PHY224H1: The Milikan Experiment

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Abstract

In this experiment, the behavior of charged oil droplets was studied in order to demonstrate the quantization of charge by experimentally finding the value of the elementary charge e. For 52 oil droplets, by measuring the terminal falling velocity of each charged droplet in an electric field-free space and the stopping voltage of the same droplet in a uniform electric field, the value of e was found to be $(1.616 \pm 0.017) \times 10^{-19}\,C$. This value was found to agree to within 0.87 % of the theoretical value of $1.602 \times 10^{-19}\,C$, thus through the experimental method employed the constant was successfully determined.

1 Introduction

A tool called an *atomizer* can be used to create a mist of oil. When we use an atomizer to spray oil, each oil droplet acquires electric charge through *triboelectric charging*. This process takes place when the oil droplets come into contact with the capillary of the atomizer and some oil droplets pick up one or more excess electrons due to friction.

When we place a negatively charged oil droplet in the uniform electric field of a parallel plate capacitor, various forces act on the droplet. For a negatively charged oil droplet with charge Q, the forces acting on the droplet are the downward gravitational force $\vec{F}_{grav} = m_{oil}\vec{g}$, the upward buoyant force $\vec{F}_b = -m_{air}\vec{g}$, and in the presence of an electric field, the electric force $\vec{F}_E = Q\vec{E}$. Here, m_{oil} is the mass of the droplet, $\vec{g}$ is acceleration due to gravity, m_{air} is the mass of air displaced by the droplet, and $\vec{E}$ is the electric field produced by the parallel plate capacitor.

If we assume that the oil droplet is perfectly spherical and in motion, *Stokes' law* tells us that a viscous force $\vec{F_v} \approx 6\pi r\eta v$ will act on it in the opposite direction of motion. Here, r is the radius of the oil droplet, η is the viscosity of air, and v is the speed of the droplet. In the absence of an electric field, the droplet maintains its *terminal velocity* due to this viscous force when it falls in air.

Using these forces, the equation of motion for an oil droplet with charge Q falling through a field-free space with terminal velocity v_t is given by:

$$g(m_{oil} - m_{air}) - 6\pi r\eta v_t = 0. \tag{1}$$

The equation of motion for the same oil droplet floating in a uniform electric field E is given by:

$$g(m_{oil} - m_{air}) - QE = 0. \tag{2}$$

In both of these cases, the net force is zero because the droplet is not accelerating. With equations (1) and (2), we can derive the following relation between v_t, V_{stop}, and Q and get:

$$Q = 2 \cdot 10^{-10}\frac{v_t^{\frac{3}{2}}}{V_{stop}}. \tag{3}$$

Please see the appendix for the derivation of equation (3).

The purpose of the Milikan Experiment is to demonstrate that charge is quantized by multiples of the elementary charge $e = 1.602 \times 10^{-19}C$, and therefore extract this value of e. Specifically, we will directly measure the voltage, V_{stop} at which a given charged oil droplet floats between the capacitor plates, and the terminal velocity, v_t, of the droplet falling in the field-free space between the capacitors by switching off the voltage completely. We will take these measurements for about 50 oil droplets. Then, using equation (3) and our measurements, we will find the charge Q of each individual droplet. With the calculated Q values, we will prove the quantization of charge, and find the value of e.

2 Materials and Methods

2.1 Materials

- Leybold-Heraeus Apparatus:
  - A parallel plate capacitor ("chamber"),
  - Oil atomizer with rubber bulb to charge oil droplets,

- – Socket pair for charging the plate capacitor connected to DC power supply with adjusting knob,
- – Light source to illuminate the chamber,
- – Microscope connected to CCD camera 8.

- Computer software LabVIEW to monitor oil droplets and gather data.

2.2 Methods

We chose to do the experiment using Method 1 in the lab manual. We prepared a set of 70 blank Excel files numbered from 1 to 70, and an Excel file to record trial numbers (or droplet numbers) and stopping voltages for each droplet. For the first two sessions, we used the setup on the left. For the third session, we used the setup on the right. We started the LabVIEW software on the desktop computer after ensuring that the positive terminal of the power supply was connected to the upper socket of the chamber. We turned off the lights in the room, and switched on the power supply. In the LabVIEW software, we slid "Gain: Value" up to around 17. We selected "Filename" to be the excel file corresponding to the trial number (droplet number).

To collect data from each oil droplet, we first sprayed oil into the chamber by pumping the atomizer a few times. We then completely switched off the voltage. We slowly adjusted the microscope and "Lower Value" on LabView until we could see the falling oil droplets. Once we saw a droplet, we adjusted the microscope and lower value to focus on that specific droplet. Once the droplet became sharply visible, we increased the voltage until the droplet was floating. We recorded the stopping voltage in the excel file next to its trial number. Then, we placed the sharply focused droplet inside a small square tracker. If the tracker contained the droplet properly, we switched off the voltage completely and immediately started saving data until the droplet reached the bottom plate of the capacitor. Before we started collecting data for a new droplet, "Filename" was switched to the next blank Excel file.

We repeated the same steps for about 65 droplets.The software tracker failed to track some droplets halfway through. The files corresponding to these measurements were deleted. For the remaining files, we added a column of integer frame numbers to the left of position data. Using the "XY Scatter" feature of Excel, we plotted position versus frame number. We kept the files that correspond to a straight line, and deleted the remaining ones. The number of successful files was 52. We then added another column to the voltage file to indicate which setup was used for each droplet. Lastly, to make the Python code easier, we made up for the deleted files by changing the name of the successful files so that they were numbered from 1 to 52 with no integers missing in between.

