GENERAL & PHYSICAL CHEMISTRY

1. VOLUMETRIC ANALYSIS

FORMULAE

1. Principel of Volumetric Analysis

   No. of gram equivalent of Acid = No. of gram equivalent of Base

   $$\text{No. of gram equivalent} = \frac{\text{Weigth of substance}}{\text{Equivalent weight of substance}} = \frac{W}{E}$$

2. Concetration

   i. $g/L = \dfrac{\text{Weigth of Solution (g)}}{\text{Volume of solution (L)}}$

   ii. $\% \dfrac{w}{v} = \dfrac{\text{Weigth of Solution (g)}}{\text{Volume of solution (mL)}} \times 100$

   iii. $\% \dfrac{w}{w} = \dfrac{\text{Weigth of Solution (g)}}{\text{Wegith of solution (g)}} \times 100$

   iv. $\text{Normality (N)} = \dfrac{\text{Weigth of Solution (g)}}{\text{Gram equivalent weigth of solution}} \times \dfrac{1}{\text{Volume of solution (mL)}} \times 1000$

   $W = \dfrac{NEV}{1000}$

   v. $\text{Molarity (M)} = \dfrac{\text{Weigth of Solution (g)}}{\text{Gram molecular weigth of solution}} \times \dfrac{1}{\text{Volume of solution (mL)}} \times 1000$

   $W = \dfrac{MMwV}{1000}$

   vi. $\text{Molality (m)} = \dfrac{\text{Weigth of Solution (g)}}{\text{Molecular weigth of solution}} \times \dfrac{1}{\text{Weight of solution (g)}} \times 1000$

   vii. $ppm = \dfrac{\text{Weigth of Solute (g)}}{\text{Volume of solution (mL)}} \times 10^6$

3. Relation between different concentration

   i. G/L = Normality × Equivalent weight of solute

   ii. G/L = Molarity × Molecular weight of solute

   iii. Normality (N) = Molarity (M) × basicify (For acid)
   = Molarity (M) × basicify (For base)

   iv. $\text{Normality (N)} = \text{Molarity} \times \dfrac{\text{Molecular weight}}{\text{Equivalent weight}}$ (for salt)

   v. $\text{Equivalent weight of acid} = \dfrac{\text{Molecular weight of acid}}{\text{Basicity}}$

   vi. $\text{Equivalent weight of base} = \dfrac{\text{Molecular weight of acid}}{\text{Acidity}}$

   vii. $\text{Equivalent weight of salt} = \dfrac{\text{Molecular weight of acid}}{\text{Total + ve charge present in basic radical}}$

   viii. $\text{Normality (N)} = \dfrac{\% \frac{w}{w} \times \text{specific gravity} \times 10}{\text{Equivalent weight}}$

   viii. $\text{Molarity (M)} = \dfrac{\% \frac{w}{w} \times \text{specific gravity} \times 10}{\text{Molecular weight}}$

4. Standard Solution

   i. Normal solution = 1N — Molar solution = 1M

   ii. Decinormal solution = $\dfrac{N}{10}$ — Decimolar = $\dfrac{M}{10}$

   iii. Seminormal solution = $\dfrac{N}{2}$ — Semimolar = $\dfrac{M}{2}$

   iv. Centinormal solution = $\dfrac{N}{100}$ — Centimolar = $\dfrac{M}{100}$

5. $\% \text{ purity} = \dfrac{\text{Calculated weight}}{\text{Given weight}} \times 100$

6. Normality equation: $V_1N_1 = V_2N_2$

7. Molarity equation: $V_1M_1 = V_2M_2$

8. Normality of resultant mixture (N) = $\dfrac{V_1N_1 + V_2N_2 + \ldots}{V_1 + V_2 + \ldots}$

9. Normality of resultant acid solution (N_mV_m) $= V_1N_1 + V_2N_2 + V_3N_3 + \ldots$

10. Molarity of resultant mixture (M) = $\dfrac{V_1M_1 + V_2M_2 + \ldots}{V_1 + V_2 + \ldots}$

FORMULAE

1. Rate of reaction $= \dfrac{\text{Decrease in conc}^n \text{ of reactant}}{\text{Time interval}} = \dfrac{\text{Increase in conc}^n \text{ of product}}{\text{Time interval}}$

2. $aA + bB \longrightarrow cC + dD$

   $\text{Equivalent rate} = -\dfrac{d[A]}{a\,dt} = -\dfrac{d[B]}{b\,dt} = \dfrac{d[C]}{c\,dt} = \dfrac{d[D]}{d\,dt}$

3. Order of reaction

   $aA + bB \longrightarrow \text{product}$

   $\text{Rate} \propto [A]^a [B]^b$ (Theoretical)

   $\text{Rate} \propto [A]^m [B]^m$ (Experimental)

4. Rate law

   $aA + bB \longrightarrow \text{product}$

   $\text{Rate} = K [A]^m [B]^m$

5. Unit of rate constant

   For zero order = mol L^{-1} s^{-1} or mol L^{-1} min^{-1}

   For 1st order = $time^{-1}$

   For 2nd order = L mol^{-1} s^{-1} or L mol^{-1} min^{-1}

   For 3rd order = L^2 mol^{-2} s^{-1} or L^2 mol^{-2} min^{-1}

6. First order reaction

   $K_1 = \dfrac{2.303}{t} \log \dfrac{a}{a-x}$

   Half life period $t_{\frac{1}{2}} = \dfrac{0.693}{K_1}$

FORMULAE

1. pH and pOH

   $pH = -\log [H^+]$

   $pH = -\log [OH^+]$

   $pH + pOH = 14$

2. Ostwald's Dilution Law

   $K_a = \dfrac{\alpha^2 C}{1-\alpha}$ where, C = Conc^n in molarity

   $\alpha = \dfrac{\text{No. of moles ionized}}{\text{Total no. of moles taken}}$ = Degree of ionization

   $K_a = \alpha^2 C \ (1 - \alpha \approx 1)$

   $\alpha = \sqrt{\dfrac{Ka}{C}}$

3. Solubility product

   $K_{sp} = [A]^x [B]^y$

   $\text{Solubility in mol } L^{-1} = \dfrac{\text{Solubility of salt in g/L}}{\text{Molecular weight of salt}}$

   Solubility of g/L = Solubility in mol L × Molecular weight

FORMULAE

1. Galvanic cell: $E° \text{ cell} = E°_R - E°_L$

FORMULAE

1. Internal Energy: $\Delta E = E_P - E_R$
2. First Law of Thermodynamics

   $q = \Delta E + w$ $w = P\Delta V$

   $\Delta V = V_{final} - V_{initial}$ $q = \Delta E + P\Delta V$
2. Enthalpy (H) = E + PV

   $\Delta H = \Delta E + P\Delta V$

   $PV = nRT$ (Ideal gas equation)

   At constant T and P

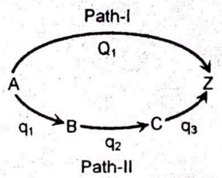
   $P\Delta V = \Delta nRT$

   So, $\Delta H = \Delta E + \Delta nRT$

   $\Delta H = H_P - H_R$
3. Hess's law of constant heat summation

   $Q_2 = q_1 + q_2 + q_3$

   $Q_1 = Q_2$



4. Bond energy

   $\Delta H = \Sigma(\text{Bond energy})_{reactant} - \Sigma(\text{Bond energy})_{product}$

   $1J = 10^7$ erg

   1 L atm = 101.3J

   1 calorine = 4.184 J

   1.987 cal = 0.0821 L atm

FORMULAE

1. Entropy (S)

   $\Delta S = S_{final} - S_{initial}$

   $= S_{product} - S_{reactant}$

   $\Delta S = \frac{q_{rev}}{T} = \frac{\Delta H}{T}$ (at constant T)

   Entropy of fusion: $\Delta S_{fus} = \frac{\Delta H_{fus}}{T_m}$

   Entropy of vapourization: $\Delta S_{vap} = \frac{\Delta H_{vap}}{T_b}$

   Entropy of sublimation: $\Delta S_{sub} = \frac{\Delta H_{sub}}{T_b}$
2. Gibb's free energy (G)

   $G = H - T\Delta S$

   $\Delta G = \Delta H - T\Delta S$

   $\Delta G = \Sigma\, \Delta G_{product} - \Sigma\, \Delta G_{reactant}$

   At standard condtion (25°C)

   $\Delta G^\circ = \Sigma\, \Delta G^\circ_{product} - \Sigma\, \Delta G^\circ_{reactant}$
3. Gibb's Helmholtz equation

   $\Delta G = \Delta H - T\Delta S$

if	$\Delta G = -ve$	spontaneous
	$\Delta G = +ve$	non spontaneous
	$\Delta G = 0$	at equilibrium

Standard free energy change and equilibrium constant

$\Delta G^\circ = -RT \ln K$

$\Delta G^\circ = -2.303\, RT \log K$

FORMULAE

1. Factorial Notation

   Factorial n, denoted by, n! or $\underline{|n}$ is given by

   $n! = 1 \cdot 2 \cdot 3 \cdot \ldots \cdot n$

   Also, $0! = 1$
2. The total number of permutations of a set of n objects taken r at a time is given by

   $${}^nP_r = P(n, r) = \frac{n!}{(n-r)!}, \; n \geq r$$
3. The total number of permutations of a set of n objects taken all at a time, when there are p objects of one kind, q objects are of the second kind and r objects are of third kind is $\frac{n!}{p!\,q!\,r!}$
4. Circular permutations of n objects = $(n - 1)!$
5. The number of permutation of n objects taken r at a time with repetition = n^r.

