

Crowdsourcing Pareto-Optimal Object Finding by Pairwise Comparisons

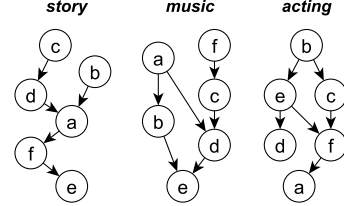
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Abstract—This is the first study on crowdsourcing Pareto-optimal object finding, which has applications in public opinion collection, group decision making, and information exploration. Departing from prior studies on crowdsourcing skyline and ranking queries, it considers the case where objects do not have explicit attributes and preference relations on objects are strict partial orders. The partial orders are derived by aggregating crowdsourcers’ responses to pairwise comparison questions. The goal is to find all Pareto-optimal objects by the fewest possible questions. It employs an iterative question-selection framework. Guided by the principle of eagerly identifying non-Pareto optimal objects, the framework only chooses candidate questions which must satisfy three conditions. This design is both sufficient and efficient, as it is proven to find a short terminal question sequence. The framework is further steered by two ideas—macro-ordering and micro-ordering. By different micro-ordering heuristics, the framework is instantiated into several algorithms with varying power in pruning questions. Experiment results using both real crowdsourcing marketplace and simulations exhibited not only orders of magnitude reductions in questions when compared with a brute-force approach, but also close-to-optimal performance from the most efficient instantiation.

I. INTRODUCTION

The growth of user engagement and functionality in crowdsourcing platforms has made computationally challenging tasks unprecedentedly convenient. The subject of our study is one such task—crowdsourcing *Pareto-optimal object finding*. Consider a set of objects O and a set of criteria C for comparing the objects. An object $x \in O$ is *Pareto-optimal* if and only if x is not dominated by any other object, i.e., $\nexists y \in O$ such that $y \succ x$. An object y dominates x (denoted $y \succ x$) if and only if x is not better than y by any criterion and y is better than x by at least one criterion, i.e., $\forall c \in C : x \not\succ_c y$ and $\exists c \in C : y \succ_c x$. If x and y do not dominate each other (i.e., $x \not\succ y$ and $y \not\succ x$), we denote it by $x \sim y$. The *preference (better-than) relation* P_c (also denoted \succ_c) for each $c \in C$ is a binary relation subsumed by $O \times O$, in which a tuple $(x, y) \in P_c$ (also denoted $x \succ_c y$) is interpreted as “ x is better than (preferred over) y with regard to criterion c ”. Hence, if $(x, y) \notin P_c$ (also denoted $x \not\succ_c y$), x is not better than y by criterion c . We say x and y are *indifferent* regarding c (denoted $x \sim_c y$), if $(x, y) \notin P_c \wedge (y, x) \notin P_c$, i.e., “ x and y are equally good or incomparable with regard to c .” We consider the setting where each P_c is a *strict partial order* as opposed to a bucket order [1] or a total order, i.e., P_c is irreflexive ($\forall x : (x, x) \notin P_c$) and transitive ($\forall x, y : (x, y) \in P_c \wedge (y, z) \in P_c \Rightarrow (x, z) \in P_c$), which together imply asymmetry ($\forall x, y : (x, y) \in P_c \Rightarrow (y, x) \notin P_c$).

Pareto-optimal object finding lends itself to applications in several areas, including public opinion collection, group decision making, and information exploration, exemplified by the following motivating examples.



(a) Preference relations (i.e., strict partial orders) on three criteria.

QUESTION	ANSWER			OUTCOME
	\succ	\sim	\prec	
$a \stackrel{?}{\succ}_s b$	1	0	4	$b \succ_s a$
$a \stackrel{?}{\succ}_s c$	0	0	5	$c \succ_s a$
$a \stackrel{?}{\succ}_s d$	0	2	3	$d \succ_s a$
$a \stackrel{?}{\succ}_s e$	4	0	1	$a \succ_s e$
$a \stackrel{?}{\succ}_s f$	3	1	1	$a \succ_s f$
$b \stackrel{?}{\succ}_s c$	1	2	2	$b \sim_s c$
$b \stackrel{?}{\succ}_s d$	1	3	1	$b \sim_s d$
$b \stackrel{?}{\succ}_s e$	5	0	0	$b \succ_s e$
$b \stackrel{?}{\succ}_s f$	4	1	0	$b \succ_s f$
$c \stackrel{?}{\succ}_s d$	3	2	0	$c \succ_s d$
$c \stackrel{?}{\succ}_s e$	4	0	1	$c \succ_s e$
$c \stackrel{?}{\succ}_s f$	3	1	1	$c \succ_s f$
$d \stackrel{?}{\succ}_s e$	3	0	2	$d \succ_s e$
$d \stackrel{?}{\succ}_s f$	3	2	0	$d \succ_s f$
$e \stackrel{?}{\succ}_s f$	1	1	3	$f \succ_s e$

(b) Deriving the preference relation for criterion *story* by pairwise comparisons. Each comparison is performed by 5 workers. $\theta = 60\%$.

Fig. 1: Finding Pareto-optimal movies by *story*, *music* and *acting*.

Example 1 (Collecting Public Opinion and Group Decision Making). Consider a set of movies $O = \{a, b, c, d, e, f\}$ and a set of criteria $C = \{\text{story}, \text{music}, \text{acting}\}$ (denoted by s, m, a in the ensuing discussion). Fig. 1a shows the individual preference relations (i.e., strict partial orders), one per criterion. Each strict partial order is graphically represented as a directed acyclic graph (DAG), more specifically a Hasse diagram. The existence of a simple path from x to y in the DAG means x is better than (preferred to) y by the corresponding criterion. For example, $(a, e) \in P_m$ ($a \succ_m e$), i.e., a is better than e by *music*. $(b, d) \notin P_s$ and $(d, b) \notin P_s$; hence $b \sim_s d$. The partial orders define the dominance relation between objects. For instance, movie c dominates d ($c \succ d$), because c is preferred than d on *story* and *music* and they are indifferent on *acting*, i.e., $c \succ_s d$, $c \succ_m d$, and $c \sim_a d$; a and b do not dominate each other ($a \sim b$), since $b \succ_s a$, $a \succ_m b$ and $b \succ_a a$. Based on the three partial orders, movie b is the only Pareto-optimal object, since no other objects dominate it and every other object is dominated by some object.

Note that tasks such as the above one may be used in both understanding the public’s preference (i.e., the preference relations are collected from a large, anonymous crowd) and making decisions for a target group (i.e., the preference relations are from a small group of people). \square

Between **A separation(2011)** and **The big Lebowski(1998)**,
which movie is better with regard to **story**?

- ☐ A separation(2011).
- ☐ The big Lebowski(1998).
- ☐ no preference.

Fig. 2: A question that asks to compare two movies by *story*.

Example 2 (Information Exploration). Consider a photography enthusiast, *Amy*, who is drown in a large number of photos she has taken and wants to select a subset of the better ones. She resorts to crowdsourcing for the task, as it has been exploited by many for similar tasks such as photo tagging, location/face identification, sorting photos by (guessed) date, and so on. Particularly, she would like to choose Pareto-optimal photos with regard to *color*, *sharpness* and *landscape*. \square

By definition, the crux of finding Pareto-optimal objects lies in obtaining the preference relations, i.e., the strict partial orders on individual criteria. Through crowdsourcing, the preference relations are derived by aggregating the crowd’s responses to *pairwise comparison* tasks. Each such comparison between objects x and y by criterion c is a question, denoted $x?_cy$, which has three possible outcomes— $x \succ_c y$, $y \succ_c x$, and $x \sim_c y$, based on the crowd’s answers. An example is as follows.

Example 3 (Deriving Preference Relations from Pairwise Comparisons by the Crowd). Fig.1b shows the hypothetical results of all 15 pairwise comparisons between the 6 movies in Example 1, by criterion $s=story$. The outcomes of all comparisons form the crowd’s preference relation on *story* (the leftmost DAG in Fig.1a). Fig.2 is the screenshot of a question form designed for one such comparison. A crowdsourcer, when facing this question, would make a choice among the three possible answers or skip a question if they do not have enough confidence or knowledge to answer it. Fig.1b shows how many crowdsourcers have selected each answer. For instance, for question $a?_sf$, three people preferred movie a , one person preferred f , and one person is indifferent. By aggregating these answers, it is derived that a is better than f with regard to *story*, since 60% of the crowdsourcers who responded to the question chose this answer. For question $b?_sc$, the result is $b \sim_sc$, since neither $b \succ_sc$ nor $b \prec_sc$ received enough votes. (Assuming a threshold $\theta=60\%$, i.e., either $b \succ_sc$ or $b \prec_sc$ should have at least 60% of votes, in order to not declare $b \sim_sc$.) \square

To the best of our knowledge, this paper is the first work on crowdsourcing Pareto-optimal object finding. The definition of Pareto-optimal objects follows the concept of *Pareto composition* of preference relations in [2]. It also resembles the definition of *skyline* objects on totally-ordered attribute domains (pioneered by [3]) and partially-ordered domains [4], [5], [6], [7]. However, except for [8], previous studies on preference and skyline queries do not use the crowd; they focus on query processing on *existing* data. On the contrary, we examine how to ask the crowd as few questions as possible in obtaining sufficient data for determining Pareto-optimal objects. Furthermore, our work differs from preference and skyline queries (including [8]) in several radical ways:

- The preference relation for a criterion is *not* governed by explicit scores or values on object attributes (e.g., sizes of houses, prices of hotels), while preference and skyline queries on both totally- and partially-ordered domains assumed explicit attribute representation. For many comparison criteria, it is difficult to model objects by explicit attributes, not to mention asking people to provide such values or scores; people’s preferences are rather based on complex, subtle personal perceptions, as demonstrated in Examples 1 and 2.
- Due to the above reason, we request crowdsourcers to perform pairwise comparisons instead of directly providing attribute values or scores. On the contrary, [8] uses the crowd to obtain missing attribute values. Pairwise comparison is extensively studied in social choice and welfare, preferences, and voting. It is known that people are more comfortable and confident with comparing objects than directly scoring them, since it is easier, faster, and less error-prone [9].
- The crowd’s preference relations are modeled as strict partial orders, as opposed to bucket orders or full orders. This is not only a direct effect of using pairwise comparisons instead of numeric scores or explicit attribute values, but also a reflection of the psychological nature of human’s preferences [10], [2], since it is not always natural to enforce a total or bucket order. Most studies on skyline queries assume total/bucket orders, except for [4], [5], [6], [7] which consider partial orders.