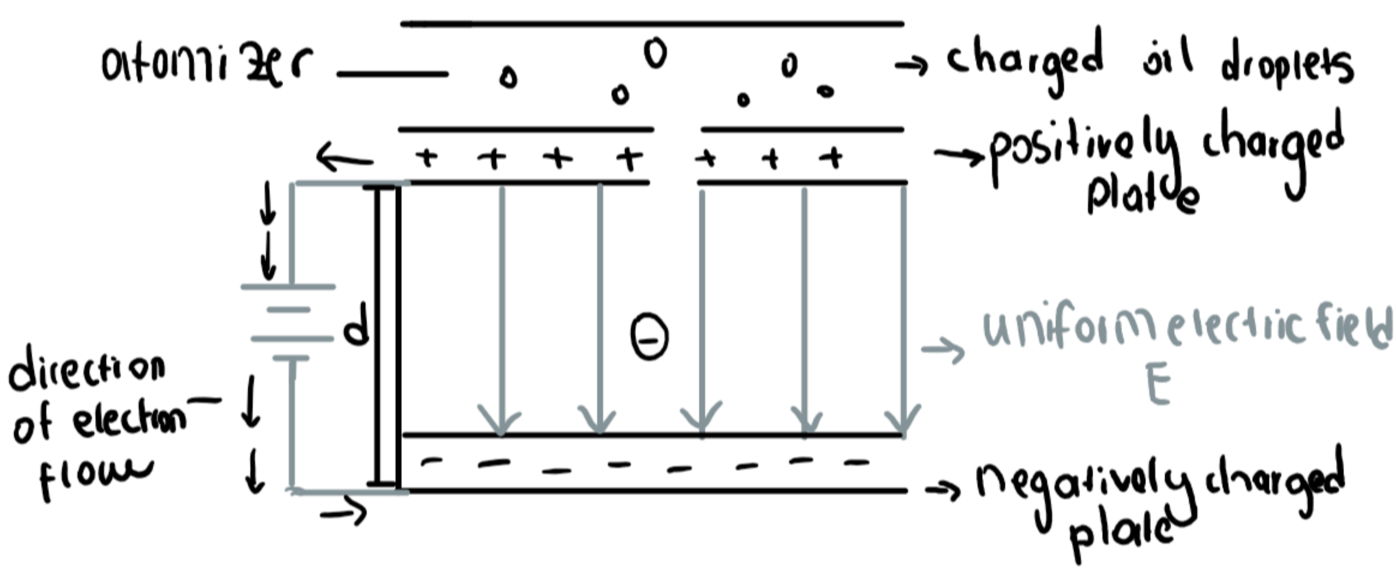
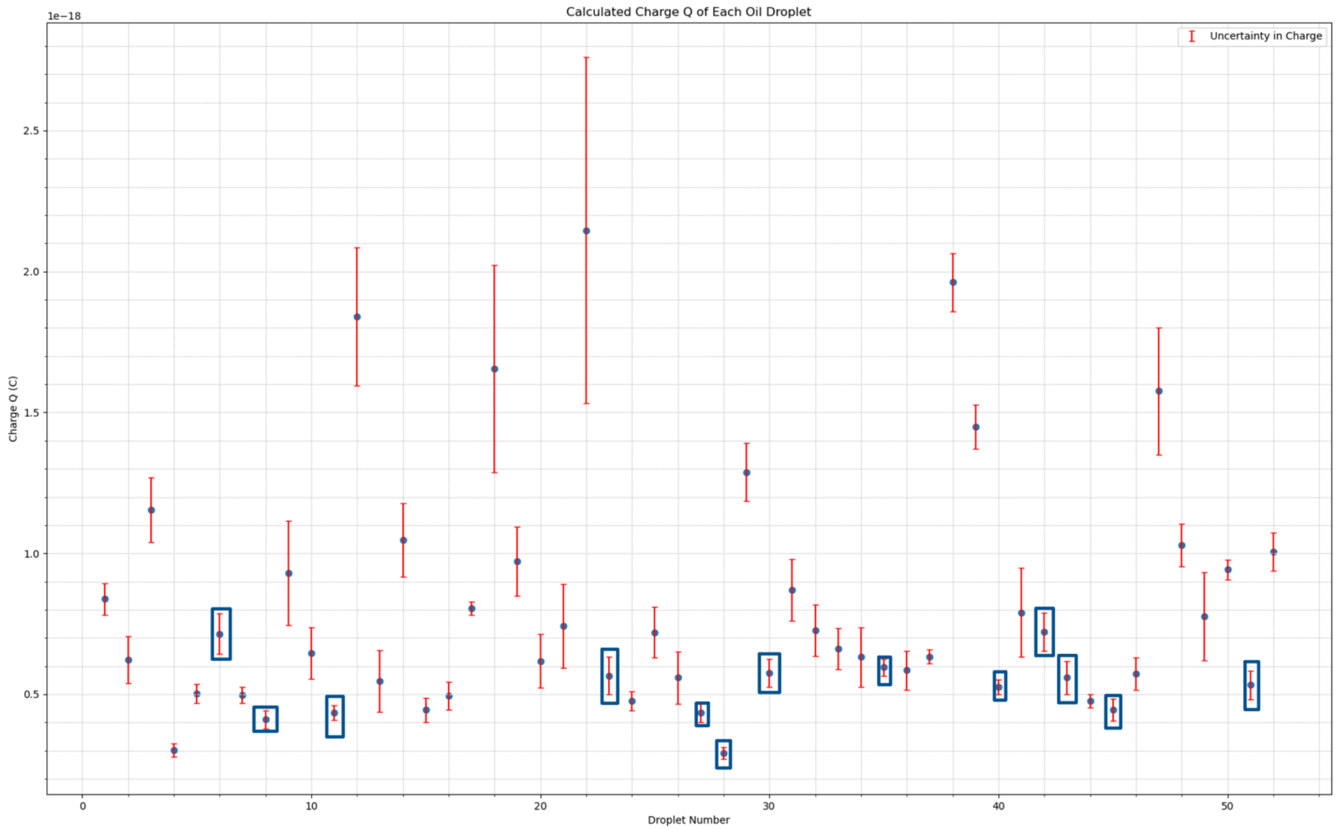


Figure (0): A negatively charged oil droplet between the capacitor plates of the Leybold-Heraeus Apparatus.

3 Data and Analysis

3.1 Data



Figure(1): The calculated charges for individual oil droplets. The boxed data points represent outlier data sets. Outlier data points will be discussed in section 3.2.

