# Querying Knowledge Graphs by Example Tuples

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***Abstract*—We witness an unprecedented proliferation of knowledge graphs that record entities and their relationships. Such data is difficult to use. The challenges lie in the gap between large and complex graphs and non-expert users. If writing structured queries over "simple" tables is difficult, complex graphs are only harder to query. As an initial step toward better usability of query systems over knowledge graphs, we propose to query such data by example entity tuples, without requiring users to form complex graph queries. To the best of our knowledge, there was no such proposal in the past. Our system, GQBE, tackles the challenges in supporting this query approach. It derives a weighted hidden maximal query graph based on input query tuples, to capture a user's query intent. It efficiently finds the top approximate answer tuples that are ranked by how well they match the query tuple. Its top-*k* query algorithm only partially evaluates the query graphs for obtaining top-*k* answers. We conducted experiments and user studies on the large Freebase and DBpedia datasets to evaluate GQBE's accuracy and efficiency.**

## I. Introduction

We witness an unprecedented proliferation of *knowledge graphs* that record entities (e.g., persons, products, organizations) and their relationships. Fig.1 is an excerpt of a knowledge graph, in which the edge labeled *founded* between nodes Jerry Yang and Yahoo! captures the fact that the person is a founder of the company. Examples of real-world knowledge graphs include DBpedia [3], YAGO [22], Freebase [4] and Probase [28]. Users and developers are tapping into knowledge graphs for numerous applications, including search, information extraction, recommendation and business intelligence.

Both users and application developers are often overwhelmed by the daunting task of understanding and using knowledge graphs. This largely has to do with the sheer size and complexity of such data. As of March 2012, the Linking Open Data community had interlinked over 52 billion RDF triples spanning over several hundred datasets. More specifically, the challenges lie in the gap between complex data and non-expert users. Knowledge graphs are often stored in relational databases, graph databases and triplestores (cf. [18] for a tutorial). In retrieving data from these databases, the norm is often to use structured query languages such as SQL, SPARQL, and those alike. However, writing structured queries requires extensive experiences in query language and data model and good understanding of particular datasets. For this reason, database usability has received considerable attention lately (cf. an overview in [12]). Graph data is not "easier" than relational data in either query language or data model. The fact it is schema-less makes it even more intangible to understand and query. *If querying "simple" tables is difficult, aren't complex graphs harder to query?*

Motivated by the aforementioned usability challenge, we build GQBE (Graph Query by Example), a system that queries knowledge graphs by example entity tuples instead of graph queries. Given a data graph and a query tuple consisting of entities, GQBE finds similar answer tuples. Consider the data graph in Fig.1. For a 2-entity query tuple ⟨Jerry Yang, Yahoo!⟩, the answer tuples can be ⟨Steve Wozniak, Apple Inc.⟩, ⟨Sergey Brin, Google⟩ and ⟨Bill Gates, Microsoft⟩, which are all founder-company pairs. If the query tuple consists of 3 or more entities (e.g., ⟨Jerry Yang, Yahoo!, San Jose⟩), the answers will be similar tuples of the same cardinality (e.g., ⟨Steve Wozniak, Apple Inc., San Jose⟩).

The paradigm of *query-by-example* (QBE) has a long history in relational databases [32], in which the idea is to express queries by filling example tables with constants and shared variables in multiple tables, which correspond to selection and join conditions, respectively. Its simplicity and improved user productivity make QBE an influential database query language. By proposing to query knowledge graphs by example tuples, which was not seen before, our conjecture is that the QBE paradigm will enjoy similar advantages on graph data. The technical challenges and approaches are vastly different, due to the fundamentally different data models. Note that query graphs or patterns are often used in the literature to graphically present queries over graphs. Underlyingly they are formed by using structured query languages or other query mechanisms such as keyword query [21], [30] and interactive query formulation [8], [13], [6], [14]. Therefore they are not what we refer to as query-by-example.

There are several challenges in building GQBE. (1) With regard to *query semantics*, since the input to GQBE is a query tuple instead of an explicit query graph, the system must derive a hidden query graph based on the query tuple, to capture user's query intent. The *query graph discovery* component (Sec.III) of GQBE fulfills this requirement and the derived graph is termed a *maximal query graph* (MQG). The edges in MQG, weighted by several frequency-based and distance-based heuristics, represent important "features" of the query tuple to be matched in answer tuples. More concretely, they capture how entities in the query tuple (i.e., nodes in a data graph) and their neighboring entities are related to each other. Answer graphs matching the MQG are projected to answer tuples, which consist of answer entities corresponding to the query tuple entities. GQBE further supports multiple query tuples as input which collectively better capture the user intent.

(2) With regard to *answer space modeling* (Sec.IV), there can be a large space of approximate answer graphs (tuples), since it is unlikely to find answer graphs exactly matching the MQG. GQBE models the space of answer tuples by a

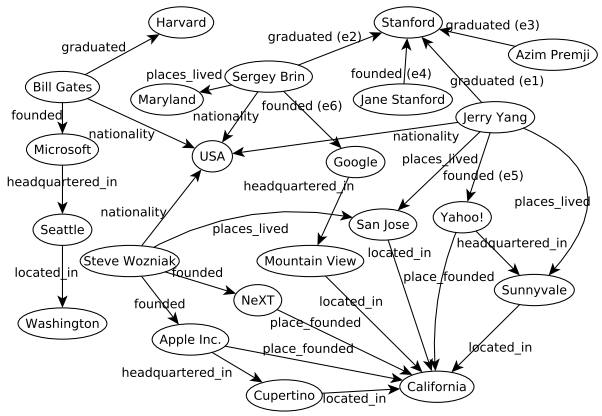



Fig. 1. An Excerpt of a Knowledge Graph

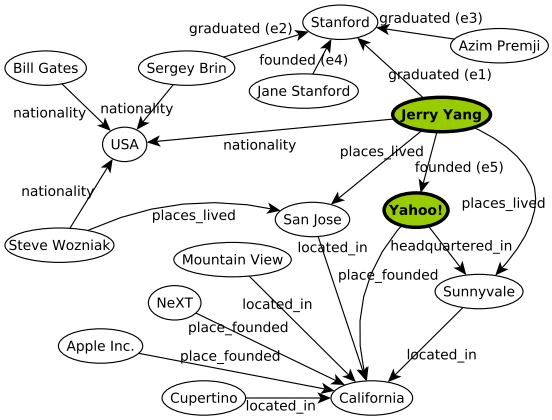



Fig. 2. Neighborhood Graph for $\langle$Jerry Yang, Yahoo!$\rangle$

*query lattice* formed by the subsumption relation between all possible query graphs. Each query graph is a subgraph of the MQG and contains all query entities. Its answer graphs are also subgraphs of the data graph and are isomorphic to the query graph. Given an answer graph, its entities corresponding to the query tuple entities form an answer tuple. Thus the answer tuples are essentially approximate answers to the MQG. For ranking answer tuples, their scores are calculated based on the edge weights in their query graphs and the match between nodes in the query and answer graphs.

(3) The query lattice can be large. To obtain top-*k* ranked answer tuples, the brute-force approach of evaluating all query graphs in the lattice can be prohibitively expensive. For *efficient query processing* (Sec.V), GQBE employs a top-$k$ lattice exploration algorithm that only partially evaluates the lattice nodes in the order of their corresponding query graphs' upper-bound scores.

We summarize the contributions of this paper as follows:

- For better usability of knowledge graph querying systems, we propose the novel approach of querying by example entity tuples, which saves users the burden of forming explicit query graphs. To the best of our knowledge, there was no such proposal in the past.
- The query graph discovery component of GQBE derives a hidden maximal query graph (MQG) based on input query tuples, to capture users' query intent. GQBE models the space of query graphs (and thus answer tuples) by a query lattice based on the MQG.
- GQBE's efficient query processing algorithm only partially evaluates the query lattice to obtain the top-*k* answer tuples ranked by how well they approximately match the MQG.
- We conducted extensive experiments and user study on the large Freebase and DBpedia datasets to evaluate GQBE's accuracy and efficiency (Sec.VI). The comparison with a state-of-the-art graph querying framework NESS [17] shows that GQBE is twice as accurate as NESS and it outperforms NESS on efficiency in most of the queries.

## II. Problem Formulation

GQBE runs queries on knowledge data graphs. A ***data graph*** is a directed multi-graph $G$ with node set $V(G)$ and edge set $E(G)$. Each node $v{\in}V(G)$ represents an entity and has a unique identifier $id(v)$. [1] Each edge $e{=}(v_i, v_j){\in}E(G)$ denotes a directed relationship from entity $v_i$ to entity $v_j$. It has a label, denoted as $label(e)$. Multiple edges can have the same label. The user input and output of GQBE are both entity tuples, called ***query tuples*** and ***answer tuples***, respectively. A tuple $t{=}\langle v_1, \ldots, v_n\rangle$ is an ordered list of entities (i.e., nodes) in $G$. The constituting entities of query (answer) tuples are called *query (answer) entities*. Given a data graph $G$ and a query tuple $t$, our goal is to find the top-*k* answer tuples $t'$ with the highest similarity scores $\mathsf{score}_t(t')$.

We define $\mathsf{score}_t(t')$ by matching the inter-entity relationships of $t$ and that of $t'$, which entails matching two graphs constructed from $t$ and $t'$, respectively. To this end, we define the *neighborhood graph* for a tuple, which is based on the concept of undirected path. An *undirected path* is a path whose edges are not necessarily oriented in the same direction. Unless otherwise stated, we will refer to undirected path simply as "path". We consider undirected path because an edge incident on a node can represent an important relationship with another node, regardless of its direction. More formally, a path $p$ is a sequence of edges $e_1, \ldots, e_n$ and we say each edge $e_i \in p$. The path connects two nodes $v_0$ and $v_n$ through intermediate nodes $v_1, \ldots, v_{n-1}$, where either $e_i{=}(v_{i-1}, v_i)$ or $e_i{=}(v_i, v_{i-1})$, for all $1{\leq}i{\leq}n$. The length of the path, $len(p)$, is $n$ and the endpoints of the path, $ends(p)$, are $\{v_0, v_n\}$. Note that there is no undirected cycle in a path, i.e., the entities $v_0, \ldots, v_n$ are all distinct.

