TUTORIAL-6

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Q1 What do you mean by minimum spanning tree? What are the applications of MST?

Ans minimum spanning tree is a subset of edges of a connected edge-weighted undirected graph that connects all the vertices together without any cycles and with minimum possible edge weighted.

Applications -

1) Consider n stations are to be linked using a communication network and lying of communication link between any two stations involves a cost. The ideal solution would be to extract a sub-graph termed as minimum cost spanning tree.

2) Design LAN

3) Suppose you want to construct highways or rail roads spanning several cities, then we can use concept of MST

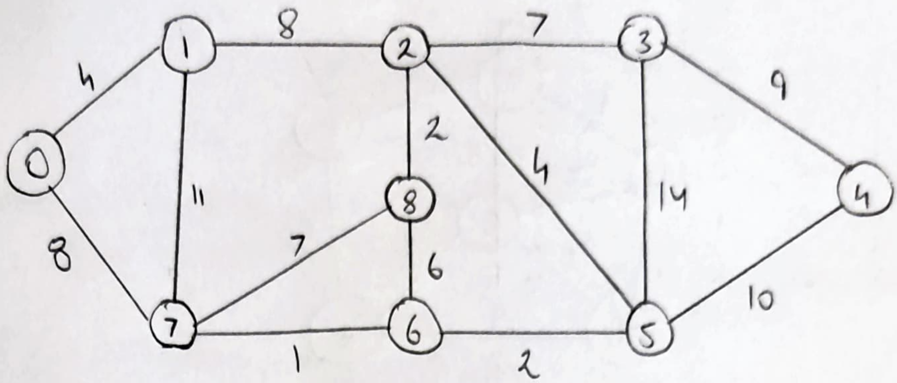
4) laying pipeline connecting offshore drilling sites, refineries and consumer markets

Q2 Analyse time and space complexity of Prim, Krushkal, Dijikshtra and Bellman Ford Algorithm.

Ans Time complexity of Prim's Algorithm : O(|E| log |V|)
Space " " " " : O|V|
Time " " Krushkal's " : O|E| log |E|
Space " " " " : O|V|

Time complexity of Dijkstra's algorithm : $O(V^2)$
space " " " " : $O(V^2)$
Time " " Bellman Ford's " : $O(VE)$
space " " " " : $O(E)$

Q3 Apply Kruskal and Prim's Algorithm on given graph to compute MST and its weight.



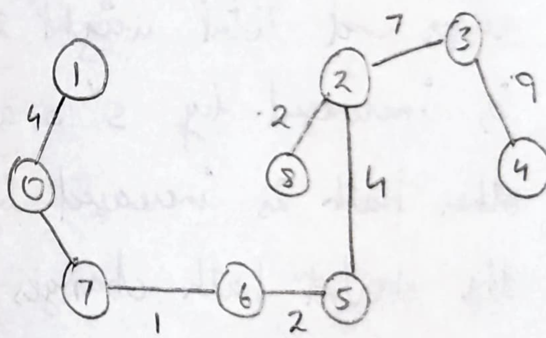
Ans

Kruskal's Algorithm

U	V	W	
6	7	1	✓
5	6	2	✓
2	8	2	✓
0	1	4	✓
2	5	4	✓
6	8	6	x
2	3	7	✓
7	8	7	x
0	7	8	✓
1	2	8	x
4	3	9	✓
4	5	10	x
1	7	11	x
3	5	14	x

Prim's Algorithm

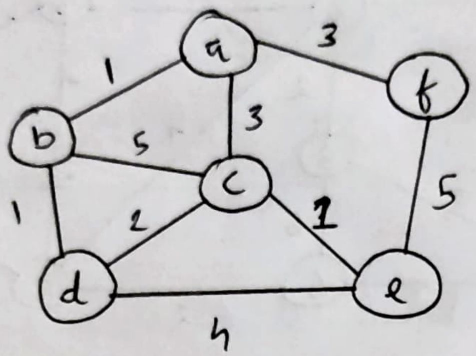
weight = 4+8+2+4+2+
+9+3
= 37



Q4 Given a directed weighted graph. You are also given the shortest path from a source vertex 's' to a destination vertex 't'. Does the shortest path remain same in following cases:

i) If weight of every edge is increased by 10 units

ii) If weight of every edge is multiplied by 10 units.

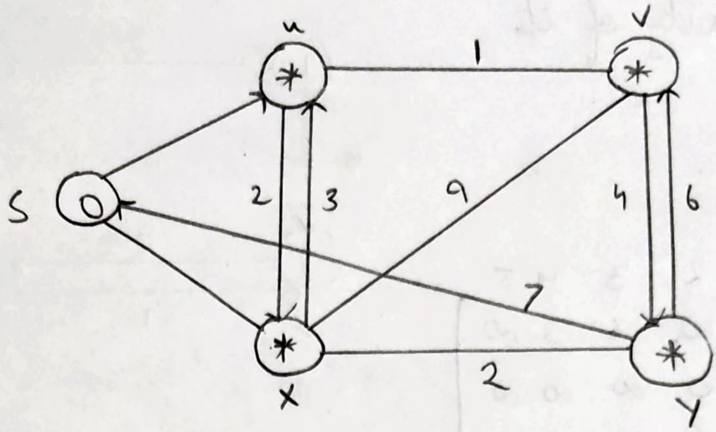


Ans i) The shortest path may change. The reason is that there maybe different no. of edges in different paths from 's' to 't'. For eg - let the shortest path of weight 15 and has edges 5. Let there be another path with 2 edges and total weight 25. The weight of shortest path is increased by 5*10 and becomes 15+50. Weight of other path is increased by 2*10 and becomes 25+20. So, the shortest path changes to other path with weight as 45.