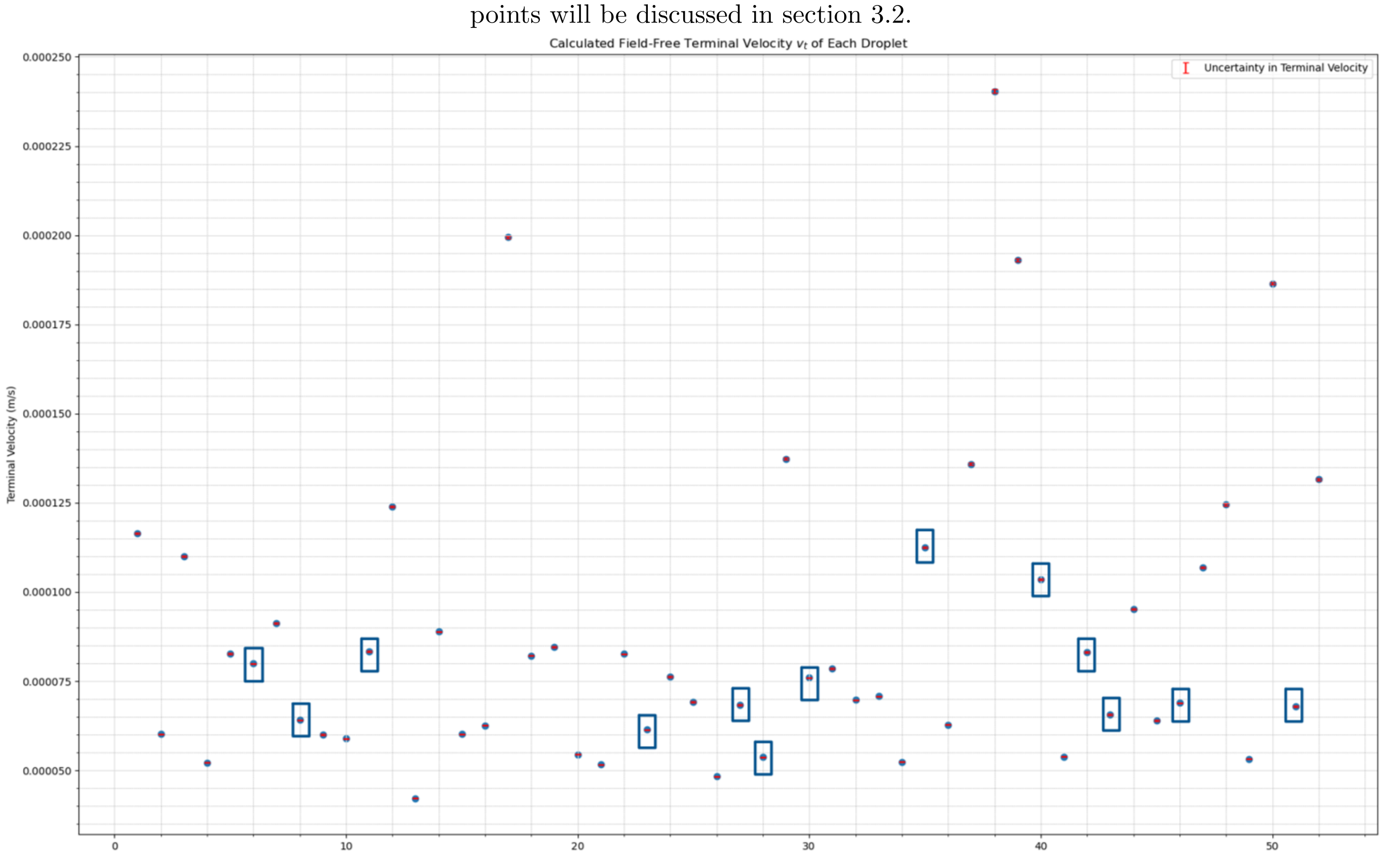


Figure (2): The calculated terminal falling velocities for individual oil droplet. The boxed data points represent outlier data sets. The error bars are almost invisible due to small uncertainties. See appendix for numerical values of uncertainties.

