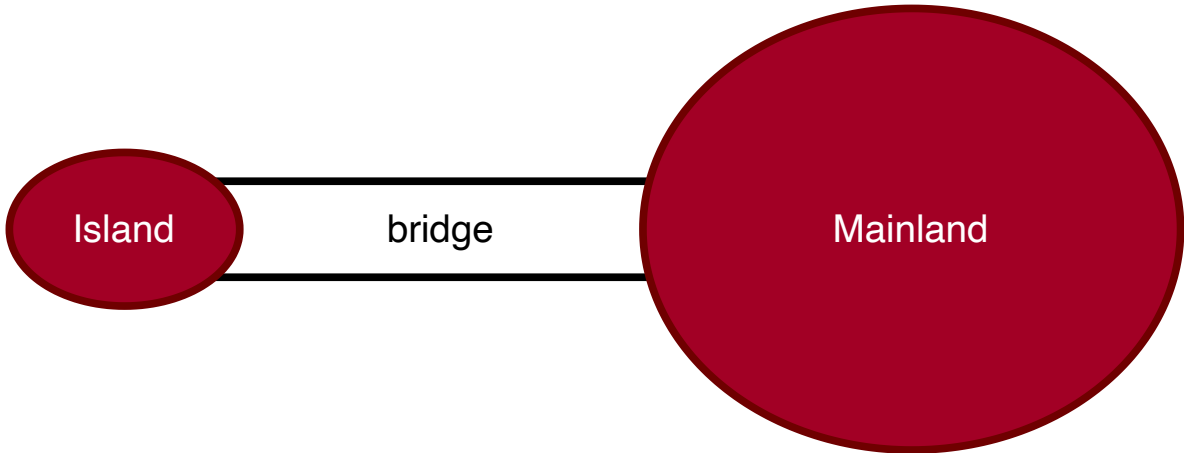
A SITUATION AS SEEN FROM THE SKY



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OUR REFINEMENT STRATEGY

- **Initial model** $\longrightarrow$ Limiting the number of cars (**FUN-2**)
- **First refinement** $\longrightarrow$ Introducing the one way bridge (**FUN-1**, **FUN-3**)
  - Our view of the system gets more accurate
  - We introduce the bridge and separate it from the island (**FUN-1**)
  - We refine the state and the events
  - We also add two new events $\longrightarrow$ `IL_in` and `IL_out`
  - We are focusing on **FUN-3** $\longrightarrow$ one-way bridge

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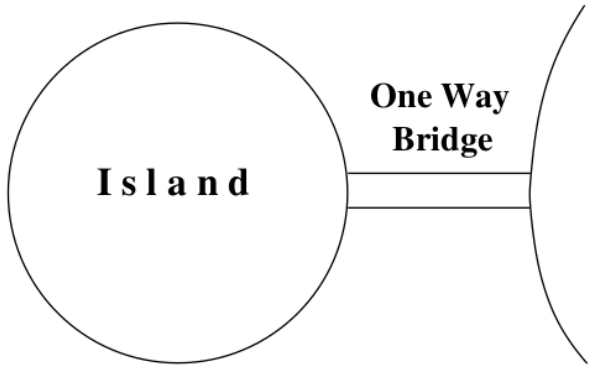
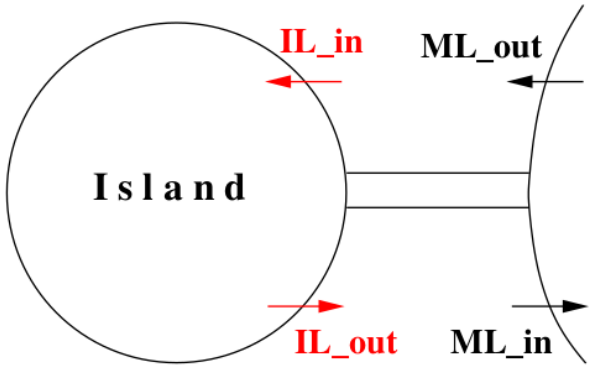
FIRST REFINEMENT

INTRODUCING A ONE-WAY BRIDGE





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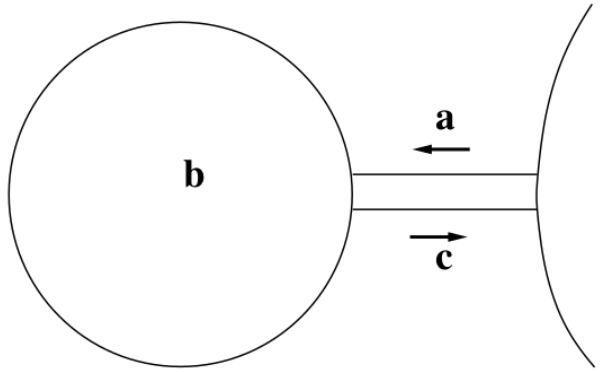
INTRODUCING THREE NEW VARIABLES



