PHY224H1: Electron Charge to Mass Ratio Experiment

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Abstract

In this experiment, the behavior of a beam of electrons in uniform magnetic fields perpendicular to the electron velocity was studied in order to determine the charge to mass ratio of an electron. By accelerating a beam of electrons through a constant potential difference, placing the beam in various uniform magnetic fields induced by different currents through two Helmholtz Coils, and measuring the diameters of electron orbits as well as the corresponding currents through Helmholtz coils, the charge to mass ratio of an electron was found to be $(1.9 \pm 0.3) \times 10^{11}\ \frac{C}{kg}$. This value was found to agree to within 6% of the theoretical value of $1.8 \times 10^{11}\ \frac{C}{kg}$.

1 Introduction

1.1 Theory

An electron moving in an induced magnetic field will experience a force $\vec{F}$ perpendicular to both the electron velocity and the magnetic field. This force is given by

$$\vec{F} = e\vec{v} \times \vec{B} \tag{1}$$

where $\vec{B}$ is the induced magnetic field, e is the charge of an electron, and $\vec{v}$ is the velocity of the electron. In the special case where the magnetic field is uniform and the electron velocity is perpendicular to the direction of the field, the electron will travel in a closed circular path. Taking the acceleration of the electron to be purely centripetal we have,

$$evB = m\frac{v^2}{r} \tag{2}$$

where r is the radius of the electron orbit.

An electron can reach velocity v when accelerated through a potential difference V. In the non-relativistic approximation, the kinetic energy of the electron is then given by

$$eV = \frac{1}{2}mv^2. \tag{3}$$

Further, we can find an expression for the curvature of the electron path by combining equations (2) and (3) and finding

$$\frac{1}{r} = \sqrt{\frac{e}{2m}}\frac{B}{\sqrt{V}}. \tag{4}$$

Additionally, a current going through a pair of Helmholtz coils distanced one radius apart induces an approximately uniform magnetic field $\vec{B}$ along the coil axis at the center. The total magnetic field at the center of the coils is then

$$B = B_c + B_e \tag{5}$$

where B_c is the field induced by the coils and B_e is the net external field due to other magnetic systems near the coils. Therefore, equation (4) becomes

$$\frac{1}{r} = \sqrt{\frac{e}{2m}}\frac{1}{\sqrt{V}}(B_c + B_e). \tag{6}$$

The central field due to two Helmholtz coils is given by

$$B_c = \left(\frac{4}{5}\right)^{3/2} \frac{\mu_o n I}{R} \tag{7}$$

where $\mu_o = 4\pi \times 10^{-7} \frac{Wb}{Am}$, R is the radius of the coils, and n is the number of turns in each coil. Then, equation (4) finally becomes

$$\frac{1}{r} = \sqrt{\frac{e}{m}}\frac{1}{\sqrt{V}}k(I - I_o) \tag{8}$$

where $k = \frac{1}{\sqrt{2}}\left(\frac{4}{5}\right)^{3/2}\frac{\mu_o n}{R}$, and $I_0 = \frac{B_e}{k}$.

Lastly, it is important to note that equation 7 becomes an insufficient approximation for the magnetic field produced by the Helmholtz coils as we get further away from the coil axis. For a distance $\rho > 0.2R$ from the axis, the magnetic field due to the Helmholtz coils becomes

$$B(\rho) = \left(1 - \frac{\rho^4}{R^4\left(0.6583 + 0.29\frac{\rho^2}{R^2}\right)^2}\right)B(0) \tag{9}$$

where $B(0)$ is the field exactly at the coil axis given by equation 7.

1.2 The Experiment

The purpose of this experiment is to experimentally determine the charge to mass ratio $\frac{e}{m}$ of an electron by analyzing the behavior of a beam of electrons moving perpendicular to various uniform magnetic fields produced by a pair of Helmholtz coils. Specifically, a beam of electrons will be accelerated through a constant potential difference V, placed between the two Helmholtz coils, and the diameters of the electron orbits as well as the various currents through the Helmholtz coils will be measured. First, these measurements will be fit using a rearranged version of equation (6) to yield a y-intercept proportional to the external magnetic field B_e. Second, in conjunction with the results from the previous fit, the same measurements will be fit with equation (8) to yield a slope in order to find the charge to mass ratio $\frac{e}{m}$.