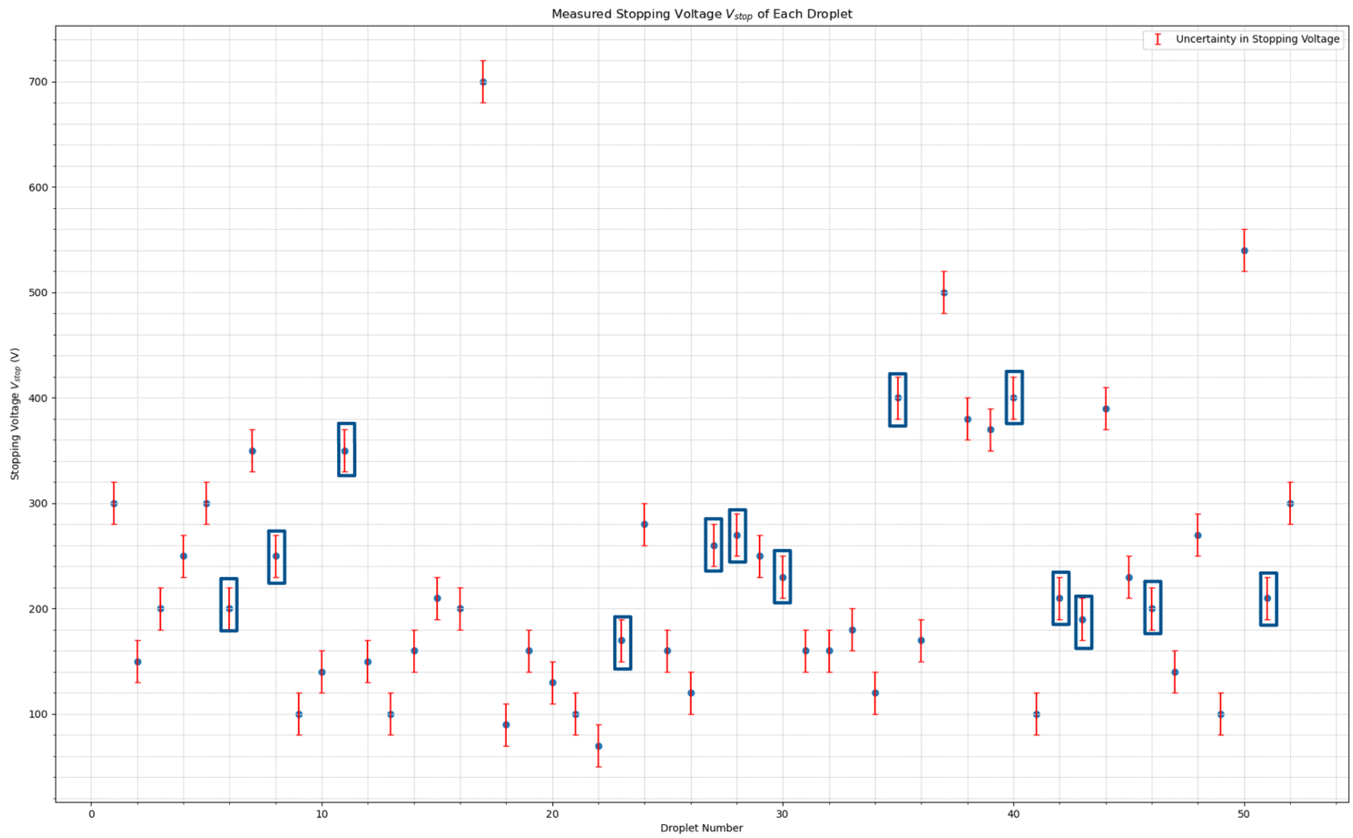


Figure (3): Calculated stopping voltage of each oil droplet. The boxed data points represent outlier data sets.

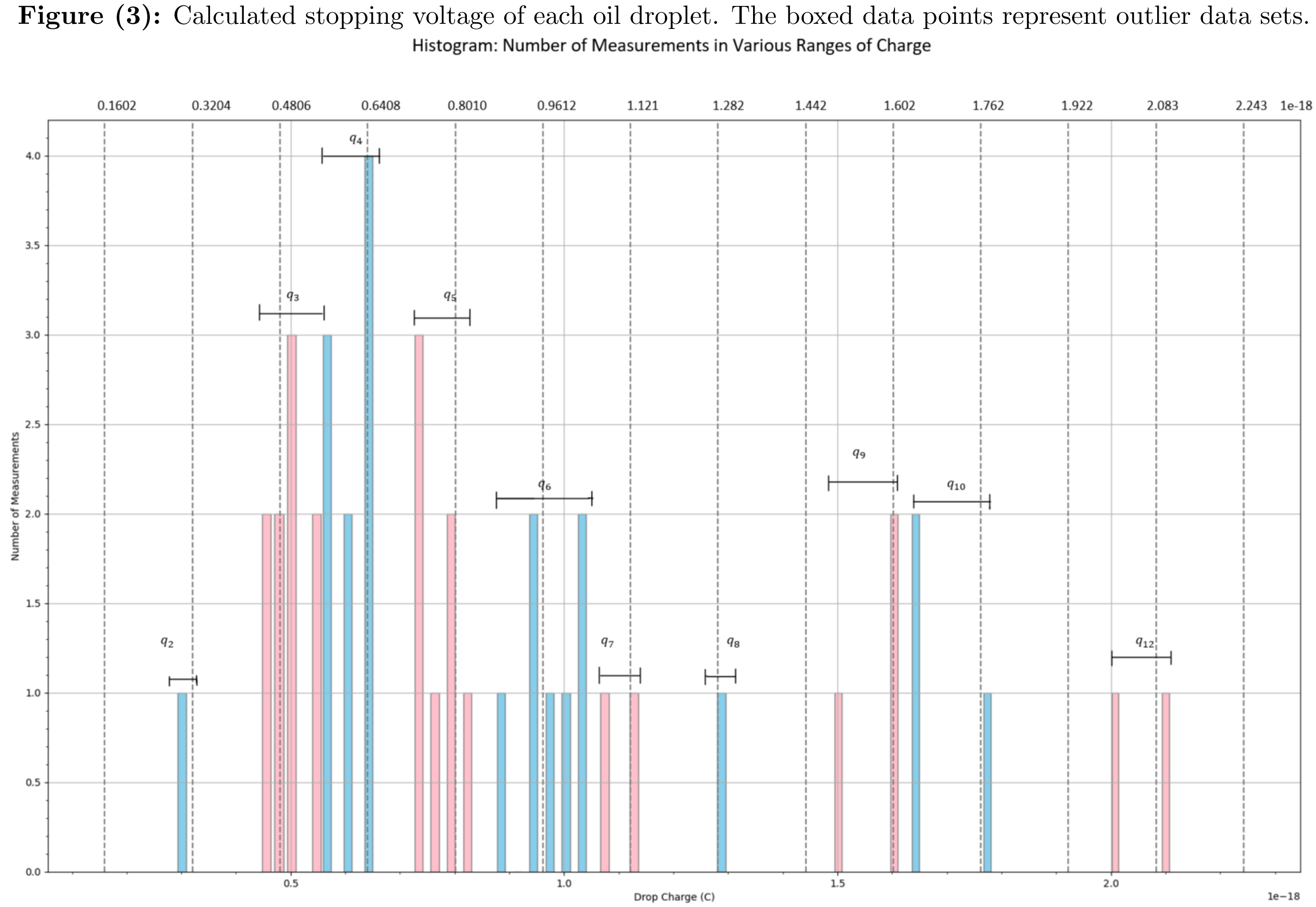


Figure (4): Histogram of the number of measurements for various ranges of oil droplet charge. The vertical dashed lines represent integer multiples of e. Each group of bins labeled as q_i represents the data points that contain $i \times e$ within their uncertainty.

	qi (Coulombs)	Δqi (+- Coulombs)	ei(Coulombs)	Δei(+- Coulombs)
q2	3.01E-19	0	1.51E-19	0
q3	4.94E-19	1.33E-20	1.65E-19	4.42E-21
q4	6.12E-19	1.30E-20	1.53E-19	3.26E-21
q5	7.71E-19	1.65E-20	1.54E-19	3.30E-21
q6	9.71E-19	2.34E-20	1.62E-19	3.90E-21
q7	1.10E-18	5.34E-20	1.57E-19	7.63E-21
q8	1.29E-18	0.00E+00	1.61E-19	0.00E+00
q9	1.56E-18	5.97E-20	1.73E-19	6.63E-21
q10	1.69E-18	7.83E-20	1.69E-19	7.83E-21
q12	2.05E-18	9.23E-20	1.71E-19	7.69E-21

Table (1): Values of q_i from Figure (4) and their uncertainties, as well as the extracted values of e_i from each group and their uncertainties. Groups q_2 and q_7 contain single data points, hence the trivial uncertainties due to zero standard deviation.