**Definition 1** The ***neighborhood graph*** of query tuple $t$, denoted $H_t$, is the *weakly connected subgraph*[2] of data graph $G$ that consists of all nodes reachable from at least one query entity by an undirected path of $d$ or less edges (including query entities themselves) and the edges on all such paths. The *path length threshold*, $d$, is an input parameter. More formally, the nodes and edges in $H_t$ are defined as follows:

$V(H_t) = \{v | v \in V(G)$ and $\exists p$ s.t. $ends(p){=}\{v_i, v\}$ where $v_i \in t$, $len(p) \leq d\}$;

[1] Without loss of generality, we use an entity's name as its identifier in presenting examples, assuming entity names are unique. [2] A directed graph is *weakly connected* if there exists an undirected path between every pair of vertices.

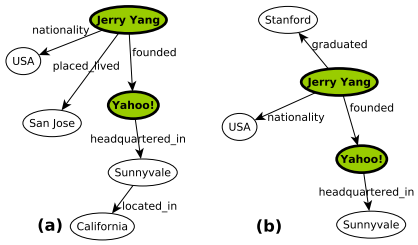



Fig. 3. Two Query Graphs in Fig.2

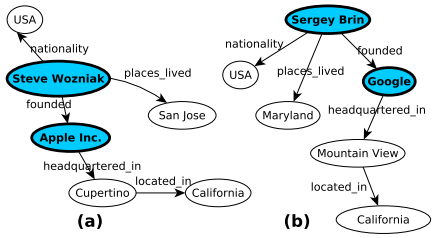



Fig. 4. Two Answer Graphs for Fig.3(a)

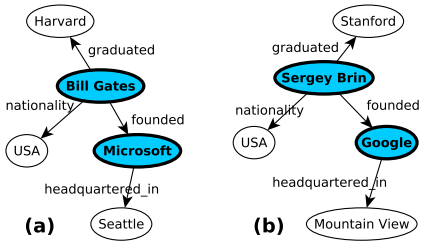



Fig. 5. Two Answer Graphs for Fig.3(b)

$E(H_t) = \{e | e \in E(G)$ and $\exists p$ s.t. $ends(p)=\{v_i, v\}$ where $v_i \in t, len(p) \leq d$, and $e \in p\}$. ■

**Example 1 (Neighborhood Graph)** Given the data graph in Fig.1, Fig.2 shows the neighborhood graph for query tuple ⟨Jerry Yang, Yahoo!⟩ with path length threshold $d$=2. The nodes in dark color are the query entities. ■

Intuitively, the neighborhood graph, by capturing how query entities and other entities in their neighborhood are related to each other, represents "features" of the query tuple that are to be matched in query answers. It can thus be viewed as a hidden query graph derived for capturing user's query intent. We are unlikely to find query answers that exactly match the neighborhood graph. It is however possible to find exact matches to its subgraphs. Such subgraphs are all query graphs and their exact matches are approximate answers that match the neighborhood graph to different extents.

**Definition 2** A ***query graph*** $Q$ is a weakly connected subgraph of $H_t$ that contains all the query entities. We use $\mathcal{Q}_t$ to denote the set of all query graphs for $t$, i.e., $\mathcal{Q}_t$=$\{Q | Q$ is a weakly connected subgraph of $H_t$ s.t. $\forall v \in t$, $v \in V(Q)\}$. ■

Continuing the running example, Fig.3 shows two query graphs for the neighborhood graph in Fig.2.

Echoing the intuition behind neighborhood graph, the definitions of answer graph and answer tuple are based on the idea that an answer tuple is similar to the query tuple if their entities participate in similar relationships in their neighborhoods.

**Definition 3** An ***answer graph*** $A$ to a query graph $Q$ is a weakly connected subgraph of $G$ that is isomorphic to $Q$. Formally, there exists a bijection $f$:$V(Q)\rightarrow V(A)$ such that:

- For every edge $e = (v_i, v_j) \in E(Q)$, there exists an edge $e' = (f(v_i), f(v_j)) \in E(A)$ such that $label(e) = label(e')$;
- For every edge $e' = (u_i, u_j) \in E(A)$, there exists $e = (f^{-1}(u_i), f^{-1}(u_j)) \in E(Q)$ such that $label(e) = label(e')$.

For a query tuple $t$=$\langle v_1, \ldots, v_n \rangle$, the ***answer tuple*** in $A$ is $t_A$=$\langle f(v_1), \ldots, f(v_n) \rangle$. We also call $t_A$ the *projection* of $A$.

We use $\mathcal{A}_Q$ to denote the set of all answer graphs of $Q$. We note that a query graph (tuple) trivially matches itself, therefore is not considered an answer graph (tuple). ■

**Example 2 (Answer Graph and Answer Tuple)** Fig.4 and Fig.5 each show two answer graphs for query graphs Fig.3(a) and Fig.3(b), respectively. The answer tuples in Fig.4 are ⟨Steve Wozniak, Apple Inc.⟩ and ⟨Sergey Brin, Google⟩. The answer tuples in Fig.5 are ⟨Bill Gates, Microsoft⟩ and ⟨Sergey Brin, Google⟩. ■

**Definition 4** The set of answer tuples for query tuple $t$ are $\{t_A | A \in \mathcal{A}_Q, Q \in \mathcal{Q}_t\}$. The ***score of an answer*** $t'$ is given by

$$\mathsf{score}_t(t') = \max_{A \in \mathcal{A}_Q, Q \in \mathcal{Q}_t} \{\mathsf{score}_Q(A) | t' = t_A\} \quad (1)$$

The score of an answer graph $A$ ($\mathsf{score}_Q(A)$) captures $A$'s similarity to query graph $Q$. Its equation is given in Sec.IV-B. ■

The same answer tuple $t'$ may be projected from multiple answer graphs, which can match different query graphs. For instance, Figs. 4(b) and 5(b), which are answers to different query graphs, have the same projection—⟨Sergey Brin, Google⟩. By Eq. (1), the highest score attained by the answer graphs is assigned as the score of $t'$, capturing how well $t'$ matches $t$.

## III. Query Graph Discovery

### A. Maximal Query Graph

The concept of neighborhood graph $H_t$ (Def.1) was formed to capture the features of a query tuple $t$ to be matched by answer tuples. Given a well-connected large data graph, $H_t$ itself can be quite large, even under a small path length threshold $d$. For example, using Freebase as the data graph, the query tuple ⟨Jerry Yang, Yahoo!⟩ produces a neighborhood graph with 800K nodes and 900K edges, for $d$=2. Such a large $H_t$ makes query semantics obscure, because there might be only few nodes and edges in it that capture important relationships in the neighborhood of $t$.

GQBE's query graph discovery component constructs a weighted *maximal query graph* (MQG) from the neighborhood graph $H_t$. MQG is expected to be drastically smaller than $H_t$ and capture only important features of the query tuple. We now define MQG and discuss its discovery algorithm.

**Definition 5** The ***maximal query graph*** $MQG_t$, given a parameter $m$, is a weakly connected subgraph of the neighborhood graph $H_t$ that maximizes total edge weight $\sum_e \mathsf{w}(e)$ while satisfying (1) it contains all query entities in $t$ and (2) it has $m$ edges. The weight of an edge $e$ in $H_t$, $\mathsf{w}(e)$, is defined in Sec.III-B.

There are two challenges in finding $MQG_t$ by directly going after the above definition. First, a weakly connected subgraph of $H_t$ with exactly $m$ edges may not exist for an arbitrary $m$. A trivial value of $m$ that guarantees the existence of the corresponding $MQG_t$ is $|E(H_t)|$, because $H_t$ itself is weakly connected. This value could be too large, which is exactly why we aim to make $MQG_t$ substantially smaller than $H_t$. Second, even if $MQG_t$ exists for an $m$, finding it requires maximizing the total edge weight, which is a hard problem as given in Theorem 1.

**Theorem 1** The decision version of finding the maximal query graph $MQG_t$ for an $m$ is NP-hard. ■

**Proof 1** We prove the NP-hardness by reduction from the NP-hard constrained Steiner network (CSN) problem [19]. The

**Algorithm 1:** Discovering the Maximal Query Graph

**Input**: neighborhood graph $H_t$, query tuple $t$, an integer $r$
**Output**: maximal query graph $MQG_t$

1 $m \leftarrow \frac{r}{|t|+1}$; $V(MQG_t) \leftarrow \phi$; $E(MQG_t) \leftarrow \phi$; $\mathcal{G} \leftarrow \phi$;
2 **foreach** $v_i \in t$ **do**
3 $G_{v_i} \leftarrow$ use DFS to obtain the subgraph containing vertices (and their incident edges) that connect to other $v_j$ in $t$ only through $v_i$;
4 $\mathcal{G} \leftarrow \mathcal{G} \cup \{G_{v_i}\}$;
5 $G_{core} \leftarrow$ use DFS to obtain the subgraph containing vertices and edges on undirected paths between query entities;
6 $\mathcal{G} \leftarrow \mathcal{G} \cup \{G_{core}\}$;
7 **foreach** $G \in \mathcal{G}$ **do**
8 $step \leftarrow 1$; $s_1 \leftarrow 0$; $s \leftarrow m$;
9 **while** $s > 0$ **do**
10 $M_s \leftarrow$ the weakly connected component found from the top-$s$ edges of $G$ that contains all of $G$'s query entities;
11 **if** $M_s$ *exists* **then**
12 **if** $|E(M_s)| = m$ **then break**;
13 **if** $|E(M_s)| < m$ **then**
14 $s_1 \leftarrow s$;
15 **if** $step = -1$ **then break**;
16 **if** $|E(M_s)| > m$ **then**
17 **if** $s_1 > 0$ **then**
18 $s \leftarrow s_1$; **break**;
19 $s_2 \leftarrow s$; $step \leftarrow -1$;
20 $s \leftarrow s + step$;
21 **if** $s = 0$ **then** $s \leftarrow s_2$;
22 $V(MQG_t) \leftarrow V(MQG_t) \cup V(M_s)$;
23 $E(MQG_t) \leftarrow E(MQG_t) \cup E(M_s)$;

interested reader is referred to our technical report [1] for the proofs of this and other theorems and properties. ■

Based on the theoretical analysis, we present a greedy method (Alg.1) to find a plausible sub-optimal graph of edge cardinality *close* to a given $m$. The value of $m$ is empirically chosen to be much smaller than $|E(H_t)|$. Consider edges of $H_t$ in descending order of weight $\mathsf{w}(e)$. We use $G_s$ to denote the graph formed by the top $s$ edges with the largest weights, which itself may not be weakly connected. We use $M_s$ to denote the weakly connected component of $G_s$ containing all query entities in $t$, if it exists. Our method finds the smallest $s$ such that $|E(M_s)|$=$m$ (Line 12). If such an $M_s$ does not exist, the method chooses $s_1$, the largest $s$ such that $|E(M_s)|$<$m$. If that still does not exist, it chooses $s_2$, the smallest $s$ such that $|E(M_s)|$>$m$, whose existence is guaranteed because $|E(H_t)|$>$m$. For each $s$ value, the method employs a depth-first search (DFS) starting from a query entity in $G_s$, if present, to check the existence of $M_s$ (Line 10).