2 Materials and Methods

2.1 Materials

Equipment:

- x2 Data Precision 2450 Digital Multimeter
- x1 10V DC source
- x1 Glass tube on stand with power supply ports containing hydrogen gas at low pressure
- x1 Electron gun
- x1 6.3V DC source

- x1 0-300V Varying voltage DC source
- x2 Helmholtz coils of radius 16.3 cm with 130 turns
- x1 Rheostat
- x8 Banana cables
- x1 Self-illuminated scale
- x1 Plastic reflector
- x1 Meter stick

2.2 Methods

First, the radius of the Helmholtz coils were measured using the meter stick provided. The value of the radius as well as the number of turns on the coils were both recorded for future calculations. Then, using eight banana cables, the apparatus was connected as shown on Figure 1. In this setup, there were three circuits each connected to the power supply ports of the stand, which are connected to the Helmholtz coils and the electron gun filament. In the first circuit, the 10V DC source, the rheostat, and the DC ammeter were connected in series to the stand. In the second, the electron gun filament mounted inside the glass tube was connected to the 6.3V DC source. In the third circuit, the 0-300V DC source, which is connected in parallel to the DC voltmeter, is connected to the stand.

The lights in the room were switched off. The 10V DC source, the DC ammeter, the DC voltmeter, and the electron gun filament were all switched on. After waiting for thirty seconds, the 0-300V source was also switched on. The voltage knob was adjusted to increase the potential until a beam of electrons were visible within the glass bulb. The potential was varied to find an appropriate value of voltage so that the diameter of the beam always stayed within the length of the scale and was visible across a decent range of currents. The voltage was finally fixed at 275 Volts. The bulb was rotated until a vertical and circular electron beam was observed, and the electron gun was moved sideways to ensure the beam was at the center of the two coils. The self-illuminated scale was switched on, pushed close to the stand, and adjusted to align with the beam and the eye level of the observer.

The current through the Helmholtz coils was varied fifteen times by adjusting the rheostat. For each current value, the diameter of the circular electron beam path was measured using the scale following the same steps as before. For each current, the current reading along with its reading error, and the diameter of the corresponding beam path were recorded.

Lastly, the voltage supplied by the 0-300V source was turned down to zero. After the 10V and 0-300V power supplies, the voltmeter, and the ammeter were all switched off, the electron gun filament was also switched off. The banana cables were disconnected from the apparatus, and returned.