3.2 Analysis

3.2.1 Finding Values of Q for Each Oil Droplet

In order to use equation (3) to find Q for each oil droplet, we had to first calculate each value of v_t. The LabVIEW software saves data as oil droplet position in pixels frame by frame with a frame rate of 10 frames per second. For a given droplet, to convert position in pixels to position in meters, we divide each position in pixels by $calibration factor \frac{px}{mm} \times 1000 \frac{mm}{m}$. The calibration factor is taken to be $(540 \pm 1)\frac{px}{mm}$ for droplets measured with the left device, and $(520 \pm 1)\frac{px}{mm}$ for droplets measured with the right device. To convert frame numbers to time in seconds, we divide each frame number by $10\,\frac{1}{s}$.

The distance travelled by each droplet is $d = d_f - d_i$ where d_f and d_i are the final and initial positions in meters, respectively. Therefore, the terminal falling velocity for each oil droplet is $v_t = \frac{d_f - d_i}{t_f} = \frac{d}{t_f}$ where t_f is the last time data point in seconds. With the calculated v_t and the measured V_{stop} corresponding to each droplet, we found Q values using equation (3). ΔQ values for each Q were found by error propagation. See appendix for how error was propagated for Q values.

3.2.2 Finding the value of e and Showing Quantization

Our method for extracting the value of e was to create a histogram where we see can see multiple normal distributions centered around integer multiples of e (Figure (4)). To create a histogram that has this property, we grouped together data points that correspond to the same integer multiple of e within their uncertainties, plotted a histogram for each group with an appropriate number of bins to show normal distribution, and overlapped these histograms on the same figure (Figure (4)). For each group, q_i was found by taking the average of data points contained in that group, and finding the uncertainty as $\frac{\sigma_i}{\sqrt{N}}$ where σ_i is the standard deviation of the group corresponding to q_i, and N is the number of data points in that group.

Before we move on, we should discuss a few things about the histogram. First; some data points contained more than one integer multiple of e within their uncertainty due to large error. Therefore, they fell into more than one group. To avoid using the same data point in more than one group, we removed it from the group whose distribution it didn't fit very well(by eyeballing), and left it in the other one. Second; some data points didn't contain *any* integer multiple of e whatsoever. These data points were removed from all of our calculations. The removed data points are explicitly shown in boxes in the plots from section 3.1. Third, very large charges had very large uncertainties, and therefore they overlapped *entirely* with other groups, and hence were entirely removed since the other groups contained the same data points. For example, q_{11} overlapped entirely with q_{10}, and was removed. Group q_{13} overlapped with q_{14} and q_{13} overlapped entirely with q_{12} and so the q_{13} and the q_{14} groups were fully removed since all of those data points are already contained in group q_{12}.

To extract values values of e from each group q_i, we used $q_i = i \times e_i$. We propagated the error Δe_i to find the values in the fifth column of Table (1). The final value of e was calculated by taking the average of the e_i values on the fourth column of Table (1). The uncertainty in the final value $\frac{\sigma}{\sqrt{N}}$ was smaller than some of the Δe_i values, therefore we used error propagation instead to find the uncertainty in the final value. Lastly, we found that $e_{final} = 1.616 \times 10^{-19}\,C \pm 0.017 \times 10^{-19}\,C$. See appendix for the Δe_i and Δe_{final} calculations.

4 Discussion and Conclusion

In this experiment, the behavior of 52 negatively charged oil droplets in the absence and presence of a uniform electric field was analyzed in order to experimentally find the value of the elementary charge e, and therefore demonstrate the quantization of charge. Specifically, we took measurements of the voltage at which each oil droplet would float between the capacitor plates of the Leybold-Heraeus Apparatus, and calculated the terminal falling velocity of the droplet at zero voltage by taking measurements of position in pixels per frame using the LabVIEW software. Using values of stopping voltage V_{stop} and terminal velocity v_t, and equation (3), we were able to calculate the charge Q of each oil droplet. Using these values, we plotted the number of measurements versus values of charge within the ranges of uncertainty in a histogram to extract the value of the elementary charge e. The final value e_{final} was found to be $1.616 \times 10^{-19}\,C \pm 0.017 \times 10^{-19}\,C$. This experimental value was found to be both accurate and precise, as it agrees with the theoretical value of $1.602 \times 10^{-19}\,C$ to within 0.87 %, and the theoretical value is within the uncertainty range of the experimental value.

The latter purpose of this experiment, showing the quantization of charge, was achieved by our already-existing notion of what the value of e should be, and by eliminating the unexpected data points based on this notion (i.e. the data points that were disregarded because they did not contain any integer multiple of e within their uncertainty). In our case, this is illogical. However, if we could gather enough data points, we would notice a quantization trend, start working only with data points that further show this trend, and eventually extract the value of e. Although this would still assume quantization, there would be no presuppositions on the value of e itself. Regardless, there is enough evidence in our results to prove quantization; every value of q_i on Table (1) is approximately an integer multiple of e_{final}.

With our values of terminal velocities and the relation between the radius r of each droplet and its terminal velocity v_t derived in section 5.1.3, we calculated the average droplet radius in our experiment to be ≈ 0.9 microns. This indicates that Stokes' resistance for is a good approximation when deriving equation (3). This is because Stokes' law assumes a large enough radius of droplets so that the droplets are continuously colliding with air molecules. The diameter of our droplets is ≈ 1.8 microns. The mean free path of air molecules at room temperature and atmospheric pressure is ≈ 0.068 microns. Evidently, the mean free path of air molecules isn't comparable to the size of our droplets, and hence the droplets are almost always colliding with air molecules. Therefore, Stokes' Law holds since the experiment works better with larger radii due to our assumptions. However, unlike the Stokes resistance force, the upward buoyancy force from the displaced air molecules isn't very significant because the density of air is about one-thousandth that of oil. Therefore, the upward buoyancy force is barely comparable to the downward gravitational force due to the mass of a given droplet, and could have been neglected in our derivations.

