

Artificial Intelligence CPSC 470/570
PS 3: Logic and Inference
12 points (CPSC 470) or 18 points (CPSC 570)
Due Wednesday, March 3RD, 11:59:59 PM

Some reminders:

- **Grading contact:** Emmanuel Adéniran (emmanuel.adeniran@yale.edu) is the point of contact for initial questions about grading for this problem set.
- **Late assignments** are not accepted without a Dean's excuse.
- **Collaboration policy:** Remains the same as in PS2.
- **Submission:** You must upload your submission electronically, **in PDF** to **Gradescope** before the deadline posted above. One way to do this is to use one of MS Word's export formats to 'save as' a PDF. Another way could be to upload this document to Google Docs, complete the assignment, and 'Download' a PDF copy of your completed assignment for submission. Alternatively, you could print the PDF version of this assignment, write out your answers with pen or pencil in the spaces provided, and then upload images of each page of your assignment.
- **Students enrolled in CPSC 570:** **Problem #4** is designed to be completed only by students enrolled in CPSC 570, as such, students taking CPSC 470 do not need to answer to problem 4.

Problem 1 (3 points)

State each of the following in First-Order Predicate Calculus (FOPC) notational conventions, using only the list of provided predicates and functions, i.e., using the syntax and semantics of First-Order Logic. You may invent any variable or constant names that you desire. **If there is exactly one unambiguous way to represent the statement, then just provide the FOPC representation using notational conventions. If however there is any ambiguity in the sentence, the interpretation, or the representation, you should write 1-3 English sentences that describe the ambiguity and provide at least 2 FOPC representations, in notational conventions, that are both accurate representations of the statements you would have made in English.**

Allowed Predicates: *Likes(x,y)*, *Bird(x)*, *Ostrich(x)*, *Penguin(x)*, *Flies(x)*, *NeedsToLove(x,y)*. *Likes(x,y)* means “x likes y” and *NeedsToLove(x,y)* means “x needs to love y”. The remaining have their obvious interpretations.

1.1 Everybody does not like something but nobody does not like Sara Lee.

1.2 All birds except Ostriches and Penguins fly.

1.3 Everybody needs somebody to love

Problem 2 (3 points)

Using propositional logic, it is possible to prove theorems by simply enumerating all possible truth values of all variables and checking that the theorem holds. Demonstrate that each of the following is a valid theorem by filling in the provided truth table with “T” for true and “F” for false.

$$2.1 \ (p \Rightarrow \neg p) \Rightarrow \neg p$$

p	$a:$ $\neg p$	$b:$ $(p \Rightarrow \neg p)$	$b \Rightarrow a$

$$2.2 \ ((p \wedge q) \wedge r) \Rightarrow (p \wedge (q \wedge r))$$

$$2.3 \quad (p \wedge (q \vee r)) \Rightarrow ((p \wedge q) \vee (p \wedge r))$$

p	q	r	a: q \vee r	b: p \wedge a	c: p \wedge q	d: p \wedge r	e: c \vee d	b \Rightarrow e

Problem 3 (6 points)

You are given the following facts:

1. Everyone who entered this country and who was not a diplomat was searched by a customs official.
2. William was a terrorist.
3. William entered this country.
4. William was searched by terrorists only.
5. No terrorist was a diplomat.

Show using first-order logic that:

Goal: There is a person who is both a terrorist and a customs official.

Your solutions should have the same format, syntax, and semantics of First-Order Logic as in the First-Order Logic class session slides.

Hints:

- Start by translating the goal into FOPC and enter it into the line marked “goal”.
- Line numbers 1-5 should be the FOPC statements that are equivalent to the English sentences 1-5 above.
- Use only the following predicates: *Entered(x)* meaning “x entered this country”, *Diplomat(x)*, *CustomsOfficial(x)*, *Terrorist(x)*, and *Searched(x,y)* meaning that “x searched y”.
- You may introduce any constants or variables that you need.
- The **Reasoning** column should contain references to an inference rule and the statements that you used to derive the new sentence. For example, “Existential elimination on 7” or “Modus ponens on 9 and 3” or “And-introduction on 1, 3, and 5” or “de Morgan’s rules on 7”.
- Your last line in the table should be the same FOPC statement as your goal.
- You may or may not need all of the lines in the table.

Problem 4 (6 points) : GRADUATE STUDENTS ONLY

Consider the following (fictional) tale:

Damola has been murdered. Atofarati, Bankole, and Chioma are suspects. Only one is guilty and the other two are innocent. The innocent ones told the truth to the police, but the guilty one may have lied.

Atofarati said that Bankole and Damola were friends and that Chioma did not like Damola. Bankole said that he was not in town at the time of the murder, and moreover, he did not know Damola. Chioma said that Atofarati and Bankole were both with Damola just before Damola was murdered.

Your job is to prove that Bankole is the murderer (i.e., $\text{murderer}(B)$).

You should do this via a proof by contradiction. You should assume $\neg \text{murderer}(B)$ and show that this leads to something of the form $P \wedge \neg P$, which is a contradiction since P cannot not be both true and false. (Your last line of the table should be something of the form $P \wedge \neg P$).

You should use the following predicates: $\text{innocent}(x)$, $\text{friends}(x,y)$, $\text{murderer}(x)$, $\text{likes}(x,y)$, $\text{inTown}(x)$, $\text{knows}(x,y)$, $\text{with}(x,y)$.