- a denotes the number of cars on bridge going to island
- b denotes the number of cars on island
- c denotes the number of cars on bridge going to mainland
- a, b, and c are the concrete variables
- They replace the abstract variable n

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REFINING THE STATE

- Variables a, b, and c denote natural numbers

VARIABLES

$a \;\; b \;\; c$

INVARIANTS

inv1_1: $a \in \mathbb{N}$

inv1_2: $b \in \mathbb{N}$

inv1_3: $c \in \mathbb{N}$

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REFINING THE STATE

INTRODUCING NEW INVARIANTS

- Relating the concrete state (a, b, c) to the abstract state (n)

  INVARIANTS

  ...

  `inv1_4:` $a + b + c = n$

- Formalizing the new invariant $\longrightarrow$ one way bridge (this is **FUN-3**)

  INVARIANTS

  ...

  `inv1_5:` $a = 0 \lor c = 0$

- Invariants `inv1_1` to `inv1_5` are called the concrete invariants
- `inv1_4` **glues** the abstract state, n, to the concrete state, a, b, c

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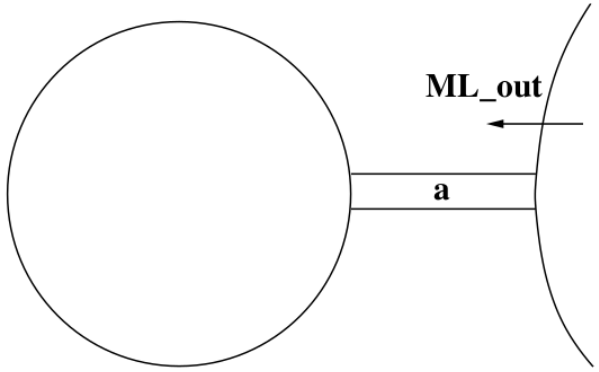
PROPOSAL FOR REFINING EVENT ML_out



ML_out $\widehat{=}$

when

grd1_1: $a + b < d$

grd1_2: $c = 0$

then

act1_1: $a := a + 1$

end

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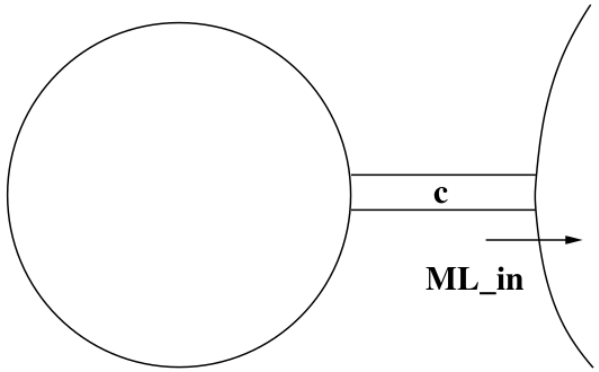
PROPOSAL FOR REFINING EVENT ML_in



ML_in $\widehat{=}$
  when
    grd1_1: $0 < c$
  then
    act1_1: $c := c - 1$
  end

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BEFORE-AFTER PREDICATES

PRESERVED VARIABLES

ML_out $\widehat{=}$
  when
    grd1_1: $a + b < d$
    grd1_2: $c = 0$
  then
    act1_1: $a := a + 1$
  end

ML_in $\widehat{=}$
  when
    grd1_1: $0 < c$
  then
    act1_1: $c := c - 1$
  end

Before-after predicates showing the unmodified variables

$$a' = a + 1 \wedge b' = b \wedge c' = c$$

$$a' = a \wedge b' = b \wedge c' = c - 1$$

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INTUITION ABOUT REFINEMENT

- The concrete model behaves as specified by the abstract model (i.e., concrete model does not exhibit any new behaviors)
- To show this we have to prove that
  1. every concrete event is simulated by its abstract counterpart (event refinement $\longrightarrow$ following slides)
  2. to every concrete initial state corresponds an abstract one (initial state refinement $\longrightarrow$ later)
- We will make these two conditions more precise and formalize them as proof obligations.