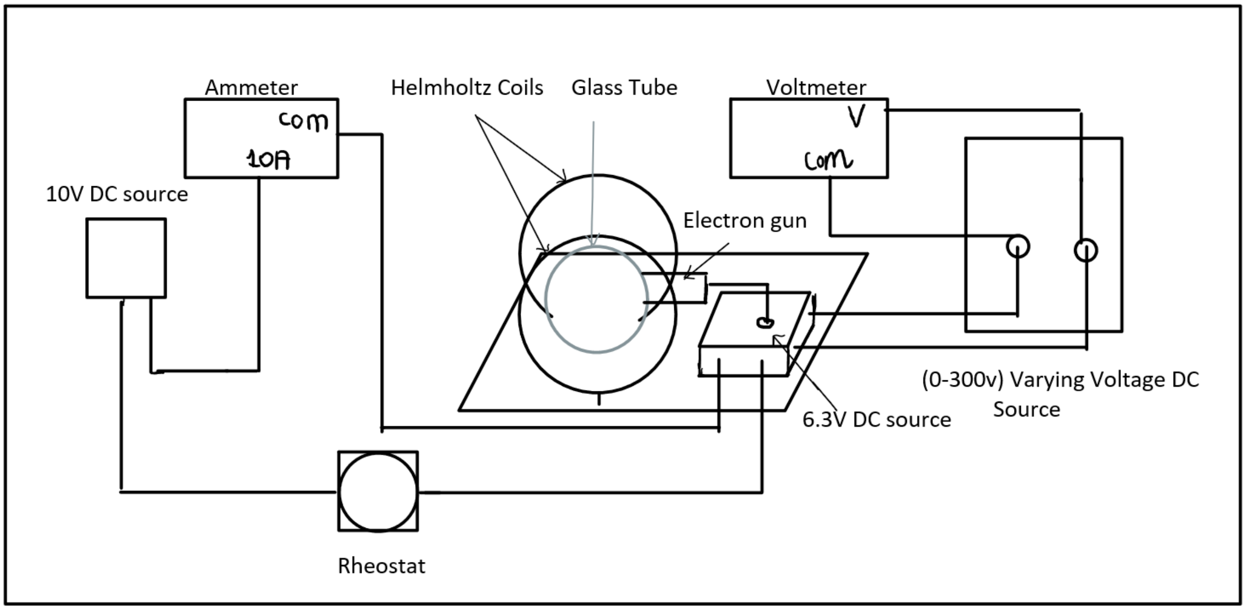

Figure (1): A diagram illustrating the apparatus setup.

3 Data and Analysis

3.1 Converting Data to the Relevant Parameters

The first step in our analysis was to convert our measurements of electron orbit diameters and Helmholtz currents to values of orbit curvatures and Helmholtz magnetic field strengths, respectively. As explained in section 2.2, before we measured the diameter of a given electron orbit, we ensured that the axis of the electron beam ring was aligned with the axis of the Helmholtz coils every time. Therefore, the distance from any point on the circular beam to the Helmholtz axis was approximately equal to the radius of the circular beam. Every single one of our diameter measurements yielded radii larger than $0.2R$. Therefore, with our current measurements, we found the Helmholtz magnetic field values using equations 7 and 9 as

$$B(\rho) = B_c = \alpha\frac{I}{R} - \frac{\alpha\rho^4 I}{R^5\left(0.6583 + 0.29\frac{\rho^2}{R^2}\right)^2} \tag{10}$$

where $\alpha = \left(\frac{4}{5}\right)^{3/2}\mu_o n$. Then, we converted our measurements of electron orbit diameters to values of curvature by finding the reciprocals of the corresponding radii. Lastly, we performed error propagation to find the uncertainties associated with our values of Helmholtz magnetic field strengths and electron orbit curvatures.

3.2 Finding the External Field

To find the external magnetic field in the room, we rearranged equation 6 as

$$B_c = \sqrt{\frac{2mV}{e}}\frac{1}{r} - B_e. \tag{11}$$

Here, V is fixed at $275\,V$, $\frac{1}{r}$ is our independent variable, and B_c is our dependent variable whose values and uncertainties were previously found with equation 10. Then, we created a model function

$$f(x, a, b) = ax + b,$$

where the fixed parameters a and b model the fixed quantities $\sqrt{\frac{2mV}{e}}$ and $-B_e$ in equation 11, respectively. Using the `curve_fit()` function in Python, we optimized parameters a and b. The optimal value of the relevant parameter b was found to be $(-1.2 \pm 0.2) \times 10^{-4}$. Using the relation $-B_e = b$, the optimal value of the external field B_e was found to be $120\,000\,nT \pm 20\,000\,nT$. The figure below shows our experimentally calculated values of B_c along with uncertainties, as well as the discussed model curve.

