PHY407F: Explanatory Notes For Lab 5

Idil Yaktubay and Souren Salehi

16 October 2022

1 Question 1 - By Souren Salehi and Idil Yaktubay

In this question, we have extracted a frequency spectrum from a time series generated from the simulation of a relativistic spring system. **(a)** First, we have used the Euler-Cromer method with zero initial velocity and three different initial positions given by $x_0 = 1m$, $x_0 = x_c$, and $x_0 = 10x_c$ to simulate three different spring systems. To ensure the stability of the Euler-Cromer method, we have used a time step of `dt = 0.0001`, and to get a good estimate of the frequency spectrum, we have generated simulations that contain at least 10 oscillations. This meant that for the $x_0 = 10x_c$ simulation, we have used a longer time segment of 150 seconds, whereas we have used 100 seconds for the other two simulations. Figure 1 depicts the the simulations corresponding to zero initial velocity and the three different initial positions.

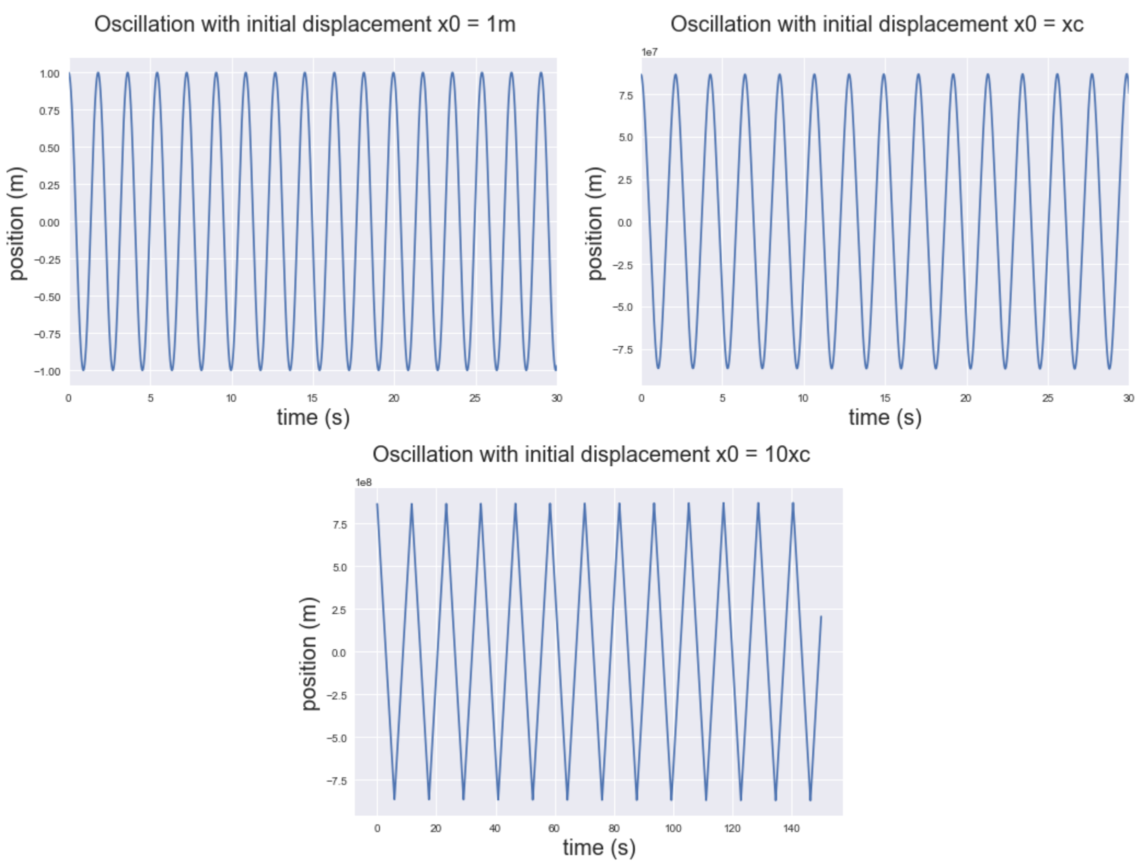


Figure 1: Position versus time plots of three different spring systems with zero initial velocity and initial positions of $x_0 = 1m$, $x_0 = x_c$, and $x_0 = 10x_c$. The plots were produced along time segments that show at least 10 oscillations (30 seconds for the top two plots and 150 seconds for the bottom plot).