FORMULAE

1. Binomial theorem for positive integral index n;

   $(a + x)^n = {}^nC_0 a^n + {}^nC_1 a^{n-1} x + {}^nC_2 a^{n-2} x^2 + {}^nC_3 a^{n-3} x^3 + \ldots + {}^nC_r a^{n-r} x^r + \ldots + {}^nC_n x^n$
2. General term:

   a. The general term of $(a + x)^n$:

   $t_{r+1} = {}^nC_r a^{n-r} x^r$

   b. The general term of $(a - x)^n$:

   $t_{r+1} = (-1)^r\, {}^nC_r a^{n-r} x^r$
3. Middle term:

   a. When n is even in $(a + x)^n$, the middle term is

   $$t_{\frac{n}{2}+1} = {}^nC_{\frac{n}{2}}\, a^{n-\frac{n}{2}} x^{\frac{n}{2}} = {}^nC_{\frac{n}{2}}\, a^{\frac{n}{2}} x^{\frac{n}{2}}$$

   b. When n is odd in $(a + x)^n$, the middle term is

   $$t_{\frac{n+1}{2}} = t_{\frac{n-1}{2}+1}$$
   $$= {}^nC_{\frac{n-1}{2}}\, a^{n-\frac{n-1}{2}}\, x^{\frac{n-1}{2}}$$
   $$= {}^nC_{\frac{n-1}{2}}\, a^{\frac{n+1}{2}}\, x^{\frac{n-1}{2}}$$
   $$t_{\frac{n+1}{2}+1} = t_{\frac{n+1}{2}}\, a^{n-\frac{n+1}{2}}\, x^{\frac{n+1}{2}}$$
   $$= {}^nC_{\frac{n+1}{2}}\, a^{\frac{n-1}{2}}\, x^{\frac{n+1}{2}}$$

FORMULAE

1. ${}^nP_r = r! \cdot {}^nC_r$
2. The total number of combinations of n objects taken r at a time is ${}^nC_r = C(n, r) = \frac{n!}{(n-r)!\,r!}$; $n \geq r$
3. i. ${}^nC_r = {}^nC_{r-1}$ (Complementary combination)

   ii. If ${}^nC_r = {}^nC_{r'}$ then either $r = r'$ or $r + r' = n$

   iii. ${}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$

FORMULAE

1. Expansion of e^x

   $$e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots$$
   $$= \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

   when $n = 1$, $e = 1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \ldots$

   when $n = -1$, $e^{-1} = 1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \ldots$
2. Expaniosn of a^x

   $$a^x = 1 + \frac{x}{1!} \log_e a + \frac{x^2}{2!} (\log_e a)^2 + \frac{x^3}{3!} (\log_e a)^3 + \ldots$$

   where a is any poitive number.

3. **The logarithmic series**

   $$\log_e (1 + x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \ldots \; (-1 < x \leq 1)$$

   and $\log_e (1 - x) = -x - \frac{x^2}{2} - \frac{x^3}{3} - \frac{x^4}{4} + \ldots \; (-1 \leq x < 1)$

   Also, $\log_e \left(\frac{1+x}{1-x}\right) = 2\left(x + \frac{x^3}{3} + \frac{x^5}{5} + \ldots\right)$

FORMULAE

1. Binary Operation on a Set

   Let S be a non empty set. A mapping f from $S \times S$ to S is said to be binary operation on the set S. The image of the ordered pair (a, b) under the mapping f is denoted by a f b. Often we use symbols $*$, $\otimes$, o, $\times$, $-$, $+$, etc. to denote binary operation on a set.
2. Let G be a non empty set with binary operation 'o'. An algebraic structure (G, o) is called a group if the operation 'o' satisfies the following properties:

   a. Closure property

   For all a, b $\in$ G implies that a o b $\in$ G.

   b. Associative property

   For all a, b, c $\in$ G implies that a o (b o c) = (a o b) o c

   c. Existence of identity element

   For each element a $\in$ G there exists an identity element denoted by e such that

   a o e = a = e o a.

   d. Existence of an inverse element

   For each element a $\in$ G there exist an inverse element of a denoted by a^{-1} in G such that

   a o a^{-1} = e = a^{-1} o a.
3. Commutative Group

   The group (G, o) is said to be commutative (abelian group) if a o b = b o a for all a, b $\in$ G.

FORMULAE AND IMPORTANT POINTS

1. Sequence
   A sequence of numbers is a set of numbers arranged in a definite order.
   An infinite sequence is a function $f : \mathbb{N} \to \mathbb{R}$ defined by $f(n) = a_n$, $n \in \mathbb{N}$ (the set of natural numbers)

2. Arithmetic progression:
   i. $t_n = a + (n - 1)d$
   ii. $S_n = \frac{n}{2}(a + l)$
   iii. $S_n = \frac{n}{2}[2a + (n - 1)d]$
   iv. A.M. between a and b $= \frac{a+b}{2}$
   v. To insert n A.M's between a and b, $d = \frac{b-a}{n+1}$

3. Geometric Progression
   i. $t_n = ar^{n-1}$
   ii. $S_n = \frac{a(r^n - 1)}{r - 1}$ if $r > 1$ and $S_n = \frac{a(1 - r^n)}{1 - r}$ if $r < 1$
   iii. $S_n = \frac{lr - a}{r - 1}$, $r \neq 1$
   iv. $S_\infty = \frac{a}{1 - r}$, $|r| < 1$
   v. If a and b are two positive numbers, G.M $= \sqrt{ab}$
   vi. To insert n G.M's between a and b, $(r) = \left(\frac{b}{a}\right)^{1/n+1}$

4. Harmonic Progression
   i. $t_n = \frac{1}{a + (n - 1)d}$
   ii. If a and b are two positive numbers, $H = \frac{2ab}{a + b}$

5. Relations between A.M., G.M. and H.M.
   a. $A \times H = G^2$
   b. $A > G > H$

6. i. $1 + 2 + 3 + \cdots + n = \sum n = \frac{n(n+1)}{2}$.
   ii. Sum of first n even natural numbers = $n(n + 1)$.
   iii. Sum of first n odd natural numbers = n^2.
   iv. $1^2 + 2^2 + 3^2 + \cdots + n^2 = \sum n^2 = \frac{n(n+1)(2n+1)}{6}$
   v. $1^3 + 2^3 + 3^3 + \ldots + n^3 = \sum n^3 = \left\{\frac{n(n+1)}{2}\right\}^2$

7. Sum of n Terms of an Arithmetico-Geometric Series
   $S_n = \frac{a}{1 - r} + d\,r\frac{(1 - r^{n-1})}{(1 - r)^2} - \frac{\{a + (n - 1)d\}r^n}{1 - r}$
   Sum to infinity:
   $S_\infty = \frac{a}{1 - r} + \frac{dr}{(1 - r)^2}$, $|r| < 1$

FORMULAE AND IMPORTANT POINTS

1. Principle of Mathematical induction
   A statement $P(n)$ is true for all $n \in \mathbb{N}$, where $\mathbb{N}$ is the set of natural numbers, provided
a. $P(1)$ is true and
b. $P(k + 1)$ is true whenever $P(k)$, $k \in \mathbb{N}$ is true.

FORMULAE AND IMPORTANT POINTS

1. Complex Number
   An ordered pair (a, b) of two real numbers a and b is said to be a complex number.
2. $i^2 = -1$
   In general:
   $i^{4n} = 1$, $i^{4n+1} = i$, $i^{4n+2} = -1$, $i^{4n+3} = -i$, where $n \in \mathbb{Z}$.
3. If $Z = a + ib$, then $\overline{Z} = a - ib$
4. If $Z = a + ib$, then $|Z| = \sqrt{a^2 + b^2}$
5. Cube Roots of Unity
   $1, \frac{-1 + i\sqrt{3}}{2}, \frac{-1 - i\sqrt{3}}{2}$ are three cube roots of unity
6. Properties of cube roots of unity
   i. Each imaginary cube root of unity is the square of the other.
   ii. The sum of the three cube roots of unity is zero.
   iii. Each complex cube root of unity is the reciprocal of the other.
   iv. The product of two imaginary cube roots of unity is 1.
7. $\omega^3 = 1$
   In general $\omega^{3n} = 1$ for $n \in \mathbb{Z}$.
8. If $z_1 = r_1(\cos\theta_1 + i\sin\theta_1)$ and $z_2 = r_2(\cos\theta_2 + i\sin\theta_2)$
   then, $z_1 z_2 = r_1 r_2\{\cos(\theta_1 + \theta_2) + i\sin(\theta_1 + \theta_2)\}$.
   and $\frac{z_1}{z_2} = \frac{r_1}{r_2}[\cos(\theta_1 - \theta_2) + i\sin(\theta_1 - \theta_2)]$.
9. amp $(z_1 z_2)$ = amp (z_1) + amp (z_2).
   amp $\left(\frac{z_1}{z_2}\right)$ = amp (z_1) − amp (z_2).
10. De-Moivre's Theorem
    For any positive integer n
    $[r(\cos\theta + i\sin\theta)]^n = r^n(\cos n\theta + i\sin n\theta)$.