Our objective is to find all Pareto-optimal objects with as few questions as possible. A brute-force approach will obtain the complete preference relations via pairwise comparisons on all object pairs by every criterion. However, without such exhaustive comparisons, the incomplete knowledge collected from a small set of questions may suffice in discerning all Pareto-optimal objects. Toward this end, it may appear that we can take advantage of the transitivity of object dominance—a cost-saving property often exploited in skyline query algorithms (e.g., [3]) to exclude dominated objects from participating in any future comparison once they are detected. But, we shall prove that object dominance in our case is *not* transitive (Property 1), due to the lack of explicit attribute representation. Hence, the aforementioned cost-saving technique is inapplicable.

Aiming at Pareto-optimal object finding by a short sequence of questions, we introduce a general, iterative algorithm framework (Sec.III). Each iteration goes through four steps—*question selection*, *outcome derivation*, *contradiction resolution*, and *termination test*. In the i -th iteration, a question $q_i=x?_cy$ is selected and its outcome is determined based on crowdsourcers’ answers. On unusual occasions, if the outcome presents a contradiction to the obtained outcomes of other questions, it is changed to the closest outcome such that the contradiction is resolved. Based on the transitive closure of the outcomes to the questions so far, the objects O are partitioned into three sets— O_{\checkmark} (objects that must be Pareto-optimal), O_{\times} (objects that must be non-Pareto optimal), and $O_{?}$ (objects whose Pareto-optimality cannot be fully discerned by the incomplete knowledge so far). When $O_{?}$ becomes empty, O_{\checkmark} contains all Pareto-optimal objects and the algorithm terminates. The

	Task	Question type	Multiple attributes	Order among objects (on each attribute)	Explicit attribute representation
[11]	full ranking	pairwise comparison	no	bucket/total order	no
[12]	top- k ranking	rank subsets of objects	no	bucket/total order	no
[13]	top- k ranking and grouping	pairwise comparison	no	bucket/total order	no
[8]	skyline queries	missing value inquiry	yes	bucket/total order	yes
This work	Pareto-optimal object finding	pairwise comparison	yes	strict partial order	no

TABLE I: Related work comparison.

question sequence so far is thus a *terminal sequence*.

There are a vast number of terminal sequences. Our goal is to find one that is as short as possible. We observe that, for a non-Pareto optimal object, knowing that it is dominated by at least one object is sufficient, and we do not need to find all its dominating objects. It follows that we do not really care about the dominance relation between non-Pareto optimal objects and we can skip their comparisons. Hence, the overriding principle of our question selection strategy is to identify non-Pareto optimal objects as early as possible. Guided by this principle, the framework only chooses from *candidate questions* which must satisfy three conditions (Sec.III-A). This design is sufficient, as we prove that an empty candidate question set implies a terminal sequence, and vice versa (Property 2). The design is also efficient, as we further prove that, if a question sequence contains non-candidate questions, there exists a shorter or equally long sequence with only candidate questions that produces the same O_\times , matching the principle of eagerly finding non-Pareto optimal objects (Theorem 1). Moreover, by the aforementioned principle, the framework selects in every iteration such a candidate question $x?_cy$ that x is more likely to be dominated by y . The selection is steered by two ideas—*macro-ordering* and *micro-ordering*. By using different micro-ordering heuristics, the framework is instantiated into several algorithms with varying power in pruning questions (Sec.IV). We also derive a lower bound on the number of questions required for finding all Pareto-optimal objects (Theorem 2).

In summary, this paper makes the following contributions:

- This is the first work on crowdsourcing Pareto-optimal object finding. Prior studies on crowdsourcing skyline queries [8] assumes explicit attribute representation and uses crowd to obtain missing attribute values. We define preference relations purely based on pairwise comparisons and we aim to find all Pareto-optimal objects by as few comparisons as possible.
- We propose a general, iterative algorithm framework (Sec.III) which follows the strategy of choosing only candidate questions that must satisfy three conditions. We prove important properties that establish the advantage of the strategy (Sec.III-A).
- We design macro-ordering and micro-ordering heuristics for finding a short terminal question sequence. Based on the heuristics, the generic framework is instantiated into several algorithms (RandomQ, RandomP, FRQ) with varying efficiency. We also derive a non-trivial lower bound on the number of required pairwise comparison questions. (Sec.IV)
- We carried out experiments by simulations to compare the amount of comparisons required by different instantiations

of the framework under varying problem sizes. We also investigated two case studies by using human judges and real crowdsourcing marketplace. The results demonstrate the effectiveness of selecting only candidate questions, macro-ordering, and micro-ordering. When these ideas are stacked together, they use orders of magnitude less comparisons than a brute-force approach. The results also reveal that FRQ is nearly optimal and the lower bound is practically tight, since FRQ gets very close to the lower bound. (Sec.V)

II. RELATED WORK

This is the first work on crowdsourcing Pareto-optimal object finding. There are several recent studies on using crowdsourcing to rank objects and answer group-by, top- k and skyline queries. Crowd-BT [11] ranks objects by crowdsourcing pairwise object comparisons. Polychronopoulos et al. [12] find top- k items in an itemset by asking human workers to rank small subsets of items. Davidson et al. [13] evaluate top- k and group-by queries by asking the crowd to answer *type* questions (whether two objects belong to the same group) and *value* questions (ordering two objects). Lofi et al. [8] answer skyline queries over incomplete data by asking the crowd to provide missing attribute values. Table I summarizes the similarities and differences between these studies and our work. The studies on full and top- k ranking [11], [12], [13] do not consider multiple attributes in modeling objects. On the contrary, the concepts of skyline [8] and Pareto-optimal objects (this paper) are defined in a space of multiple attributes. [8] assumes explicit attribute representation. Therefore, they resort to the crowd for completing missing values, while other studies including our work request the crowd to compare objects. Our work considers strict partial orders among objects on individual attributes. Differently, other studies assume a bucket/total order [11], [12], [13] or multiple bucket/total orders on individual attributes [8].

Besides [11], there were multiple studies on ranking objects by pairwise comparisons, which date back to decades ago as aggregating the preferences of multiple agents has always been a fundamental problem in social choice and welfare [14]. The more recent such studies include [15], [16], [17], [18], [19], [20], [21], [22], [23], [24], [25]. Different from our work, none of them is about Pareto-optimal objects, since they all assume a bucket/total order among objects; they either do not consider multiple attributes [15], [16], [17], [21], [23], [24], [25] or require explicit feature representation [18], [19], [20], [22]. Moreover, except [22], [23], [24], they all assume comparison results are already obtained before their algorithms kick in. In contrast, we aim at minimizing the pairwise comparison questions to ask in finding Pareto-optimal objects.

III. GENERAL FRAMEWORK

By the definition of Pareto-optimal objects, the key to finding such objects is to obtain the preference relations, i.e., the strict partial orders on individual criteria. Toward this end, the most basic operation is to perform *pairwise comparison*—given a pair of objects x and y and a criterion c , determine whether one is better than the other (i.e., $(x, y) \in P_c$ or $(y, x) \in P_c$) or they are indifferent (i.e., $(x, y) \notin P_c \wedge (y, x) \notin P_c$).

The problem of crowdsourcing Pareto-optimal object finding is thus essentially crowdsourcing pairwise comparisons. Each comparison task between x and y by criterion c is presented to the crowd as a question q (denoted $x?_cy$). The outcome to the question (denoted $rlt(q)$) is aggregated from the crowd's answers. Given a set of questions, the outcomes thus contain an (incomplete) knowledge of the crowd's preference relations for various criteria. Fig.2 illustrates the screenshot of one such question (comparing two movies by *story*) used in our empirical evaluation. We note that there are other viable designs of question, e.g., only allowing the first two choices ($x \succ_c y$ and $y \succ_c x$). Our work is agnostic to the specific question design.

Given n objects and r criteria, a brute-force approach will perform pairwise comparisons on all object pairs by every criterion, which leads to $r \cdot n \cdot (n-1)/2$ comparisons. The corresponding question outcomes amount to the complete underlying preference relations. The quadratic nature of the brute-force approach renders it wasteful. The bad news is that, in the worst case, we cannot do better than it. To understand this, consider the scenario where all objects are indifferent by every criterion. If any comparison $x?_cy$ is skipped, we cannot determine if x and y are indifferent or if one dominates another.

In practice, though, the outlook is much brighter. Since we look for only Pareto-optimal objects, it is an overkill to obtain the complete preference relations. Specifically, for a Pareto-optimal object, knowing that it is not dominated by any object is sufficient, and we do not need to find all the objects dominated by it; for a non-Pareto optimal object, knowing that it is dominated by at least one object is sufficient, and we do not need to find all its dominating objects. Hence, without exhausting all possible comparisons, the incomplete knowledge on preference relations collected from a set of questions may suffice in fully discerning all Pareto-optimal objects.