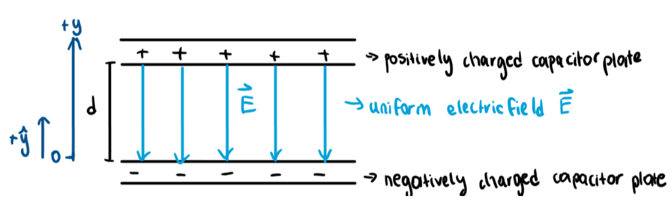
Another topic of discussion is that it was harder to prove quantization with larger values of charge since these data points fell into multiple groups of q_i due to their much larger uncertainties. The larger uncertainties for larger charges comes from the fact that the stopping voltage V_{stop} is much smaller for such droplets because the electric force due the capacitor plates pushing the droplet upward is naturally large due to the abundance of electrons on the droplet, and thus we don't need a high voltage to stop it. From the equation in section 5.3 for ΔQ, we see that the smaller the value of V_{stop}, the larger the uncertainty in Q, and hence harder it is to prove quantization.

A possible source of error is the large reading error due to the unstable arm of the voltmeter on the power supply. It was very hard to tell whether the arm had finally stopped, or that it was directly above a marking on the voltmeter. We could have reduced this reading error by using a multimeter that gives us a digital display of the voltage. Lastly, the fact that we removed 13 data points because they were unexpected also reduces the reliability of our results. However, there was no way of us finding out that they were unexpected without analyzing them individually. Regardless, the experiment was successfully able to analyze various charged oil droplets to experimentally determine the value of the elementary charge and demonstrate quantization.

5 Appendix

5.1 "Exercise 1" - Derivation of Equation (3)

5.1.1 Electric Field due to a Parallel Plate Capacitor



The above figure is a diagram of the uniform electric field produced by a parallel plate capacitor whose plates are a distance d apart. We have labeled the coordinate system on the left. If we take our reference point to be $y = 0$, the potential due to the parallel plate capacitors as shown above is:

$$V = -\int_0^d \vec{E} \cdot d\vec{l} = -\int_0^d -E\hat{\mathbf{y}} \cdot dy\hat{\mathbf{y}} = \int_0^d E dy = Ed.$$

Therefore, the electric field between the charged plates is $E = \frac{V}{d}$.

5.1.2 Forces on a Charged Oil Droplet floating in Electric Field E

The diagram to the left is a free body diagram of an oil droplet with negative charge Q floating in an electric field E that points in the downward direction. Since the droplet is floating, then $V = V_{stop}$. From the result in 2.1.1, we then have $E = \frac{V_{stop}}{d}$. From equation (2), the equation motion is:

$$g(m_{oil} - m_{air}) - Q\frac{V_{stop}}{d} = 0.$$

But $m_{oil} = \mathcal{V}\rho_{oil}$ and $m_{air} = \mathcal{V}\rho_{air}$ where $\mathcal{V}$ is the volume of the oil droplet. We can re-write,

$$\mathcal{V}(\rho_{oil} - \rho_{air})g - Q\frac{V_{stop}}{d} = 0$$

$$\boxed{\tfrac{4}{3}\pi r^3(\rho_{oil} - \rho_{air})g - Q\tfrac{V_{stop}}{d} = 0.}$$

5.1.3 Forces on a Charged Oil Droplet Falling in a Field-Free Space

The diagram to the left is a free body diagram of an oil droplet with negative charge Q falling through a field-free space with terminal veloity v_t. From equation (1), the equation of motion is:

$$\boxed{\tfrac{4}{3}\pi r^3(\rho_{oil} - \rho_{air})g - 6\pi r\eta v_t = 0.}$$

We can manipulate this equation and write r in terms of the other constants as follows:

$$\frac{4}{3}\pi r^2(\rho_{oil} - \rho air)g = 6\pi\eta v_t$$

$$r^2 = \frac{9}{2}\frac{\eta}{g(\rho_{oil} - \rho air)}v_t$$

$$r = \sqrt{\frac{9}{2}\frac{\eta}{g(\rho_{oil} - \rho air)}v_t}.$$

5.1.4 Combining the Results from 2.1.2 and 2.1.3

We can set the equations of motion from 2.1.2 and 2.1.3 equal to each other and solve for Q:

$$\frac{4}{3}\pi r^3(\rho_{oil} - \rho_{air})g - 6\pi r\eta v_t = \frac{4}{3}\pi r^3(\rho_{oil} - \rho_{air})g - Q\frac{V_{stop}}{d}$$

$$6\pi r\eta v_t = Q\frac{V_{stop}}{d}$$

$$Q = \frac{6\pi\eta d}{V_{stop}}v_t r = \frac{6\pi\eta d}{V_{stop}}v_t\sqrt{\frac{9}{2}\frac{\eta}{g(\rho_{oil} - \rho_{air})}v_t}$$

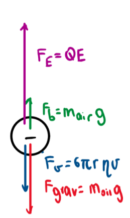
$$Q = 6\pi d\sqrt{\frac{9}{2}\frac{\eta^3}{g(\rho_{oil} - \rho_{air})}}\frac{v_t^{\frac{3}{2}}}{V_{stop}}$$

Lastly, we can plug in the numerical values of the constants for $t^o = 20^oC$:

$$Q = 6\pi(0.006m)\sqrt{\frac{9}{2}\frac{(1.827\times10^{-5}\,Pa\,s)^3}{(9.80\frac{m}{s^2})(875.3\frac{kg}{m^3} - 1.204\frac{kg}{m^3})}}\frac{v_t^{\frac{3}{2}}}{V_{stop}}$$

$$\boxed{Q = 2.0\cdot10^{-10}\tfrac{v_t^{\frac{3}{2}}}{V_{stop}}}$$

5.2 "Exercise 2" - Derivation of Q for "Method 2"



For an oil droplet with negative charge Q moving upward at a constant velocity v in a uniform electric field E between capacitor plates at a definite potential V_{up}, the equation of motion is:

$$\boxed{\tfrac{4}{3}\pi r^3(\rho_{oil} - \rho_{air})g - Q\tfrac{V_{up}}{d} + 6\pi rv\eta = 0}$$