FORMULAE AND IMPORTANT POINTS

1. A function *f(x)* defined by
$f(x) = a_0x^n + a_1x^{n-1} + a_2x^{n-2} + a_3x^{n-3} + \cdots + a_{n-1}x + a_n, (a_0 \neq 0)$
is said to be a polynomial of degree n in x, where $a_0, a_1, a_2, \cdots, a_n \in \mathbb{R}$ are all constants, n is a non-negative integer.
2. The roots of the quadratic equation $ax^2 + bx + c = 0$ are
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
3. Nature of the Roots of Quadratic Equations

a. i. If $b^2 - 4ac > 0$, then $\sqrt{b^2 - 4ac}$ is real, hence the roots are real and distinct, for $a, b, c \in \mathbb{R}$.

ii. If $b^2 - 4ac > 0$ and a perfect square, then the roots are rational and distinct for a, b, c rational.

iii. If $b^2 - 4ac > 0$ and not a perfect square, then the roots are irrational and distinct.

b. If $b^2 - 4ac = 0$, then $\sqrt{b^2 - 4ac} = 0$, the roots are real and equal for $a, b, c \in \mathbb{R}$.

c. If $b^2 - 4ac < 0$, then $\sqrt{b^2 - 4ac}$ is an imaginary number. So the roots are imaginary and distinct.

4. i. If a quadratic equation has an irrational root, then other root will be its conjugate. That is, if $p + \sqrt{q}$ be one root, the other root will be the conjugate irrational quantity $p - \sqrt{q}$ and conversely.

ii. Imaginary roots always occur in conjugate pair. That is, if $p + iq$ be one root, the other root will be the conjugate imaginary quantity $p - iq$ and conversely.

5. Relation between Roots and Coefficients of quadratic equation $ax^2 + bx + c = 0$

Sum of the roots $= -\dfrac{b}{a} = -\dfrac{\text{coefficient of } x}{\text{coefficient of } x^2}$

Product of the roots $= \dfrac{c}{a} = \dfrac{\text{constant term}}{\text{coefficient of } x^2}$.

6. Formulation of Quadratic Equation
$x^2 - (\text{sum of roots})\, x + (\text{product of roots}) = 0$
7. Condition for one root common
$(bc_1 - b_1c)(ab_1 - a_1b) = (ca_1 - c_1a)^2$
The common root is
$\dfrac{bc_1 - b_1c}{ca_1 - c_1a}$ or, $\dfrac{ca_1 - c_1a}{ab_1 - a_1b}$
8. Condition for two roots common
$\dfrac{a}{a_1} = \dfrac{b}{b_1} = \dfrac{c}{c_1}$

FORMULAE AND IMPORTANT POINTS

1. If the system has one solution, the system is said to be consistent and independent. If the system has no solution, the system is said to be inconsistent and independent. If the system has an infinite number of solutions, the system is said to be consistent and dependent.
2. Row Equivalent Matrix
Let $a_1x + b_1y + c_1z = d_1$
$a_2x + b_2y + c_2z = d_2$
$a_3x + b_3y + c_3z = d_3$
Augmented matrix.
$$\begin{bmatrix} a_1 & b_1 & c_1 & : & d_1 \\ a_2 & b_2 & c_2 & : & d_2 \\ a_3 & b_3 & c_3 & : & d_3 \end{bmatrix}$$
of the system. By using elementary row operations, the augmented matrix is reduced to the matrix of the form
$$\begin{bmatrix} 1 & 0 & 0 & : & p \\ 0 & 1 & 0 & : & q \\ 0 & 0 & 1 & : & r \end{bmatrix}$$
Then $x = p$, $y = q$ and $z = r$
3. We follow the following indicated path to reduce the augmented matrix.
$$\begin{bmatrix} 1 & 0 & : & p \\ \downarrow & \uparrow & & \\ 0 \rightarrow & 1 & : & q \end{bmatrix}$$
$\therefore$ $x = p$ and $y = q$
$$\begin{bmatrix} 1 & 0 \leftarrow & 0 & : & p \\ \downarrow & & \uparrow & & \\ 0 & 1 & 0 & : & q \\ \downarrow \nearrow & \downarrow & \uparrow & & \\ 0 & 0 \rightarrow & 1 & : & r \end{bmatrix} \text{ or } \begin{bmatrix} 1 & 0 & 0 & : & p \\ \downarrow & \uparrow & \uparrow & & \\ 0 & 1 & 0 & : & q \\ \downarrow \nearrow & \downarrow \searrow & \uparrow & & \\ 0 & 0 & 1 & : & r \end{bmatrix} \text{ etc.}$$
$\therefore$ $x = p$, $y = q$ and $z = r$.

FORMULAE AND IMPORTANT POINTS

1. The general solution of $\sin x = 0$ is $x = n\pi$, $n \in \mathbb{Z}$.
2. The general solution of $\cos x = 0$ is $x = (2n + 1)\frac{\pi}{2}$, $n \in \mathbb{Z}$.
3. The general solution of $\tan x = 0$ is $x = n\pi$, $n \in \mathbb{Z}$.
4. The general solution of $\cot x = 0$ is $x = (2n + 1)\frac{\pi}{2}$, $n \in \mathbb{Z}$.
5. The general solution of $\sin x = k$, $(-1 \leq k \leq 1)$ is $x = n\pi + (-1)^n\theta$, $n \in \mathbb{Z}$.
6. The general solution of $\cos x = k$, $(-1 \leq k \leq 1)$ is $x = 2n\pi \pm \theta$, $n \in \mathbb{Z}$.
7. The general solution of $\tan x = k$ is $x = n\pi + \theta$, $n \in \mathbb{Z}$.

FORMULAE

Tables for Different Types of Ellipse

Ellipse	Center	Vertex	Focus	Major axis	Minor axis	Eccentricity (e)	Length of Latus ratum	Directrix
$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, $a>b>0$	(0, 0)	$(\pm a, 0)$	$(\pm ae, 0)$	2a	2b	$\sqrt{1-\frac{b^2}{a^2}}$	$\frac{2b^2}{a}$	$x=\pm\frac{a}{e}$
$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, $b>a>0$	(0, 0)	$(0, \pm b)$	$(0, \pm be)$	2b	2a	$\sqrt{1-\frac{a^2}{b^2}}$	$\frac{2a^2}{b}$	$y=\pm\frac{b}{e}$
$\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$, $a>b>0$	(h, k)	$(h\pm a, k)$	$(h\pm ae, k)$	2a	2b	$\sqrt{1-\frac{b^2}{a^2}}$	$\frac{2b^2}{a}$	$x=h\pm\frac{a}{e}$
$\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$, $b>a>0$	(h, k)	$(h, k\pm b)$	$(h, k\pm be)$	2b	2a	$\sqrt{1-\frac{a^2}{b^2}}$	$\frac{2a^2}{b}$	$y=k\pm\frac{b}{e}$

Length of conjugate axis	2b	2a	2b	2a
Equation of directrices	$x=\pm\frac{a}{e}$	$y=\pm\frac{b}{e}$	$x=h\pm\frac{a}{e}$	$x=k\pm\frac{b}{e}$
Eccentricity (e)	$\sqrt{1+\frac{b^2}{a^2}}$	$\sqrt{1+\frac{a^2}{b^2}}$	$\sqrt{1+\frac{b^2}{a^2}}$	$\sqrt{1+\frac{a^2}{b^2}}$
Latus rectum	$\frac{2b^2}{a}$	$\frac{2a^2}{b}$	$\frac{2b^2}{a}$	$\frac{2a^2}{b}$

FORMULAE

Tables for Different Types of Hyperbola

Equation of hyperbola	$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$	$\frac{y^2}{b^2}-\frac{x^2}{a^2}=1$	$\frac{(x-h)^2}{a^2}-\frac{(y-k)^2}{b^2}=1$	$\frac{(x-h)^2}{a^2}-\frac{(y-k)^2}{b^2}=1$
Center	(0, 0)	(0, 0)	(h, k)	(h, k)
Vertices	$(\pm a, 0)$	$(0, \pm b)$	$(h\pm a, k)$	$(h, k\pm b)$
Foci	$(\pm ae, 0)$	$(0, \pm be)$	$(h\pm a, k)$	$(h, k\pm be)$
Equation of transverse axis	y = 0	x = 0	x = h	y = k
Equation of conjugate axis	x = 0	y = 0	y = k	x = h
Length of transverse axis	2a	2b	2a	2b