Our objective is to find all Pareto-optimal objects with as few questions as possible. By pursuing this goal, we are applying a very simple cost model—the cost of a solution only depends on its number of questions. Although the cost of a task in a crowdsourcing environment may depend on monetary cost, latency and other factors, the number of questions is a generic, platform-independent cost measure and arguably proportionally correlates with the real cost. Therefore, we assume a sequential execution model which asks the crowd an ordered sequence of questions $Q = \langle q_1, \dots, q_n \rangle$ —it only asks q_{i+1} after $rlt(q_i)$ is obtained. Thereby, we do not consider asking multiple questions concurrently. Furthermore, in discussion of our approach, the focus shall be on how to find a short question sequence instead of the algorithms' complexity.

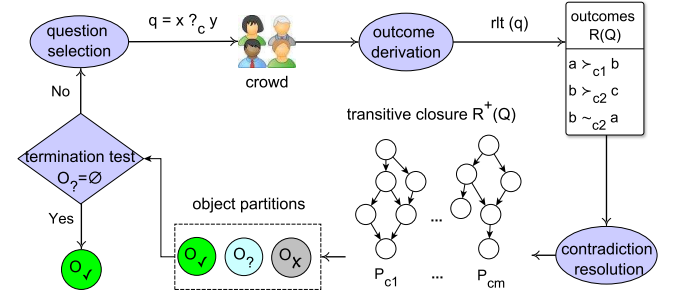


Fig. 3: The general framework.

Algorithm 1: The general framework

Input: O : the set of objects
Output: O_{\checkmark} : Pareto-optimal objects of O

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1  $R(Q) \leftarrow \emptyset$ ; /* question outcomes */
2 repeat
3    $x?_cy \leftarrow$  question selection;
4    $rlt(x?_cy) \leftarrow$  outcome derivation; /* resolve conflict, if any */
5    $R(Q) \leftarrow R(Q) \cup \{rlt(x?_cy)\}$ ;
6    $(O_{\checkmark}, O_{\times}, O_{?}) \leftarrow$  partitioning objects based on  $R^+(Q)$ ;
   /*  $R^+(Q)$  is the transitive closure of  $R(Q)$  */
7 until  $O_{?} = \{\}$ ;
8 return  $O_{\checkmark}$ ;
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To find a short sequence, we design a general algorithm framework, as displayed in Fig.3. Alg.1 shows the framework's pseudo-code. Its execution is iterative. Each iteration goes through four steps—*question selection*, *outcome derivation*, *contradiction resolution*, and *termination test*. In the i -th iteration, a question $q_i = x?_cy$ is selected and presented to the crowd. The question outcome $rlt(q_i)$ is derived from the crowd's aggregated answers. On unusual occasions, if the outcome presents a contradiction to the obtained outcomes of other questions so far, it is changed to the closest outcome to resolve contradiction. By computing $R^+(Q_i)$, the *transitive closure* of $R(Q_i)$ —the obtained outcomes to questions so far $\langle q_1, \dots, q_i \rangle$, the outcomes to certain questions are derived and such questions will never be asked. Based on $R^+(Q_i)$, if every object is determined to be either Pareto-optimal or non-Pareto optimal without uncertainty, the algorithm terminates.

Below, we discuss outcome derivation and termination test. Sec.III-A examines the framework's key step—question selection, and Sec.III-B discusses contradiction resolution.

Outcome derivation Given a question $x?_cy$, its outcome $rlt(x?_cy)$ must be aggregated from multiple crowdsourcers, in order to reach a reliable result with confidence. Particularly, one of three mutually-exclusive outcomes is determined based on k crowdsourcers' answers to the question:

$$rlt(x?_cy) = \begin{cases} x \succ_c y & \text{if } \frac{\#x}{k} \geq \theta \\ y \succ_c x & \text{if } \frac{\#y}{k} \geq \theta \\ x \sim_c y \ (x \not\succ_c y \wedge y \not\succ_c x) & \text{otherwise} \end{cases} \quad (1)$$

where θ is such a predefined threshold that $\theta > 50\%$, $\#x$ is the number of crowdsourcers (out of k) preferring x over y on criterion c , and $\#y$ is the number of crowdsourcers preferring y over x on c . Fig.1b shows the outcomes of all 15 questions according to Equation (1) for comparing movies by *story*

using $k=5$ and $\theta=60\%$. Other conceivable definitions may be used in determining the outcome of $x?_cy$. For example, the outcome may be defined as the choice (out of the three possible choices) that receives the most votes from the crowd. The ensuing discussion is agnostic to the specific definition.

The current framework does not consider different levels of confidence on question outcomes. The confidence on the outcome of a question may be represented as a probability value based on the distribution of crowdsourcers' responses. An interesting direction for future work is to find Pareto-optimal objects in probabilistic sense. The confidence may also reflect the crowdsourcers' quality and credibility [26].

Termination test In each iteration, Alg.1 partitions the objects into three sets by their Pareto-optimality based on the transitive closure of question outcomes so far. If every object's Pareto-optimality has been determined without uncertainty, the algorithm terminates. Details are as follows.

Definition 1 (Transitive Closure of Outcomes). Given a set of questions $Q=\langle q_1, \dots, q_n \rangle$, the transitive closure of their outcomes $R(Q)=\{rlt(q_1), \dots, rlt(q_n)\}$ is $R^+(Q)=\{x \sim_c y \mid x \sim_c y \in R(Q)\} \cup \{x \succ_c y \mid (x \succ_c y \in R(Q)) \vee (\exists w_1, w_2, \dots, w_m : w_1=x, w_m=y \wedge (\forall 0 < i < m : w_i \succ_c w_{i+1} \in R(Q)))\}$. \square

In essence, the transitive closure dictates $x \succ_c z$ without asking the question $x?_cz$, if the existing outcomes $R(Q)$ (and recursively the transitive closure $R^+(Q)$) contains both $x \succ_c y$ and $y \succ_c z$. Based on $R^+(Q)$, the objects O can be partitioned into three sets:

$O_\vee = \{x \in O \mid \forall y \in O : (\exists c \in C : x \succ_c y \in R^+(Q)) \vee (\forall c \in C : x \sim_c y \in R^+(Q))\}$;

$O_\times = \{x \in O \mid \exists y \in O : (\forall c \in C : y \succ_c x \in R^+(Q)) \vee (x \sim_c y \in R^+(Q)) \wedge (\exists c \in C : y \succ_c x \in R^+(Q))\}$;

$O_? = O \setminus (O_\vee \cup O_\times)$.

O_\vee contains objects that must be Pareto-optimal, O_\times contains objects that cannot possibly be Pareto-optimal, and $O_?$ contains objects for which the incomplete knowledge $R^+(Q)$ is insufficient for discerning their Pareto-optimality. The objects in $O_?$ may turn out to be Pareto-optimal after more comparison questions. If the set $O_?$ for a question sequence Q is empty, O_\vee contains all Pareto-optimal objects and the algorithm terminates. We call such a Q a *terminal sequence*, defined below.

Definition 2 (Terminal Sequence). A question sequence Q is a terminal sequence if and only if, based on $R^+(Q)$, $O_? = \emptyset$.

A. Question Selection

Given objects O and criteria C , there can be a huge number of terminal sequences. Our goal is to find a sequence as short as possible. As Fig.3 and Alg.1 show, the framework is an iterative procedure of object partitioning based on question outcomes. It can also be viewed as the process of moving objects from $O_?$ to O_\vee and O_\times . Once an object is moved to O_\vee or O_\times , it cannot be moved again. With regard to this process, we make two important observations, as follows.

- *In order to declare an object x not Pareto-optimal, it is sufficient to just know x is dominated by another object.* It immediately follows that we do not really care about the

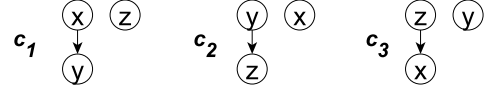


Fig. 4: Intransitivity of object dominance: $x \succ y$, $y \succ z$, $z \succ x$.

dominance relationship between objects in O_\times and thus can skip the comparisons between such objects. Once we know $x \in O_?$ is dominated by another object, it cannot be Pareto-optimal and is immediately moved to O_\times . Quickly moving objects into O_\times can allow us skipping many comparisons between objects in O_\times .

- *In order to declare an object x Pareto-optimal, it is necessary to know that no object can dominate x .* This means we may need to compare x with all other objects including non Pareto-optimal objects. As an extreme example, it is possible for x to be dominated by only a non-Pareto optimal object y but not by any other object (not even the objects dominating y). This is because object dominance based on preference relations is intransitive, which is formally stated in Property 1.

Property 1 (Intransitivity of Object Dominance). Object dominance based on the preference relations over a set of criteria is not transitive. Specifically, if $x \succ y$ and $y \succ z$, it is not necessarily true that $x \succ z$. In other words, it is possible that $x \sim z$ or even $z \succ x$. \square

We show the intransitivity of object dominance by an example. Consider objects $O=\{x, y, z\}$, criteria $C=\{c_1, c_2, c_3\}$, and the preference relations in Fig.4. Three dominance relationships violate transitivity: (i) $x \succ y$ (based on $x \succ_{c_1} y$, $x \sim_{c_2} y$, $x \sim_{c_3} y$), (ii) $y \succ z$ (based on $y \sim_{c_1} z$, $y \succ_{c_2} z$, $y \sim_{c_3} z$), and (iii) $z \succ x$ (based on $z \sim_{c_1} x$, $z \sim_{c_2} x$, $z \succ_{c_3} x$). As another example, in Fig.1a, $b \succ c$ (since $b \sim_s c$, $b \sim_m c$, $b \succ_a c$, where $s=story$, $m=music$, $a=acting$) and $c \succ a$ (since $c \succ_s a$, $c \sim_m a$, $c \sim_a a$), but $a \sim b$ (since $b \succ_s a$, $a \succ_m b$, $b \succ_a a$) where transitivity does not hold.