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INTUITION ABOUT REFINEMENT

ML_out $\widehat{=}$ //abstract
when
grd0_1: $n < d$
then
act0_1: $n := n + 1$
end

ML_out $\widehat{=}$ //concrete
when
grd1_1: $a + b < d$
grd1_2: $c = 0$
then
act1_1: $a := a + 1$
end

- The concrete version is not contradictory with the abstract one
- When the concrete version is enabled then so is the abstract one
- Executions seem to be compatible

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INTUITION ABOUT REFINEMENT

```
ML_in ≙ //abstract
  when
    grd0_1: 0 < n
  then
    act0_1: n := n − 1
  end
```

```
ML_in ≙ //concrete
  when
    grd1_1: 0 < c
  then
    act1_1: c := c − 1
  end
```

- Same remarks as in the previous slide
- But this has to be confirmed by well-defined proof obligations

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PROOF OBLIGATIONS FOR REFINEMENT

- The concrete guard is stronger than the abstract one
- Each concrete action is compatible with its abstract counterpart

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PROVING CORRECT REFINEMENT

THE SITUATION

- Constants c with axioms $A(c)$
- Abstract variables v with abstract invariant $I(c, v)$
- Concrete variables w with concrete invariant $J(c, v, w)$
- Abstract event with guards $G(c, v) \longrightarrow G_1(c, v), G_2(c, v), \ldots$
- Abstract event with before-after predicate $v' = E(c, v)$
- Concrete event with guards $H(c, w)$ and b-a predicate $w' = F(c, w)$

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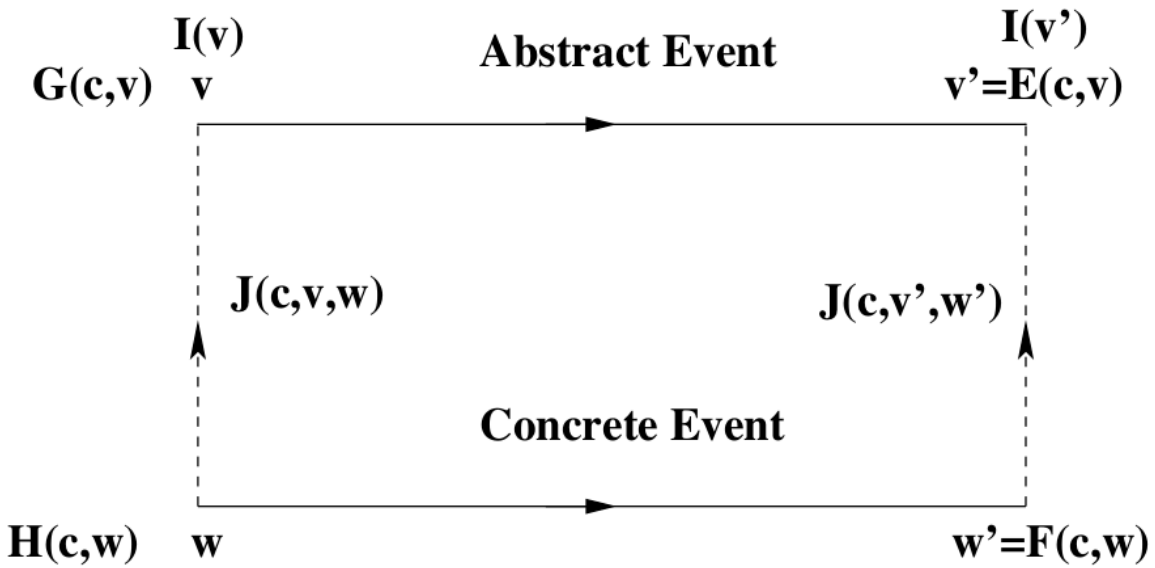
CORRECTNESS OF EVENT REFINEMENT



1. The concrete guard is stronger than the abstract one (Guard Strengthening, following slides)

2. Each concrete action is simulated by its abstract counterpart (Concrete Invariant Preservation, later)

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PROOF OBLIGATION

GUARD STRENGTHENING

Axioms	$A(c)$	
Abstract Invariants	$I(c, v)$	
Concrete Invariants	$J(c, v, w)$	GRD
Concrete Guard	$H(c, w)$	
$\vdash$	$\vdash$	
Abstract Guard	$G_i(c, v)$	

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APPLYING GUARD STRENGTHENING TO EVENT ML_out

PROOF OF ML_out/GRD

ML_out $\widehat{=}$ //abstract
  when
    grd0_1: $n < d$
  then
    act0_1: $n := n + 1$
  end

ML_out $\widehat{=}$ //concrete
  when
    grd1_1: $a + b < d$
    grd1_2: $c = 0$
  then
    act1_1: $a := a + 1$
  end

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APPLYING GUARD STRENGTHENING TO EVENT ML_out

PROOF OF ML_out/GRD

$$
\begin{array}{l}
d \in \mathbb{N} \\
0 < d \\
n \in \mathbb{N} \\
n \leq d \\
a \in \mathbb{N} \\
b \in \mathbb{N} \\
c \in \mathbb{N} \\
a + b + c = n \\
a = 0 \lor c = 0 \\
a + b < d \\
c = 0 \\
\vdash \\
n < d
\end{array}
$$