FORMULAE AND IMPORTANT POINTS

1. For the equations $a_1x + b_1y = c_1$ and $a_2x + b_2y = c_2$.

$$D = \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}, D_1 = \begin{vmatrix} c_1 & b_1 \\ c_2 & b_2 \end{vmatrix}, D_2 = \begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix}$$

$$x = \frac{D_1}{D} \text{ and } y = \frac{D_2}{D}$$

2. For the equations,

$$a_1x + b_1y + c_1z = d_1$$
$$a_2x + b_2y + c_2z = d_2$$
$$a_3x + b_3y + c_3z = d_3$$

$$D = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}, \quad D_1 = \begin{vmatrix} d_1 & b_1 & c_1 \\ d_2 & b_2 & c_2 \\ d_3 & b_3 & c_3 \end{vmatrix},$$

$$D_2 = \begin{vmatrix} a_1 & d_1 & c_1 \\ a_2 & d_2 & c_2 \\ a_3 & d_3 & c_3 \end{vmatrix} \& \quad D_3 = \begin{vmatrix} a_1 & b_1 & d_1 \\ a_2 & b_2 & d_2 \\ a_3 & b_3 & d_3 \end{vmatrix}$$

$$x = \frac{D_1}{D}, y = \frac{D_2}{D}, z = \frac{D_3}{D}$$

3. If D = 0, we can't apply Cramer's rule.

FORMULAE AND IMPORTANT POINTS

1. Inverse Matrix Method

Let $a_{11}x + a_{12}y + a_{13}z = b_1$

$a_{21}x + a_{22}y + a_{23}z = b_2$

$a_{31}x + a_{32}y + a_{33}z = b_3$

The system of equations can be written in the matrix form as

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} \quad \ldots(1)$$

The equation (1) can again be written as a single matrix equation as

$$AX = B, \quad \ldots(2)$$

Thus, the solution can be obtained by finding out A^{-1} and multiplying it with B, where,

$$A^{-1} = \frac{1}{|A|} \text{Adj } A.$$

FORMULAE

1. $2\sin A\cos B = \sin(A+B) + \sin(A-B)$
$2\cos A\sin B = \sin(A+B) - \sin(A-B)$
$2\cos A\cos B = \cos(A+B) + \cos(A-B)$
$2\sin A\sin B = \cos(A-B) - \cos(A+B)$

2. $\sin C + \sin D = 2\sin\frac{C+D}{2}\cos\frac{C-D}{2}$

$\sin C - \sin D = 2\cos\frac{C+D}{2}\sin\frac{C-D}{2}$

$\cos C + \cos D = 2\cos\frac{C+D}{2}\cos\frac{C-D}{2}$

$\cos C - \cos D = 2\sin\frac{C+D}{2}\sin\frac{D-C}{2}$

$= -2\sin\frac{C+D}{2}\sin\frac{C-D}{2}$

3. Domain and range of inverse circular functions

Functions	Domain (x)	Range (y)
$y=\sin^{-1}x$	$-1\le x\le 1$	$-\frac{\pi}{2}\le y\le\frac{\pi}{2}$
$y=\cos^{-1}x$	$-1\le x\le 1$	$0\le y\le\pi$
$y=\tan^{-1}x$	$(-\infty,\infty)$	$-\frac{\pi}{2}<y<\frac{\pi}{2}$
$y=\cot^{-1}x$	$(-\infty,\infty)$	$0<y<\pi$
$y=\sec^{-1}x$	$x\ge 1$, or $x\le -1$	$0\le y\le\pi,\ y\ne\frac{\pi}{2}$
$y=\operatorname{cosec}^{-1}x$	$x\ge 1$, or $x\le -1$	$-\frac{\pi}{2}\le y\le\frac{\pi}{2},\ y\ne 0$

4. Self adjust property
$\sin\sin^{-1}x = x = \sin^{-1}\sin x$
$\cos\cos^{-1}x = x = \cos^{-1}\cos x$
$\tan\tan^{-1}x = x = \tan^{-1}\tan x$

5. Reciprocal property
$\sin^{-1}x = \operatorname{cosec}^{-1}\frac{1}{x}$ $\cos^{-1}x = \sec^{-1}\frac{1}{x}$

$\tan^{-1}x = \cot^{-1}\frac{1}{x}$

6. Conversion property
$\sin^{-1}x = \cos^{-1}\sqrt{1-x^2}$ $\sin^{-1}x = \tan^{-1}\frac{x}{\sqrt{1-x^2}}$

$\sin^{-1}x = \sec^{-1}\frac{1}{\sqrt{1-x^2}}$ $\sin^{-1}x = \cot^{-1}\frac{\sqrt{1-x^2}}{x}$

7. For any numerical value of x,
i. $\sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}$ ii. $\tan^{-1}x + \cot^{-1}x = \frac{\pi}{2}$

iii. $\operatorname{cosec}^{-1}x + \sec^{-1}x = \frac{\pi}{2}$

8. For given numerical value of x.
a. $\sin^{-1}(-x) = -\sin^{-1}x$ b. $\cos^{-1}(-x) = \pi - \cos^{-1}x$
c. $\tan^{-1}(-x) = -\tan^{-1}x$ d. $\cot^{-1}(-x) = \pi - \cot^{-1}x$

9. $\tan^{-1}x + \tan^{-1}y = \tan^{-1}\left(\frac{x+y}{1-xy}\right)$, $xy<1$

10. $\sin^{-1}x + \sin^{-1}y = \sin^{-1}\{x\sqrt{1-y^2} + y\sqrt{1-x^2}\}$

11. $\cos^{-1}x + \cos^{-1}y = \cos^{-1}(xy - \sqrt{1-x^2}\sqrt{1-y^2})$

FORMULAE

1. Scalar Product

If $\vec{a} = (a_1, a_2)$ and $\vec{b} = (b_1, b_2)$ then

$\vec{a}\cdot\vec{b} = a_1b_1 + a_2b_2$

If $\vec{a} = (a_1, a_2, a_3)$, $\vec{b} = (b_1, b_2, b_3)$ then

$\vec{a}\cdot\vec{b} = a_1b_1 + a_2b_2 + a_3b_3$

2. The angle between two vectors $\vec{a}$ and $\vec{b}$ is given by $\cos^{-1}\left(\frac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|}\right)$.

3. Projection of $\vec{a}$ on $\vec{b} = \frac{\vec{a}\cdot\vec{b}}{|\vec{b}|}$

Projection of $\vec{b}$ on $\vec{a} = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}|}$

4. Perpendicular or orthogonal vectors: Two vectors $\vec{a}$ and $\vec{b}$ are orthogonal if and only if $\vec{a}\cdot\vec{b} = 0$

5. $\vec{i}\cdot\vec{j} = \vec{j}\cdot\vec{k} = \vec{k}\cdot\vec{i} = 0$
$\vec{i}\cdot\vec{i} = \vec{j}\cdot\vec{j} = \vec{k}\cdot\vec{k} = 1$

6. If $\vec{a} = (a_1, a_2, a_3)$, $\vec{b} = (b_1, b_2, b_3)$, then

$\vec{a}\times\vec{b} = (a_2b_3 - a_3b_2)\vec{i} - (a_1b_3 - a_3b_1)\vec{j} + (a_1b_2 - a_2b_1)\vec{k}$

$$= \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

7. $\vec{a}\times\vec{b} = (|\vec{a}||\vec{b}|\sin\theta)\hat{n} = ab\sin\theta\,\hat{n}$

8. Angle between Two Vectors

$\theta = \sin^{-1}\frac{|\vec{a}\times\vec{b}|}{|\vec{a}||\vec{b}|}$

9. Vector perpendicular to $\vec{a}$ and $\vec{b} = \vec{a}\times\vec{b}$.

Unit vector perpendicular to $\vec{a}$ and $\vec{b} = \frac{\vec{a}\times\vec{b}}{|\vec{a}\times\vec{b}|}$.