Differently, transitivity of object dominance holds in skyline analysis [3]. The contradiction is due to the lack of explicit attribute representation—in our case two objects are considered equally good on a criterion if they are indifferent, while in skyline analysis they are equally good regarding an attribute if they bear identical values. Skyline query algorithms exploit the transitivity of object dominance to reduce execution cost, because an object can be immediately excluded from further comparison once it is found dominated by any other object. However, due to Property 1, we cannot leverage such pruning anymore.

Based on these observations, the overriding principle of our question selection strategy (shown in Alg.2) is to identify non-Pareto optimal objects as early as possible. At every iteration of the framework (Alg.1), we choose to compare x and y by criterion c (i.e., ask question $x?_cy$) where $x?_cy$ belongs to *candidate questions*. Such candidate questions must satisfy three conditions (Definition 3). There can be many candidate questions. In choosing the next question, by the aforementioned principle, we select such $x?_cy$ that x is more likely to be dominated by y . More specifically, we design two ordering heuristics—*macro-ordering* and *micro-ordering*. Given the three object partitions O_\vee , O_\times and $O_?$, the macro-ordering

Algorithm 2: Question selection**Input:** $R^+(Q_i), O_{\checkmark}(Q_i), O_{?}(Q_i), O_{\times}(Q_i)$ **Output:** Q_{can}^1 or Q_{can}^2

```

1  $Q_{can} \leftarrow \{x?_cy \mid rlt(x?_cy) \notin R^+(Q_i) \wedge x \in O_{?}(Q_i) \wedge (\nexists c' \in C : x \succ_{c'} y \in R^+(Q_i))\};$ 
2  $Q_{can}^1 \leftarrow \{x?_cy \mid x?_cy \in Q_{can}, y \notin O_{\times}(Q_i)\};$ 
3  $Q_{can}^2 \leftarrow \{x?_cy \mid x?_cy \in Q_{can}, y \in O_{\times}(Q_i)\};$ 
  /* Macro-ordering: consider  $Q_{can}^1$  before  $Q_{can}^2$ . */
4 if  $Q_{can}^1 \neq \emptyset$  then
5   | return micro-ordering( $Q_{can}^1$ );
6 else if  $Q_{can}^2 \neq \emptyset$  then
7   | return micro-ordering( $Q_{can}^2$ );

```

idea is simply that we choose x from $O_{?}$ (required by one of the conditions on candidate questions) and y from $O_{\checkmark} \cup O_{?}$ (if possible) or O_{\times} (otherwise). The reason is that it is less likely for an object in O_{\times} to dominate x . Micro-ordering further orders all candidate questions satisfying the macro-ordering heuristic. In Sec.IV, we instantiate the framework into a variety of solutions with varying power in pruning questions, by using different micro-ordering heuristics.

Definition 3 (Candidate Question). Given Q , the set of asked questions so far, $x?_cy$ is a *candidate question* if and only if it satisfies the following conditions:

- (i) The outcome of $x?_cy$ is unknown yet, i.e., $rlt(x?_cy) \notin R^+(Q)$;
- (ii) x must belong to $O_{?}$;
- (iii) Based on $R^+(Q)$, the possibility of $y \succ x$ must not be ruled out yet, i.e., $\nexists c' \in C : x \succ_{c'} y \in R^+(Q)$.

We denote the set of candidate questions by Q_{can} . Thus, $Q_{can} = \{x?_cy \mid rlt(x?_cy) \notin R^+(Q) \wedge x \in O_{?} \wedge (\nexists c' \in C : x \succ_{c'} y \in R^+(Q))\}$. \square

If no candidate question exists, the question sequence Q is a terminal sequence. The reverse statement is also true, i.e., upon a terminal sequence, there is no candidate question left. This is formalized in the following property.

Property 2. $Q_{can} = \emptyset$ if and only if $O_{?} = \emptyset$.

Proof: It is straightforward that $O_{?} = \emptyset \Rightarrow Q_{can} = \emptyset$, since an empty $O_{?}$ means no question can satisfy condition (ii). We prove $Q_{can} = \emptyset \Rightarrow O_{?} = \emptyset$ by proving its equivalent contrapositive $O_{?} \neq \emptyset \Rightarrow Q_{can} \neq \emptyset$. Assume $O_{?} \neq \emptyset$, i.e., $O_{?}$ contains at least one object x (condition (ii) satisfied). Since x does not belong to O_{\checkmark} , there exists at least an object y that may turn out to dominate x (condition (iii) satisfied). Since x is not in O_{\times} , we cannot conclude yet that y dominates x . Hence, there must exist a criterion c for which we do not know the outcome of $x?_cy$ yet, i.e., $rlt(x?_cy) \notin R^+(Q)$ (condition (i) satisfied). The question $x?_cy$ would be a candidate question because it satisfies all three conditions. Hence $O_{can} \neq \emptyset$. \blacksquare

Questions violating the three conditions may also lead to terminal sequences. However, choosing only candidate questions matches our objective of quickly identifying non-Pareto optimal objects. Below we justify the conditions.

Condition (i): This is straightforward. If $R(Q)$ or its transitive closure already contains the outcome of $x?_cy$, we do not ask the same question again.

Condition (ii): This condition essentially dictates that at least one of the two objects in comparison is from $O_{?}$. (If only one of them belongs to $O_{?}$, we make it x .) Given a pair x and y , if neither is from $O_{?}$, there are three scenarios—(1) $x \in O_{\checkmark}, y \in O_{\checkmark}$, (2) $x \in O_{\checkmark}, y \in O_{\times}$ or $x \in O_{\times}, y \in O_{\checkmark}$, (3) $x \in O_{\times}, y \in O_{\times}$. Once we know an object is in O_{\checkmark} or O_{\times} , its membership in such a set will never change. Hence, we are not interested in knowing the dominance relationship between objects from O_{\checkmark} and O_{\times} only. In all these three scenarios, comparing x and y is only useful for indirectly determining (by transitive closure) the outcome of comparing other objects. Intuitively speaking, such indirect pruning is not as efficient as direct pruning.

Condition (iii): This condition requires that, when $x?_cy$ is chosen, we cannot rule out the possibility of y dominating x . Otherwise, if y cannot possibly dominate x , the outcome of $x?_cy$ cannot help prune x . Note that, in such a case, comparing x and y by c may help prune y , if y still belongs to $O_{?}$ and x may dominate y . Such possibility is not neglected and is covered by a different representation of the same question— $y?_cx$, i.e., swapping the positions of x and y in checking the three conditions. If it is determined x and y cannot dominate each other, then their further comparison is only useful for indirectly determining the outcome of comparing other objects. Due to the same reason explained for condition (ii), such indirect pruning is less efficient.

The following simple Property 3 helps to determine whether $y \succ x$ is possible: If x is better than y by any criterion, then we can already rule out the possibility of $y \succ x$, without knowing the outcome of their comparison by every criterion. This allows us to skip further comparisons between them. Its correctness is straightforward based on the definition of object dominance.

Property 3 (Non-Dominance Property). At any given moment, suppose the set of asked questions is Q . Consider two objects x and y for which the comparison outcome is not known for every criterion, i.e., $\exists c$ such that $rlt(x?_cy) \notin R^+(Q)$. It can be determined that $y \not\succ x$ if $\exists c \in C$ such that $x \succ_c y \in R^+(Q)$. \square

In justifying the three conditions in defining candidate questions, we intuitively explained that indirect pruning is less efficient—if it is known that x does not belong to $O_{?}$ or y cannot possibly dominate x , we will not ask question $x?_cy$. We now justify this strategy theoretically and precisely. Consider a question sequence $Q = \langle q_1, \dots, q_n \rangle$. We use $O_{\checkmark}(Q), O_{?}(Q), O_{\times}(Q)$ to denote object partitions according to $R^+(Q)$. For any question q_i , the subsequence comprised of its preceding questions is denoted $Q_{i-1} = \langle q_1, \dots, q_{i-1} \rangle$. If q_i was not a candidate question when it was chosen (i.e., after $R(Q_{i-1})$ was obtained), we say it is a *non-candidate*. The following Theorem 1 states that, if a question sequence contains non-candidate questions, we can replace it by a shorter or equally long sequence without non-candidate questions that produces the same set of dominated objects O_{\times} . Recall that the key to our framework is to recognize dominated objects and move them into O_{\times} as early as possible. Hence, the new sequence will likely lead to less cost when the algorithm terminates. Hence, it is a good idea to only select among candidate questions.

Theorem 1. If Q contains non-candidate questions, there exists a question sequence Q' without non-candidate questions such that $|Q'| \leq |Q|$ and $O_\times(Q') = O_\times(Q)$.

Proof: We prove by demonstrating how to transform Q into such a Q' . Given any non-candidate question $q_i = x?_cy$ in Q , we remove it and, when necessary, replace several questions. The decisions and choices are partitioned into the following three mutually exclusive scenarios, which correspond to violations of the three conditions in Definition 3.

Case (i): q_i violates condition (i), i.e., $rlt(q_i) \in R^+(Q_{i-1})$. We simply remove q_i from Q , which does not change O_\times , since the transitive closure already contains the outcome.

Case (ii): q_i conforms to condition (i) but violates condition (ii), i.e., $rlt(q_i) \notin R^+(Q_{i-1})$ and $x \notin O_\times(Q_{i-1})$. The proof for this case is similar to a subset of the proof for the following case (iii). We omit the complete proof, due to space limitations.

Case (iii): q_i conforms to conditions (i) and (ii) but violates condition (iii), i.e., $rlt(q_i) \notin R^+(Q_{i-1})$, $x \in O_\times(Q_{i-1})$, $\exists c' \in C : x \succ_{c'} y$. There are three possible subcases, as follows.