The $M_s$ found by this method may be unbalanced. Query entities with more neighbors in $H_t$ likely have more prominent representation in the resulting $M_s$. A balanced graph should instead have a fair number of edges associated with each query entity. Therefore, we further propose a divide-and-conquer mechanism to construct a balanced $MQG_t$. The idea is to break $H_t$ into $n$+1 weakly connected subgraphs. One is the *core graph*, which includes all the $n$ query entities in $t$ and all undirected paths between query entities. Other $n$ subgraphs are for the $n$ query entities individually, where the subgraph for entity $v_i$ includes all entities (and their incident edges) that connect to other query entities only through $v_i$. The subgraphs are identified by a DFS starting from each query entity (Lines 2-6 of Alg.1). During the DFS from $v_i$, all edges on the undirected paths reaching any other query entity within distance $d$ belong to the core graph, and other edges belong to $v_i$'s individual subgraph. The method then applies the aforementioned greedy algorithm to find $n$+1 weakly connected components, one for each subgraph, that contain the query entities in corresponding subgraphs. Since the core graph connects all query entities, the $n$+1 components altogether form a weakly connected subgraph of $H_t$, which becomes the final $MQG_t$. For an empirically chosen small $r$ as the target size of $MQG_t$, we set the target size for each individual component to be $\frac{r}{n+1}$, aiming at a balanced $MQG_t$.

**Complexity Analysis** The complexity analysis of Alg.1 can be found in the technical report [1].

### *B. Edge Weighting*

The definition of $MQG_t$ (Def.5) depends on edge weights. There can be various plausible weighting schemes. We propose a weighting function based on several heuristic ideas. The weight of an edge $e$ in $H_t$, $\mathsf{w}(e)$, is proportional to its inverse edge label frequency ($\mathsf{ief}(e)$) and inversely proportional to its participation degree ($\mathsf{p}(e)$), given by

$$\mathsf{w}(e) = \mathsf{ief}(e) \ / \ \mathsf{p}(e) \tag{2}$$

**Inverse Edge Label Frequency** Edge labels that appear frequently in the entire data graph $G$ are often less important. For example, edges labeled *founded* (for a company's founders) can be rare and more important than edges labeled *nationality* (for a person's nationality). We capture this by the *inverse edge label frequency*.

$$\mathsf{ief}(e) = \log\left(|E(G)| \ / \ \#label(e)\right) \tag{3}$$

where $|E(G)|$ is the number of edges in $G$, and $\#label(e)$ is the number of edges in $G$ with the same label as $e$.

**Participation Degree** The *participation degree* $p(e)$ of an edge $e$=$(u, v)$ is the number of edges in $G$ that share the same label and one of $e$'s end nodes. Formally,

$$\mathsf{p}(e) = \mid \{e'\text{=}(u', v') \mid label(e)\text{=}label(e'), u'\text{=}u \vee v'\text{=}v\} \mid \tag{4}$$

While $\mathsf{ief}(e)$ captures the global frequencies of edge labels, $\mathsf{p}(e)$ measures their local frequencies—an edge is less important if there are other edges incident on the same node with the same label. For instance, *employment* might be a relatively rare edge globally but not necessarily locally to a company. Specifically, consider the edges representing the *employment* relationship between a company and its *many* employees and the edges for the *board member* relationship between the company and its *few* board members. The latter edges are more significant.

Note that $\mathsf{ief}(e)$ and $\mathsf{p}(e)$ are precomputed offline, since they are query-independent and only rely on the data graph $G$.

### *C. Preprocessing: Reduced Neighborhood Graph*

The discussion so far focuses on discovering $MQG_t$ from $H_t$. The neighborhood graph $H_t$ may have clearly unimportant edges. As a preprocessing step, GQBE removes such edges from $H_t$ before applying Alg.1. The reduced size of $H_t$ not

only makes the execution of Alg.1 more efficient but also helps prevent clearly unimportant edges from getting into $MQG_t$.

Consider the neighborhood graph $H_t$ in Fig.2, based on the data graph excerpt in Fig.1. Edge $e_1$=(Jerry Yang, Stanford) and $label(e_1)$=*graduated*. Two other edges labeled *graduated*, $e_2$ and $e_3$, are also incident on node Stanford. The neighborhood graph from a complete real-world data graph may contain many such edges for people graduated from Stanford University. Among these edges, $e_1$ represents an important relationship between Stanford and query entity Jerry Yang, while other edges represent relationships between Stanford and other entities, which are deemed unimportant with respect to the query tuple.

We formalize the definition of *unimportant edges* as follows. Given an edge $e$=$(u, v) \in E(H_t)$, $e$ is unimportant if it is unimportant from the perspective of its either end, $u$ or $v$, i.e., if $e \in UE(u)$ or $e \in UE(v)$. Given a node $v \in V(H_t)$, $E(v)$ denotes the edges incident on $v$ in $H_t$. $E(v)$ is partitioned into three disjoint subsets—the important edges $IE(v)$, the unimportant edges $UE(v)$ and the rest—defined as follows:

$IE(v)$=
$\{e \in E(v) \mid \exists v_i \in t, p$ s.t. $e \in p, ends(p)=\{v, v_i\}, len(p) \leq d\}$;

$UE(v)$=
$\{e \in E(v) \mid e \notin IE(v), \exists e' \in IE(v)$ s.t. $label(e)=label(e'),$
$(e=(u, v) \wedge e'=(u', v)) \vee (e=(v, u) \wedge e'=(v, u'))\}$.

An edge $e$ incident on $v$ belongs to $IE(v)$ if there exists a path between $v$ and any query entity in the query tuple $t$, through $e$, with path length at most $d$. For example, edge $e_1$ in Fig.2 belongs to $IE$(Stanford). An edge $e$ belongs to $UE(v)$ if (1) it does not belong to $IE(v)$ (i.e., there exists no such aforementioned path) and (2) there exists $e' \in IE(v)$ such that $e$ and $e'$ have the same label and they are both either incoming into or outgoing from $v$. By this definition, $e_2$ and $e_3$ belong to $UE(v)$ in Fig.2, since $e_1$ belongs to $IE(v)$. In the same neighborhood graph, $e_4$ is in neither $IE(v)$ nor $UE(v)$.

All edges deemed unimportant by the above definition are removed from $H_t$. The resulting graph may not be weakly connected anymore and may have multiple weakly connected components. [3] Theorem 2 states that one of the components—called the *reduced neighborhood graph*, denoted $H'_t$—contains all query entities in $t$. In other words, $H'_t$ is the largest weakly connected subgraph of $H_t$ containing all query entities and no unimportant edges. Alg.1 is applied on $H'_t$ (instead of $H_t$) to produce $MQG_t$. Since the techniques in the ensuing discussion only operate on $MQG_t$, the distinction between $H_t$ and $H'_t$ will not be further noted.

**Theorem 2** Given the neighborhood graph $H_t$ for a query tuple $t$, the reduced neighborhood graph $H'_t$ always exists. ■

### D. Multi-tuple Queries

A single query tuple might not be sufficient for capturing user's query intent. While the experiment results in Sec.VI show that a single-tuple query obtains excellent accuracy in many cases, the results also exhibit that allowing multiple query tuples often helps improve query answer accuracy. This is because important relationships commonly associated with multiple query tuples express the user intent more precisely. For instance, suppose a user has provided two query tuples together—⟨Jerry Yang, Yahoo!⟩ and ⟨Steve Wozniak, Apple Inc.⟩. The query entities in both tuples share common properties such as *places_lived* in San Jose and *headquartered_in* a city in California, as shown in Fig.1. This might indicate that the user is interested in finding people from San Jose who founded technology companies in California.

Given a set of tuples $T$, GQBE aims at finding top-$k$ answer tuples similar to $T$ collectively. To accomplish this, GQBE produces a merged and re-weighted MQG that captures the importance of edges with respect to their presence across multiple MQGs for $T$. The merged MQG is then processed by the same method for single-tuple queries. Due to space limitations, we leave the details to the technical report [1].

## IV. Answer Space Modeling

Given the maximal query graph $MQG_t$ for a tuple $t$, we model the space of possible query graphs by a lattice. We further discuss the scoring of answer graphs by how well they match query graphs.