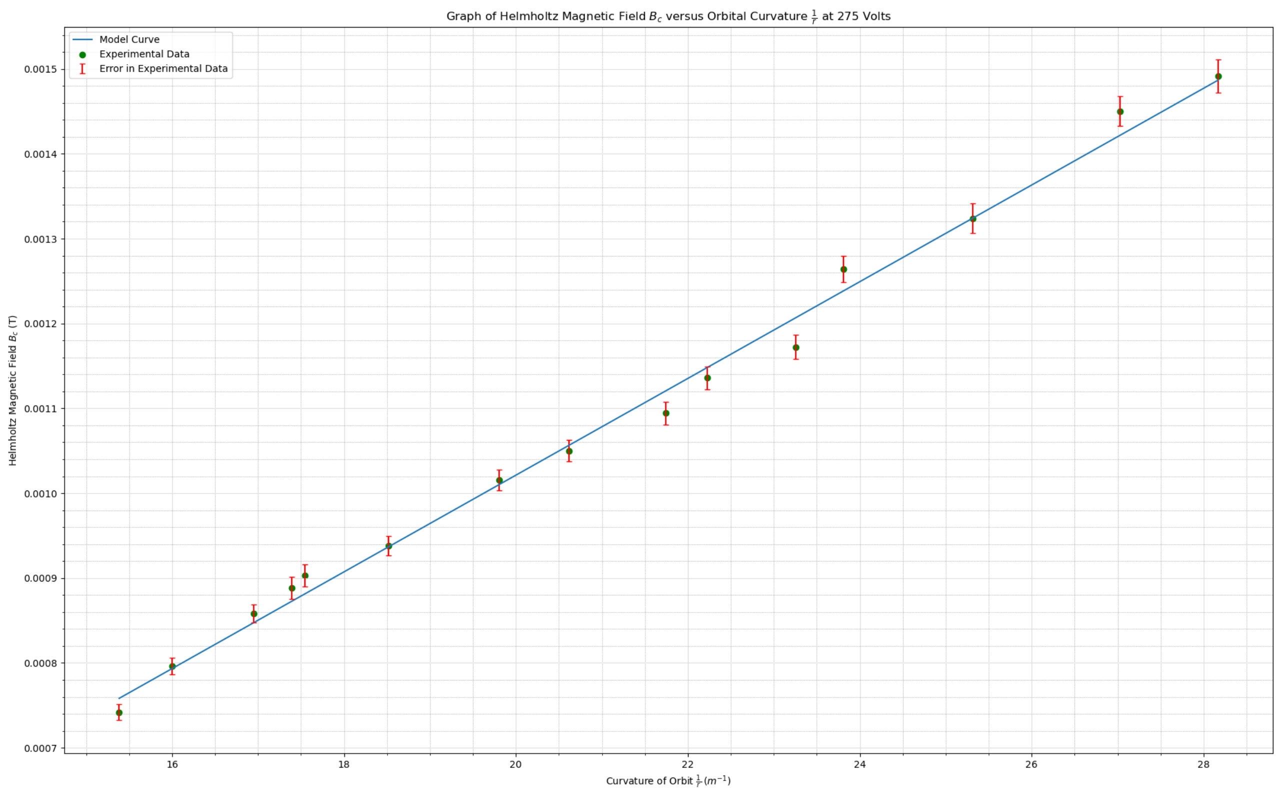

Figure (3): Helmholtz Magnetic field B_c vs. Orbital Curvature $\frac{1}{r}$ at 275 V.

3.3 Finding the Electron Charge to Mass Ratio

To find the charge to mass ratio of an electron, we performed linear fitting on equation 8. In equation 8, V is again fixed at $275\,V$, I is our independent variable, and $\frac{1}{r}$ is our dependent variable. We created a model function

$$f(x, a, b) = a(x + b)$$

where the fixed parameters a and b model the fixed quantities $\sqrt{\frac{e}{m}}\frac{1}{\sqrt{V}}k$ and $-I_o$ in equation 8, respectively. This time, we provided guesses to the `curve_fit()` function as to what parameters a and b should be. Our guess for a was calculated using the accepted values of e and m, the previously defined value of k, and the fixed voltage at $275V$. Our guess for b was $-I_o = \frac{-B_e}{k}$ where B_e is the optimal value calculated in the previous section. The optimal value of the relevant parameter a was found to be 13 ± 1. Using the relation $a = \sqrt{\frac{e}{mV}}k$ and performing error propagation, the final value of the charge to mass ratio of an electron was found to be $(1.9 \pm 0.3) \times 10^{11}\,\frac{C}{kg}$. The figure below shows our experimentally calculated values of curvature $\frac{1}{r}$ along with uncertainties, as well as the discussed model curve.