(b) For each of the three simulations, we have found the dominant Fourier coefficients, and therefore the dominant amplitudes of oscillation, by Fourier transforming each of the position functions from Figure 1. To show the coefficient amplitudes corresponding to the constituent frequencies of the three oscillations, we have produced a plot of relative amplitude versus frequency for each simulation, depicted by Figure 2. These amplitudes are relative in the sense that they have all been divided by the appropriate maximum amplitude of each simulation to fit in the same figure. As we can see, the dominant frequency of the $x_0 = 10x_c$ simulation is much smaller than those of the other simulations. This is because the initial position is much larger, and therefore it takes the system a lot longer to complete a single oscillation. In other words, the third system has a longer period than the other two. This comparison can also be applied to $x_0 = 1m$ and $x_0 = x_c$ because the frequency of the latter is smaller than that of the former. Further, notice that the $x_0 = 10x_c$ and $x_0 = x_c$ simulations have small peaks at different frequencies, whereas the first simulation has a pure frequency. This makes sense since the first simulation represents a classical spring system whereas the remaining two represent relativistic spring systems.

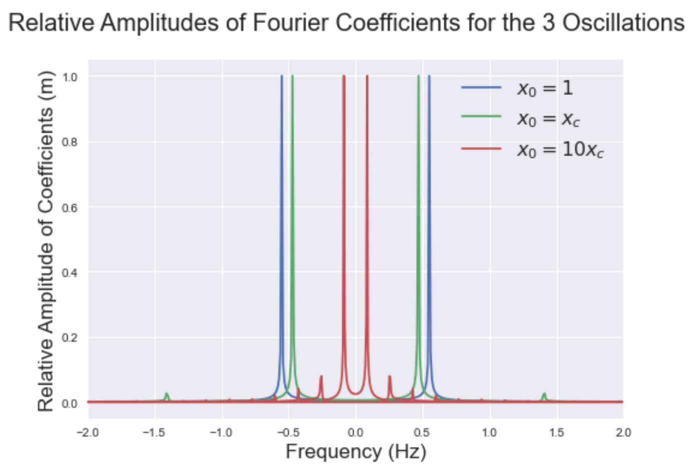
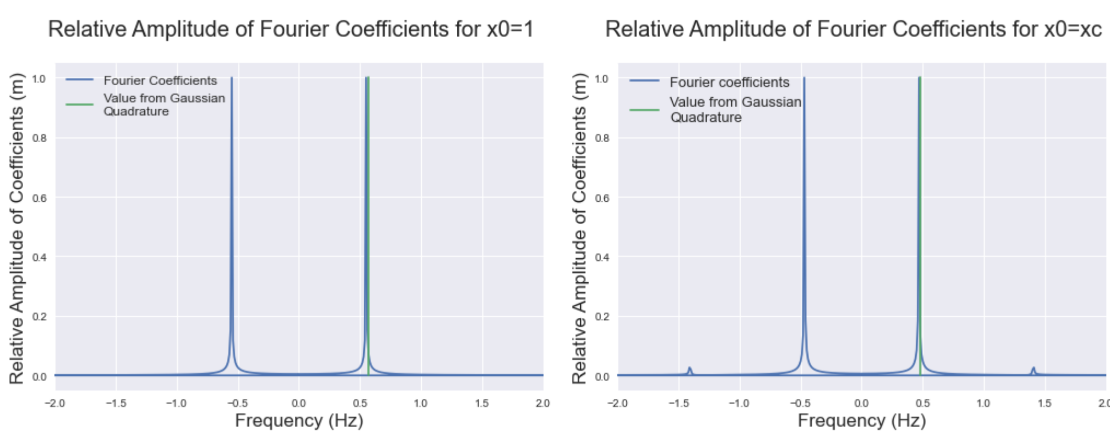
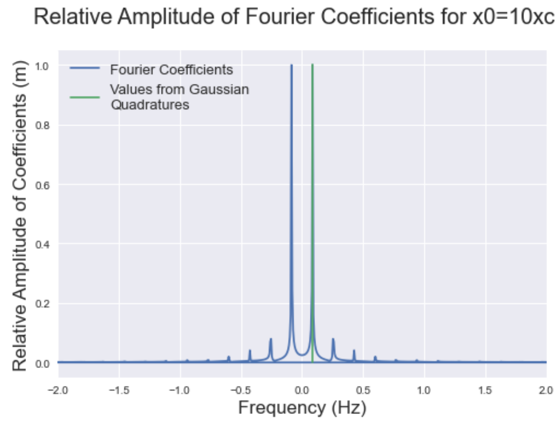