10. Vector Products of $\vec{i}, \vec{j}, \vec{k}$

a. $\vec{i}\times\vec{j} = \vec{k}$, $\vec{j}\times\vec{k} = \vec{i}$, $\vec{k}\times\vec{i} = \vec{j}$

b. $\vec{j}\times\vec{i} = -\vec{k}$, $\vec{k}\times\vec{j} = -\vec{i}$, $\vec{i}\times\vec{k} = -\vec{j}$

c. $\vec{i}\times\vec{i} = \vec{j}\times\vec{j} = \vec{k}\times\vec{k} = 0$

11. $\vec{a}\times\vec{b} \ne \vec{b}\times\vec{a}$ but $\vec{a}\times\vec{b} = -\vec{b}\times\vec{a}$

12. Area of the parallelogram $= |\vec{a}\times\vec{b}| = ab\sin\theta$

Area of the triangle $= \frac{1}{2}|\vec{a}\times\vec{b}| = \frac{1}{2}ab\sin\theta$

13. Two vectors $\vec{a}$ and $\vec{b}$ are parallel if $\vec{a}\times\vec{b} = 0$.

FORMULAE

1. Intercept Form of the Equation of a Plane
   $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$
2. Normal Form of the Equation of a Plane
   $lx + my + nz = p$
3. General equation of a plane that passes through a given point (x_1, y_1, z_1) is
   $a(x - x_1) + b(y - y_1) + c(z - z_1) = 0$
4. Equation of the plane passing through three given non collinear points (x_1, y_1, z_1), (x_2, y_2, z_2) and (x_3, y_3, z_3) is

$$\begin{vmatrix} x & y & z & 1 \\ x_1 & y_1 & z_1 & 1 \\ x_2 & y_2 & z_2 & 1 \\ x_3 & y_3 & z_3 & 1 \end{vmatrix} = 0$$

5. Angle between Two Planes

$$\cos\theta = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}}$$

   i. If the planes are parallel, then $\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$

   ii. If the planes are perpendicular, then $a_1a_2+b_1b_2 + c_1c_2=0$
6. Plane through the Intersection of Two Planes
   The plane through the intersection of two planes
   $a_1x + b_1y + c_1z + d_1 = 0, a_2x + b_2y + c_2z + d_2 = 0$ is
   $(a_1x + b_1y + c_1z + d_1) + k\,(a_2x + b_2y + c_2z + d_2) = 0$
   where k is same constant.
7. Angle Between a Plane and a Line

$$\cos(90° - \theta) = \sin\theta = \frac{Aa_1 + Bb_1 + Cc_1}{\sqrt{A^2 + B^2 + C^2}\sqrt{a_1^2 + b_1^2 + c_1^2}}$$

   where, the plane is $Ax + By + Cz + D = 0$ and *a*, *b*, *c* are the direction ratios of the line.

FORMULAE

1. Karl Pearson's Coefficient of Correlation

   i. $r = \frac{\text{Cov}(X, Y)}{\sqrt{\text{Var}(X)}\sqrt{\text{Var}(Y)}}$

   ii. $r = \frac{n\Sigma XY - \Sigma X\,\Sigma Y}{\sqrt{n\Sigma X^2 - (\Sigma X)^2}\sqrt{n\Sigma Y^2 - (\Sigma Y)^2}}$

   iii. $r = \frac{\Sigma xy}{n\sigma_x\sigma_y}$

   iv. $r = \frac{n\Sigma XY - \Sigma X\,\Sigma Y}{\sqrt{n\Sigma X^2 - (\Sigma X)^2}\sqrt{n\Sigma Y^2 - (\Sigma Y)^2}}$

   v. $r = \frac{n\Sigma uv - \Sigma u\Sigma v}{\sqrt{n\Sigma u^2 - (\Sigma u)^2}\sqrt{n\Sigma v^2 - (\Sigma v)^2}}$
2. The value of r lies between –1 and +1.

FORMULAE

1. Regression equation of Y on X
   $Y - \bar{Y} = b_{YX}(X - \bar{X})$,
   where, $b_{YX} = \frac{n\Sigma XY - \Sigma X\,\Sigma Y}{n\Sigma X^2 - (\Sigma X)^2}$
2. Regression equation of X on Y
   $X - \bar{X} = b_{XY}(Y - \bar{Y})$
   where, $b_{XY} = \frac{n\Sigma XY - \Sigma X\,\Sigma Y}{n\Sigma Y^2 - (\Sigma Y)^2}$
3. Relation between r and b
   The correlation coefficient r is the geometric mean of two regression coefficients
   $r = \sqrt{b_{YX}b_{XY}}$.
   Also, the regression coefficients can be expressed in terms of r and σ as,
   $b_{XY} = r\frac{\sigma_X}{\sigma_Y}$ and $b_{YX} = r\frac{\sigma_Y}{\sigma_X}$.

FORMULAE

1. $P(E) = p = \frac{\text{Favourable number of cases}}{\text{Total number of cases}} = \frac{m}{n}$
2. Properties of Probability
   If p denotes the probability of happening of an event, then the following properties of the probability are always true.
   i. $p + q = 1$
   ii. p(certain event) = 1
   iii. p(impossible event) = 0
   iv. $0 \le p \le 1$.
3. Addition Theorem
   $P(A \cup B) = P(A \text{ or } B) = P(A) + P(B) - P(A \cap B)$
4. Multiplication Theorem
   $P(A \cap B) = P(A \text{ and } B) = P(A) \times P(B)$,
   where A and B are independent events.
5. Conditional Probability
   $P(A/B) = \frac{P(A \cap B)}{P(B)}$, provided $P(B) \neq 0$

   $P(B/A) = \frac{P(A \cap B)}{P(A)}$, provided $P(A) \neq 0$
6. $P(\bar{A}) = 1 - P(A)$

FORMULAE

1. Total probability for r success in n independent trials
   $P(r) = {}^nC_r\,p^r\,q^{n-r}$
2. Mean of the distribution is given by np.
3. Variance of the distribution is given by npq.
   Then, S.D. $= \sqrt{npq}$
4. Binomial distribution $= (q + p)^n$

FORMULAE

1. Distance between point (x_1, y_1, z_1) and (x_2, y_2, z_2)
$= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$
2. Section formula:

i. $(x, y, z) = \left(\frac{m_1x_2 + m_2x_1}{m_1 + m_2}, \frac{m_1y_2 + m_2y_1}{m_1 + m_2}, \frac{m_1z_2 + m_2z_1}{m_1 + m_2}\right)$

(internal division)

ii. $(x, y, z) = \left(\frac{m_1x_2 - m_2x_1}{m_1 - m_2}, \frac{m_1y_2 - m_2y_1}{m_1 - m_2}, \frac{m_1z_2 - m_2z_1}{m_1 - m_2}\right)$

(external division)

3. Centroid of a Triangle:
$(x, y, z) = \left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}, \frac{z_1 + z_2 + z_3}{3}\right)$
4. Mid point formula:
$(x, y, z) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2}\right)$
5. Direction Cosines of a Line
Let α, β, γ be the angles which a given directed line makes with the positive direction of the axes. Then $\cos\alpha$, $\cos\beta$, $\cos\gamma$, are called the direction cosines (dc's) of the line. The direction cosines of a line are usually denoted by l, m, n so that $l = \cos\alpha$, $m = \cos\beta$, $n = \cos\gamma$.
Also, $l^2 + m^2 + n^2 = 1$
6. Direction Ratios
Any three numbers a, b, c which are proportional to the direction cosines are called direction ratios (dr's.) of the given line. That is, if a, b, c are the direction ratios or the direction numbers of the line, then
$\frac{l}{a} = \frac{m}{b} = \frac{n}{c}$

$\therefore \quad l = \frac{a}{\sqrt{a^2 + b^2 + c^2}}$

$m = \frac{b}{\sqrt{a^2 + b^2 + c^2}}$

$n = \frac{c}{\sqrt{a^2 + b^2 + c^2}}$

7. The dcs. of x axis are (1, 0, 0), dcs. of y axis are (0, 1, 0) and dcs. of z axis are (0, 0, 1).
8. Projection of the join of two points on a line
Let PQ be the line joining two given points $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$. Let C'D' be a line whose direction cosines are l, m, n.
Projection of PQ on C'D' $= (x_2 - x_1)l + (y_2 - y_1)m + (z_2 - z_1)n$
9. Direction Ratios and Direction Cosines of Line Joining Two Points
The line PQ, joining $P(x_1, y_1, z_1)$, $Q(x_2, y_2, z_2)$ has its direction ratios: $x_2 - x_1$, $y_2 - y_1$ and $z_2 - z_1$ and direction cosines: $\frac{x_2 - x_1}{PQ}, \frac{y_2 - y_1}{PQ}, \frac{z_2 - z_1}{PQ}$
10. Angle Between Two Lines

a. $\theta = \cos^{-1}(l_1l_2 + m_1m_2 + n_1n_2)$
The lines are perpendicular if $l_1l_2 + m_1m_2 + n_1n_2 = 0$ and the lines are parallel if $\frac{l_1}{l_2} = \frac{m_1}{m_2} = \frac{n_1}{n_2}$

b. $\theta = \cos^{-1}\left(\frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}}\right)$
The lines are perpendicular if $a_1a_2 + b_1b_2 + c_1c_2 = 0$ and the lines are parallel if $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

FORMULAE

1. $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$
2. Right Hand and Left Hand Derivatives