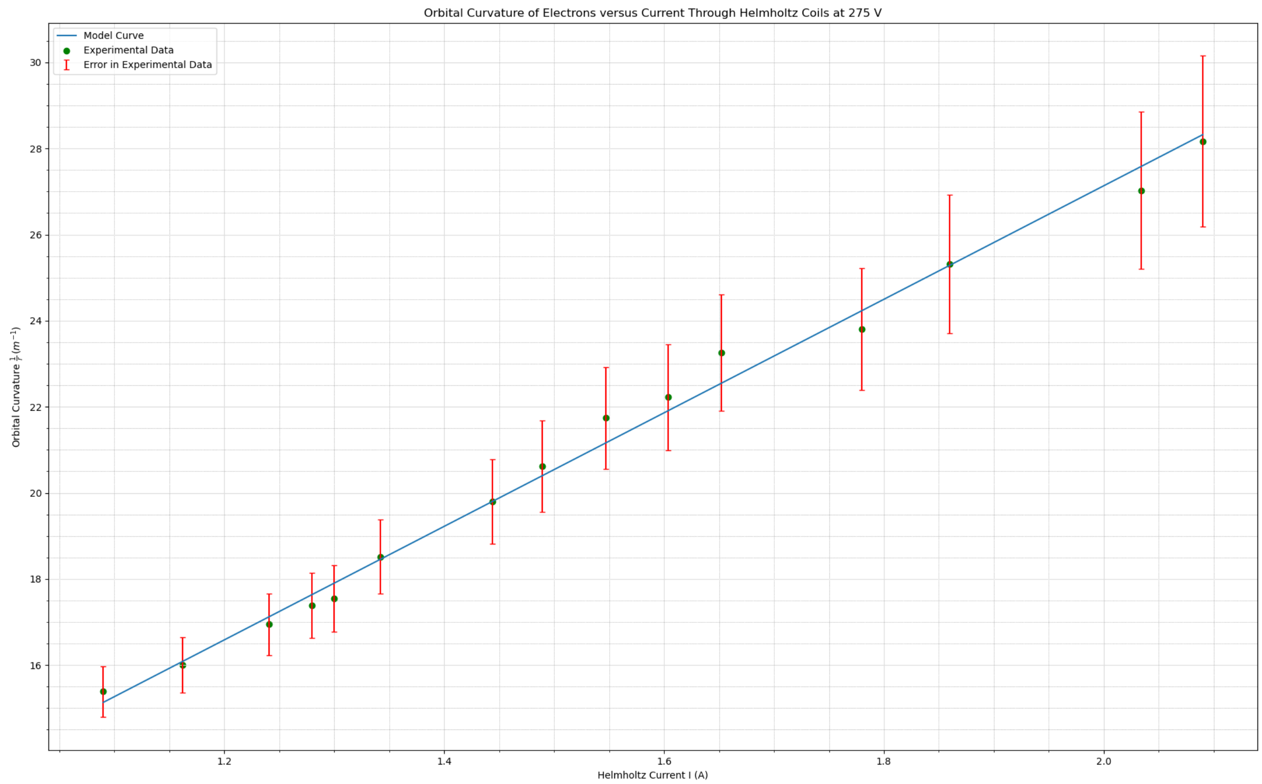

Figure (4): Orbital Curvature of Electrons vs. Helmholtz Current at 275 V

3.3.1 Reduced χ^2 Values

Lastly, our reduced χ^2 value for the linear fitting performed on equation 11 was found to be 2.0. Our reduced χ^2 for the linear fitting performed on equation 8 was found to be 0.1.

4 Discussion and Conclusion

4.1 Goodness of Fits: χ^2 Values

As discussed in section 3, we first performed curve fitting on equation 11 to find the external magnetic field in the room, then on equation 8 to find the charge to mass ratio of an electron. The reduced χ^2 value for the fit corresponding to equation 11 was 2.0, which deviates from the ideal value of 1.0. Looking at Figure 3, we see that this is because the vertical distance from most of the data points to the model curve is larger than the corresponding uncertainties. Hence, due to the relatively small uncertainties in values of B_c, we have that $\chi^2 > 1$. However, since 2.0 is still close to the ideal value, the model is a decent fit.