Figure 2: Absolute values of Fourier coefficients with respect to the Fourier frequencies for the three spring system simulations from Figure 1. For each system, the coefficients have been divided by the maximum coefficient for the corresponding system. The dominant coefficients represent the dominant amplitudes of oscillation for each system, which correspond to the dominant frequencies.

(c) Lastly, we have compared the three frequencies obtained by Figure 2 to the frequencies obtained by inverting the periods calculated with Gaussian Quadrature integration in Lab03. Previously, we had found the periods corresponding to the $x_0 = 1m$, $x_0 = x_c$ and $x_0 = 10x_c$ oscillations to be $T \approx 1.7707$, $T_c \approx 2.08755$, and $T_{c1} \approx 11.628$, respectively. Figure 3 depicts plots of the inverted Gaussian quadrature periods laid on top of the frequencies depicted on Figure 2. As we can see, the two frequency estimations are very close to each other for all three systems.



(First part of Figure 3)



(Second part of Figure 3)

Figure 3: Absolute valus of Fourier coefficients with respect to the Fourier frequencies with comparisons to the corresponding Gaussian Quadrature frequencies for the three spring system simulations from Figure 1. As before, the plots have been scaled to a maximum amplitude of 1. The green vertical lines on each plot represent the Gaussian Quadrature frequencies for each simulation.

2 Question 2 - By Idil Yaktubay

In this question, we have applied a *low pass filter* to the stereo file `GraviteaTime.wav`, which has two sound channels that have the same number of data points and sampling frequencies. **(b)** Figure 4 depicts the plots for each channel with respect to time in seconds. To apply a low pass filter to both channels, we have filtered out all frequencies greater than 880 Hz in the two signals by setting the corresponding Fourier coefficients to zero. We have done this by Fourier transforming the channel signals to frequency domains, setting the appropriate coefficients to zero, and lastly, transforming the filtered coefficients back to the time domain by performing inverse Fourier transforms. **(d)** Figure 5 depicts the absolute values of the original and filtered coefficient amplitudes for channels 0 and 1. Notice that we have not included the amplitudes in the range $\frac{N}{2} \to N$ on Figure 5 because such coefficients correspond to the complex conjugates of the coefficients in the range $0 \to \frac{N}{2}$ (the ones depicted on the figure), and thus have identical absolute values. Further, Figure 6 depicts the original and filtered time series over 30 milliseconds for channels 0 and 1. As we can see, the filtered time series for both channels are smoother versions of the original time series. **(e)** Lastly, we have created a new file called `GraviteaTime_lpf.wav` that contains the audio corresponding to the filtered data for both channels, which is submitted alongside this report. We note that the filtered audio sounds a lot lower than the original audio due to the elimination of high frequencies.