MON $\Longrightarrow$

$$
\begin{array}{l}
a + b + c = n \\
a + b < d \\
c = 0 \\
\vdash \\
n < d
\end{array}
$$

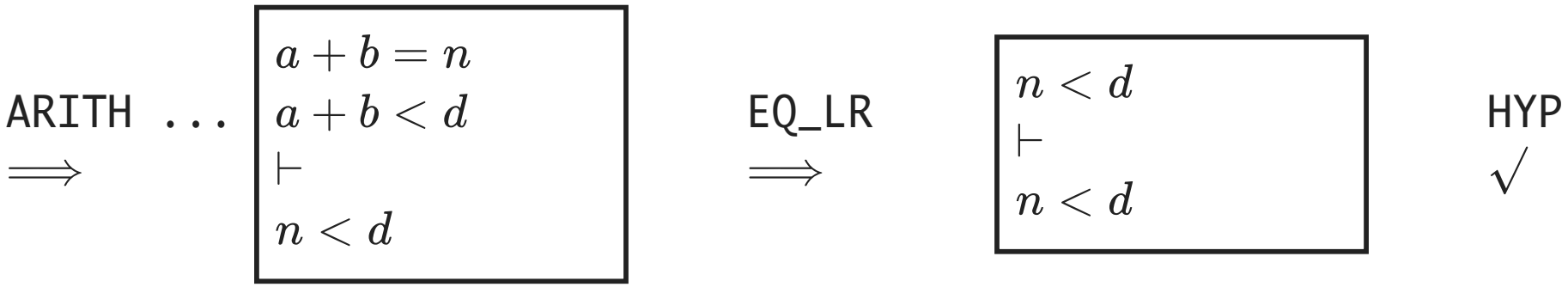
EQ_LR $\Longrightarrow$

$$
\begin{array}{l}
a + b + 0 = n \\
a + b < d \\
\vdash \\
n < d
\end{array}
$$

ARITH ... $\Longrightarrow$

APPLYING GUARD STRENGTHENING TO EVENT ML_out

PROOF OF ML_out/GRD



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APPLYING GUARD STRENGTHENING TO EVENT ML_in

PROOF OF ML_in/GRD

ML_in $\widehat{=}$ //abstract
  when
    grd0_1: $0 < n$
  then
    act0_1: $n := n - 1$
  end

ML_in $\widehat{=}$ //concrete
  when
    grd1_1: $0 < c$
  then
    act1_1: $c := c - 1$
  end

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APPLYING GUARD STRENGTHENING TO EVENT ML_in

PROOF OF ML_in/GRD

$$
\begin{array}{|l}
\hline
d \in \mathbb{N} \\
0 < d \\
n \in \mathbb{N} \\
n \leq d \\
a \in \mathbb{N} \\
b \in \mathbb{N} \\
c \in \mathbb{N} \\
a + b + c = n \\
a = 0 \lor c = 0 \\
a + b < d \\
0 < c \\
\vdash \\
0 < n \\
\hline
\end{array}
$$

MON $\Longrightarrow$

$$
\begin{array}{|l}
\hline
b \in \mathbb{N} \\
a + b + c = n \\
a = 0 \lor c = 0 \\
0 < c \\
\vdash \\
0 < n \\
\hline
\end{array}
$$

OR_L $\Longrightarrow$

$$
\begin{array}{|l}
\hline
b \in \mathbb{N} \\
a + b + c = n \\
a = 0 \\
0 < c \\
\vdash \\
0 < n \\
\hline
\end{array}
$$

EQ_LR ... $\Longrightarrow$

$$
\begin{array}{|l}
\hline
b \in \mathbb{N} \\
a + b + c = n \\
c = 0 \\
0 < c \\
\vdash \\
0 < n \\
\hline
\end{array}
$$