$Rf'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}, h > 0$

$Lf'(x) = \lim_{h \to 0} \frac{f(x-h) - f(x)}{-h}, h > 0$

FORMULAE

1. i. $\frac{d(x^n)}{dx} = nx^{n-1}$ ii. $\frac{d}{dx}(e^x) = e^x$

iii. $\frac{d}{dx}(a^x) = a^x \ln a$ iv. $\frac{d}{dx}(\ln x) = \frac{1}{x}$

v. $\frac{d}{dx}(\sin x) = \cos x$ vi. $\frac{d}{dx}(\cos x) = -\sin x$

vii. $\frac{d}{dx}(\tan x) = \sec^2 x$ viii. $\frac{d}{dx}(\sec x) = \sec x \tan x$

ix. $\frac{d}{dx}(\cot x) = -\operatorname{cosec}^2 x$

x. $\frac{d}{dx}(\operatorname{cosec} x) = -\operatorname{cosec} x \cdot \cot x$

2. i. $\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}$ ii. $\frac{d}{dx}(\cos^{-1} x) = \frac{-1}{\sqrt{1-x^2}}$

iii. $\frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2}$ iv. $\frac{d}{dx}(\cot^{-1} x) = \frac{-1}{1+x^2}$

v. $\frac{d}{dx}(\sec^{-1} x) = \frac{1}{x\sqrt{x^2-1}}$ vi. $\frac{d}{dx}(\operatorname{cosec}^{-1} x) = \frac{-1}{x\sqrt{x^2-1}}$

3. i. $\frac{d[uv]}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$ ii. $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

4. Derivative of hyperbolic function

$\frac{d}{dx}(\sinh x) = \cosh x$

$\frac{d}{dx}(\cosh x) = \sinh x$

$\frac{d}{dx}(\tanh x) = \operatorname{sech}^2 x$

$\frac{d}{dx}(\coth x) = -\operatorname{cosech}^2 x$

$\frac{d}{dx}(\operatorname{sech} x) = -\operatorname{sech} x \tanh x$

$\frac{d}{dx}(\operatorname{cosech} x) = -\operatorname{cosech} x \coth x$

5. Derivative of inverse hyperbolic functions

$\frac{d(\sinh^{-1} x)}{dx} = \frac{1}{\sqrt{1+x^2}}$

$\frac{d}{dx}(\cosh^{-1} x) = \frac{1}{\sqrt{x^2-1}}$ $(x > 1)$

$\frac{d}{dx}(\tanh^{-1} x) = \frac{1}{1-x^2}$ $-1 < x < 1$

$\frac{d}{dx}(\coth^{-1} x) = \frac{-1}{x^2-1}$, $|x| > 1$

$\frac{d}{dx}(\operatorname{sech}^{-1} x) = \frac{-1}{x\sqrt{1-x^2}}$, $|x| < 1$

$\frac{d}{dx}(\operatorname{cosech}^{-1} x) = \frac{-1}{|x|\sqrt{1+x^2}}$, $x \in \mathbb{R} - \{0\}$

FORMULAE

1. Differentials

a. $\Delta y = f(x + \Delta x) - f(x)$ is the actual change in dependent variable y.

b. The differential of independent variable x, denoted by dx, is defined by $dx = \Delta x$.

c. The differential of dependent variable y, denoted by dy, is defined by $dy = f'(x)\,dx$; which is the approximate change in y.

d. Error = |Actual change – Approximate change|.

e. Percentage error $= \left|\frac{\Delta y - dy}{y}\right| \times 100$.

2. Equation of Tangent

$y - y_1 = -\left(\frac{dy}{dx}\right)_{(x,y)=(x_1,y_1)} (x - x_1)$

3. Equation of Normal

$y - y_1 = -\left(\frac{dx}{dy}\right)_{(x,y)=(x_1,y_1)} (x - x_1)$

4. The tangent to $y = f(x)$ at P is horizontal if and only if $\frac{dy}{dx} = 0$ at P.

5. The tangent to $y = f(x)$ at P is vertical if and only if $\frac{dx}{dy} = 0$ at P.

FORMULAE

If $\phi(x)$ and $\psi(x)$ and their derivatives $\phi'(x)$ and $\psi'(x)$ are continuous at $x = a$ and if $\phi(a) = \psi(a) = 0$, then

$\operatorname{Lt}_{x \to a} \frac{\phi(x)}{\psi(x)} = \operatorname{Lt}_{x \to a} \frac{\phi'(x)}{\psi'(x)} = \frac{\phi'(a)}{\psi'(a)}$ provided $\psi'(a) \neq 0$

FORMULAE

1. Rolle's Theorem

If f(x) be a function defined on [a, b] such that

i. f(x) is continuous in [a, b]

ii. f(x) is derivable in (a, b)

iii. $f(a) = f(b)$,

then there exists at least one $c \in (a, b)$ such that $f'(c) = 0$.

2. Lagrange's Mean Value Theorem

Let f(x) be a function defined in [a, b] such that

i. f(x) is continuous in [a, b]

ii. f(x) is derivable in (a, b)

then there exists at least one value $c \in (a, b)$ such that $f'(c) = \frac{f(b) - f(a)}{b - a}$

FORMULAE

1. i. $\int x^n\,dx = \dfrac{x^{n+1}}{n+1} + c,\ n \neq -1$

 ii. $\int \dfrac{1}{x}\,dx = \ln|x| + c,\ x \neq 0$

 iii. $\int e^x\,dx = e^x + c$

 iv. $\int a^x\,dx = \dfrac{a^x}{\ln a} + c$

2. i. $\int \sin x\,dx = -\cos x + c$

 ii. $\int \cos x\,dx = \sin x + c$

 iii. $\int \tan x\,dx = \ln \sec x + c$

 iv. $\int \cot x\,dx = \ln \sin x + c$

 v. $\int \sec x\,dx = \ln|\sec x + \tan x| + c$

 $= \ln\left|\tan\left(\dfrac{\pi}{4} + \dfrac{x}{2}\right)\right| + c$

 vi. $\int \operatorname{cosec} x\,dx = \ln|\operatorname{cosec} x - \cot x| + c$

 $= \ln \tan\dfrac{x}{2} + c$

 vii. $\int \sec^2 x\,dx = \tan x + c$

 viii. $\int \operatorname{cosec}^2 x\,dx = -\cot x + c$

 ix. $\int \sec x \tan x\,dx = \sec x + c$

 x. $\int \operatorname{cosec} x \cot x\,dx = -\operatorname{cosec} x + c$

3. i. $\int \dfrac{1}{\sqrt{1-x^2}}\,dx = \sin^{-1} x + c$

 ii. $\int \dfrac{1}{1+x^2}\,dx = \tan^{-1} x + c$

 iii. $\int \dfrac{1}{|x|\sqrt{x^2-1}}\,dx = \sec^{-1} x + c$

4. i. $\int (ax+b)^n\,dx = \dfrac{1}{a}\,\dfrac{(ax+b)^{n+1}}{n+1} + c,\ n \neq -1.$

 ii. $\int \dfrac{1}{ax+b}\,dx = \dfrac{1}{a}\ln|ax+b| + c$

5. $\int (uv)\,dx = u\int v\,dx - \int\left\{\dfrac{d}{dx}(u)\int v\,dx\right\}dx$

6. i. $\int \dfrac{1}{x^2-a^2}\,dx = \dfrac{1}{2a}\ln\left|\dfrac{x-a}{x+a}\right| + c$

 ii. $\int \dfrac{1}{a^2-x^2}\,dx = \dfrac{1}{2a}\ln\left|\dfrac{a+x}{a-x}\right| + c$

 iii. $\int \dfrac{1}{x^2+a^2}\,dx = \dfrac{1}{a}\tan^{-1}\dfrac{x}{a} + c$

7. i. $\int \dfrac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\dfrac{x}{a} + c$

 ii. $\int \dfrac{1}{\sqrt{x^2-a^2}}\,dx = \ln\left|x + \sqrt{x^2-a^2}\right| + c$

 iii. $\int \dfrac{1}{\sqrt{a^2+x^2}}\,dx = \ln\left|x + \sqrt{x^2+a^2}\right| + c$

8. i. $\int \sqrt{a^2-x^2}\,dx = \dfrac{x}{2}\sqrt{a^2-x^2} + \dfrac{a^2}{2}\sin^{-1}\dfrac{x}{a} + c$

 ii. $\int \sqrt{x^2-a^2}\,dx = \dfrac{x}{2}\sqrt{x^2-a^2} - \dfrac{a^2}{2}\ln\left|x + \sqrt{x^2-a^2}\right| + c$

 iii. $\int \sqrt{x^2+a^2}\,dx = \dfrac{x}{2}\sqrt{x^2+a^2} + \dfrac{a^2}{2}\ln\left|x + \sqrt{x^2+a^2}\right| + c$