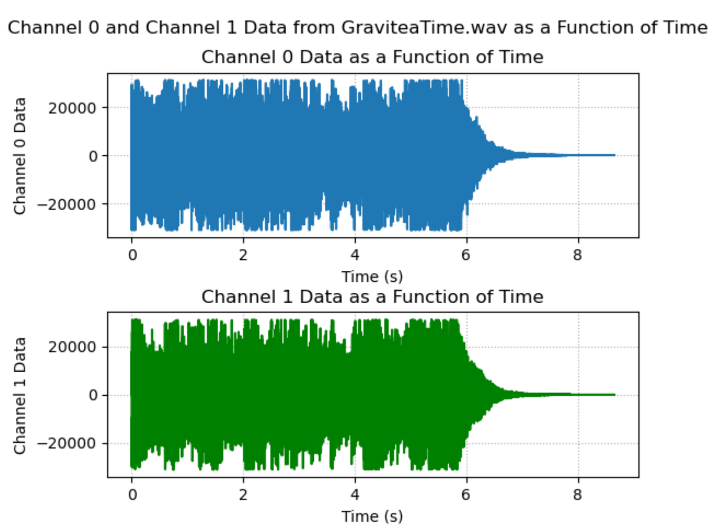


Figure 4: Plots of channels 0 and 1 from the data file `GraviteaTime.wav` with respect to time in seconds. Each channel has 381700 data points and a sampling frequency of 44100 Hz. The total time interval is approximately 8.7 seconds.

