Tutorial-4

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Q1 $T(n) = 3T(n/2) + n^2$

$T(n) = aT(n/b) + f(n^2)$

$a \geq 1, b \geq 1$

On comparing

$a=3, b=2, f(n)=n^2$

Now, $c = \log_b a = \log_2 3 = 1.584$

$n^c = n^{1.584} < n^2$

$f(n) > n^c$

$T(n) = O(n^2)$

Q2 $T(n) = 4T(n/2) + n^2$

$a \geq 1, b > 1$

$a=4, b=2, f(n)=n^2$

$c = \log_2 4 = 2$

$n^c = n^2 \Rightarrow f(n) = n^2$

$T(n) = O(n^2 \log n)$

Q3 $T(n) = T(n/2) + 2^n$

$a=1, b=2$

$f(n) = 2^n$

$c = \log_b a = \log_2 0 = 0$

$n^c = n^0 = 1$

$f(n) > n^c$

$T(n) = O(2^n)$

Q4 $T(n) = 2^n T(n/2) + n^2$

$a = 2^n$

$b = 2$, $f(n) = n^2$

$c = \log_b a = \log_2 2^n = n$

$n^c = n^n$

$f(n) = n^c$

$f(n) = \Theta(n^2 \log n)$

Q5 $T(n) = 16\, T(n/a) + n$

$a = 16$, $b = 4$

$f(n) = n$

$c = \log_4 16 = \log_4 (4)^2 = 2 \log_4 4 = 2$

$n^c = n^2$

$f(n) < n^c$

$T(n) = \Theta(n^2)$

Q6 $T(n) = 2T(n/2) + n\log n$

$a = 2$, $b = 2$

$f(n) = n\log n$

$c = \log_2 2 = 1$

$n^c = n^2 = n$

$n\log n > n^c$

$f(n) > n^c$

$= \Theta(n\log n)$

Q7 $T(n) = 2T(n/2) + n\log n$

Ans $a=2, b=2, f(n) = n\log n$

$c = \log_2 2 = 1$

$n^c = n^1 = n$

$\therefore \frac{n}{\log n} < n$

$\therefore f(n) < n^c$

$\therefore T(n) = \theta(n)$

Q8 $T(n) = 2T(n/4) + n^{0.51}$

Ans $a=2, b=4, f(n) = n^{0.51}$

$c = \log_b a = \log_4 2 = 0.5$

$n^c < n^{0.51}$

$f(n) > n^c$

$\therefore T(n) = \theta(n^{0.51})$

Q9 $T(n) = 0.5T(n/2) + 1/n$

Ans $a=0.5, b=2$

$a \geq 1$ but here a is 0.5

So we cannot apply master's theorem

Q10 $T(n) = 16T(n/4) + n!$

$a=16, b=4, f(n)=n!$

$c = \log_b a = \log_4 16 = 2$

$n^c = n^2$

As $n! > n^2$

$\therefore T(n) = \theta(n!)$

Q10 $T(n) = 16T(n/4) + n!$

$a = 16, b = 4, f(n) = n!$

$\therefore c = \log_b a = \log_4 16 = 2$

$n^c = n^2$

As $n! > n^2$

$T(n) = \Theta(n!)$

Q11 $4T(n/2) + \log n$

$a = 4, b = 2, f(n) = \log n$

$c = \log_b a = \log_2 4 = 2$

$n^c = n^2$

$f(n) = \log n$

$\because \log n < n^2$

$f(n) < n^c$

$T(n) = \Theta(n^c)$

Q12 $T(n) = \sqrt{n} + T(n/2) + \log n$

Ans $a = \sqrt{n}, b = 2$

$c = \log_b a = \log_2 \sqrt{n} = \frac{1}{2} \log_2 n$

$\therefore \frac{1}{2} \log_2 n < \log(n)$

$f(n) > n^c$

$T(n) = \Theta(f(n))$

$T(n) = \Theta(\log(n))$

Q13 $T(n) = 3T(n/2) + n$

Ans $a = 3, b = 2, f(n) = n$

$c = \log_b a = \log_2 3 = 1.5849$

$n^c = n^{1.5489}$

$n < n^{1.5489}$

$\Rightarrow f(n) < n^c$

$T(n) = \theta(n^{1.5849})$

Q14 $T(n) = 3T(n/3) + \text{sqrt}(n)$

Ans $a = 3, b = 3$

$c = \log_b a = \log_3 3 = 1$

$n^c = n^1 = n$

As $\text{sqrt}(n) < n$

$f(n) < n^c$

$T(n) = \theta(n)$

Q15 $T(n) = 4T(n/2) + n$

Ans $a = 4, b = 2$

$c = \log_b a = \log_2 4 = 2$

$n < n^2$

$f(n) < n^2$

$f(n) = \theta(n^2)$

Q16 $T(n) = 3T(n/4) + n\log n$

Ans $a = 3, b = 4, f(n) = n\log n$

$c = \log_b a = \log_4 3 = 0.792$

$n^c = n^{0.792}$

$n^{0.792} < n\log n$

$T(n) = \theta(n\log n)$

Q17 $T(n) = 3T(n/3) + n/2$

Ans $a=3, b=3$

$c = \log_b a = \log_3 3 = 1$

$f(n) = n/2$

$n^c = n^1 = n$

As $n/2 < n$

$f(n) < n^c$

$\therefore T(n) = \theta(n)$

Q18 $T(n) = 6T(n/3) + n^2 \cdot \log n$

Ans $a=6, b=3$

$c = \log_b a = \log_3 6 = 1.6309$

$n^c = n^{1.6309}$

As $n^{1.6309} < n^2 \log n$

$T(n) = \theta(n^2 \log n)$

Q19 $T(n) = 4T(n/2) + n\log n$

Ans $a=4, b=2, f(n) = \frac{n}{\log n}$

$c = \log_b a = \log_2 4 = 2$

$n^c = n^2$

$\frac{n}{\log n} < n^2$

$T(n) = \theta(n^2)$

Q20 $T(n) = 64T(n/8) - n^2 \log n$

Ans $a=64, b=8,$

$c = \log_b a = \log_8 64 = \log_8 8^2 = 2$

$n^c = n^2$

$n^2 \log n > n^2$

$T(n) = \theta(n^2 \log n)$

Q21 $T(n) = 7T(n/3) + n^2$

Ans $a = 7, b = 3, f(n) = n^2$

$c = \log_b a = \log_3 7 = 1.77$

$n^c = n^{1.7712}$

$n^{1.7712} < n^2$

$T(n) = \theta(n^2)$

Q22 $T(n) = T(n/2) + n(2 - \cos n)$

Ans $a = 1, b = 2$

$c = \log_b a = \log_2 1 = 0$

$n^c = n^0 = 1$

$n(2 - \cos n) > \Omega\, n^c$

$T(n) = \theta(n(2 - \cos n))$