(iii-1): $rlt(q_i) = x \sim_c y$. We simply remove q_i from Q and thus remove $x \sim_c y$ from $R^+(Q)$. (We shall explain one exception, for which we replace q_i instead of removing it.) The removal of q_i does not change object partitioning and thus does not change O_\times , as explained below. The difference between $R^+(Q)$ and $R(Q)$ is due to transitivity. Since $x \sim_c y$ does not participate in transitivity, we only need to consider the direct impact of removing $x \sim_c y$ from $R^+(Q)$. Therefore, (1) with respect to any object z that is not x or y , $x \sim_c y$ does not have any impact on z since it does not involve z . (2) With regard to x , if $x \notin O_\times(Q)$, removing $x \sim_c y$ from $R^+(Q)$ will not move x into $O_\times(Q')$; if $x \in O_\times(Q)$, then Q must contain comparisons between x and $z \neq y$ such that $z \succ x$. ($z \neq y$, because case (iii) violates condition (iii), i.e., $y \succ x$ is impossible.) Removing $x \sim_c y$ from $R^+(Q)$ does not affect the comparisons between z and x and thus does not affect O_\times . (3) For y , if $y \notin O_\times(Q)$, removing $x \sim_c y$ from $R^+(Q)$ will not move y into $O_\times(Q')$; if $y \in O_\times(Q)$, then there are three possible situations—(a) If $y \in O_\times(Q_{i-1})$, removing $x \sim_c y$ will not move y out of Q_\times and thus will not change Q_\times . (b) If $y \in O_\times(Q_{i-1})$ and $x \succ y$ was ruled out before q_i , then Q must contain comparisons between y and $z \neq x$ such that z dominates y . Removing $x \sim_c y$ from $R^+(Q)$ does not affect Q_\times since it does not affect the comparisons between z and y ; (c) If $y \in O_\times(Q_{i-1})$ and $x \succ y$ was not ruled out before q_i , then we replace q_i (instead of removing q_i) by $y?_cx$. Note that $y?_cx$ and $x?_cy$ (i.e., q_i) are the same question but different with regard to satisfying the three conditions for candidate questions. Different from q_i , $y?_cx$ is a candidate question when it is chosen (i.e., with regard to Q_{i-1})— $rlt(y?_cx) \notin R^+(Q_{i-1})$, $y \in O_\times(Q_{i-1})$, $x \succ y$ is not ruled out.

(iii-2): $rlt(q_i) = x \succ_c y$. We remove q_i and replace some questions in Q . Consider $S = O_\times(Q) \setminus O_\times(Q \setminus \{q_i\})$, i.e., the objects that would be in $O_\times(Q)$ but instead are in $O_\times(Q \setminus \{q_i\})$ due to the removal of $x \succ_c y$ from $R^+(Q)$. The question replacements are for maintaining O_\times intact. Fig.5 eases our explanation of this case. Suppose $S_1 = \{y\} \cup \{v \mid y \succ_c v \in R^+(Q)\}$

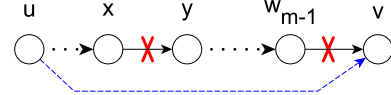


Fig. 5: Question removal and replacement.

and $S_2 = \{x\} \cup \{u \mid u \succ_c x \in R^+(Q)\}$. We can derive that (1) $S \subseteq S_1$. The reason is that the outcome $rlt(q_i) = x \succ_c y$ may have impact on whether other objects dominate v only if $v = y$ or $y \succ_c v$, i.e., $v \in S_1$. For an object v not in S_1 , the removal of q_i cannot possibly move v from $O_\times(Q)$ into $O_\times(Q \setminus \{q_i\})$. (2) $\forall v \in S, \exists u \in S_2$ such that $u \succ v$ and $u \succ_c v$. This is because, if there does not exist such a u , removing q_i cannot possibly move v from $O_\times(Q)$ into $O_\times(Q \setminus \{q_i\})$, which contradicts with $v \in S$.

According to the above results, $\exists w_1, \dots, w_{m-1}, w_m$ such that $w_1 = u, w_m = v, \forall 0 < j < m, w_j \succ_c w_{j+1} \in R(Q)$. In order to make sure v stays in O_\times , we replace the question $w_{m-1}?_cv$ by $v?_cu$. If removing $w_{m-1}?_cv$ moves any object from O_\times into O_\times , we recursively deal with it as we do for q_i . Note that $v?_cu$ is a candidate question when it succeeds Q_{i-1} , since $rlt(v?_cu) \notin R^+(Q_{i-1})$ (otherwise v does not belong to S), $v \in O_\times(Q_{i-1})$ (again, since $v \in S$), and $u \succ v$ cannot be ruled out.

(iii-3): $rlt(q_i) = y \succ_c x$. This case is symmetric to (iii-2) and so is the proof. We thus omit the details. ■

B. Resolving Unusual Contradictions in Question Outcomes

A preference relation can be more accurately derived, if more input is collected from the crowd. However, under practical constraints on budget and time, the limited responses from the crowd (k answers per question) may present two types of contradicting preferences.

(i) Suppose $rlt(x?_cy) = x \succ_c y$ and $rlt(y?_cz) = y \succ_c z$ have been derived, i.e., they belong to $R(Q)$. They together imply $x \succ_c z$, since a preference relation must be transitive. Therefore the question $x?_cz$ will not be asked. If the crowd is nevertheless asked to further compare x and z , the result $rlt(x?_cz)$ might be possibly $z \succ_c x$, which presents a contradiction.

(ii) Suppose $rlt(x?_cy) = x \sim_c y$ and $rlt(y?_cz) = y \succ_c z$ have been derived from the crowd. If the crowd is asked to further compare x and z , the result $rlt(x?_cz)$ might be possibly $z \succ_c x$. The outcomes $y \succ_c z$ and $z \succ_c x$ together imply $y \succ_c x$, which contradicts with $x \sim_c y$. (A symmetric case is $rlt(x?_cy) = x \sim_c y, rlt(y?_cz) = z \succ_c y$, and the crowd might respond with $rlt(x?_cz) = x \succ_c z$, which also leads to contradiction with $x \sim_c y$. The following discussion applies to this symmetric case, which is thus not mentioned again.)

In practice, such contradictions are rare, even under just modest number of answers per question (k) and threshold (θ). This is easy to understand intuitively—as long as the underlying preference relation is transitive, the collective wisdom of the crowds will reflect it. We can find indirect evidence of it in [27], [28], which confirmed that preference judgments of relevance in document retrieval are transitive. Our empirical results also directly verified it. We asked Amazon Mechanical Turk workers to compare 10 photos by *color*, *sharpness* and *landscape*, and we asked students at our institution to compare 10 U.S. cities with regard to *weather*, *living expenses*, and

job opportunities. In both experiments, we asked all possible questions—comparing every pair of objects by every criterion. For each criterion, we considered the graph representing the outcomes of questions, where a directed edge represents “better-than” and an undirected edge represents “indifferent”. If there is such a “cycle” that it contains zero or one undirected edge and all its directed edges are in the same direction, the outcomes in the cycle form a contradiction. We adapted depth-first search to detect all elementary cycles. (A cycle is elementary if no vertices in the cycle (except the start/end vertex) appear more than once.) The number of elementary cycles amounts to only 2.9% and 2.2% of the number of question outcomes in the two experiments. These values would be smaller if we had used larger k and θ .

Nevertheless, contradictions still occur. Type (i) contradictions can be prevented by enforcing the following simple Rule 1 to assume transitivity and thus skip certain questions. They will never get into the derived preference relations. In fact, in calculating transitive closure (Definition 1) and defining candidate questions (Sec.III-A), we already apply this rule.

Rule 1 (Contradiction Prevention by Skipping Questions). Given objects x, y, z and a criterion c , if $rlt(x?_cy)=x>_cy$ and $rlt(y?_cz)=y>_cz$, we assume $rlt(x?_cz)=x>_cz$ and thus will not ask the crowd to further compare x and z by criterion c . \square

To resolve type (ii) contradictions, we enforce the following simple Rule 2.

Rule 2 (Contradiction Resolution by Choosing Outcomes). Consider objects x, y, z and a criterion c . Suppose $rlt(x?_cy)=x\sim cy$ and $rlt(y?_cz)=y>_cz$ are obtained from the crowd. If $rlt(x?_cz)=z>_cx$ is obtained from the crowd afterwards, we replace the outcome of this question by $x\sim cz$. (Note that we do not replace it by $x>_cz$, since $z>_cx$ is closer to $x\sim cz$.) \square

IV. MICRO-ORDERING IN QUESTION SELECTION

At every iteration of Alg.1, we choose a question $x?_cy$ from the set of candidate questions. By macro-ordering, when available, we choose a candidate question in which $y \notin O_\times$, i.e., we choose from Q_{can}^1 . Otherwise, we choose from Q_{can}^2 . The size of Q_{can}^1 and Q_{can}^2 can be large. Micro-ordering is for choosing from the many candidates. As discussed in Sec.III, in order to find a short question sequence, the overriding principle of our question selection strategy is to identify non-Pareto optimal objects as early as possible. Guided by this principle, we discuss several micro-ordering strategies in this section. Since the strategies are the same for Q_{can}^1 and Q_{can}^2 , we will simply use the term “candidate questions”, without distinction between Q_{can}^1 and Q_{can}^2 .