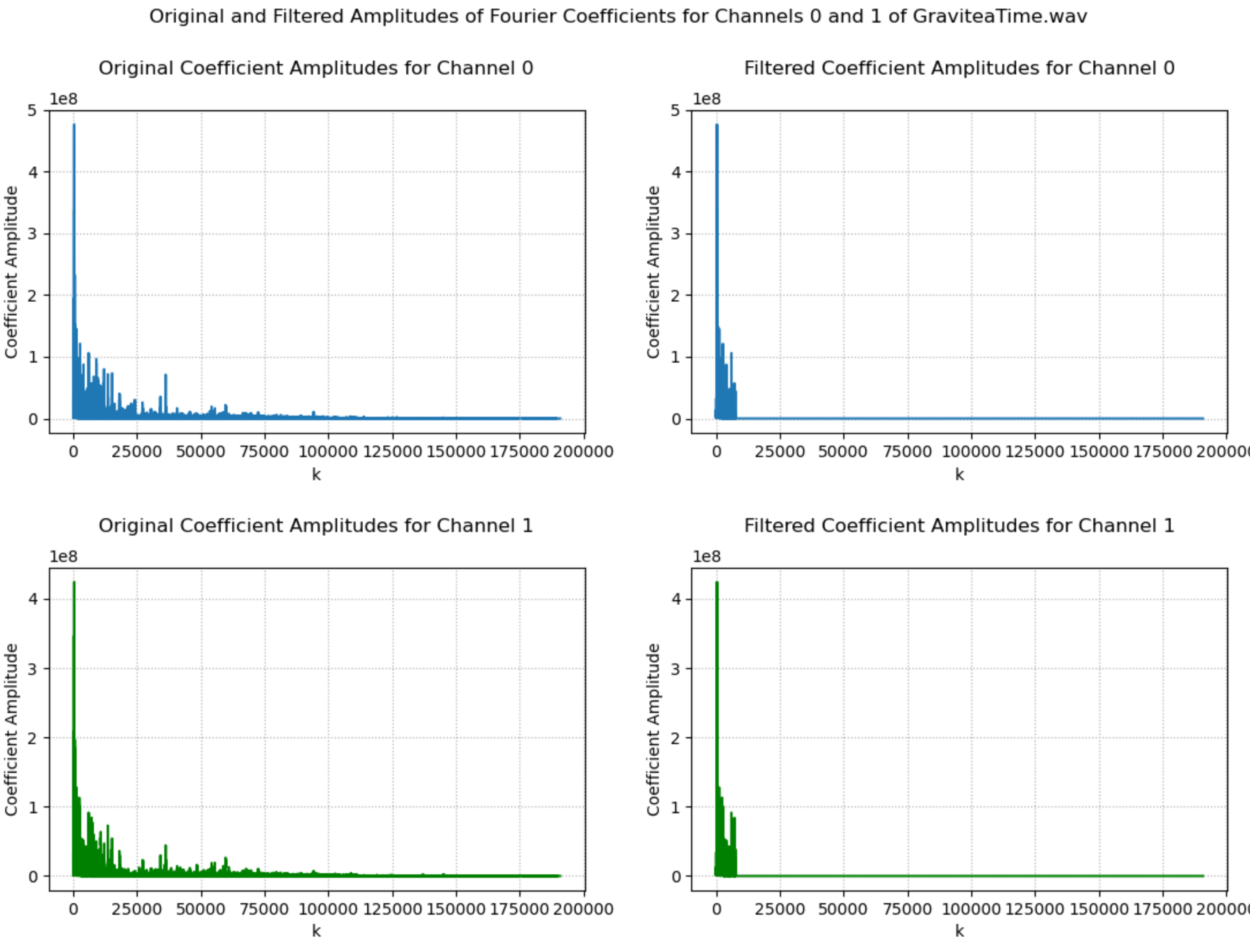
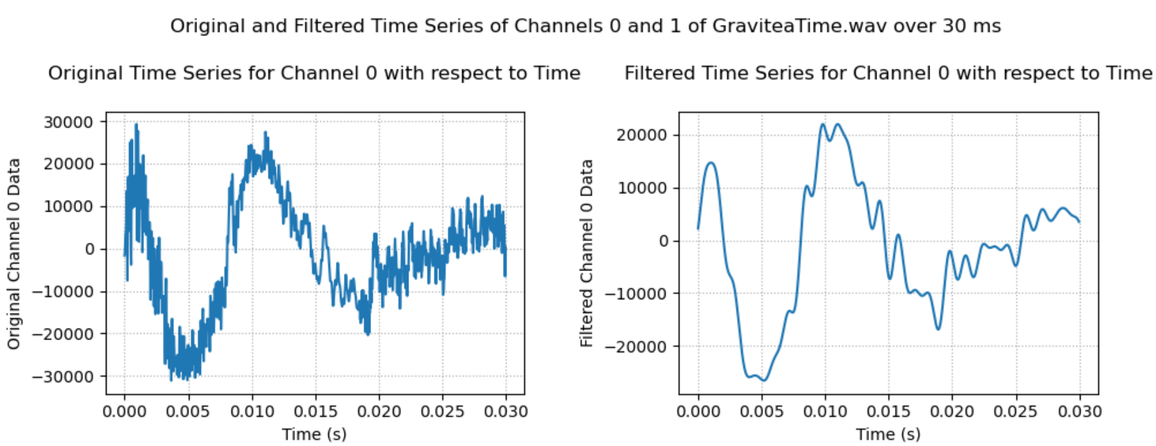


Figure 5: Original and filtered absolute Fourier coefficient amplitudes for channels 0 and 1 of the data file `GraviteaTime.wav` with respect to the coefficient index k. The filtering was done by setting the Fourier coefficients that correspond to frequencies higher than 880 Hz to zero.