The reduced χ^2 value for the fit corresponding to equation 8 was 0.1, which deviates from the ideal value in the opposite direction. Looking at Figure 4, we see that the uncertainties associated with the data points are much larger than the vertical distances from the points to the model curve. This is due to the large reading error associated with measuring the diameter of an electron beam. Hence, we have that $\chi^2 < 1$. This small value also indicates that the model is fitting the external magnetic field in the room as well as the Helmholtz magnetic field.

4.2 The External Magnetic Field

The net magnetic field applied on the electron beams was a result of the Helmholtz magnetic field, the Earth's magnetic field, and various magnetic fields due to the ferromagnetic materials in the room. As discussed in section 3.2, we found the net external magnetic field B_e, not produced by the Helmholtz coils, to be $120\,000\,nT \pm 20\,000\,nT$. As the Earth's magnetic field strength is approximately between $25\,000\,nT$ and $65\,000\,nT$, we notice that this value is larger than the Earth's magnetic field within its uncertainty. This result is expected as the experiment setup itself consists of other sources of magnetic fields. The current flowing through other parts of the setup such as the banana cables also induces a magnetic field in the vicinity of the beam. For example, if we approximate the current through the banana cables as steady, and the distance from a given banana cable to the beam as ≈ 20 cm, the magnetic field produced by the current is $\approx 2000\,nT$ in the range of the current values we worked with.

There were also magnetic fields produced by other ferromagnetic materials in the room such as cellphones. To test this, we brought a cellphone near the glass bulb, and saw that the electron trajectory was disturbed as the beam became a cloudy ring as opposed to the sharply defined circular beam it was previously. Considering the drastic effect of the cellphone in this case, we can conclude that the effect of the fields due to multiple electronic devices, at most half a meter from the beam, was significant.

4.3 The Anomalous Beam Behavior at Extremes

As we were trying to choose the value at which we would fix the accelerating voltage, we noticed that in the case of a voltage below $\approx 60\,V$ and a high current of $\approx 2\,A$, the electron beam became less visible at every part of its trajectory. This is because the low kinetic energy electrons, faced with a very large magnetic field, are unable to excite the hydrogen gas with collisions. Therefore, we avoided using a low accelerating voltage to minimize our reading errors.

4.4 Additional Sources of Error

A challenge we faced while taking measurements was to reduce the problems caused by parallax when taking measurements of the orbit diameters. The problem of parallax occurs when the line of sight to your scale is at an angle. To avoid this we had to ensure that we were reading the self-illuminated scale at eye-level. This was achieved by ensuring that the self-illuminated scale stayed at its adjusted height and remained at the same distance from the viewer and the electron curvature for each measurement.

Another source of error was that since it took us a while to find the appropriate accelerating voltage and take our measurements, the voltmeter used in the experiment had enough time to accumulate charge. This results in voltage output value fluctuations. Hence, the fixed voltage value we have used in all of our calculations might have been slightly off. We could have minimized this error by turning all the devices off, letting the voltmeter neutralize, then *immediately* tuning the voltage up to $275\,V$ and taking measurements instead of spending time trying to find an appropriate voltage.

4.5 Summary and Conclusion

In this experiment, the behavior of a beam of electrons moving perpendicular to various uniform magnetic fields was studied to determine the charge to mass ratio of an electron. Specifically, a beam of electrons was accelerated through a fixed potential difference of $275\ V$ and placed in a uniform magnetic field produced by a pair of Helmholtz coils. The magnetic field was varied fifteen times by changing the current through the Helmholtz coils, and measurements of these currents as well as the diameters of the corresponding circular electron paths were taken. By fitting the appropriately converted data to equation 8, the charge to mass ratio of an electron was found to be $(1.9 \pm 0.3) \times 10^{11}\,\frac{C}{kg}$. This experimental value was found to be both accurate and precise, as it agrees with the theoretical value of $1.8 \times 10^{11}\,\frac{C}{kg}$ to within 6%, and the theoretical value is within the uncertainty range of the experimental value. Therefore, through the experimental method employed the value of the constant was successfully determined.