A. Random Question (RandomQ)

RandomQ, as its name suggests, simply selects a random candidate question. Table II shows an execution of the general framework under RandomQ for Example 1. For each iteration i , the table shows the question outcome $rlt(q_i)$. Following the question form $x?_cy$ in Definition 3, the object “ x ” in a question is underlined when we present the question outcome. The column “derived results” displays derived question outcomes

i	$rlt(q_i)$	Derived Results	O_\checkmark	$O_?$	O_\times
1-9	<u>b</u> > _m e, <u>c</u> ~ _a d, <u>a</u> ~ _m c <u>c</u> > _s e, <u>b</u> ~ _s d, <u>b</u> > _a a <u>d</u> > _m e, <u>b</u> ~ _m d, <u>b</u> > _s f		\emptyset	{a,b,c,d,e,f}	\emptyset
10	<u>a</u> > _m d	a> _m e			
11	<u>c</u> > _a a				
12	<u>b</u> ~ _s c				
13	<u>c</u> > _m d	c> _m e			
14-19	<u>d</u> > _s e, <u>e</u> ~ _a c, <u>d</u> ~ _a f <u>a</u> ~ _a d, <u>f</u> > _a a, <u>b</u> > _a e				
20	<u>b</u> > _a d	b>d	\emptyset	{a,b,c,e,f}	{d}
21-23	<u>c</u> > _s f, <u>a</u> > _s e, <u>f</u> ~ _m b				
24	<u>a</u> > _s f	a~f			
25	<u>e</u> > _a f	b> _a f, b> _a a e> _a a, b> _a f	\emptyset	{a,b,c,e}	{d,f}
26	<u>b</u> > _a c				
27	<u>a</u> > _m b				
28	<u>b</u> > _s e	b>e	\emptyset	{a,b,c}	{d,e,f}
29	<u>c</u> > _s a	c> _s e, c> _a a	\emptyset	{b,c}	{a,d,e,f}
30	<u>b</u> ~ _m c	b>c	{b}	\emptyset	{a,c,d,e,f}

TABLE II: RandomQ on Example 1.

by transitive closure (e.g., $a>_me$ based on $rlt(q_7)=d>_me$ and $rlt(q_{10})=a>_md$) and derived object dominance (e.g., $b>d$ after q_{20}). The table also shows the object partitions (O_\checkmark , $O_?$ and O_\times) when the execution starts and when the partitions are changed after an iteration. Multiple iterations may be presented together if other columns are the same for them.

As Table II shows, this particular execution under RandomQ requires 30 questions. When the execution terminates, it finds the only Pareto-optimal object b . This simplest micro-ordering strategy (or rather no strategy at all) already avoids many questions in the brute-force approach. The example clearly demonstrates the benefits of choosing candidate questions only and applying macro-strategy.

B. Random Pair (RandomP)

RandomP randomly selects a pair of objects x and y and keeps asking questions to compare them ($x?_cy$ or $y?_cx$) until there is no such candidate question, upon which it randomly picks another pair of objects. This strategy echoes our principle of eagerly identifying non-Pareto optimal objects. In order to declare an object x non-Pareto optimal, we must identify another object y such that y dominates x . If we directly compare x and y , it requires comparing them by every criterion in C in order to make sure $y>x$. By skipping questions according to transitive closure, we do not need to directly compare them by every criterion. However, Property 4 below states that we still need at least $|C|$ questions involving x —some are direct comparisons with y , others are comparisons with other objects which indirectly lead to outcomes of comparisons with y . When there is a candidate question $x?_cy$, it means y may dominate x . In such a case, the fewer criteria remain for comparing them, the more likely y will dominate x . Hence, by keeping comparing the same object pair, RandomP aims at finding more non-Pareto objects by less questions.

Property 4. Given a set of criteria C and an object $x \in O$, at least $|C|$ pairwise comparison questions involving x are required in order to find another object y such that $y>x$.

Proof: By the definition of object dominance, if $y>x$, then $\forall c \in C$, either $y>_cx \in R^+(Q)$ or $x\sim_c y \in R^+(Q)$, and $\exists c \in C$ such

i	$rlt(q_i)$	Derived Results	O_{\checkmark}	$O_{?}$	O_{\times}
1	$c \succ_s f$		\emptyset	$\{a,b,c,d,e,f\}$	\emptyset
2	$f \succ_m c$	$f \sim c$			
3 – 4	$a \succ_s e, a \succ_m e$				
5	$e \succ_a a$	$a \sim e$			
6 – 7	$c \succ_s e, c \succ_m e$				
8	$e \sim_a c$	$c \succ e$	\emptyset	$\{a,b,c,d,f\}$	$\{e\}$
9	$b \succ_s a$	$b \succ_s e$			
10	$a \succ_m b$	$a \sim b$			
11	$d \succ_s f$				
12	$f \succ_m d$	$f \sim d$			
13	$d \succ_s a$	$d \succ_s e$			
14	$a \succ_m d$	$a \sim d$			
15 – 16	$b \sim_s c, b \sim_m c$				
17	$b \succ_a c$	$b \succ c$	\emptyset	$\{a,b,d,f\}$	$\{c,e\}$
18 – 19	$d \sim_s b, d \sim_m b$				
20	$b \succ_a d$	$b \succ d$	\emptyset	$\{a,b,f\}$	$\{c,d,e\}$
21	$a \succ_s f$	$b \succ_s f$			
22	$a \sim_m f$				
23	$f \succ_a a$	$a \sim f$			
24	$b \sim_m f$				
25	$b \succ_a f$	$b \succ_a a, b \succ f$	$\{b\}$	$\{a\}$	$\{c,d,e,f\}$
26 – 27	$c \succ_s a, a \sim_m c$				
28	$c \succ_a a$	$c \succ a$	$\{b\}$	\emptyset	$\{a,c,d,e,f\}$

TABLE III: RandomP on Example 1.

that $y \succ_c x \in R^+(Q)$. Given any particular c , if $x \sim_c y \in R^+(Q)$, then $x \sim_c y \in R(Q)$, i.e., a question $x?_c y$ or $y?_c x$ belongs to the sequence Q , because indifference of objects on a criterion cannot be derived by transitive closure. If $y \succ_c x \in R^+(Q)$, then $y \succ_c x \in R(Q)$ or $\exists w_1, \dots, w_m \in O$ such that $y \succ_c w_1 \in R(Q), \dots, w_i \succ_c w_{i+1} \in R(Q), \dots, w_m \succ_c x \in R(Q)$. Either way, at least one question involving x on each criterion c is required. Thus, it takes at least $|C|$ questions involving x to determine $y \succ x$. ■

Table III illustrates an execution of RandomP Example 1. The initial two questions are between c and f . Afterwards, it is concluded that $c \sim f$ by Property 3. Therefore, RandomP moves on to ask 3 questions between a and e . In total, the execution requires 28 questions. Although it is shorter than Table II by only 2 questions due to the small size of the example, it clearly moves objects into O_{\times} more quickly. (In Table II, O_{\times} is empty until the 20th question. In Table III, O_{\times} already has 3 objects after 20 questions.) The experiment results in Sec.V exhibit significant performance gain of RandomP over RandomQ on larger data.

C. Pair Having the Fewest Remaining Questions (FRQ)

Similar to RandomP, once a pair of objects x and y are chosen, FRQ keeps asking questions between x and y until there is no such candidate questions. Different from RandomP, instead of randomly picking a pair of objects, FRQ always chooses a pair with the fewest remaining questions. There may be multiple such pairs. To break ties, FRQ chooses such a pair that x has dominated the fewest other objects and y has dominated the most other objects. Furthermore, in comparing x and y , FRQ orders their remaining questions (and thus criteria) by how likely x is worse than y on the criteria. Below we explain this strategy in more detail.

Selecting Object Pair Consider a question sequence Q_i so far and FRQ is to select the next question Q_{i+1} . We use $C_{x,y}$ to denote the set of criteria c such that $x?_c y$ is a candidate question, i.e., $C_{x,y} = \{c \in C \mid x?_c y \in Q_{can}^1\}$. (We assume Q_{can}^1 is not empty. Otherwise, FRQ chooses from Q_{can}^2 in the same way;

cf. Alg.2.) By Definition 3, the outcomes of these questions are unknown, i.e., $\forall c \in C_{x,y} : rlt(x?_c y) \notin R^+(Q_i)$. Furthermore, if any remaining question (whose outcome is unknown) between x and y is a candidate question, then all remaining questions between them are candidate questions. FRQ chooses a pair with the fewest remaining candidate questions, i.e., a pair belonging to $S_1 = \arg \min_{(x,y)} |C_{x,y}|$.

The reason to choose such a pair is intuitive. It requires at least $|C_{x,y}|$ candidate questions to determine $y \succ x$. (The proof would be similar to that of Property 4.) Therefore, $\min_{(x,y)} |C_{x,y}|$ is the minimum number of candidate questions to further ask, in order to determine that an object is dominated, i.e., non-Pareto optimal. Thus, a pair in S_1 may lead to a dominated object by the fewest questions, matching our goal of identifying non-Pareto optimal objects as soon as possible.

We can further justify this strategy in a probabilistic sense. For $y \succ x$ to be realized, it is necessary that none of the remaining questions has an outcome $x \succ_c y$, i.e., $\forall c \in C_{x,y} : rlt(x?_c y) \neq x \succ_c y$. Make the simplistic assumption that every question $x?_c y$ has an equal probability p of not having outcome $x \succ_c y$, i.e., $\forall x?_c y \in Q_{can}^1, P(rlt(x?_c y) \neq x \succ_c y) = p$. Further assuming independence of question outcomes, the probability of satisfying the aforementioned necessary condition is $p^{|C_{x,y}|}$. By taking a pair belonging to S_1 , we have the largest probability of finding a dominated object. We note that, for $y \succ x$ to be realized, in addition to the above necessary condition, another condition must be satisfied—if $\nexists c$ such that $y \succ_c x \in R^+(Q_i)$, the outcome of at least one remaining question should be $y \succ_c x$, i.e., $\exists c \in C_{x,y} : rlt(x?_c y) = y \succ_c x$. Our informal probability-based analysis does not consider this extra requirement.