EQ_LR ... $\Longrightarrow$

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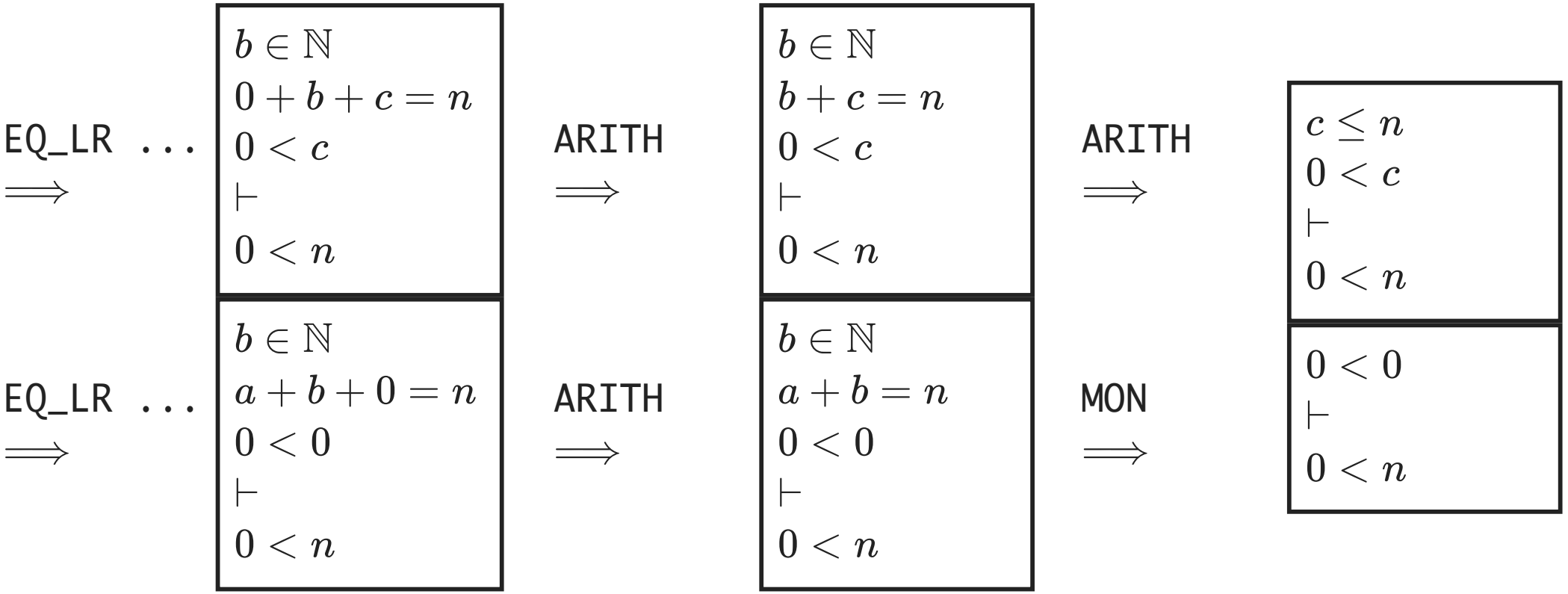
APPLYING GUARD STRENGTHENING TO EVENT ML_in

PROOF OF ML_in/GRD



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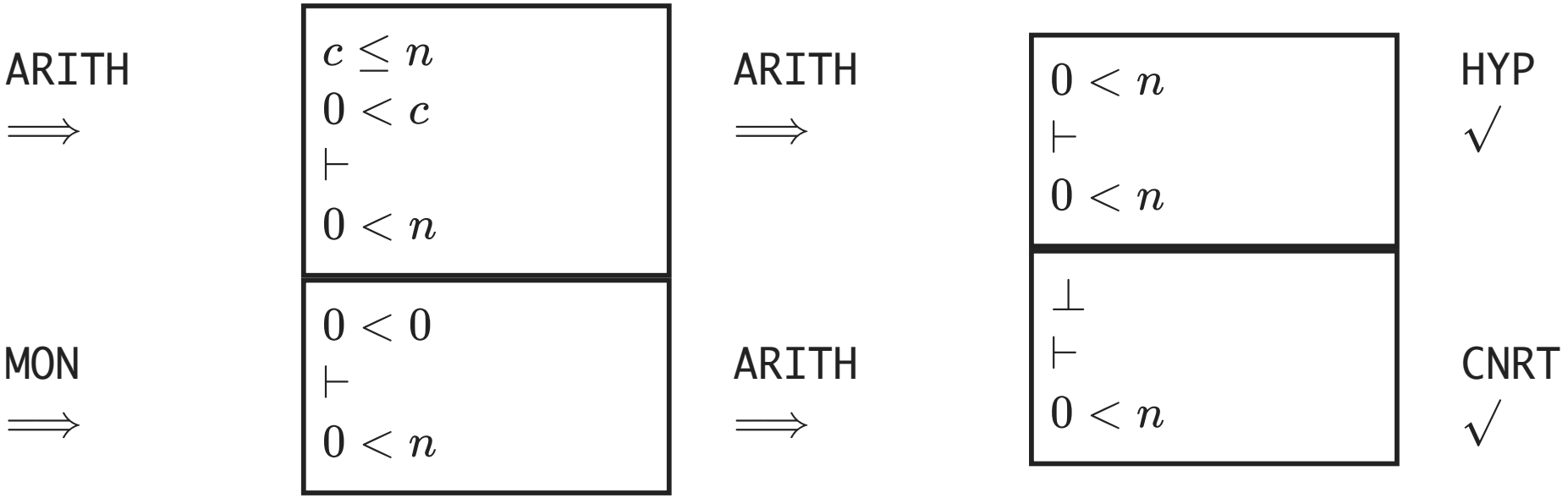
APPLYING GUARD STRENGTHENING TO EVENT ML_in

