MATH 3046, Differential Equations with Computer Lab
Spring 2017

Lab 11

The focus of this lab is solving spring-mass equations using `ode45`.

1. Write the unforced spring-mass equation

$$mx'' + bx' + kx = 0$$

as a system of first-order differential equations, and create a function file that allows the passing of parameters m, b, and k. Plot the solution given the initial conditions $x(0) = 1$ m and $x'(0) = 0$ m/s over the interval $[0, 10]$ for the following cases:

   (a) Undamped: $m = 1, b = 0$, and $k = 16$

   (b) Underdamped: $m = 1, b = 2$, and $k = 16$

   (c) Critically damped: $m = 1, b = 8$, and $k = 16$

   (d) Overdamped: $m = 1, b = 10$, and $k = 16$

2. Suppose that the spring in an unforced spring-mass system without damping loses elasticity with time. A possible model for an "aging" spring-mass system is

$$x'' + 2e^{-0.12t}x = 0.$$

   Suppose that the spring is stretched two units from equilibrium and released from rest.

   Use `ode45` to approximate the solution of the equation, and plot the position for $t \in [0, 25]$. Does the solution make sense physically?

3. Consider an unforced spring-mass system with damping and a "hard" spring (such that the restoring force of the spring is proportional to $x + x^3$ instead of x) modeled by the equation

$$x'' + 3x' + x + x^3 = 0.$$

   Use `ode45` to approximate the solution of the equation if the mass starts at the equilibrium position with initial velocity 3 m/s, and plot the solution for $t \in [0, 20]$ for each initial velocity.

4. Consider an unforced spring-mass system with damping and a "soft" spring (such that the restoring force of the spring is proportional to $x - x^3$ instead of x) modeled by the equation

$$x'' + 3x' + x - x^3 = 0.$$

   Use `ode45` to approximate the solution of the equation if the mass starts at the equilibrium position with initial velocity 3 m/s, and plot the solution for $t \in [0, 20]$ for each initial velocity. Comparing the solutions of Problems 3 and 4, can you give a physical justification for the differences in the solutions?