Breaking Ties There can be multiple object pairs with the fewest remaining questions, i.e., $|S_1| > 1$. To break ties, FRQ chooses such an x that has dominated the fewest other objects, since it is more likely to be dominated. If there are still ties, FRQ further chooses such a y that has dominated the most other objects, since it is more likely to dominate x . More formally, FRQ chooses a pair belonging to $S_2 = \{(x,y) \in S_1 \mid \nexists (x',y') \in S_1 \text{ such that } d(x') > d(x) \vee (d(x') = d(x) \wedge d(y') > d(y))\}$, where the function $d(\cdot)$ returns the number of objects so far dominated by an object, i.e., $\forall x, d(x) = |\{y \mid x \succ y \text{ based on } R^+(Q_i)\}|$. This heuristic follows the principle of detecting non-Pareto optimal objects as early as possible. Note that S_2 may still contain multiple object pairs. In such a case, FRQ chooses an arbitrary pair.

Selecting Comparison Criterion Once a pair (x,y) is chosen, FRQ has to select a criterion for the next question. FRQ orders the remaining criteria $C_{x,y}$ based on the heuristic that the sooner it understands $y \succ x$ will not happen, the lower cost it pays. As discussed before, $|C_{x,y}|$ questions are required in order to conclude that $y \succ x$; on the other hand, only one question (if asked first) can be enough for ruling it out. Consider the case that x is better than y by only one remaining criterion, i.e., $\exists c \in C_{x,y} : rlt(x?_c y) = x \succ_c y$ and $\forall c' \in C_{x,y}, c' \neq c : rlt(x?_{c'} y) = x \not\succ_{c'} y$. If FRQ asks $x?_c y$ after all other remaining questions, it takes $|C_{x,y}|$ questions to

i	$rlt(q_i)$	Derived Results	$(x,y), C_{x,y}$	O_{\checkmark}	$O_{?}$	O_{\times}
			$(a,b), \{s, m, a\}$	\emptyset	$\{a,b,c,d,e,f\}$	\emptyset
1	$b \succ_s a$		$(a,b), \{m, a\}$			
2	$a \succ_m b$	$a \sim b$	$(a,c), \{s, a, m\}$			
3	$c \succ_s a$		$(a,c), \{a, m\}$			
4	$c \sim_a a$		$(a,c), \{m\}$			
5	$c \succ_m a$	$c \succ a$	$(b,c), \{a, s, m\}$	\emptyset	$\{b,c,d,e,f\}$	$\{a\}$
6	$b \sim_a c$		$(b,c), \{s, m\}$			
7	$b \sim_s c$		$(b,c), \{m\}$			
8	$b \succ_m c$	$b \succ_m a, b \succ c$	$(d,b), \{a, s, m\}$	\emptyset	$\{b,d,e,f\}$	$\{a,c\}$
9	$b \sim_a d$		$(d,b), \{s, m\}$			
10	$b \sim_s d$		$(d,b), \{m\}$			
11	$b \succ_m d$	$b \succ d$	$(e,b), \{a, s, m\}$	\emptyset	$\{b,e,f\}$	$\{a,c,d\}$
12	$b \succ_a e$		$(e,b), \{s, m\}$			
13	$b \sim_s e$		$(e,b), \{m\}$			
14	$b \succ_m e$	$a \succ_m e, b \succ e$	$(f,b), \{a, s, m\}$	\emptyset	$\{b,f\}$	$\{a,c,d,e\}$
15	$b \succ_a f$		$(f,b), \{s, m\}$			
16	$b \sim_s f$		$(f,b), \{m\}$			
17	$b \succ_m f$	$b \succ f$		$\{b\}$	\emptyset	$\{a,c,d,e,f\}$

TABLE IV: FRQ on Example 1.

understand y does not dominate x ; but if $x \succ_c y$ is asked first, no more questions are necessary, because there will be no more candidate questions in the form of $x \succ_c y$.

Therefore, FRQ orders the criteria $C_{x,y}$ by a scoring function that reflects the likelihood of x 's superiority than y by the corresponding criteria. More specifically, for each $c \in C_{x,y}$, its score is $r_c(x,y) = r_c(y) + r'_c(y) - r''_c(y) - (r_c(x) + r'_c(x) - r''_c(x))$ where $r_c(y) = |\{z \mid z \succ_c y \in R^+(Q_i)\}|$, $r'_c(y) = |\{z \mid y \sim_c z \in R^+(Q_i)\}|$, and $r''_c(y) = |\{z \mid y \succ_c z \in R^+(Q_i)\}|$. In this scoring function, $r_c(y)$ is the number of objects preferred over y by criterion c , $r'_c(y)$ is the number of objects equally good (or bad) as y by c , and $r''_c(y)$ is the number of objects to which y is preferred with regard to c . FRQ asks the remaining questions in decreasing order of the corresponding criteria's scores. This way, it may find such a question that $rlt(x \succ_c y) = x \succ_c y$ earlier than later.

Table IV presents the framework's execution for Example 1, by applying the FRQ policy. In addition to the same columns in Tables II and III, Table IV also includes an extra column to show, at each iteration, the chosen object pair for the next question (x,y) and the set of remaining comparison criteria between them ($C_{x,y}$). The criteria in $C_{x,y}$ are ordered by the aforementioned ranking function $r(\cdot)$. At the beginning of the execution, the object pair is arbitrarily chosen and the criteria are arbitrarily ordered. In the example, we assume $a \succ_s b$ is chosen as the first question. After q_2 , FRQ can derive that $a \sim b$. Hence, there is no more candidate question between them and FRQ chooses the next pair (a,c) . Three questions are asked for comparing them. At the end of q_5 , multiple object pairs have the fewest remaining questions. By breaking ties, (b,c) is chosen as the next pair, since only c has dominated any object so far. The remaining criteria $C_{b,c}$ are ordered as $\{a, s, m\}$, because $r_a(b,c) > r_s(b,c)$ and $r_a(b,c) > r_m(b,c)$. The execution sequence terminates after 17 questions, much shorter than the 30 and 28 questions by RandomQ and RandomP, respectively.

To conclude the discussion on micro-ordering, we derive a lower bound on the number of questions required for finding all Pareto-optimal objects (Theorem 2). The experiment results in Sec.V reveal that FRQ is nearly optimal and the lower bound is practically tight, since the number of questions used by FRQ is very close to the lower bound.

Theorem 2. Given objects O and criteria C , to find all Pareto-optimal objects in O , at least $(|O|-k) \times |C| + (k-1) \times 2$ pairwise comparison questions are necessary, where k is the number of Pareto-optimal objects in O .

Proof: Suppose the non-Pareto optimal objects are O_1 and the Pareto-optimal objects are O_2 ($O_1 \cup O_2 = O$ and $O_1 \cap O_2 = \emptyset$). We first separately consider n_1 (the minimum number of questions involving objects in O_1) and n_2 (the minimum number of questions comparing objects within O_2 only).

(1) By Property 4 (and its proof), for every non-Pareto optimal object $x \in O_1$, at least $|C|$ questions involving x are required. There exists at least an object y such that $y \succ x$. The required $|C|$ questions lead to outcome either $y \sim_c x$ or $z \succ_c x$ such that $y \succ_c \dots \succ_c z \succ_c x$ (z can be y) for each $c \in C$. For different x , the $|C|$ questions cannot overlap—for a question with outcome $z \succ_c x$, the x is different; for a question with outcome $y \sim_c x$, the same question cannot be part of both the $|C|$ questions for x and the $|C|$ questions for y to detect both as non-Pareto optimal, because it is impossible that $x \succ y$ and $y \succ x$. Hence, $n_1 = (|O|-k) \times |C|$.

(2) Given any Pareto-optimal object $x \in O_2$, for any other $y \in O_2$, either (a) $x \sim_c y$ for all criteria $c \in C$ or (b) there exist at least two criteria c_1 and c_2 such that $x \succ_{c_1} y$ and $y \succ_{c_2} x$. Among the $k-1$ other objects in O_2 , suppose k_a and k_b of them belong to cases (a) and (b), respectively ($k_a + k_b = k-1$). Under case (a), each of the k_a objects requires $|C|$ questions. Under case (b), there must be a question leading to outcome $z \succ_{c_1} y$, where $z = x$ or $x \succ_{c_1} \dots \succ_{c_1} z \succ_{c_1} y$. Similarly, there must be a question with outcome $y \succ_{c_2} z$ such that $z = x$ or $y \succ_{c_2} z \succ_{c_2} \dots \succ_{c_2} x$. Therefore, each of the k_b objects requires at least 2 questions. Clearly, such required questions for comparing x with the $k-1$ other objects in O_2 are all distinct. They are also all different from the questions involving non-Pareto optimal objects (case (1)). Hence, $n_2 = k_a \times |C| + k_b \times 2 \geq (k-1) \times 2$.

Summing up n_1 and n_2 , a lower bound on the number of required questions is thus $(|O|-k) \times |C| + (k-1) \times 2$. Note that, when $k=0$, a trivial, tighter lower bound is $|O| \times |C|$. (One example in which $k=0$ is Fig.4.) ■

V. EXPERIMENTS AND CASE STUDIES

We designed and conducted experiments to compare the efficiency of different instantiations of the general framework under varying problem sizes, by simulations on a large dataset. We also investigated two case studies by using a set of human judges and a real crowdsourcing marketplace.