PROOF OF ML_in/GRD



- In the previous proof, we have used and additional inference rule
- It says that a false hypothesis entails any goal $\bot \vdash P$ CNTR

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PROOF OBLIGATION

INVARIANT REFINEMENT

Axioms	$A(c)$	
Abstract Invariants	$I(c, v)$	
Concrete Invariants	$J(c, v, w)$	INV
Concrete Guard	$H(c, w)$	
$\vdash$	$\vdash$	
Modified Concrete Invariant	$J_j(c, E(c, v), F(c, w))$	

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APPLYING INVARIANT REFINEMENT TO EVENT ML_out

PROOF OF ML_out/inv1_4/INV

ML_out $\widehat{=}$ //abstract
  when
    grd0_1: $n < d$
  then
    act0_1: $n := n + 1$
  end

ML_out $\widehat{=}$ //concrete
  when
    grd1_1: $a + b < d$
    grd1_2: $c = 0$
  then
    act1_1: $a := a + 1$
  end

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APPLYING INVARIANT REFINEMENT TO EVENT ML_out

PROOF OF ML_out/inv1_4/INV

$$
\begin{array}{l}
d \in \mathbb{N} \\
0 < d \\
n \in \mathbb{N} \\
n \leq d \\
a \in \mathbb{N} \\
b \in \mathbb{N} \\
c \in \mathbb{N} \\
a + b + c = n \\
a = 0 \lor c = 0 \\
a + b < d \\
c = 0 \\
\vdash \\
a + 1 + b + c = n + 1
\end{array}
$$

MON $\Longrightarrow$

$$
\begin{array}{l}
a + b + c = n \\
\vdash \\
a + 1 + b + c = n + 1
\end{array}
$$

ARITH $\Longrightarrow$

$$
\begin{array}{l}
a + b + c = n \\
\vdash \\
a + b + c + 1 = n + 1
\end{array}
$$

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APPLYING INVARIANT REFINEMENT TO EVENT ML_out

PROOF OF ML_out/inv1_4/INV

$$\boxed{\begin{array}{l} a+b+c=n \\ \vdash \\ a+b+c+1=n+1 \end{array}}$$

EQ_LR

$\Longrightarrow$

$$\boxed{\begin{array}{l} \vdash \\ n+1=n+1 \end{array}}$$

EQL

$\surd$

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REFINING THE INITIALIZATION EVENT `init`

- Concrete initialization

  EVENTS

  `INITIALISATION` $\widehat{=}$

  $a := 0$

  $b := 0$

  $c := 0$

- Corresponding after predicate

  $a' = 0 \wedge b' = 0 \wedge c' = 0$

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PROOF OBLIGATION

INITIALIZATION REFINEMENT

- Constants c with axioms $A(c)$
- Concrete invariant $J(c, v, w)$
- Abstract initialization with after predicate $v' = K(c)$
- Concrete initialization with after predicate $w' = L(c)$

$$\begin{array}{lll} \text{Axioms} & A(c) & \\ \vdash & \vdash & \texttt{INV} \\ \text{Modified concrete invariants} & J_j(c, K(c), L(c)) & \end{array}$$

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PROOF OBLIGATION

INITIALIZATION REFINEMENT

$$
\begin{array}{l}
d \in \mathbb{N} \\
0 < d \\
\vdash \\
0 + 0 + 0 = 0 \\
(a + b + c = n)
\end{array}
$$

$$
\begin{array}{l}
d \in \mathbb{N} \\
0 < d \\
\vdash \\
0 = 0 \lor 0 = 0 \\
(a = 0 \lor c = 0)
\end{array}
$$

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ADDING NEW EVENTS

- new events add transitions that have no abstract counterpart
- can be seen as a kind of internal steps (w.r.t. abstract model)
- can only be seen by an observer who is *"zooming in"*
- temporal refinement: refined model has a finer time granularity

