Tutorial - 1

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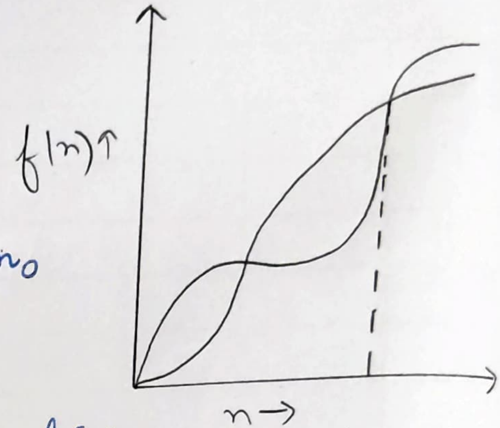
Q1 What do you understand by asymptotamatic notations, define different with example.

Ans i) Big $O(n)$

$f(n) = O\,g(n)$



if $f(n) \leq g(n) \times c \;\forall\; n \geq n_0$

for some constant, $c > 0$

$g(n)$ is 'tight' upper bound of $f(n)$

Eg → $f(n) = n^2 + n$

$g(n) = n^3$

$n^2 + n \leq c * n^3$

$n^2 + n = O(n^3)$

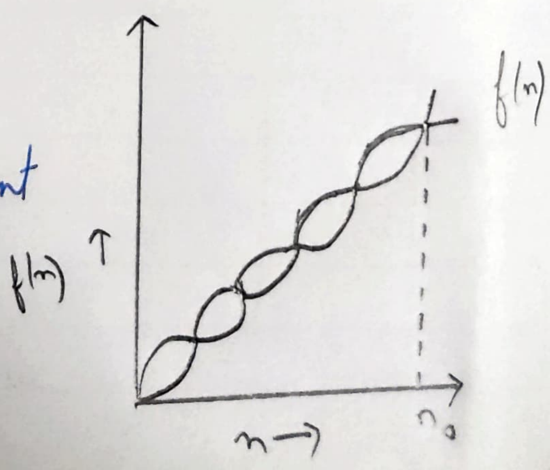
ii) Big Omega (Ω)

When $f(n) = \Omega(g(n))$, means $g(n)$ is tight lower bound of $f(n)$ i.e. $f(n)$ can be going beyond $g(n)$ i.e

$f(n) = \Omega\, g(n)$

iff $f(n) \geq g(n)$

$\forall\; n_2 > n_0$ & c is constant



Eg $f(n) = n^3 + 4n^2$

$g(n) = n^2$

i.e. $f(n) \geq c^* g(n)$

$n^3 + 4n^2 \geq c^* n^2$

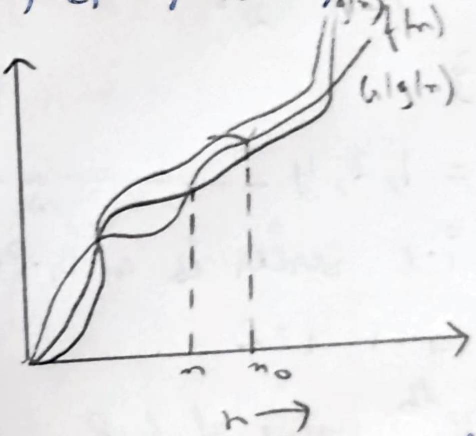
$n^3 + 4n^2 = \Omega n^2$

iii) Big Theta Θ

when $f(n) = \Theta(g(n))$ gives the tight upper bound and lower bound both i.e $f(n) = \Theta(g(n))$ iff $c_1^* g(n_1) \leq f(n) \leq c_2^* g(n_2)$ for all $n \geq \max(n_1, n_2)$ some constant $c_1 > 0$ & $c_2 > 0$ i.e. $f(n)$ can never go beyond $c_2(g(n))$ and will never come down of $c_1(g(n))$