A. Efficiency and Scalability Test by Simulation

In this experiment we studied the efficiency and scalability of various instantiations of the general framework. Given the large number of questions required for such a study, we cannot afford using a real crowdsourcing marketplace. Hence, we performed the following simulation. Each object is an NBA player in a particular year. The objects are compared by 10 criteria, i.e., performance categories such as *points*, *rebounds*, *assists*, etc. We simulated the corresponding 10 preference relations

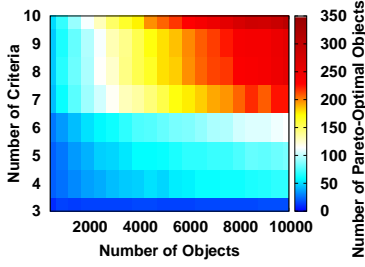
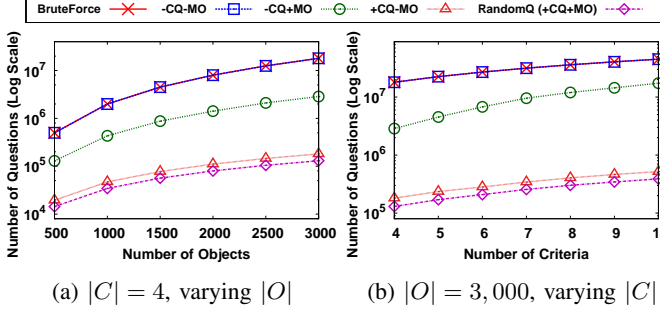


Fig. 6: Number of Pareto-optimal objects by $|O|$ and $|C|$.

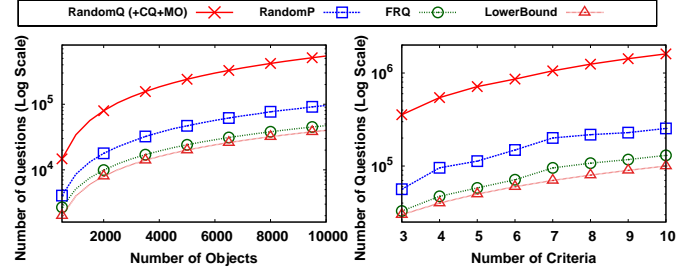


(a) $|C| = 4$, varying $|O|$ (b) $|O| = 3,000$, varying $|C|$

Fig. 7: Numbers of questions by BruteForce and four basic methods.

based on the players' real performance in individual years, as follows. Consider a performance category c and two objects $x=(\text{player1}, \text{year1})$ and $y=(\text{player2}, \text{year2})$. Suppose $x.c > y.c$, where $x.c$ is player1's per-game performance on category c in year1 (similarly for $y.c$). We generated a uniform random number v in $[0, 1]$. If $v < 1 - e^{-(x.c - y.c)}$, we set $x \succ_c y$, otherwise we set $x \sim_c y$. This way, we introduced a perturbation into the preference relations in order to make sure they are partial orders, as opposed to directly using real performance statistics (which would imply bucket orders). Fig.6 shows that the number of Pareto-optimal objects increases by the sizes of both object set O (objects are randomly selected) and criteria set C (the first $|C|$ criteria of the aforementioned 10 criteria).

1) *Effectiveness of candidate questions and macro-ordering:* To verify the effectiveness of candidate questions and macro-ordering, we compared five methods—BruteForce, -CQ-MO, -CQ+MO, +CQ-MO, and +CQ+MO. The notation +/- before CQ and MO indicates whether a method only selects candidate questions (CQ) and whether it applies the macro-ordering strategy (MO), respectively. In all these five methods, qualifying questions are randomly selected, i.e., no particular micro-ordering heuristics are applied. For instance, +CQ+MO selects only candidate questions and applies macro-ordering. Hence, it is equivalent to RandomQ. Fig.7 shows the numbers of required pairwise comparisons (in logarithmic scale) for each method, varying by object set size ($|O|$ from 500 to 3,000 for $|C|=4$) and criterion set size ($|C|$ from 4 to 10 for $|O|=3,000$). The figure clearly demonstrates the effectiveness of both CQ and MO, as taking out either feature leads to significantly worse performance than RandomQ. Particularly, the gap between +CQ-MO and -CQ+MO suggests that choosing only candidate questions has more fundamental impact than macro-ordering. If neither is applied (i.e., -CQ-MO), the performance is equally poor as that of BruteForce. (-CQ-MO uses slightly less questions than



(a) $|C| = 4$, varying $|O|$ (b) $|O| = 10,000$, varying $|C|$

Fig. 8: Numbers of questions by different micro-ordering heuristics.

Between each pair of cities, which has the better weather overall?

☐ Dallas(TX)
 ☐ no preference
 ☐ Chicago(IL)
 ☐ not sure

Fig. 9: A question in Case Study 1.

BruteForce, since it can terminate before exhausting all questions. However, the difference is negligible for practical purpose, as their curves overlap under logarithmic scale.)

2) *Effectiveness of micro-ordering:* Fig.8 presents the numbers of pairwise comparisons required by different micro-ordering heuristics (RandomQ, i.e., +CQ+MO, RandomP, FRQ) and LowerBound (cf. Theorem 2) under varying sizes of the object set ($|O|$ from 500 to 10,000 for $|C|=4$) and the criteria set ($|C|$ from 4 to 10 for $|O|=10,000$). In all these instantiations of the general framework, CQ and MO are applied. The results are averaged across 30 executions. All these methods outperformed BruteForce by orders of magnitude. (BruteForce is not shown in Fig.8 since it is off scale, but its number can be calculated by equation $|C| \times |O| \times (|O| - 1) / 2$.) For instance, for 5,000 objects and 4 criteria, the ratio of pairwise comparisons required by even the naive RandomQ to that used by BruteForce is already as low as 0.0048. This clearly shows the effectiveness of CQ and MO, as discussed for Fig.7. The ratios for RandomP and FRQ are further several times smaller (0.00094 and 0.00048, respectively). The big gain by FRQ justifies the strategy of choosing object pairs with the fewest remaining questions. Especially, FRQ has nearly optimal performance, because it gets very close to LowerBound in Fig.8. The figure further suggests excellent scalability of FRQ as its number of questions grows linearly by both $|C|$ and $|O|$.

B. Case Studies

1) *Case Study 1: collecting opinions from a small group of people (cf. Example 1):* We asked a small number of students at our institution to compare 10 U.S. cities by *overall weather*, *job opportunities*, and *living expenses*. The 135 possible pairwise comparison questions were partitioned into 15 question forms, each of which contains 9 questions on a criterion. Fig.9 shows one such question. We requested responses from 10 people for each form. Some people skipped various questions, allowed by the “not sure” option in Fig.9. Eventually we collected at least 6 responses to each question. The question outcomes were derived by setting $\theta=60\%$ (cf. Equation (1)).

Table V shows the numbers of questions taken by various

	LowerBound	FRQ	RandomP	RandomQ	BruteForce
Case Study 1	24	48	76	90	135
Case Study 2	25	40	52	77	135

TABLE V: Numbers of questions by various methods in case studies.

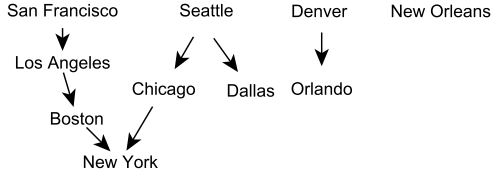


Fig. 10: Hasse diagram for the 10 U.S. cities in Case Study 1.

methods in this case study, averaged over 30 executions. While the performance gap between different methods is far less than in Sec.V-A, we note that it is due to the very small object set. In fact, based on the question outcomes, there are 4 Pareto-optimal cities out of the 10 cities, which makes it less likely for cities to dominate each other and thus requires more questions. Fig.10 shows the Hasse diagram for representing the dominance relation between the cities.

2) *Case Study 2: information exploration using a crowdsourcing marketplace (cf. Example 2)*: This study was conducted in similar manner as Case Study 1. We used a real crowdsourcing marketplace—Amazon Mechanical Turk—to compare 10 photos of our institution with regard to *color*, *sharpness* and *landscape*. The 135 possible questions were also partitioned into 15 tasks, each containing 9 questions on a criterion. We did not provide the “skip” option, because the questions require no special knowledge. We did basic quality control by including in each task two validation questions that expect certain answers. For instance, one such question asks the crowd to compare a colorful photo and a dull photo by criterion *color*. A crowdsourcer’s input is discarded if their response to a validation question deviates from our expectation. (5 crowdsourcers failed on this.) The parameters in Equation (1) were set to be $k=10$ and $\theta=0.6$. (Thus each of the 15 tasks was taken by 10 crowdsourcers that passed the validation.) As Table V shows, the numbers of questions by various methods are similar to those in Case Study 1. Based on the question outcomes, 3 of the 10 photos are Pareto-optimal. Fig.11 shows the Hasse diagram for the photos.

VI. CONCLUSIONS

This paper is the first study on how to use crowdsourcing to find Pareto-optimal objects when objects do not have explicit attributes and preference relations are strict partial orders. The partial orders are obtained by pairwise comparison questions to the crowd. It introduces an iterative question-selection framework that is instantiated into different methods by exploiting the ideas of candidate questions, macro-ordering and micro-ordering. Experiment were conducted by simulations on large object sets and case studies were carried out using both human judges and a real crowdsourcing marketplace. The results exhibited not only orders of magnitude reductions in questions when compared with a brute-force approach, but also close-to-optimal performance from the most efficient method.

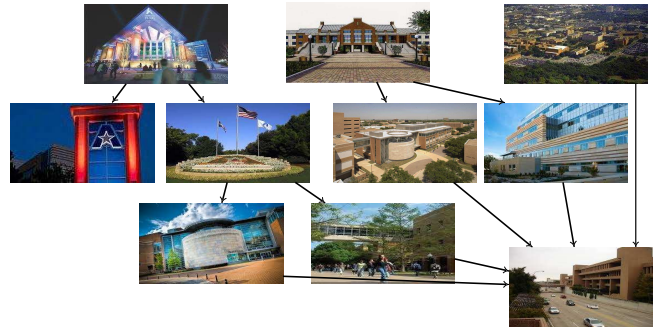


Fig. 11: Hasse diagram for the 10 photos in Case Study 2.

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