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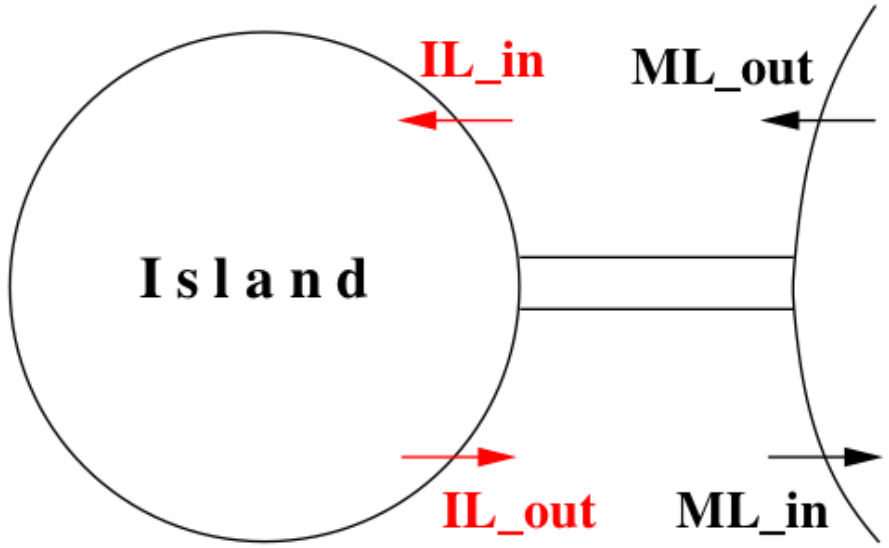
ADDING NEW EVENTS



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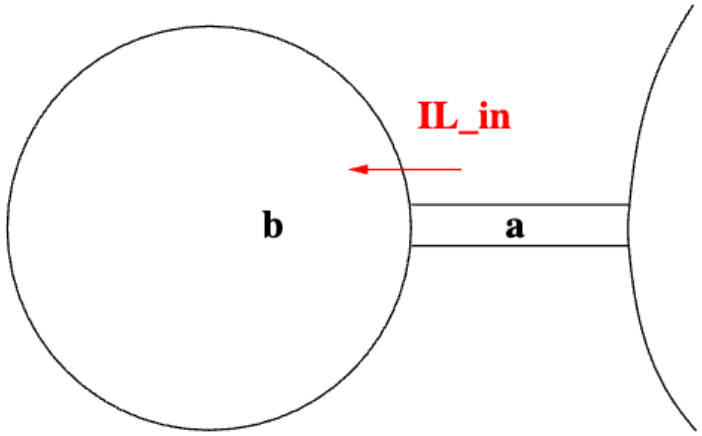
NEW EVENT IL_in



IL_in $\widehat{=}$

when

grd1_1: $0 < a$

then

act1_1: $a := a - 1$

act1_2: $b := b + 1$

end

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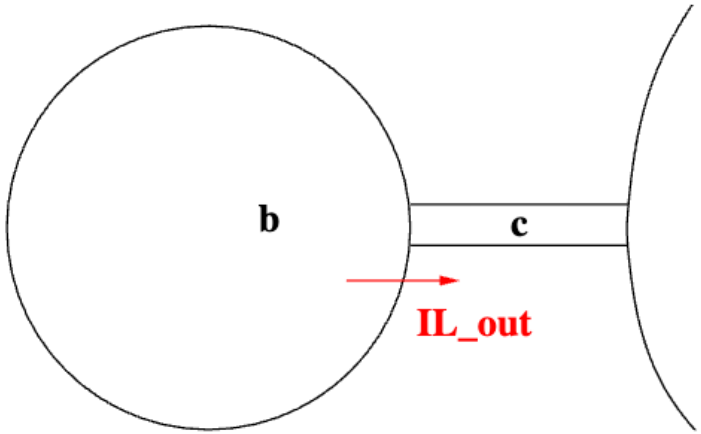
NEW EVENT IL_out



IL_out $\widehat{=}$
  when
    grd1_1: $0 < b$
    grd1_2: $a = 0$
  then
    act1_1: $b := b - 1$
    act1_2: $c := c + 1$
  end

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BEFORE-AFTER PREDICATES

IL_in $\widehat{=}$
  when
    grd1_1: $0 < a$
  then
    act1_1: $a := a - 1$
    act1_2: $b := b + 1$
  end

IL_out $\widehat{=}$
  when
    grd1_1: $0 < b$
    grd1_2: $a = 0$
  then
    act1_1: $b := b - 1$
    act1_2: $c := c + 1$
  end

$$a' = a - 1 \wedge b' = b + 1 \wedge c' = c$$
$$a' = a \wedge b' = b - 1 \wedge c' = c + 1$$

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THE EMPTY ASSIGNMENT `skip`

- The before-after predicate of `skip` in the initial model

$$n' = n$$

- The before-after predicate of `skip` in the first refinement

$$a' = a \wedge b' = b \wedge c' = c$$

- The guard of the `skip` event is `true`.