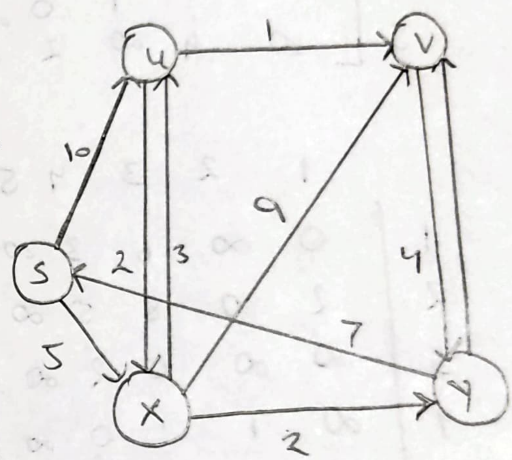
If we multiply all edges weight by 10, the shortest path does not change. The reason is that weight of all paths from 's' to 't' gets multiplied by same unit. The number of edges on path doesn't matter.

Q5 Apply Dijkstra and Bellman Ford algorithm on graph given right side to compute shortest path to all nodes from node S.

Ans



Dijkstra's Algorithm



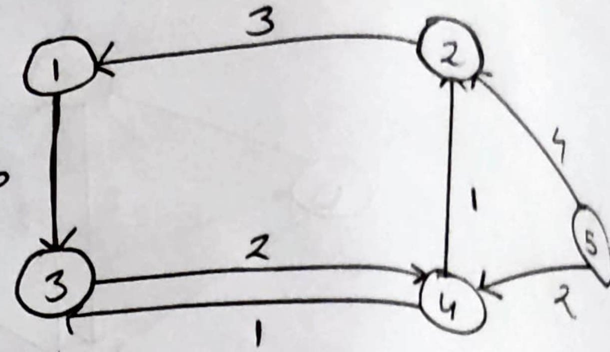
NODE	SHORTEST DISTANCE FROM SOURCE NODE
u	8
x	5
v	9
y	7

Bellman Ford Algorithm -

	S	u	v	x	y
1st →	0	~~∞~~ 10	∞	~~∞~~ 5	∞
2nd →	0	10	~~∞~~ 11	5	∞
3rd →	0	8	~~11~~ 9	5	~~∞~~ 7
4th →	0	8	9	5	7

Graph does not have negative cycle

Q6 Apply all pair shortest path algorithm. Floyd Warshall on below mentioned graph. Also analyse space and time complexity of it.



$$
\begin{array}{c}
 \\ 1 \\ 2 \\ 3 \\ 4 \\ 5
\end{array}
\begin{array}{c}
\begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\
\begin{bmatrix}
0 & \infty & 6 & 3 & \infty \\
2 & 0 & \infty & \infty & \infty \\
\infty & \infty & 0 & 2 & \infty \\
\infty & 1 & 1 & 0 & \infty \\
\infty & 4 & \infty & 2 & 0
\end{bmatrix}
\end{array}
$$

$$
\begin{array}{c}
 \\ 1 \\ 2 \\ 3 \\ 4 \\ 5
\end{array}
\begin{array}{c}
\begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\
\begin{bmatrix}
0 & \infty & 6 & 3 & \infty \\
2 & \infty & 8 & 5 & \infty \\
\infty & \infty & 0 & 2 & \infty \\
\infty & 1 & 1 & 0 & \infty \\
\infty & 4 & \infty & 2 & 0
\end{bmatrix}
\end{array}
$$

$$
\begin{array}{c}
 \\ 1 \\ 2 \\ 3 \\ 4 \\ 5
\end{array}
\begin{array}{c}
\begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\
\begin{bmatrix}
0 & \infty & 6 & 3 & \infty \\
2 & 0 & 8 & 5 & \infty \\
\infty & \infty & 0 & 2 & \infty \\
3 & 1 & 1 & 0 & \infty \\
6 & 4 & 2 & 2 & 0
\end{bmatrix}
\end{array}
$$

$$\begin{array}{cc} & \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} & \begin{bmatrix} 0 & \infty & 6 & 3 & \infty \\ 2 & 0 & 8 & 5 & \infty \\ \infty & \infty & 0 & 2 & \infty \\ 3 & 1 & 1 & 0 & \infty \\ 6 & 4 & 12 & 2 & 0 \end{bmatrix} \end{array}$$

Ans

Time complexity $\rightarrow O(|V|^3)$

Space complexity $\rightarrow O(|V|^3)$