Eg $3n+2 = \Theta(n)$ as $3n+2 \geq 3n$ &

$3n+2 \leq 4n$ for n, $c_1 = 3$, $c_2 = 4$, $n_0 = 2$



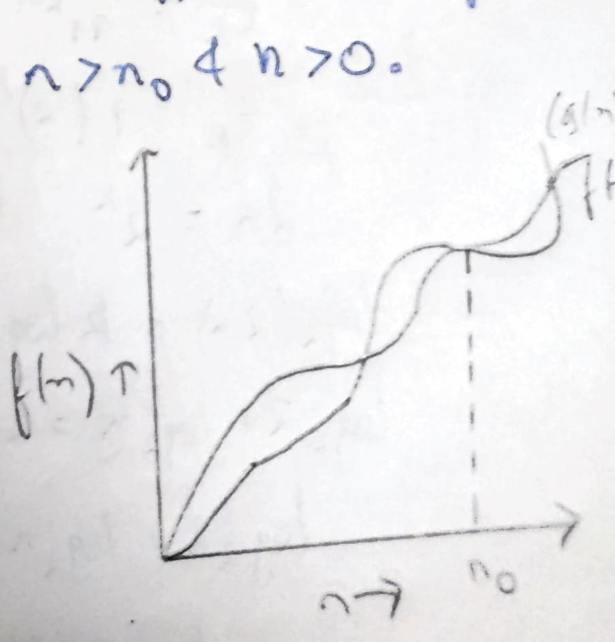
iv) Small o (θ)

when $f(n) = o(g(n))$ gives the upper bound $f(n) = o\,g(n)$ iff $f(n) < c^* g(n)$ $\forall$ $n > n_0$ & $n > 0$.

Eg $f(n) = n^2$; $g(n) = n^3$

$f(n) < c^* g(n)$

$n^2 = o(n^3)$



v) Small Omega (ω)

It gives the 'lower bound' i.e $f(n) = \omega(g(n))$ where $g(n)$ i.e lower bound of $f(n)$ iff $f(n) > c^* g(n) \ \forall \ n > n_0$ & $c > 0$

Q2 What should be the time complexity of

```
for (int i = 1 to n)
   {
      i = i * 2;
   }
```

Ans for $i = 1, 2, 4$ ------------- n times

i.e series is a G.P.

So $a = 1, r = 2$

K^{th} value of G.P

$t_k = ar^{k-1}$

$t_k = 1(2)^{k-1}$

$2n = 2^k$

$\log_2(2n) = k \log 2$

$\log_2 2 + \log_2 n = k$

$\log_2 2 + \log_2 n = k$

$\log_2 n + 1 = k$ So time complexity is $T(n) \Rightarrow O(\log_2 n)$

Q_3 $T(n) = \{ 3T(n-1)$ if $n>0$ otherwise $1 \}$

Ans
$$T(n) \Rightarrow 3T(n-1) \quad -①$$
$$T(n) = 1$$
put $n = n-1$ in ①
$$T(n-1) = 3T(n-1-1) \quad — ②$$
put ② in ①
$$T(n) = 3(3T(n-2))$$
$$T(n) = 9T(n-2) \quad -③$$
put it in ③
$$T(n) = 27\,T(n-3) \quad — ④$$
so $T(K) = 3^K T(n-k) \quad — ⑤$

for K^{th} term, let $n-k=1$

$K = n-1$ put in ⑤
$$T(n) = 3^{n-1}\,T(1)$$
$$T(n) = 3^{n-1}$$
$$T(n) = O(3^n)$$

Q_4 $T(n) = \{ 2T(n-1)-1$ if $n>0$, otherwise $1 \}$

Ans $T(n) = 2T(n-1)-1 \quad — ①$

put $n = n-1$
$$T(n-1) = 2T(n-2) - 1 \quad — ②$$
put in ①
$$T(n) = 8T(n-3) - 4 - 2 - 1 \quad — ④$$

So $T(n) = 2^k [n-k - 2^{k-1} - 2^{k-2} \text{———} 2^n]$

$\Rightarrow$ k^{th} term

let $n-k = 1$

$k = n-1$

$$T(n) = 2^{n-1} T(1) - 2^k \left(\frac{1}{2} + \frac{1}{2^2} + \text{———} \frac{1}{2^n} \right)$$

$$= 2^{n-1} - 2^{n-1} \left(\frac{1}{2} + \frac{1}{2^2} + \text{———} \frac{1}{2^n} \right)$$

It is a G.P.

$a = 1/2$, $r = 1/2$

So $T(n) = 2^{n-1} \left(1 - \frac{1/2 \left(1 - (1/2)^{n-1}\right)}{1 - \frac{1}{2}} \right)$

$$T(n) = \frac{2^{n-1}}{2^{n-1}}$$

$$T(n) = O(1)$$

Q5 int i=1, s=1;

```
while (s<=n)
{
    i++;
    s= s+i;
printf ("#");
}
```

Ans $i = 1, 2, 3, 4$ ————

$s = 1+3+6+10+15$ ————

sum of $s = 1+3+6+10$ ———— n — ①

Also $s = 1+3+6+10+$ ———— $+ T_{n-1} + T_n$ — ②

$O = 1+2+3+4 \text{ --------- } n - T_m$

$T_K = 1+2+3+4 \text{ --------- } k$

$T_K = \frac{1}{2} K(K+1)$

for K, $1+2+3+ \text{ ---------- } K \Leftarrow n$

$$\frac{K(K+1)}{2} <= n$$

$$\frac{K^2+K}{2} <= n$$

$$O(K^2) \leq n$$

$$K = O(\sqrt{n})$$

$$T(n) = O(\sqrt{n})$$

Q6

```
Void f(int n)
{
  int i, count = 0;
  for( int i=1; i* i<=n; ++i)
  }
```