5 Appendix

5.1 Error Propagation for Values of Curvature

When measuring the electron orbit diameters, it was quite hard to read the self-illuminated scale due to parallax even after adjusting the scale stand to minimize this issue. Because of this source of large reading error, we chose the uncertainty in each measurement to be 5 times the distance between the smallest markings on the scale. The smallest markings were each 1 mm apart, thus we chose the error in each measurement to be ± 5 mm.

The error propagation equation for the multiplication or division of two independent quantities is

$$\left(\frac{\Delta z}{z}\right)^2 = \left(\frac{\Delta x}{x}\right)^2 + \left(\frac{\Delta y}{y}\right)^2. \tag{12}$$

For

$$r = \frac{d}{2},$$

where r is the orbital radius and d is the orbital diameter, we have

$$\Delta r = \frac{\Delta d}{d} r. \tag{13}$$

For

$$c = \frac{1}{r},$$

where c is the curvature of orbit, and r is the radius of orbit, we have

$$\Delta c = \frac{\Delta r}{r} c. \tag{14}$$

In our Python code, we used equations 11 and 12 to propagate error for values of curvature.

5.2 Error Propagation for Values of the Helmholtz Magnetic Field

We used equation 10 to find values of B_c, and we see that the parameters I, R, and ρ have uncertainties associated with them.

The calculus error propagation equation is

$$\Delta z^2 = \sum_i \left(\frac{\partial f}{\partial x_i} \Delta x_i\right)^2. \tag{15}$$

Therefore, we propagated the error on equation 10 as

$$\Delta B_c = \sqrt{\left(\frac{\partial f}{\partial I} \Delta I\right)^2 + \left(\frac{\partial f}{\partial R} \Delta R\right)^2 + \left(\frac{\partial f}{\partial \rho} \Delta \rho\right)^2} \tag{16}$$

where $f = B_c$. In order to find the partial derivatives, we used an online derivative calculator. The equations for these derivatives can be found in the Python code attached to the submission of this report.

5.3 Error Propagation for the Charge to Mass Ratio

The relationship between parameter a and the ratio $\frac{e}{m}$ in section 3.2 is given by

$$\frac{e}{m} = (\frac{1}{k^2}a^2)V. \tag{17}$$

Then, using equation 15, the error in the charge to mass ratio is

$$\Delta(\frac{e}{m}) = \sqrt{\left(\frac{2aV}{k^2}\Delta a\right)^2 + \left(\frac{a^2}{k^2}\Delta V\right)^2 + \left(\frac{-2a^2V}{k^3}\Delta k\right)^2} \tag{18}$$

where Δa is the computationally calculated error in parameter a, ΔV is a reading error of $\pm 1\,V$ due to large fluctuations, and $\Delta k = \frac{\Delta R}{R}k$

5.4 Data Table

Current (A)	Diameter (cm)	Current Uncertainty (A)	Diameter Uncertainty (cm)
2.09	7.1	0.01	0.5
2.034	7.4	0.001	0.5
1.86	7.9	0.01	0.5
1.780	8.4	0.001	0.5
1.652	8.6	0.001	0.5
1.604	9.0	0.001	0.5
1.547	9.2	0.001	0.5
1.489	9.7	0.001	0.5
1.444	10.1	0.001	0.5
1.342	10.8	0.001	0.5
1.30	11.4	0.01	0.5
1.28	11.5	0.01	0.5
1.241	11.8	0.001	0.5
1.162	12.5	0.001	0.5
1.090	13.0	0.001	0.5

Figure (5): Data collected at a fixed potential of $275V \pm 1V$ on the 0-300V DC source.

5.5 Code Output

```
FINDING THE EXTERNAL FIELD
The optimized parameter b is -0.00011834170675440328 +-
1.8363961947778015e-05
Therefore, the calculated value of the external field B_e
is: 118341.70675440328 nT +- 18363.961947778014 nT.

FINDING THE CHARGE TO MASS RATIO
The optimized value of parameter a is 13.187234082468445 +-
1.0350502775632842
The charge to mass ratio of an electron is:
185981152750.3183 C/kg +- 29557197888.354557 C/kg.

Recuced chi squared for the fit used to calculate B_e is
2.0
Reduced chi squared for the fit used to calculate e/m is
0.1
```

Figure (6): Code output from the Python analysis.