9. i. $\int \sinh x\,dx = \cosh x + c$

 ii. $\int \cosh x\,dx = \sinh x + c$

 iii. $\int \tanh x\,dx = \ln \cosh x + c$

 iv. $\int \coth x\,dx = \ln \sinh x + c$

 v. $\int \operatorname{sech} x\,dx = \tan^{-1}|\sinh x| + c$

 vi. $\int \operatorname{cosech} dx = \ln \tanh\dfrac{x}{2} + c$

 vii. $\int \operatorname{sech}^2 x\,dx = \tanh x + c$

 viii. $\int \operatorname{cosech}^2 x\,dx = -\coth x + c$

 ix. $\int \operatorname{sech} x \tanh x\,dx = -\operatorname{sech} x + c$

 x. $\int \operatorname{cosech} x \coth x\,dx = -\operatorname{cosech} x + c$

FORMULAE

1. Separation of variables

 If the equation $M\,dx + N\,dy = 0$ can be put in the form $f_1(x)\,dx + f_2(y)\,dy = 0$, then it can be solved by integrating each term separately. Thus, the solution of the above equation is $\int f_1(x)\,dx + \int f_2(y)\,dy = c$

2. Homogeneous Equations

 $\dfrac{dy}{dx} = F\left(\dfrac{y}{x}\right)$

 To solve such equations, we put $y = vx$, where v is a function of x. Then,

 $\dfrac{dx}{x} = \dfrac{dv}{F(v) - v}$

3.
 i. $y\,dx + x\,dy = d(xy)$

 ii. $\dfrac{y\,dx - x\,dy}{y^2} = d\left(\dfrac{x}{y}\right)$

 iii. $\dfrac{x\,dy - y\,dx}{x^2} = d\left(\dfrac{y}{x}\right)$

 iv. $\dfrac{y\,dx - x\,dy}{x^2+y^2} = \dfrac{\dfrac{y\,dx - x\,dy}{y^2}}{1 + \left(\dfrac{x}{y}\right)^2} = d\left[\tan^{-1}\dfrac{x}{y}\right]$

 v. $\dfrac{x\,dy - y\,dx}{x^2+y^2} = \dfrac{\dfrac{x\,dy - y\,dx}{x^2}}{1 + \left(\dfrac{y}{x}\right)^2} = d\left[\tan^{-1}\dfrac{y}{x}\right]$

4. Linear equation

 The expression $e^{\int P dx}$ is called the Integrating Factor (I.F.) of linear equation $\dfrac{dy}{dx} + P\,y = Q$. The solution of this equation is of the form:

 $y(\text{I.F.}) = \int Q(\text{I.F.})\,dx + c$

FORMULAE

Summary of the Simplex Method (Maximization)

1. Convert the constraints to equations by adding slack variables.
2. Create the initial simplex tableau.
3. Locate the most negative entry in the last row. The column for this entry is called the **entering column**. If ties occur, we can use any one column for pivot column.
4. The **departing row** corresponds to the smallest non-negative ratio $\frac{b_i}{a_{ij}}$. If all entries in the entering column are 0 or negative, then there is no maximum value.
   For ties, choose either entry. The element in the entering column and departing row is pivot.
5. Use elementary row operations to make pivot 1 and all other entries in the entering column 0.
6. **If all the entries in the last row are non-negative then the optimal solution is obtained. If not, repeat step 3 to 5 until the entries in the last row are all non-negative.**

The Simplex Method (Minimization)

A LPP of minimizing the objective function must have the constraints in the form of $\geq$. We solve minimization problem by converting it into maximization problem and use the method stated above.

A minimization problem in standard form is

$z = c_1x_1 + c_2x_2 + \ldots + c_nx_n$ (objective function)

subject to the constraints

$$a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n \geq b_1$$
$$a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n \geq b_2$$
$$\vdots \quad \vdots \quad \vdots \quad \vdots$$
$$a_{m1}x_1 + a_{m2}x_2 + \ldots + a_{mn}x_n \geq b_m$$

where $x_i \geq 0$, $i = 1, 2 \ldots n$

$b_i \geq 0$, $i = 1, 2 \ldots m$

Step I: Write the augmented matrix

$$A = \left[\begin{array}{cccc|c} a_{11} & a_{12} & \ldots & a_{1n} & b_1 \\ a_{21} & a_{22} & \ldots & a_{2n} & b_2 \\ \vdots & \vdots & & \vdots & \vdots \\ a_{m1} & a_{m2} & \ldots & a_{mn} & b_m \\ \hline c_1 & c_2 & \ldots & c_n & 0 \end{array}\right]$$

Step II: Find the transpose of the matrix A.

$$A^T = \left[\begin{array}{cccc|c} a_{11} & a_{12} & \ldots & a_{m1} & c_1 \\ a_{12} & a_{22} & \ldots & a_{m2} & c_2 \\ \vdots & \vdots & & \vdots & \vdots \\ a_{1n} & a_2 & \ldots & a_{mn} & c_n \\ \hline b_1 & b_2 & \ldots & b_m & 0 \end{array}\right]$$

Step III: Form the dual maximization problem as follows

Maximize: $w = b_1y_1 + b_2y_2 + \ldots + b_ny_n$

subject to

$$a_{11}y_1 + a_{21}y_2 + \ldots + am_1y_m \leq c_1$$
$$a_{12}y_1 + a_{22}y_2 + \ldots + am_2y_m \leq c_2$$
$$\vdots \quad \vdots \quad \vdots \quad \vdots$$
$$a_{1n}y_1 + a_{2n}y_2 + \ldots + a_{mn}y_m \leq c_n$$

where $y_i \geq 0$, $i = 1, 2 \ldots m$

Step IV: Apply the simplex method to the dual maximization problem. The maximum value of w will be the minimum value z. The values $x_1, x_2 \ldots x_n$ will occur in last row of the final simplex tableau.

FORMULAE

1. Momentum = mv
   Change in momentum = mv – mu
   Average rate of change of momentum = $\frac{m(v-u)}{t}$
2. Upward force: $R = m(a + g)$
   Downward force: $R = m(g - a)$
3. Impulsive force: $F = \frac{d(mv)}{dt}$, as m is constant
4. Principle of conservation of linear momentum
   $mv - MV = 0$
   $m_1v_1 + m_2v_2 = (m_1 + m_2)v$

FORMULAE

1. Equations of Motion of a Projectile
   For the vertical motion,
   $v \sin\theta = u \sin\alpha - gt$
   and for the horizontal motion,
   $v \cos\theta = u \cos\alpha$
2. Time to reach the greatest height
   $t = \frac{u \sin\alpha}{g}$
3. Time of Flight and Range
   $T = \frac{2u \sin\alpha}{g}$ $\qquad$ $R = \frac{u^2 \sin 2\alpha}{g}$
4. Maximum horizontal range = $\frac{u^2}{g}$
5. Greatest height: $H = \frac{u^2 \sin^2\alpha}{2g}$

FORMULAE

1. A system of linear equation is said to be **consistent** if it has either one solution or infinitely many solutions and system is said to **inconsistent** if it has no solution.

WAVES & OPTICS

6. WAVE MOTION

FORMULAE

1. Some Relations:
   i. $v = f\lambda$ ii. $\lambda = vT$
   iii. $k = \frac{2\pi}{\lambda} = \frac{2\pi f}{v} = \frac{\omega}{v}$ iv. $\omega = 2\pi f = \frac{2\pi}{T}$
2. The forms of progressive wave equations are
   $y = a \sin(\omega t - \phi)$ $y = a \sin 2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right)$
   $y = a \sin(\omega t - kx)$ $y = a \sin\frac{2\pi}{\lambda}(vt - x)$
3. Stationary wave: $y = A \sin \omega t$, where $A = 2a \cos kx$ is the amplitude of the stationary wave.

7. MECHANICAL WAVES

FORMULAE

1. Speed of sound, $v = \sqrt{\frac{E}{\rho}} = \frac{\text{Elasticity}}{\text{Density}}$
2. Speed of longitudinal wave,
   In solid, $v = \sqrt{\frac{E}{\rho}}$, In liquid, $v = \sqrt{\frac{K}{\rho}}$,
   In gas, $v = \sqrt{\frac{\gamma P}{\rho}}$, $\gamma = \frac{C_p}{C_v}$
3. Factors affecting the speed of sound in gas,
   i. Temperature, $v \propto \sqrt{T}$, so $\frac{v_1}{v_2} = \sqrt{\frac{T_1}{T_2}}$
   ii. Pressure, no effect at constant temperature
   iii. Density, $v \propto \frac{1}{\sqrt{\rho}}$, so $\frac{v_1}{v_2} = \sqrt{\frac{\rho_1}{\rho_2}}$ (for different gases)
   iv. Molar mass, $v \propto \frac{1}{\sqrt{M}}$, so $\frac{v_1}{v_2} = \sqrt{\frac{M_2}{M_1}}$
   v. Humidity, $v_{humid} > v_{dry}$
4. Temperature coefficient of speed of sound,
   $$\frac{v_t}{v_0} = \sqrt{\frac{273 + \theta}{273}} = v_0\left(1 + \frac{1}{2}\alpha\theta\right)$$
   Where, α temperature coefficient of speed of sound
5. Speed of transverse wave in a stretched string, $v = \sqrt{\frac{T}{\mu}}$,
   where T = tension on string and μ = mass per unit length