### A. Query Lattice

**Definition 6** The ***query lattice*** $\mathcal{L}$ is a partially ordered set (poset) $(\mathcal{QG}_t, \prec)$, where $\prec$ represents the subgraph-supergraph subsumption relation and $\mathcal{QG}_t$ is the subset of query graphs (Def.2) that are subgraphs of $MQG_t$, i.e., $\mathcal{QG}_t$=$\{Q | Q \in \mathcal{Q}_t$ and $Q \preceq MQG_t\}$. The top element (root) of the poset is thus $MQG_t$. When represented by a Hasse diagram, the poset is a directed acyclic graph, in which each node corresponds to a distinct query graph in $\mathcal{QG}_t$. Thus we shall use the terms *lattice node* and *query graph* interchangeably. The *children* (*parents*) of a lattice node $Q$ are its subgraphs (supergraphs) with one less (more) edge, as defined below.

$$\text{Children}(Q) = \{Q' | Q' \in \mathcal{QG}_t, Q' \prec Q, |E(Q)| - |E(Q')| = 1\}$$
$$\text{Parents}(Q) = \{Q' | Q' \in \mathcal{QG}_t, Q \prec Q', |E(Q')| - |E(Q)| = 1\}$$

The leaf nodes of $\mathcal{L}$ constitute of the *minimal query trees*, which are those query graphs that cannot be made any simpler and yet still keep all the query entities connected.

**Definition 7** A query graph $Q$ is a ***minimal query tree*** if none of its subgraphs is also a query graph. In other words, removing any edge from $Q$ will disqualify it from being a query graph—the resulting graph either is not weakly connected or does not contain all the query entities. Note that such a $Q$ must be a tree. ■

**Example 3 (Query Lattice and Minimal Query Tree)** Fig.6(a) shows a maximal query graph $MQG_t$, which contains two query entities in shaded circles and five edges $F, G, H, L,$ and $P$. Its corresponding query lattice $\mathcal{L}$ is in Fig.6(b). The root node of $\mathcal{L}$, denoted $FGHLP$, represents $MQG_t$ itself. The two bottom nodes, $F$ and $HL$, are the two minimal query trees. Each lattice node is a distinct subgraph of $MQG_t$. For example, the node $FLP$ represents a query graph with only edges $F, L$ and $P$. Note that there is no

[3] A weakly connected component of a directed graph is a maximal subgraph where an undirected path exists for every pair of vertices.

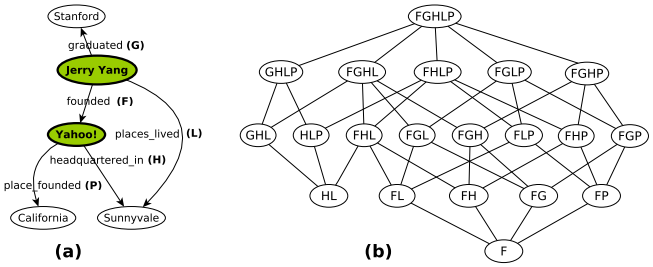



Fig. 6. Maximal Query Graph and Query Lattice

lattice node for $GLP$, which is not a valid query graph since it is not connected. ■

The construction of the query lattice, i.e., the generation of query graphs corresponding to its nodes, is integrated with its exploration. In other words, the lattice is built in a "lazy" manner—a lattice node is not generated until the query algorithm (Sec.V) must evaluate it. The lattice nodes are generated in a bottom-up way. A node is generated by adding exactly one appropriate edge to the query graph for one of its children. The generation of bottom nodes, i.e., the minimal query trees, is described below.

By definition, a minimal query tree can only contain edges on undirected paths between query entities. Hence, it must be a subgraph of the weakly connected component $M_s$ found from the core graph described in Sec.III-A. To generate all minimal query trees, our method enumerates all distinct spanning trees of $M_s$ by the technique in [10] and then trim them. Specifically, given one such spanning tree, all non-query entities (nodes) of degree one along with their edges are deleted. The deletion is performed iteratively until there is no such node. The result is a minimal query tree. Only distinct minimal query trees are kept. Enumerating all spanning trees in a large graph is expensive. However, in our experiments on the Freebase dataset, the $MQG_t$ discovered by the approach in Sec.III mostly contains less than 15 edges. Hence, the $M_s$ from the core graph is also empirically small, for which the cost of enumerating all spanning trees is negligible.

### B. Answer Graph Scoring Function

The score of an answer graph $A$ ($\mathsf{score}_Q(A)$) captures $A$'s similarity to the query graph $Q$. It is defined below and is to be plugged into Eq. (1) for defining answer tuple score.

$$\begin{aligned} \mathsf{score}_Q(A) &= \mathsf{s\_score}(Q) + \mathsf{c\_score}_Q(A) \\ \mathsf{s\_score}(Q) &= \sum_{e \in E(Q)} \mathsf{w}(e) \\ \mathsf{c\_score}_Q(A) &= \sum_{\substack{e=(u,v)\in E(Q) \\ e'=(f(u),f(v))\in E(A)}} \mathsf{match}(e,e') \end{aligned} \tag{5}$$

In Eq. (5), $\mathsf{score}_Q(A)$ sums up two components—the *structure score* of $Q$ ($\mathsf{s\_score}(Q)$) and the *content score* for $A$ matching $Q$ ($\mathsf{c\_score}_Q(A)$). $\mathsf{s\_score}(Q)$ is the total edge weight of $Q$. It measures the important structure in $MQG_t$ that is captured by $Q$ and thus by $A$. $\mathsf{c\_score}_Q(A)$ is the total extra credit for identical nodes among the matching nodes in $A$ and $Q$ given by $f$—the bijection between $V(Q)$ and $V(A)$ as in Def.3. For instance, among the 6 pairs of matching nodes between Fig.3(a) and Fig.4(a), the identical matching nodes are USA, San Jose and California. The rationale for the extra credit is that although node matching is not mandatory, the more nodes are matched, the more similar $A$ and $Q$ are.

The extra credit is defined by the following function $\mathsf{match}(e, e')$. Note that it does not award an identical matching node excessively. Instead, only a fraction of $\mathsf{w}(e)$ is awarded, where the denominator is either $|E(u)|$ or $|E(v)|$. ($E(u)$ are the edges incident on $u$ in $MQG_t$.) This heuristic is based on that, when $u$ and $f(u)$ are identical, many of their neighbors can be also identical matching nodes.

$$\mathsf{match}(e,e') = \begin{cases} \frac{\mathsf{w}(e)}{|E(u)|} & \text{if } u{=}f(u) \\ \frac{\mathsf{w}(e)}{|E(v)|} & \text{if } v{=}f(v) \\ \frac{\mathsf{w}(e)}{min(|E(u)|,|E(v)|)} & \text{if } u{=}f(u), v{=}f(v) \\ 0 & \text{otherwise} \end{cases} \tag{6}$$

In discovering $MQG_t$ from $H_t$ by Alg.1, the weights of edges in $H_t$ are defined by Eq. (2) which does not consider an edge's distance from the query tuple. The rationale behind the design is to obtain a balanced $MQG_t$ which includes not only edges incident on query entities but also those in the larger neighborhood. For scoring answers by Eq. (5) and Eq. (6), however, our empirical observations show it is imperative to differentiate the importance of edges in $MQG_t$ with respect to query entities, in order to capture how well an answer graph matches $MQG_t$. Edges closer to query entities convey more meaningful relationships than those farther away. Hence, we define edge depth ($\mathsf{d}(e)$) as follows. The larger $\mathsf{d}(e)$ is, the less important $e$ is.

**Edge Depth** The depth $\mathsf{d}(e)$ of an edge $e$=$(u, v)$ is its smallest distance to any query entity $v_i \in t$, i.e.,

$$\mathsf{d}(e) = \min_{v_i \in t} \min_{u,v} \{\mathsf{dist}(u, v_i), \mathsf{dist}(v, v_i)\} \tag{7}$$

Here, $\mathsf{dist}(.,.)$ is the shortest length of all undirected paths in $MQG_t$ between the two nodes.

In summary, GQBE uses Eq. (2) as the definition of $\mathsf{w}(e)$ in weighting edges in $H_t$. After $MQG_t$ is discovered from $H_t$ by Alg.1, it uses the following Eq. (8) as the definition of $\mathsf{w}(e)$ in weighting edges in $MQG_t$. Eq. (8) incorporates $\mathsf{d}(e)$ into Eq. (2). The answer graph scoring functions Eq. (5) and Eq. (6) are based on Eq. (8).

$$\mathsf{w}(e) = \mathsf{ief}(e) \;/\; (\mathsf{p}(e) \times \mathsf{d}^2(e)) \tag{8}$$

## V. Query Processing

The query processing component of GQBE takes the maximal query graph $MQG_t$ (Sec.III) and the query lattice $\mathcal{L}$ (Sec.IV) and finds answer graphs matching the query graphs in $\mathcal{L}$. Before we discuss how $\mathcal{L}$ is evaluated (Sec.V-B), we introduce the storage model and query plan for processing one query graph (Sec.V-A).

### A. Processing One Query Graph

The abstract data model of knowledge graph can be represented by the Resource Description Framework (RDF)—the standard Semantic Web data model. In RDF, a data graph

is parsed into a set of triples, each representing an edge $e$=$(u, v)$. A triple has the form (subject, property, object), corresponding to $(u, label(e), v)$. Among different schemes of RDF data management, one important approach is to use relational database techniques to store and query RDF graphs. To store a data graph, we adopt this approach and, particularly, the vertical partitioning method [2]. This method partitions a data graph into multiple two-column tables. Each table is for a distinct edge label and stores all edges bearing that label. The two columns are $(subj, obj)$, for the edges' source and destination nodes, respectively. For efficient query processing, two in-memory search structures (specifically, hash tables) are created on the table, using $subj$ and $obj$ as the hash keys, respectively. The whole data graph is hashed in memory by this way, before any query comes in.

Given the above storage scheme, to evaluate a query graph is to process a multi-way join query. For instance, the query graph in Fig.6(a) corresponds to SELECT F.subj, F.obj FROM F,G,H,L,P WHERE F.subj=G.sbj AND F.obj=H.subj AND F.subj=L.subj AND F.obj=P.subj AND H.obj=L.obj. We use right-deep hash-joins to process such a query. Consider the topmost join operator in a join tree for query graph $Q$. Its left operand is the *build relation* which is one of the two in-memory hash tables for an edge $e$. Its right operand is the *probe relation* which is a hash table for another edge or a join subtree for $Q'$=$Q-e$ (i.e., the resulting graph of removing $e$ from $Q$). For instance, one possible join tree for the aforementioned query is $G\bowtie(F\bowtie(P\bowtie(H\bowtie L)))$. With regard to its topmost join operator, the left operand is $G$'s hash table that uses $G.sbj$ as the hash key, and the right operand is $(F\bowtie(P\bowtie(H\bowtie L)))$. The hash-join operator iterates through tuples from the probe relation, finds matching tuples from the build relation, and joins them to form answer tuples.