Setting this equal to the equation of motion from 2.1.3 and solving for Q:

$$\frac{4}{3}\pi r^3(\rho_{oil}-\rho_{air})g - Q\frac{V_{up}}{d} + 6\pi r v\eta = \frac{4}{3}\pi r^3(\rho_{oil}-\rho_{air})g - 6\pi r\eta v_t$$

$$Q\frac{V_{up}}{d} = 6\pi r\eta(v_t+v)$$

$$Q = 6\pi\eta d r\frac{(v_t+v)}{V_{up}}$$

$$Q = 6\pi d\sqrt{\frac{9}{2}\frac{\eta^3}{g(\rho_{oil}-\rho air)}}\frac{(v_t+v)}{V_{up}}\sqrt{v_t}$$

Plugging in the same constants as before:

$$\boxed{Q = 2.0\cdot 10^{-10}\tfrac{(v_t+v)}{V_{up}}\sqrt{v_t}}$$

5.3 Error Propagation for ΔQ

With error propagation, the uncertainty in each calculated velocity is $\Delta v_t = \frac{\Delta d}{d}v_t$ with $\Delta d = \frac{\Delta c}{c}d$ where $\Delta c = \pm 1$ is the uncertainty in calibration factor c of the corresponding camera. Therefore, $\Delta v_t = \frac{\Delta c}{c}v_t$. We assigned $\Delta V_{stop} = \pm 20V$, which is twice the distance between two markings on the DC power supply. We chose this large uncertainty for V_{stop} because it was quite hard to read the voltage due to the unsteady arm of the power supply voltmeter. With error propagation, the uncertainty of each value of Q is:

$$\Delta Q = \sqrt{\left(\frac{3\cdot 10^{-10}\cdot v_t^{1/2}}{V_{stop}}\Delta v_t\right)^2 + \left(\frac{2\cdot 10^{-10}\cdot v_t^{3/2}}{V_{stop}^2}\Delta V_{stop}\right)^2}$$

$$= \sqrt{\left(\frac{3\cdot 10^{-10}\cdot v_t^{1/2}}{V_{stop}}\frac{\Delta c}{c}v_t\right)^2 + \left(\frac{2\cdot 10^{-10}\cdot v_t^{3/2}}{V_{stop}^2}(20V)\right)^2}.$$

5.4 Error Propagation for e_i and e_{final}

To extract values values of e_i from each group q_i, we used $q_i = i\times e_i$. We then get the relation $e_i = \frac{q_i}{i}$. From this, we did error propagation as:

$$\left(\frac{\Delta e_i}{e_i}\right)^2 = \left(\frac{\Delta q_i}{q_i}\right)^2$$

$$\Delta e_i = \frac{\Delta q_i}{q_i}e_i.$$

Table (1) shows the values of e_i and their uncertainties. Lastly, we propagated the error Δe_{final} as follows:

$$e_{final} = \frac{(e_2+e_3+e_4+e_5+e_6+e_7+e_8+e_9+e_{10}+e_{12})}{10} = \frac{x}{y}$$

$$\left(\frac{\Delta e_{final}}{e_{final}}\right)^2 = \left(\frac{\Delta x}{x}\right)^2 + \left(\frac{\Delta y}{y}\right)^2 = \left(\frac{\Delta x}{x}\right)^2$$

$$\Delta e_{final} = \frac{\sqrt{\Delta e_2^2+\Delta e_3^2+\Delta e_4^2+\Delta e_5^2+\Delta e_6^2+\Delta e_7^2+\Delta e_8^2+\Delta e_9^2+\Delta e_{10}^2+\Delta e_{12}^2}}{e_2+e_3+e_4+e_5+e_6+e_7+e_8+e_9+e_{10}+e_{12}}e_{final}.$$

5.5 Data Table

The highlighted data points were removed, as discussed in the analysis.

Trial	Camera	V_stop (V)	ΔV_stop (+- V	v_t (m/s)	Δv_t (+- m/s)	Q (c)	ΔQ (+- C)
1	R	300	20	1.16E-04	2.24E-07	8.38E-19	5.59E-20
2	L	150	20	6.02E-05	1.11E-07	6.23E-19	8.30E-20
3	L	200	20	1.10E-04	2.04E-07	1.15E-18	1.15E-19
4	L	250	20	5.21E-05	9.66E-08	3.01E-19	2.41E-20
5	L	300	20	8.28E-05	1.53E-07	5.02E-19	3.35E-20
6	L	200	20	7.99E-05	1.48E-07	7.14E-19	7.15E-20
7	R	350	20	9.12E-05	1.75E-07	4.98E-19	2.85E-20
8	R	250	20	6.41E-05	1.23E-07	4.11E-19	3.29E-20
9	R	100	20	6.00E-05	1.15E-07	9.30E-19	1.86E-19
10	L	140	20	5.89E-05	1.09E-07	6.46E-19	9.23E-20
11	L	350	20	8.33E-05	1.54E-07	4.35E-19	2.49E-20
12	L	150	20	1.24E-04	2.30E-07	1.84E-18	2.45E-19
13	L	100	20	4.22E-05	7.81E-08	5.47E-19	1.09E-19
14	L	160	20	8.89E-05	1.65E-07	1.05E-18	1.31E-19
15	L	210	20	6.01E-05	1.11E-07	4.44E-19	4.23E-20
16	R	200	20	6.26E-05	1.20E-07	4.95E-19	4.95E-20
17	R	700	20	1.99E-04	3.84E-07	8.05E-19	2.31E-20
18	L	90	20	8.21E-05	1.52E-07	1.65E-18	3.68E-19
19	L	160	20	8.45E-05	1.57E-07	9.72E-19	1.21E-19
20	R	130	20	5.44E-05	1.05E-07	6.18E-19	9.50E-20
21	L	100	20	5.16E-05	9.56E-08	7.42E-19	1.48E-19
22	R	70	20	8.26E-05	1.59E-07	2.15E-18	6.13E-19
23	R	170	20	6.14E-05	1.18E-07	5.66E-19	6.66E-20
24	R	280	20	7.63E-05	1.47E-07	4.76E-19	3.40E-20
25	L	160	20	6.92E-05	1.28E-07	7.19E-19	8.99E-20
26	R	120	20	4.83E-05	9.29E-08	5.59E-19	9.32E-20
27	R	260	20	6.84E-05	1.31E-07	4.35E-19	3.35E-20
28	L	270	20	5.37E-05	9.95E-08	2.92E-19	2.16E-20
29	L	250	20	1.37E-04	2.54E-07	1.29E-18	1.03E-19
30	L	230	20	7.60E-05	1.41E-07	5.76E-19	5.01E-20
31	L	160	20	7.86E-05	1.45E-07	8.70E-19	1.09E-19
32	L	160	20	6.97E-05	1.29E-07	7.28E-19	9.10E-20
33	R	180	20	7.08E-05	1.36E-07	6.61E-19	7.35E-20
34	L	120	20	5.24E-05	9.70E-08	6.32E-19	1.05E-19
35	L	400	20	1.12E-04	2.08E-07	5.96E-19	2.99E-20
36	L	170	20	6.28E-05	1.16E-07	5.85E-19	6.89E-20
37	R	500	20	1.36E-04	2.61E-07	6.34E-19	2.54E-20
38	R	380	20	1.36E-04	4.62E-07	1.96E-18	1.03E-19
39	R	370	20	1.93E-04	3.71E-07	1.45E-18	7.85E-20
40	R	400	20	1.04E-04	1.99E-07	5.27E-19	2.64E-20
41	R	100	20	5.38E-05	1.04E-07	7.90E-19	1.58E-19
42	R	210	20	8.31E-05	1.60E-07	7.21E-19	6.87E-20
43	R	190	20	6.56E-05	1.26E-07	5.60E-19	5.89E-20
44	R	390	20	9.52E-05	1.83E-07	4.77E-19	2.45E-20
45	R	230	20	6.40E-05	1.23E-07	4.45E-19	3.87E-20
46	R	200	20	6.89E-05	1.33E-07	5.72E-19	5.72E-20
47	R	140	20	1.07E-04	2.05E-07	1.58E-18	2.25E-19
48	R	270	20	1.24E-04	2.39E-07	1.03E-18	7.63E-20
49	R	100	20	5.32E-05	1.02E-07	7.76E-19	1.55E-19
50	R	540	20	1.86E-04	3.59E-07	9.43E-19	3.50E-20
51	R	210	20	6.79E-05	1.31E-07	5.33E-19	5.08E-20
52	R	300	20	1.32E-04	2.53E-07	1.01E-18	6.71E-20