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REFINEMENT PROOF OBLIGATIONS FOR NEW EVENTS

- A new event must refine an implicit event, made of a `skip` action
  - Guard strengthening is trivial
  - Need to prove invariant refinement
- The new events must not diverge
  - To prove this we have to exhibit a variant
  - The variant yields a natural number (could be more complex)
  - Each new event must decrease this variant

EVENT IL_in REFINES skip

IL_in $\widehat{=}$

when

grd1_1: $0 < a$

then

act1_1: $a := a - 1$

act1_2: $b := b + 1$

end

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PROOF OF IL_in/inv1_4/INV

$$
\begin{array}{l}
d \in \mathbb{N} \\
0 < d \\
n \in \mathbb{N} \\
n \leq d \\
a \in \mathbb{N} \\
b \in \mathbb{N} \\
c \in \mathbb{N} \\
a + b + c = n \\
a = 0 \vee c = 0 \\
0 < a \\
\vdash \\
a - 1 + b + 1 + c = n
\end{array}
$$

MON $\Longrightarrow$

$$
\begin{array}{l}
a + b + c = n \\
\vdash \\
a - 1 + b + 1 + c = n
\end{array}
$$

ARITH $\Longrightarrow$

$$
\begin{array}{l}
a + b + c = n \\
\vdash \\
a + b + c = n
\end{array}
$$

HYP $\surd$

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PROOF OBLIGATION

CONVERGENCE OF NEW EVENTS

- Axioms $A(c)$, invariants $I(c, v)$, concrete invariant $J(c, v, w)$
- New event with guard $H(c, w)$
- Variant $V(c, w)$

Axioms	$A(c)$	
$\text{Abstract Invariants}$	$I(c, v)$	
$\text{Concrete Invariants}$	$J(c, v, w)$	NAT
Concrete Guard	$H(c, w)$	
$\vdash$	$\vdash$	
$\text{Variant} \in \mathbb{N}$	$V(c, w) \in \mathbb{N}$	

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PROOF OBLIGATION

CONVERGENCE OF NEW EVENTS

- Axioms $A(c)$, invariants $I(c, v)$, concrete invariant $J(c, v, w)$
- New event with guard $H(c, w)$ and b-a predicate $w' = F(c, w)$
- Variant $V(c, w)$

Axioms	$A(c)$	
$\text{Abstract Invariants}$	$I(c, v)$	
$\text{Concrete Invariants}$	$J(c, v, w)$	VAR
Concrete Guard	$H(c, w)$	
$\vdash$	$\vdash$	
$\text{Modified Var} < \text{Var}$	$V(c, F(c, w)) < V(c, w)$	

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PROPOSED VARIANT

Weighted sum of a and b

VARIANT

variant_1: $2 \times a + b$

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DECREASING OF THE VARIANT BY EVENT IL_in

IL_in $\widehat{=}$
  when
    grd1_1: $0 < a$
  then
    act1_1: $a := a - 1$
    act1_2: $b := b + 1$
  end

$$
\begin{array}{l}
d \in \mathbb{N} \\
0 < d \\
n \in \mathbb{N} \\
n \leq d \\
a \in \mathbb{N} \\
b \in \mathbb{N} \\
c \in \mathbb{N} \\
a + b + c = n \\
a = 0 \lor c = 0 \\
0 < a \\
\vdash \\
2 \times (a - 1) + (b + 1) < 2 \times a + b
\end{array}
$$

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DECREASING OF THE VARIANT BY EVENT IL_out

IL_out $\widehat{=}$
  when
    grd1_1: $0 < b$
    grd1_2: $a = 0$
  then
    act1_1: $b := b - 1$
    act1_2: $c := c + 1$
  end

$$
\begin{array}{l}
d \in \mathbb{N} \\
0 < d \\
n \in \mathbb{N} \\
n \leq d \\
a \in \mathbb{N} \\
b \in \mathbb{N} \\
c \in \mathbb{N} \\
a + b + c = n \\
a = 0 \lor c = 0 \\
0 < a \\
\vdash \\
2 \times a + (b - 1) < 2 \times a + b
\end{array}
$$

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PLAN

Retour au plan - Retour à l'accueil

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SUPERPOSITION
(NOT COVERED IN THIS TUTORIAL)

Abstract_Event $\widehat{=}$
when
$G(c, u, v)$
then
$u := E(c, u, v)$
$v := M(c, u, v)$
end

Concrete_Event $\widehat{=}$
when
$H(c, v, w)$
then
$v := N(c, v, w)$
$w := F(c, v, w)$
end

Axioms
Abstract Invariants
Concrete Invariants
Concrete Guard
$\vdash$
Same actions in common variables

$A(c)$
$I(c, v)$
$J(c, v, w)$
$H(c, w)$
$\vdash$
$M(c, u, v) = N(c, v, w)$

SIM

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MERCI

Version PDF des slides

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