### *B. Best-first Exploration of Query Lattice*

Given a query lattice, a brute-force approach is to evaluate all lattice nodes (query graphs) to find all answer tuples. Its exhaustive nature leads to clear inefficiency, since we only seek top-*k* answers. Moreover, the potentially many queries are evaluated separately, without sharing of computation. Suppose query graph $Q$ is evaluated by the aforementioned hash-join between the build relation for $e$ and the probe relation for $Q'$. By definition, $Q'$ is also a query graph in the lattice, if $Q'$ is weakly connected and contains all query entities. In other words, in processing $Q$, we would have processed one of its children query graph $Q'$ in the lattice.

We propose Alg.2, which allows sharing of computation. It explores the query lattice in a *bottom-up* way, starting with the minimal query trees, i.e., the bottom nodes. After a query graph is processed, its answers are materialized in files. To process a query $Q$, at least one of its children $Q'$=$Q-e$ must have been processed. The materialized results for $Q'$ form the probe relation and a hash table on $e$ is the build relation.

While any topological order would work for the bottom-up exploration, Alg.2 employs a *best-first* strategy that always chooses to evaluate the most promising lattice node $Q_{best}$ from a set of candidate nodes. The gist is to process the lattice nodes in the order of their upper-bound scores and $Q_{best}$ is the candidate with the highest upper-bound score (Line 3). If processing $Q_{best}$ does not yield any answer graph, $Q_{best}$ and all its ancestors are pruned (Line 6) and the upper-bound scores of other candidate nodes are recalculated (Line 7). The algorithm terminates, without fully evaluating all lattice nodes, when it has obtained at least *k* answer tuples with scores higher than the highest possible upper-bound score among all unevaluated nodes (Line 10).

For an arbitrary query graph $Q$, its upper-bound score is given by the best possible score $Q$'s answer graphs can attain. Deriving such upper-bound score based on $\mathsf{score}_Q(A)$ in Eq. (5) leads to loose upper-bound. $\mathsf{score}_Q(A)$ sums up the structure score of $Q$ ($\mathsf{s\_score}(Q)$) and the content score for $A$ matching $Q$ ($\mathsf{c\_score}_Q(A)$). While $\mathsf{s\_score}(Q)$ only depends on $Q$ itself, $\mathsf{c\_score}_Q(A)$ captures the matching nodes in $A$ and $Q$. Without evaluating $Q$ to get $A$, we can only assume perfect $\mathsf{match}(e, e')$ in Eq. (5), which is clearly an over-optimism. Under such a loose upper-bound, it can be difficult to achieve an early termination of lattice evaluation.

To alleviate this problem, GQBE takes a two-stage approach. Its query algorithm first finds the top-$k'$ answers ($k'$>$k$) based on the structure score $\mathsf{s\_score}(Q)$ only, i.e., the algorithm uses a simplified answer graph scoring function $\mathsf{score}_Q(A) = \mathsf{s\_score}(Q)$. In the second stage, GQBE re-ranks the top-$k'$ answers by the full scoring function Eq. (5) and returns the top-*k* answer tuples based on the new scores. Our experiments showed the best accuracy for $k$ ranging from 10 to 25 when $k'$ was set to around 100. Lesser values of $k'$ lowered the accuracy and higher values increased the running time of the algorithm. In the ensuing discussion, we will not further distinct $k'$ and $k$.

Below we provide the algorithm details.

### *C. Details of the Best-first Exploration Algorithm*

#### *(1) Selecting* $\mathbf{Q_{best}}$

At any given moment during query lattice evaluation, the lattice nodes belong to three mutually-exclusive sets—the evaluated, the unevaluated and the pruned. A subset of the unevaluated nodes, denoted the *lower-frontier* ($\mathcal{LF}$), are candidates for the node to be evaluated next. At the beginning, $\mathcal{LF}$ contains only the minimal query trees (Line 1 of Alg.2). After a node is evaluated, all its parents are added to $\mathcal{LF}$ (Line 9). Therefore, the nodes in $\mathcal{LF}$ either are minimal query trees or have at least one evaluated child:

$$\mathcal{LF} = \{Q|\ Q \text{ is not pruned}, \mathsf{Children}(Q)=\emptyset \text{ or } (\exists Q' \in \mathsf{Children}(Q) \text{ s.t. } Q' \text{ is evaluated})\}.$$

To choose $Q_{best}$ from $\mathcal{LF}$, the algorithm exploits two important properties, dictated by the query lattice's structure.

**Property 1** If $Q_1 \prec Q_2$, then $\forall A_2 \in \mathcal{A}_{Q_2}$, $\exists A_1 \in \mathcal{A}_{Q_1}$ s.t. $A_1 \prec A_2$ and $t_{A_1}$=$t_{A_2}$. ■

Property 1 says, if an answer tuple $t_{A_2}$ is projected from answer graph $A_2$ to lattice node $Q_2$, then every descendent of $Q_2$ must have at least one answer graph subsumed by $A_2$ that projects to the same answer tuple. Putting it in an informal way, an answer tuple (graph) to a lattice node can always be

**Algorithm 2:** Best-first Exploration of Query Lattice

**Input**: query lattice $\mathcal{L}$, query tuple $t$, and an integer $k$
**Output**: top-$k$ answer tuples

1 lower frontier $\mathcal{LF} \leftarrow$ leaf nodes of $\mathcal{L}$; *Terminate* $\leftarrow$ **false**;
2 **while not** *Terminate* **do**
3  $Q_{best} \leftarrow$ node with the highest upper-bound score in $\mathcal{LF}$;
4  $\mathcal{A}_{Q_{best}} \leftarrow$ evaluate $Q_{best}$; (Sec.V-A)
5  **if** $\mathcal{A}_{Q_{best}}=\emptyset$ **then**
6  prune $Q_{best}$ and all its ancestors from $\mathcal{L}$;
7  recompute upper-bound scores of nodes in $\mathcal{LF}$ (Alg. 3);
8  **else**
9  insert Parents$(Q_{best})$ into $\mathcal{LF}$;
10  **if** top-$k$ answer tuples found [Theorem 4] **then** *Terminate*$\leftarrow$**true** ;

"grown" from its descendant nodes and thus ultimately from the minimal query trees.

**Property 2** If $Q_1 \prec Q_2$, then s_score$(Q_1)$<s_score$(Q_2)$. ■

Property 2 says that, if a lattice node $Q_2$ is an ancestor of $Q_1$, $Q_2$ has a higher structure score. This can be directly proved by referring to the definition of s_score$(Q)$ in Eq. (5).

For each unevaluated candidate node $Q$ in $\mathcal{LF}$, we define an *upper-bound score*, which is the best score $Q$'s answer tuples can possibly attain. The chosen node, $Q_{best}$, must have the highest upper-bound score among all the nodes in $\mathcal{LF}$. By the two properties, if evaluating $Q$ returns an answer graph $A$, $A$ has the potentia to grow into an answer graph $A'$ to an ancestor node $Q'$, i.e., $Q \prec Q'$ and $A \prec A'$. In such a case, $A$ and $A'$ are projected to the same answer tuple $t_A$=$t_{A'}$. The answer tuple always gets the better score from $A'$, under the simplified answer scoring function score$_Q(A)$ = s_score$(Q)$, which Alg.2 adopts as mentioned in Sec. V-B. Hence, $Q$'s upper-bound score depends on its *upper boundary*— $Q$'s unpruned ancestors that have no unpruned parents.

**Definition 8** The ***upper boundary*** of a node $Q$ in $\mathcal{LF}$, denoted $\mathcal{UB}(Q)$, consists of nodes $Q'$ in the ***upper-frontier*** ($\mathcal{UF}$) that subsume or equal to $Q$:

$$\mathcal{UB}(Q) = \{Q' |\ Q' \succeq Q, Q' \in \mathcal{UF}\}, \text{where}$$

$\mathcal{UF}$ is the set of unpruned nodes without unpruned parents: $\mathcal{UF}$=$\{Q|$ $Q$ is not pruned, $\nexists Q' \succ Q$ s.t. $Q'$ is not pruned$\}$. ■

**Definition 9** The ***upper-bound score*** of a node $Q$ is the maximum score of any query graph in its upper boundary:

$$U(Q) = \max_{Q' \in \mathcal{UB}(Q)} \text{s\_score}(Q') \quad (9)$$

***(2) Pruning and Lattice Recomputation***

A lattice node that does not have any answer graph is referred to as a *null node*. If the most promising node $Q_{best}$ turns out to be a null node after evaluation, all its ancestors are also null nodes based on Property 3 below which follows directly from Property 1.