5.6 Code Outputs

```
This array contains the charge of each droplet: [8.38006659e-19
6.22548424e-19 1.15436211e-18 3.01256877e-19
 5.01876881e-19 7.14495894e-19 4.97611487e-19 4.10585010e-19
 9.30240772e-19 6.45682718e-19 4.34700442e-19 1.84019880e-18
 5.47378988e-19 1.04756560e-18 4.44125851e-19 4.94706134e-19
 8.04670984e-19 1.65436680e-18 9.71653832e-19 6.17555536e-19
 7.41650873e-19 2.14624588e-18 5.66221440e-19 4.76019238e-19
 7.19226260e-19 5.59190358e-19 4.34923461e-19 2.91522842e-19
 1.28769345e-18 5.76167826e-19 8.70443361e-19 7.27787274e-19
 6.61260715e-19 6.31842501e-19 5.96095303e-19 5.85079805e-19
 6.33690046e-19 1.96158171e-18 1.44954448e-18 5.26979783e-19
 7.89875787e-19 7.21348453e-19 5.59603432e-19 4.76694927e-19
 4.45134358e-19 5.72039211e-19 1.57621635e-18 1.02870815e-18
 7.76417910e-19 9.42967579e-19 5.33240287e-19 1.00594550e-18]

This array contains the uncertainty in the charge of each droplet:
[5.59193841e-20 8.30244682e-20 1.15480738e-19 2.41150740e-20
 3.34874899e-20 7.14771495e-20 2.84711496e-20 3.28681468e-20
 1.86067505e-19 9.22578241e-20 2.48693570e-20 2.45413080e-19
 1.09486356e-19 1.30978029e-19 4.23156875e-20 4.94911914e-20
 2.31074767e-20 3.67665788e-19 1.21486715e-19 9.50252433e-20
 1.48344481e-19 6.13244360e-19 6.66343081e-20 3.40290895e-20
 8.99254781e-20 9.32123510e-20 3.34791661e-20 2.16094627e-20
 1.03077557e-19 5.01271066e-20 1.08832282e-19 9.09958690e-20
 7.34981693e-20 1.05321708e-19 2.98507247e-20 6.88521022e-20
 2.54134281e-20 1.03396088e-19 7.84652467e-20 2.63928027e-20
 1.57991588e-19 6.87313577e-20 5.89277385e-20 2.44845373e-20
 3.87286275e-20 5.72277158e-20 2.25219665e-19 7.62583607e-20
 1.55299733e-19 3.50304919e-20 5.08080787e-20 6.71257829e-20]

This array contains the values of q_i: [3.0125687652798655e-19,
4.936375802591823e-19, 6.115810101732934e-19, 7.710908210174508e-19,
9.710749707394531e-19, 1.1009638567017503e-18, 1.2876934545719352e-18,
1.560042544029113e-18, 1.690260649865263e-18, 1.804256956543601e-18,
2.053913792518246e-18]

This array contains the uncertainties in the values of q_i [0.0,
1.3266268146094475e-20, 1.3046556707647091e-20, 1.6488415776426253e-20,
2.338745439187464e-20, 5.339825494390167e-20, 0.0, 5.96775783359853e-20,
7.828997222371305e-20, 1.2672733648150484e-19, 9.233208406036868e-20]

This array contains the values of e_i extracted from each group q_i:
[1.5062843826399327e-19, 1.645458600863941e-19, 1.5289525254332336e-19,
1.5421816420349018e-19, 1.618458284565755e-19, 1.572805509573929e-19,
1.609616818214919e-19, 1.7333806044767922e-19, 1.690260649865263e-19,
1.7115948270985382e-19]

This array contains the uncertainties in e_i values [0.0,
4.422089382031492e-21, 3.261639176911773e-21, 3.297683155285251e-21,
3.89790906531244e-21, 7.628322134843097e-21, 0.0, 6.630842037331701e-21,
7.828997222371306e-21, 7.694340338364056e-21]

THE FINAL VALUE OF THE ELEMENTARY CHARGE: 1.6158993844767205e-19 C +-
1.6700808644032616e-21 C

The typical droplet in our experiment had a radius of approximately 0.9
microns.
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