Ans $\quad$ As $i^2 = n$

$i = \sqrt{n}$

$i = 1, 2, 3, 4 \text{ ---------- } \sqrt{n}$

$\sum_{i=1} 1+2+3+4 \text{ ------- } \sqrt{n}$

$$T(n) = \frac{\sqrt{n} * (\sqrt{n}+1)}{2}$$

$$T(n) = \frac{n * \sqrt{n}}{2}$$

$$T(n) = O(n)$$

Q7 Void f(int n)

```
{
 int i, j, k, count = 0;
 for (int i = n/2; i <= n; j = j * 2)
 for (K = 1; K <= n; K = K * 2)
      count ++;
}
```

Ans Since for $n = K^2$

$K = 1, 2, 4, 8 \text{-------} n$

∵ Series is in G.P.

So $a = 1, r = 2$

$$\frac{1(2^K - 1)}{1} = n$$

$$n = 2^K - 1$$

$$n + 1 = 2^K$$

$$\log_2(n) = K$$

i	j	K
1	log(n)	log(n) * log(n)
2	log(n)	log(n) * log(n)
⋮	⋮	⋮
n	log n	log(n) * log(n)

T.C $\Rightarrow O(n * \log n * \log n)$

$\Rightarrow O(n \log^2(n))$

Q8 Void function (int n)

```
{
  if (n==1) return;
  for (i=1 to n) {
    for (j=1 to n) {
      printf("*");
    }
  } function(n-3);
}
```

Ans for (i=1 to n)

we get j = n times every time

$i * j = n^2$

K^{th} ~~loop~~ now,

$T(n) = n^2 + T(n-3);$

$T(n-3) = (n^2-3)^2 + T(n-6);$

$T(n-6) = (n^2-6)^2 + T(n=9);$

and $T(1) = 1;$

Now these values in $T(n)$

$T(n) = n^2 + (n-3)^2 + (n-6)^2 + \cdots$

let $n - 3K = 1$

$K = (n-1)/3$

Total terms, $K+1$

$T(n) = n^2 + (n-3)^2 + (n-6)^2 + \cdots 1$

$T(n) = Kn^2$

$T(n) = (K-1)/3^* n^3$

So $T(n) = O(n^3)$

Q) void f (int n) {

for (int i=1 to n) {

for (int j=1; j<=n; j=j+1) {

printf (" * ");

}

}

}

Ans for $i=1$ $\quad j = 1+2+3 \text{ ------- } (n \geq j+i)$

$i=2$ $\quad j = 1+3+5 \text{ ------- } (n \geq j+i)$

$i=3$ $\quad j = 1+4+7 \text{ ------- } (n \geq j+i)$

n^{th} term of A.P is

$$T(n) = a + d^* m$$

$$T(n) = 1 + d^* m$$

$$(n-1)/d = n$$

for $i=1$ $\quad (n-1)/1$ times

$i=2$ $\quad (n-2)/2$ times

$i=n-1$

$$T(n) = i_1 j_1 + i_2 j_2 + \text{ ------- } i_{n-1} j_{n-1}$$

$$= \frac{(n-1)}{2} + \frac{(n-2)}{2} + \text{ ------- } + 1$$

$$= n + \frac{n}{2} + \frac{n}{3} + \text{ ------- } n/n-1 + n>=1$$

$$= n(1 + 1/2 + 1/3 + \text{ ------- } 1/(n-1)) - n^* 1$$

$$= n^* \log n - n + 1$$

since $\int 1/n = \log n$

$$T(n) = O(n \log n)$$

Q10 For the function n^k & c^n, what is the asymptotic relationship between these functions. Assume that $k >= 1$ & $c > 1$ are constants. Find out the value of c & no. of which relationship holds.

Ans As given n^k & c^n

Relationship between n^k & c^n is

$$n^k = O(c^n)$$

$$n^k \leq a(c^n)$$

$\forall n \geq n_0$, constant, $a > 0$

for $n_0 = 1$; $c = 2$

$$1^k = a^2$$

$n_0 = 1$ & $c = 2$