**Property 3 (Upward Closure)** If $\mathcal{A}_{Q_1} = \emptyset$, then $\forall Q_2 \succ Q_1$, $\mathcal{A}_{Q_2} = \emptyset$. ■

Based on Property 3, when $Q_{best}$ is evaluated to be a null node, Alg.2 prunes $Q_{best}$ and its ancestors, which changes the upper-frontier $\mathcal{UF}$. It is worth noting that $Q_{best}$ itself may be an upper-frontier node, in which case only $Q_{best}$ is pruned. In general, due to the evaluation and pruning of nodes, $\mathcal{LF}$ and

**Algorithm 3:** Recomputing Upper-bound Scores

**Input**: query lattice $\mathcal{L}$, null node $Q_{best}$, and lower-frontier $\mathcal{LF}$
**Output**: $U(Q)$ for all $Q$ in $\mathcal{LF}$

1 **foreach** $Q \in \mathcal{LF}$ **do**
2  $\mathcal{NB} \leftarrow \phi$; // set of new upper boundary candidates of $Q$.
3  **foreach** $Q' \in \mathcal{UB}(Q) \cap \mathcal{UB}(Q_{best})$ **do**
4  $\mathcal{UB}(Q) \leftarrow \mathcal{UB}(Q) \setminus \{Q'\}$;
5  $\mathcal{UF} \leftarrow \mathcal{UF} \setminus \{Q'\}$;
6  $V(Q'') \leftarrow V(Q')$;
7  **foreach** $e \in E(Q_{best}) \setminus E(Q)$ **do**
8  $E(Q'') \leftarrow E(Q') \setminus \{e\}$;
9  find $Q_{sub}$, the weakly-connected component of $Q''$, containing all query entities;
10  $\mathcal{NB} \leftarrow \mathcal{NB} \cup \{Q_{sub}\}$;
11  **foreach** $Q_{sub} \in \mathcal{NB}$ **do**
12  **if** $Q_{sub} \not\prec$ *(any node in* $\mathcal{UF}$ *or* $\mathcal{NB}$*)* **then**
13  $\mathcal{UB}(Q) \leftarrow \mathcal{UB}(Q) \cup \{Q_{sub}\}$, $\mathcal{UF} \leftarrow \mathcal{UF} \cup \{Q_{sub}\}$;
14  recompute $U(Q)$ using Eq. (9);

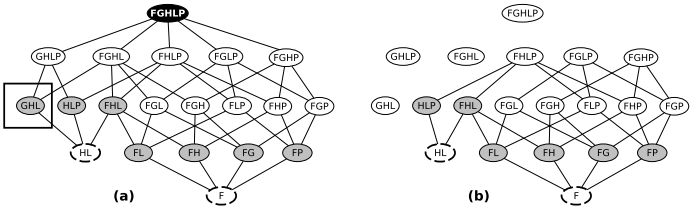



Fig. 7. Recomputing Upper Boundary of Dirty Node $AE$

$\mathcal{UF}$ might overlap. For nodes in $\mathcal{LF}$ that have at least one upper boundary node among the pruned ones, the change of $\mathcal{UF}$ leads to changes in their upper boundaries and, sometimes, their upper-bound scores too. We refer to such nodes as *dirty nodes*. The rest of this section presents an efficient method (Alg. 3) to recompute the upper boundaries, and if changed, the upper-bound scores of the dirty nodes.

Consider all the pairs $\langle Q, Q' \rangle$ such that $Q$ is a dirty node in $\mathcal{LF}$, and $Q'$ is one of its pruned upper boundary nodes. Three properties of a potential new upper boundary node for $Q$ are that it is (1) a supergraph of $Q$, (2) a subgraph of $Q'$ and (3) not a supergraph of $Q_{best}$. If there are $q$ edges in $Q_{best}$ but not in $Q$, we create a set of $q$ distinct graphs $Q''$. Each $Q''$ contains all edges in $Q'$ excepting exactly one of the aforementioned $q$ edges (Line 8 in Alg. 3). For each $Q''$, we find $Q_{sub}$ which is the weakly connected component containing all the query entities (Lines 9-10). Lemma 1 and 2 show that $Q_{sub}$ must be one of the unevaluated nodes after pruning the ancestor nodes of $Q_{best}$ from $\mathcal{L}$.

**Lemma 1** $Q_{sub}$ is a *query graph* and it does not belong to the pruned nodes of lattice $\mathcal{L}$. ■

**Lemma 2** $Q \preceq Q_{sub}$. ■

If $Q_{sub}$ (a candidate new upper boundary node of $Q$) is not subsumed by any node in the upper-froniter or other candidate nodes, we add $Q_{sub}$ to $\mathcal{UB}(Q)$ and $\mathcal{UF}$ (Lines 11-13). Finally, we recompute $Q$'s upper-bound score (Line 14). Theorem 3 justifies the correctness of the above procedure.

**Theorem 3** The aforementioned procedure identifies all new upper boundary nodes for every dirty node $Q$. ■

**Example 4 (Recomputing Upper Boundary)** Consider the

lattice in Fig.7(a) where the bold-dashed nodes belong to the evaluated, the lightly shaded nodes belong to $\mathcal{LF}$ and the darkly shaded node belongs to $\mathcal{UF}$. If node *GHL* is the currently evaluated null node $Q_{best}$ and *FGHLP* is $Q'$, let *FG* be the dirty node $Q$ whose upper boundary is to be recomputed. The edges in $Q_{best}$ that are not present in $Q$ are *H* and *L*. A new upper boundary node $Q''$ contains all edges in $Q'$ excepting exactly either *H* or *L*. This leads to two new upper boundary nodes, *FGHP* and *FGLP*, by removing *L* and *H* from *FGHLP*, respectively. Since *FGHP* and *FGLP* do not subsume each other and are not subgraphs of any other upper-frontier node, they are now part of $\mathcal{UB}(Q)$ and the new $\mathcal{UF}$. Fig.7(b) shows the modified lattice where the pruned nodes are disconnected. *FHLP* is another node in $\mathcal{UF}$ that is discovered using dirty nodes such as *FL* and *HLP*. ■

**Complexity Analysis** The complexity analysis of Alg.3 can be found in the technical report [1].

***(3) Termination***

After $Q_{best}$ is evaluated, its answer tuples are $\{t_A | A \in \mathcal{A}_{Q_{best}}\}$. For a $t_A$ projected from answer graph $A$, the score assigned by $Q_{best}$ to $A$ (and thus $t_A$) is s_score$(Q_{best})$, based on score$_Q(A)$=s_score$(Q)$—the simplified scoring function adopted by Alg.2. If $t_A$ was also projected from already evaluated nodes, it has a current score. By Def.4, the final score of $t_A$ will be from its best answer graph. Hence, if s_score$(Q_{best})$ is higher than its current score, then its score is updated. In this way, all found answer tuples so far are kept and their current scores are maintained to be the highest scores they have received. The algorithm terminates when the current score of the $k^{th}$ best answer tuple so far is greater than the upper-bound score of the next $Q_{best}$ chosen by the algorithm, by Theorem 4.

**Theorem 4** Terminating the lattice evaluation at the aforementioned condition guarantees that the current top-$k$ answer tuples have scores higher than s_score$(Q)$ for any unevaluated query graph $Q$. ■

## VI. Experiments

This section presents our experiment results on the accuracy and efficiency of GQBE. The experiments were conducted on a double quad-core 24 GB Memory 2.0 GHz Xeon server.

**Datasets** The experiments were conducted over two large real-world knowledge graphs, the Freebase [4] and DBpedia [3] datasets. We preprocessed the graphs so that the kept nodes are all named entities (e.g., Stanford University) and abstract concepts (e.g., Jewish people). The resulting Freebase graph contains 28M nodes, 47M edges, and 5,428 distinct edge labels. The DBpedia graph contains 759K nodes, 2.6M edges and 9,110 distinct edge labels.

**Methods Compared** GQBE was compared with a Baseline and NESS [17]. NESS is a graph querying framework that finds approximate matches of query graphs with unlabeled nodes which correspond to query entity nodes in MQG. Note that, like other systems, NESS must take a query graph (instead of a query tuple) as input. Hence, we feed the MQG discovered by GQBE as the query graph to NESS. For each node $v$ in the query graph, a set of candidate nodes in the data graph are identified. Since, NESS does not consider edge-labeled graphs, we adapted it by requiring each candidate node $v'$ of $v$ to have at least one incident edge in the data graph bearing the same label of an edge incident on $v$ in the query graph. The score of a candidate $v'$ is the similarity between the neighborhoods of $v$ and $v'$, represented in the form of vectors, and further refined using an iterative process. Finally, one unlabeled query node is chosen as the pivot $p$. The top-$k$ candidates for multiple unlabeled query nodes are put together to form answer tuples, if they are within the neighborhood of $p$'s top-$k$ candidates. Similar to the best-first method (Sec.V), Baseline explores a query lattice in a bottom-up manner and prunes ancestors of null nodes. However, differently, it evaluates the lattice by breadth-first traversal instead of in the order of upper-bound scores. There is no early-termination by top-*k* scores, as Baseline terminates when every node is either evaluated or pruned. We implemented GQBE and Baseline in Java and we obtained the source code of NESS from the authors.

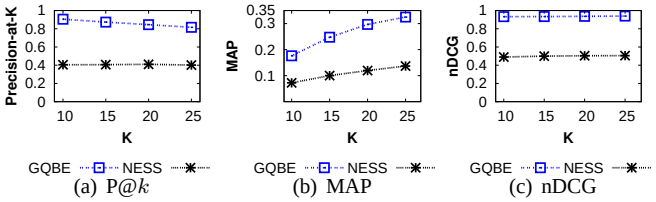
| Query | Query Tuple | Table Size |
|---|---|---|
| $F_1$ | ⟨Donald Knuth, Stanford University, Turing Award⟩ | 18 |
| $F_2$ | ⟨Ford Motor, Lincoln, Lincoln MKS⟩ | 25 |
| $F_3$ | ⟨Nike, Tiger Woods⟩ | 20 |
| $F_4$ | ⟨Michael Phelps, Sportsman of the Year⟩ | 55 |
| $F_5$ | ⟨Gautam Buddha, Buddhism⟩ | 621 |
| $F_6$ | ⟨Manchester United, Malcolm Glazer⟩ | 40 |
| $F_7$ | ⟨Boeing, Boeing C-22⟩ | 89 |
| $F_8$ | ⟨David Beckham, A. C. Milan⟩ | 94 |
| $F_9$ | ⟨Beijing, 2008 Summer Olympics⟩ | 41 |
| $F_{10}$ | ⟨Microsoft, Microsoft Office⟩ | 200 |
| $F_{11}$ | ⟨Jack Kirby, Ironman⟩ | 25 |
| $F_{12}$ | ⟨Apple Inc, Sequoia Capital⟩ | 300 |
| $F_{13}$ | ⟨Beethoven, Symphony No. 5⟩ | 600 |
| $F_{14}$ | ⟨Uranium, Uranium-238⟩ | 26 |
| $F_{15}$ | ⟨Microsoft Office, C++⟩ | 300 |
| $F_{16}$ | ⟨Dennis Ritchie, C⟩ | 163 |
| $F_{17}$ | ⟨Steven Spielberg, Minority Report⟩ | 40 |
| $F_{18}$ | ⟨Jerry Yang, Yahoo!⟩ | 8349 |
| $F_{19}$ | ⟨C⟩ | 1240 |
| $F_{20}$ | ⟨TomKat⟩ | 16 |
| $D_1$ | ⟨Alan Turing, Computer Scientist⟩ | 52 |
| $D_2$ | ⟨David Beckham, Manchester United⟩ | 273 |
| $D_3$ | ⟨Microsoft, Microsoft Excel⟩ | 300 |
| $D_4$ | ⟨Steven Spielberg, Catch Me If You Can⟩ | 37 |
| $D_5$ | ⟨Boeing C-40 Clipper, Boeing⟩ | 118 |
| $D_6$ | ⟨Arnold Palmer, Sportsman of the year⟩ | 251 |
| $D_7$ | ⟨Manchester City FC, Mansour bin Zayed Al Nahyan⟩ | 40 |
| $D_8$ | ⟨Bjarne Stroustrup, C++⟩ | 964 |