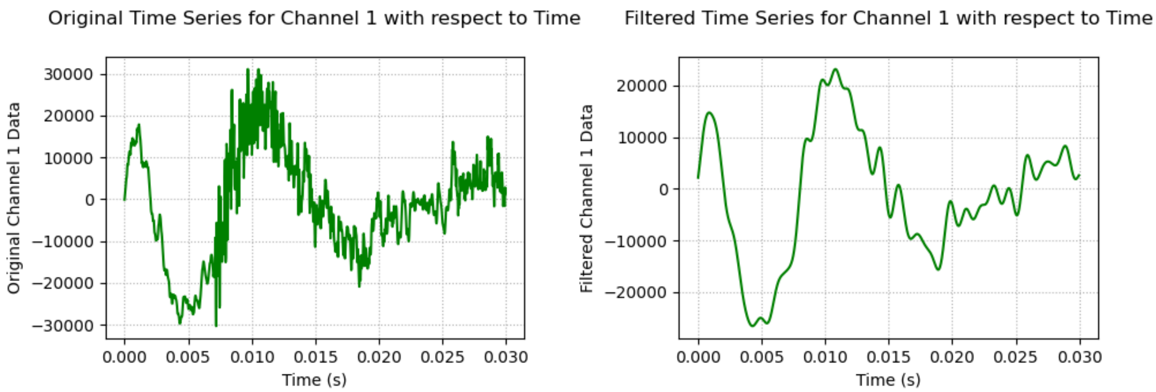


Figure 6: Original and filtered time Series for channels 0 and 1 of the data file `GraviteaTime.wav` over 30 milliseconds. We have filtered out frequencies higher than 880 Hz and therefore made the sound lower.

3 Question 3 - By Souren Salehi

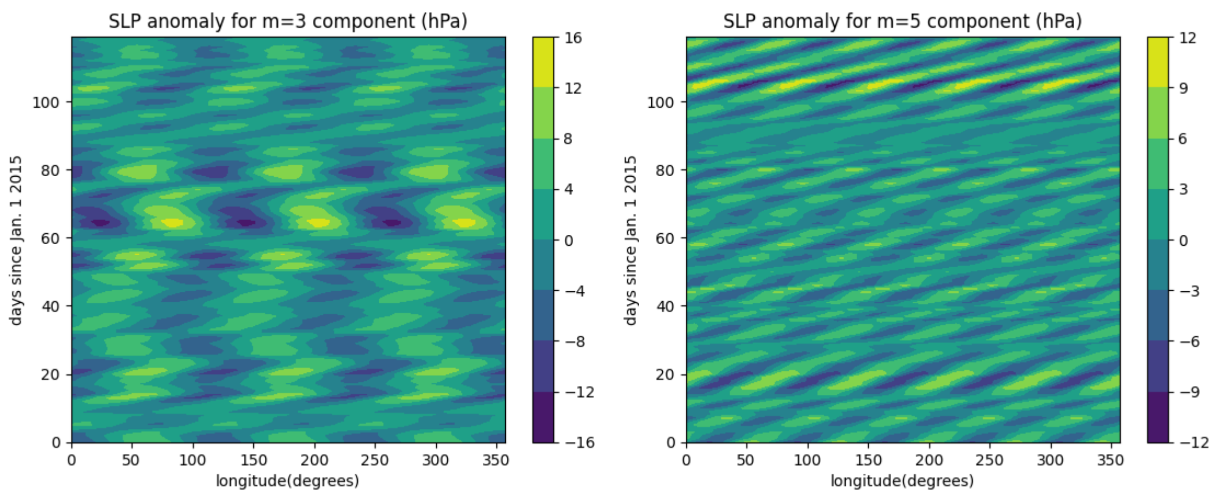


Figure 7: Contour plot of the m=3 and m=5 wavenumber (left and right respectively) SLP anomaly extracted using Fourier-decomposition in the longitudinal direction over 120 days.

For the m = 3 decomposition we see the pressure at a given day, that seem to oscillate between regions of high and low pressure as we change the longitude. We can also see that at a given longitude, the pressure seems to oscillate as well with the regions of high and low pressure moving back and forth in the longitudinal direction but slowly moving towards higher longitudes.

For the m=3 decomposition we see the same trend at constant times, where the sea level pressures oscillate as we increase the longitude. However we see that over time the high and low sea level pressures seem to be moving to higher longitudes. If we follow one path of low pressure, we can see it travels 360^o in around 40 days.

We thus see our waves agree with the theory of atmospheric wave propagation, where our shorter wavelength wave (m=5) seems to be propagating eastwards much faster than the m=3 wave.