TABLE I
Queries and Ground Truth Table Size

**Queries and Ground Truth** Two groups of queries are used on the two datasets, respectively. The Freebase queries $F_1$– $F_{20}$ are based on Freebase tables such as http://www.freebase.com/view/computer/programming_language_designer?instances, except $F_1$ and $F_6$ which are from Wikipedia tables such as http://en.wikipedia.org/wiki/List_of_English_football_club_owners. The DBpedia queries $D_1$– $D_8$ are based on DBpedia tables such as the values for property is dbpedia-owl:author of on page http://dbpedia.org/page/Microsoft. Each such table is a collection of tuples, in which each tuple consists of one, two, or three entities. For each table, we used one or more tuples as query tuples and the remaining tuples as the ground truth for query answers. All the 28 queries and their corresponding table sizes are summarized in Table I. They cover diverse domains, including people, companies, movies, sports,

| Query Tuple | Top-3 Answer Tuples |
|---|---|
| ⟨Donald Knuth, Stanford, Turing Award⟩ | ⟨D. Knuth, Stanford, V. Neumann Medal⟩<br>⟨J. McCarthy, Stanford, Turing Award⟩<br>⟨N. Wirth, Stanford, Turing Award⟩ |
| ⟨Jerry Yang, Yahoo!⟩ | ⟨David Filo, Yahoo!⟩<br>⟨Bill Gates, Microsoft⟩<br>⟨Steve Wozniak, Apple Inc.⟩ |
| ⟨C⟩ | ⟨Java⟩<br>⟨C++⟩<br>⟨C Sharp⟩ |

TABLE II
CASE STUDY: TOP-3 RESULTS FOR SELECTED QUERIES



(a) P@$k$ (b) MAP (c) nDCG

Fig. 8. Accuracy of GQBE and NESS on Freebase Queries

awards, religions, universities and automobiles.

**Sample Answers** Table II only lists the top-3 results found by GQBE for 3 queries ($F_1$, $F_{18}$, $F_{19}$), due to space limitations.

**(A) Accuracy Based on Ground Truth**

We measured the accuracy of GQBE and NESS by comparing their query results with the ground truth. The accuracy on a single query is captured by three widely-used measures—Precision-at-$k$ (P@$k$), Average Precision (AvgP), and Normalized Discounted Cumulative Gain (nDCG). The accuracy on a set of queries is the average of accuracy on individual queries. Particularly the measure based on AvgP for a set of queries is called Mean Average Precision (MAP). The definition of these measures can be found in our technical report [1] and the literature (e.g., [20]).

Fig.8 shows these measures for different values of $k$ on the Freebase queries. GQBE has high accuracy. For instance, its P@25 is over 0.8. The absolute value of MAP is not high, merely because Fig.8(b) only shows the MAP for at most top-25 results, while the ground truth size (i.e., the denominator in calculating MAP) for many queries is much larger. Moreover, GQBE outperforms NESS substantially, as its accuracy in all three measures is almost always twice as better. This is because GQBE gives priority to query entities and important edges in MQG, while NESS gives equal importance to all nodes and edges except the pivot. Furthermore, the way NESS handles edge labels does not explicitly require answer entities to be connected by the same paths between query entities.

Table III further shows the accuracy of GQBE on individual DBpedia queries at $k$=10. It exhibits high accuracy on all queries, including perfect precision in several cases.

**(B) Accuracy Based on User Study**

We conducted an extensive user study through Amazon Mechanical Turk (MTurk, https://www.mturk.com/mturk/) to evaluate GQBE's accuracy on Freebase queries, measured by Pearson Correlation Coefficient (PCC). For each of the 20 queries, we obtained the top-30 answers from GQBE and generated 50 random pairs of these answers. We presented each pair to 20 MTurk workers and asked for their preference between the two answers in the pair. Hence, in total, 20,000 opinions were

| Query | P@$k$ | nDCG | AvgP | Query | P@$k$ | nDCG | AvgP |
|---|---|---|---|---|---|---|---|
| $D_1$ | 1.00 | 1.00 | 0.20 | $D_2$ | 1.00 | 1.00 | 0.04 |
| $D_3$ | 1.00 | 1.00 | 0.03 | $D_4$ | 0.80 | 0.94 | 0.19 |
| $D_5$ | 0.90 | 1.00 | 0.08 | $D_6$ | 1.00 | 1.00 | 0.04 |
| $D_7$ | 0.90 | 0.98 | 0.22 | $D_8$ | 1.00 | 1.00 | 0.01 |

TABLE III
ACCURACY OF GQBE ON DBPEDIA QUERIES, $k$=10

| Query | PCC | Query | PCC | Query | PCC | Query | PCC |
|---|---|---|---|---|---|---|---|
| $F_1$ | 0.79 | $F_2$ | 0.78 | $F_3$ | 0.60 | $F_4$ | 0.80 |
| $F_5$ | 0.34 | $F_6$ | 0.27 | $F_7$ | 0.06 | $F_8$ | 0.26 |
| $F_9$ | 0.33 | $F_{10}$ | 0.77 | $F_{11}$ | 0.58 | $F_{12}$ | undefined |
| $F_{13}$ | undefined | $F_{14}$ | 0.62 | $F_{15}$ | 0.43 | $F_{16}$ | 0.29 |
| $F_{17}$ | 0.64 | $F_{18}$ | 0.30 | $F_{19}$ | 0.40 | $F_{20}$ | 0.65 |

TABLE IV
PEARSON CORRELATION COEFFICIENT (PCC) BETWEEN GQBE AND AMAZON MTURK WORKERS, $k$=30

obtained. We then constructed two value lists per query, $X$ and $Y$, which represent GQBE and MTurk workers' opinions, respectively. Each list has 50 values, for the 50 pairs. For each pair, the value in $X$ is the difference between the two answers' ranks given by GQBE, and the value in $Y$ is the difference between the numbers of workers favoring the two answers. The PCC value for a query is $(\mathrm{E}(XY) - \mathrm{E}(X)\mathrm{E}(Y))/(\sqrt{\mathrm{E}(X^2) - (\mathrm{E}(X))^2}\sqrt{\mathrm{E}(Y^2) - (\mathrm{E}(Y))^2})$. The value indicates the degree of correlation between the pairwise ranking orders produced by GQBE and the pairwise preferences given by MTurk workers. The value range is from $-1$ to 1. A PCC value in the ranges of [0.5,1.0], [0.3,0.5) and [0.1,0.3) indicates a strong, medium and small positive correlation, respectively [7]. PCC is undefined, by definition, when $X$ and/or $Y$ contain all equal values.

Table IV shows the PCC values for $F_1$-$F_{20}$. Out of the 20 queries, GQBE attained strong, medium and small positive correlation with MTurk workers on 9, 5 and 3 queries, respectively. Only query $F_7$ shows no correlation. Note that PCC is undefined for $F_{12}$ and $F_{13}$, because all the top-30 answer tuples have the same score and thus the same rank, resulting in all zero values in $X$, i.e., GQBE's list.

**(C) Accuracy on Multi-tuple Queries**

We investigated the effectiveness of the multi-tuple querying approach (Sec.III-D). In aforementioned single-tuple query experiment (A), GQBE attained perfect P@25 for 13 of the 20 Freebase queries. We thus focused on the remaining 7 queries. For each query, Tuple1 refers to the query tuple in Table I, while Tuple2 and Tuple3 are two tuples from its ground truth. Table V shows the accuracy of top-25 GQBE answers for the three tuples individually, as well as for the first two and three tuples together by merged MQGs, which are denoted Combined(1,2) and Combined(1,2,3), respectively. $F_4$ attained perfect precision after Tuple2 was included. Therefore, Tuple3 was not used for $F_4$. The results show that, in most cases, Combined(1,2) had better accuracy than individual tuples and Combined(1,2,3) further improved accuracy.

**(D) Efficiency Results**

We compared the efficiency of GQBE, NESS and Baseline on Freebase queries. The total run time for a query tuple is spent on two components—query graph discovery and query processing. Fig.9 compares the three methods' query processing time, in logarithmic scale. For each edge cardinality

| Query | Tuple1 | | | Tuple2 | | | Combined (1,2) | | | Tuple3 | | | Combined (1,2,3) | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | P@$k$ | nDCG | AvgP | P@$k$ | nDCG | AvgP | P@$k$ | nDCG | AvgP | P@$k$ | nDCG | AvgP | P@$k$ | nDCG | AvgP |
| $F_1$ | **0.36** | 0.76 | 0.32 | **0.36** | **1.00** | **0.50** | 0.12 | 0.38 | 0.02 | **0.36** | 0.73 | 0.22 | 0.12 | 0.49 | 0.02 |
| $F_2$ | 0.76 | **1.00** | 0.79 | 0.00 | 0.00 | 0.00 | **0.80** | **1.00** | 0.80 | 0.12 | 0.70 | 0.05 | **0.80** | **1.00** | **0.91** |
| $F_4$ | 0.32 | 0.73 | 0.09 | 0.40 | 0.65 | 0.08 | **1.00** | **1.00** | **0.45** | N/A | N/A | N/A | N/A | N/A | N/A |
| $F_6$ | 0.24 | 0.89 | 0.16 | 0.28 | 0.89 | 0.18 | **0.40** | 0.87 | 0.16 | 0.36 | 0.98 | **0.22** | 0.12 | **0.94** | 0.07 |
| $F_8$ | 0.92 | 0.79 | 0.20 | **1.00** | **1.00** | **0.27** | 0.96 | 0.98 | 0.24 | 0.48 | 0.86 | 0.08 | **1.00** | **1.00** | **0.27** |
| $F_9$ | 0.68 | 0.72 | 0.23 | 0.56 | 0.66 | 0.17 | 0.80 | 0.86 | 0.35 | **1.00** | **1.00** | 0.62 | **1.00** | **1.00** | **0.66** |
| $F_{17}$ | 0.32 | **1.00** | 0.33 | 0.64 | 0.83 | 0.25 | 0.32 | **1.00** | 0.32 | 0.56 | 0.84 | 0.23 | **0.68** | **1.00** | **0.46** |

TABLE V
ACCURACY OF GQBE ON MULTI-TUPLE QUERIES, $k$=25

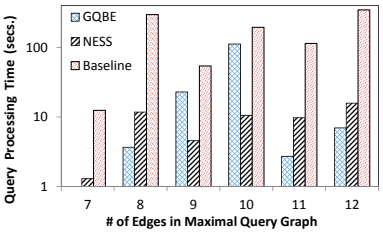


Fig. 9. Query Processing Time

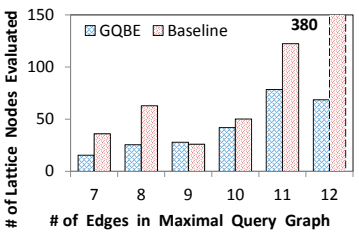


Fig. 10. Num. of Lattice Nodes Evaluated

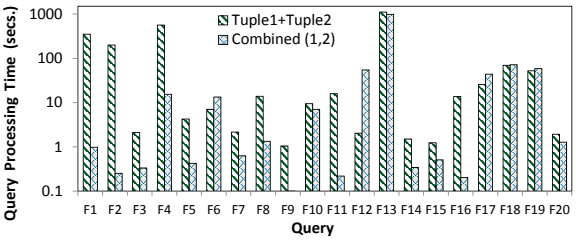


Fig. 11. Query Processing Time of 2-tuple Queries

of MQG, the figure shows the average time on queries with the same edge cardinality. The query cost does not appear to increase by edge cardinality, regardless of the query method. For GQBE and Baseline, this is because query graphs are evaluated by joins and join selectivity plays a more significant role in evaluation cost than number of edges. NESS finds answers by intersecting postings lists on feature vectors. Hence, in evaluation cost, intersection size matters more than edge cardinality. GQBE outperformed NESS on 16 of the 20 queries and was more than 3 times faster in 10 of them. It finished within 10 seconds on 17 queries. However, it performed very poorly on a 9-edge MQG and a 10-edge MQG, taking 51 and 552 seconds, respectively. This indicates that the edges in the two MQGs lead to poor join selectivity. Baseline clearly suffered, due to its inferior pruning power compared to the best-first exploration employed by GQBE. This is evident in Fig.10 which shows the numbers of lattice nodes evaluated, under varying edge cardinality of MQG. GQBE evaluated considerably less nodes in most cases and about 6 times less on queries with 12 edges in MQG. (The value for Baseline is 380, which is off the chart and listed explicitly.)

MQG discovery precedes the query processing step and is shared by all three methods. Column $MQG_1$ in Table VI lists the time spent on discovering MQG for each Freebase query. This time component varies across individual queries, depending on the sizes of query tuples' neighborhood graphs. Compared to the values shown in Fig.9, the time taken to discover an MQG in average is comparable to the time spent by GQBE in evaluating it.

Fig.11 shows GQBE's query processing time, in logarithmic scale, on the merged MQGs of 2-tuple queries in Table V, denoted by Combined(1,2). It also shows the total time for evaluating the two tuples' MQGs individually, denoted Tuple1+Tuple2. The time for Combined(1,2) is 1-3 orders of magnitude less in 8 out of 20 queries and is significantly less in 5 other queries. This suggests that the merged MQGs gave higher weights to more selective edges, resulting in faster lattice evaluation. Meanwhile, these selective edges are also more important edges common to the two query tuples, leading to improved answer accuracy as shown in Table V. Table VI further shows the time taken to discover $MQG_1$ and $MQG_2$ for the two tuples, along with the time for merging them. The latter is negligible compared to the former.

| Query | $MQG_1$ | $MQG_2$ | Merge | Query | $MQG_1$ | $MQG_2$ | Merge |
|---|---|---|---|---|---|---|---|
| $F_1$ | 73.141 | 73.676 | 0.034 | $F_2$ | 0.049 | 0.029 | 0.006 |
| $F_3$ | 12.566 | 4.414 | 0.024 | $F_4$ | 5.731 | 7.083 | 0.024 |
| $F_5$ | 9.982 | 2.522 | 0.079 | $F_6$ | 6.082 | 4.654 | 0.039 |
| $F_7$ | 0.152 | 0.107 | 0.007 | $F_8$ | 10.272 | 2.689 | 0.032 |
| $F_9$ | 62.285 | 2.384 | 0.041 | $F_{10}$ | 2.910 | 5.933 | 0.030 |
| $F_{11}$ | 59.541 | 65.863 | 0.032 | $F_{12}$ | 1.977 | 0.021 | 0.006 |
| $F_{13}$ | 9.481 | 5.624 | 0.034 | $F_{14}$ | 0.038 | 0.015 | 0.004 |
| $F_{15}$ | 0.154 | 5.143 | 0.021 | $F_{16}$ | 54.870 | 6.928 | 0.057 |
| $F_{17}$ | 60.582 | 69.961 | 0.041 | $F_{18}$ | 58.807 | 75.128 | 0.053 |
| $F_{19}$ | 0.224 | 0.076 | 0.003 | $F_{20}$ | 0.025 | 0.017 | 0.002 |

TABLE VI
TIME FOR DISCOVERING AND MERGING MQGS (SECS.)

## VII. RELATED WORK

Our work is the first to query knowledge graphs by example entity tuples. In the literature on graph query, the input to a query system in most cases is a structured query, which is often graphically presented as a query graph or a query pattern. Such is not what we refer to as query-by-example, because underlyingly the query graphs and patterns are formed by using structured query languages or other query mechanisms. In fact, substantial progress has been made on more user-friendly query mechanisms that do not require explicit query graphs or help users construct query graphs. Such mechanisms include keyword search (e.g., [15]), keyword-based query formulation [30], natural language questions [29], interactive and form-based query formulation [8], and visual interface for query graph construction [6], [14].

PathSim [23] finds the top-$k$ similar entities that are connected to a query entity, based on a user-defined meta-path semantics in a heterogeneous network. In [31], given a query graph as input, the system finds structurally isomorphic answer graphs with semantically similar entity nodes. In both works, a user should know the network schema to specify a meta-path or a query graph. In contrast, the query-by-example approach in GQBE only requires a user to provide an entity tuple, without knowing the underlying schema.

The goal of *set expansion* is to grow a set of objects starting from seed objects. Example systems include [27], [11], and the now defunct Google Sets and Squared services (http://en.wikipedia.org/wiki/List_of_Google_products). Chang et al. [5] identify top-$k$ correlated keyword terms from an information network given a set of terms, where each term can be an entity. These systems, except [5], do not operate on data graphs. Instead, they find existing answers within structures in web pages such as HTML tables and lists. Furthermore, all these systems except Google Squared and [11] take a set of individual entities as input. GQBE is more general in that each query tuple contains multiple entities.

Several works [25], [16], [9] identify the best subgraphs/paths in a data graph to describe how several input nodes are related. The query graph discovery component of GQBE is different in important ways– (1) The graphs in [25] contain nodes of the same type and edges representing the same relationship, e.g., social networks capturing friendship between people. The graphs in GQBE and others [16], [9] have many different types of entities and relationships. (2) The paths discovered by their techniques only connect the input nodes. REX [9] has the further limitation of allowing only two input entities. Differently the maximal query graph in GQBE includes edges incident on individual query entities. (3) GQBE uses the discovered query graph to find answer graphs and answer tuples, which is not within the focus of the aforementioned works.

There are many studies on approximate/inexact subgraph matching in large graphs, such as G-Ray [26], TALE [24] and NESS [17]. GQBE's query processing component is different from them on several aspects. First, GQBE only requires to match edge labels and matching node identifiers is not mandatory. This is equivalent to matching a query graph with all unlabeled nodes and thereby significantly increases the problem complexity. Only a few previous methods (e.g., NESS [17]) allow unlabeled query nodes. Second, in GQBE, the top-$k$ query algorithm centers around query entities. More specifically, the weighting function gives more importance to query entities and the minimal query trees mandate the presence of entities corresponding to query entities. On the contrary, previous methods give equal importance to all nodes in a query graph, since the notion of query entity does not exist there. Our empirical results show that this difference makes NESS produce less accurate answers than GQBE.

## VIII. Conclusion

We introduce GQBE, a system that queries knowledge graphs by example entity tuples. As an initial step toward better usability of graph query systems, GQBE saves users the burden of forming explicit query graphs. To the best of our knowledge, there has been no such proposal in the past. Its query graph discovery component derives a hidden query graph based on example tuples. The query lattice based on this hidden graph may contain a large number of query graphs. GQBE's query algorithm only partially evaluates query graphs for obtaining the top-$k$ answers. Experiments on Freebase and DBpedia datasets show that GQBE outperforms the state-of-the-art system NESS on both accuracy and efficiency.

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