11. INTERFERENCE

FORMULAE

1. The resultant amplitude in interference of two waves,
   $a = \sqrt{a_1^2 + a_2^2 + 2a_1a_2 \cos\phi}$
2. Intensity of resultant wave, $I = I_1 + I_2 + 2\sqrt{I_1I_2}\cos\phi$,
   Where ϕ is the phase difference between two waves
3. Constructive interference is obtained, when $\phi = 2n\pi$ and $x = n\lambda$, where, n = 0, 1, 2, ...
4. Destructive interference is obtained, when $\phi = (2n - 1)\pi$ and $x = (2n - 1)\frac{\lambda}{2}$, where n = 1, 2, 3, ...
5. Relation between optical path and geometric path:
   Optical path (L) = refractive index (η) $\times$ geometric path (x)
6. $\phi = \frac{2\pi}{\lambda}x$
7. The angular width of a fringe produced by Young's double slit experiment,
   $\theta = \frac{\beta}{D} = \frac{\alpha}{D} = \frac{\lambda D}{dD} = \frac{\lambda}{d}$
8. The fringe width for both bright and dark pattern is
   $\alpha = \beta = \frac{\lambda D}{d}$

9. ACOUSTIC PHENOMENA

FORMULAE

1. Intensity
   i. $I = \frac{\text{Power transfer}}{\text{surface area}} = \frac{P}{A} = 2a^2f^2a^2\rho v = \frac{P^2_{max}}{2\rho v}$
   ii. $\frac{I_1}{I_2} = \frac{r_2^2}{r_1^2}$ (inverse square law)
   iii. Threshold of hearing, $L_0 = \log I_0$, $I_0 = 10^{-12}\ Wm^{-2}$
   iv. Intensity level, $\beta = 10 \log\frac{I}{I_0}$ (decibel)
   v. Comparison, $\Delta\beta = \beta_1 - \beta_2 = 20 \log\left(\frac{r_2}{r_1}\right)$.
   vi. Energy density, $U = \frac{\text{Energy transfer}}{\text{volume}}$
   viii. Intensity, point source (spherical wave) $I = \frac{P}{4\pi r^2}$, $a \propto \frac{1}{\sqrt{r}}$
2. Beats: The beat frequency, $f_b = f_1 - f_2$ (for $f_1 > f_2$) and $f_b = f_2 - f_1$ (for $f_2 > f_1$)
3. Doppler's effect:
   i. Apparent frequency heard by the observer, $f' = \frac{v \pm v_0}{v \pm v_s}$
   ii. Apparent frequency heard by driver moving towards a hill (i.e. reflector), $f' = \frac{v + v_0}{v - v_s} \times f$
   iii. Apparent frequency of echo of horn of his car heard by driver moving towards the hill (i.e. reflector)
   $f' = \frac{v + v_0}{v - v_s} \times f$
   iv. Doppler's effect in light: Apparent frequency of light received by an observer, $f' = \left(\frac{c \pm v}{c}\right) f$
   Positive sign is chosen when the source and the observer are approaching each other and negative sign is chosen when the source and the observer are receding away.

8. WAVE IN PIPES AND STRINGS

FORMULAE

1. Waves in Pipe:
   i. The fundamental frequency in closed organ pipe,
   $f_1 = \frac{v}{4l} = \frac{1}{4l}\sqrt{\frac{\gamma P}{\rho}} = \frac{1}{4l}\sqrt{\frac{\gamma RT}{M}}$
   ii. The fundamental frequency of organ pipe,
   $f_1 = \frac{v}{2l} = \frac{1}{2l}\sqrt{\frac{\gamma P}{\rho}} = \frac{1}{2l}\sqrt{\frac{\gamma RT}{M}}$
   iii. End correction:
   a. (i) $e = \frac{l_2 - 3l_1}{2}$
   (ii) e = 0.3d, d = diameter of resonance tube
   b. Fundamental frequency in closed organ pipe,
   $f_1 = \frac{v}{4(l + e)}$
2. Waves in string:
   i. The fundamental frequency, $f_1 = \frac{1}{2l}\sqrt{\frac{T}{\mu}}$
   The speed of transverse vibration, $v = \sqrt{\frac{T}{\mu}}$
   ii. For a cylindrical wire,
   $\mu = A.\rho = \frac{\pi d^2}{4}.\rho$ or $\pi r^2.\rho$,
   where ρ= Mass per unit length

10. NATURE AND PROPAGATION OF LIGHT

FORMULAE

1. Refractive index: $\eta = \frac{\text{wavelength of light in vacuum}}{\text{wavelength of light in medium}} = \frac{\lambda}{\lambda'}$

11. INTERFERENCE

FORMULAE

1. The resultant amplitude in interference of two waves, $a = \sqrt{a_1^2 + a_2^2 + 2a_1a_2\cos\phi}$
2. Intensity of resultant wave, $I = I_1 + I_2 + 2\sqrt{I_1I_2}\cos\phi$, Where ϕ is the phase difference between two waves
3. Constructive interference is obtained, when $\phi = 2n\pi$ and $x = n\lambda$, where, $n = 0, 1, 2, \ldots$
4. Destructive interference is obtained, when $\phi = (2n - 1)\pi$ and $x = (2n - 1)\frac{\lambda}{2}$, where $n = 1, 2, 3, \ldots$
5. Relation between optical path and geometric path: Optical path (L) = refractive index (η) $\times$ geometric path (x)
6. $\phi = \frac{2\pi}{\lambda}x$
7. The angular width of a fringe produced by Young's double slit experiment, $\theta = \frac{\beta}{D} = \frac{\alpha}{D} = \frac{\lambda D}{dD} = \frac{\lambda}{d}$
8. The fringe width for both bright and dark pattern is $\alpha = \beta = \frac{\lambda D}{d}$

12. DIFFRACTION

FORMULAE

1. Condition of n^{th} minimum is, $d\sin\theta = n\lambda$, where $n = 1, 2, 3, \ldots$
2. Condition of n^{th} secondary maximum, $d\sin\theta = (2n + 1)\frac{\lambda}{2}$, where $n = 1, 2, 3, \ldots$
3. The angular position of n^{th} minimum, $\theta_n = \frac{n\lambda}{d}$
4. The distance of n^{th} minimum from the center of the screen, $x_n = \frac{n\lambda D}{d}$
5. The angular position of n^{th} secondary maximum, $\theta_n' = (2n + 1)\frac{\lambda}{2d}$
6. Distance of n^{th} secondary maximum from the center of the screen $x_n' = \left(\frac{2n+1}{2}\right)\frac{\lambda D}{d}$
7. Width of central maximum, width $\beta_0 = 2\beta = \frac{2\lambda D}{d}$
8. Angular spread of central maximum on either side, $\theta = \pm\frac{\lambda}{d}$.
9. Total angular spread of central maximum $2\theta = \frac{2\lambda}{d}$.
10. The resolving power $= \frac{1}{\text{limit of resolution}}$.
11. Limit of resolution, $\theta = \frac{1.22\lambda}{D}$.

    The resolving power $= \frac{D}{1.22\lambda}$.
12. Grating equation, $\sin\theta_n = Nn\lambda$, where, $N = \frac{1}{(a + b)}$.

13. POLARIZATION

FORMULAE

1. Malus law: $I \propto \cos^2\theta$
2. Brewster's law: $\eta = \tan\theta_P$

   Where, $\eta = \frac{c}{v}$ = refractive index and θ_P = polarizing angle

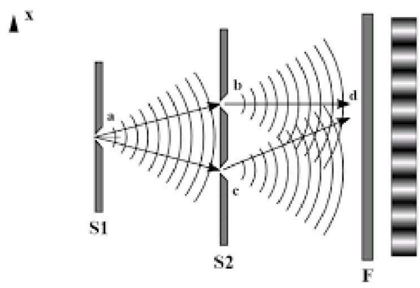
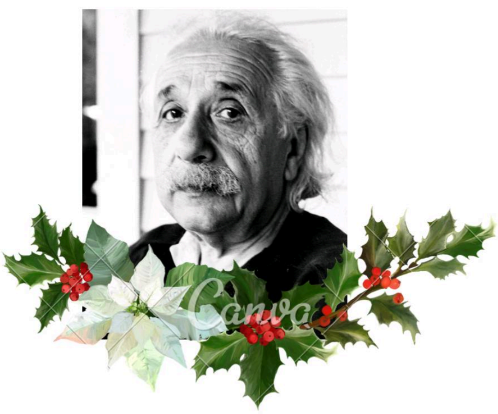


Fig 1: Interference patterns of light obtained after passing light through two slits.



RIP

MECHANICS

1. ROTATIONAL DYNAMICS

FORMULAE

1. The counterpart symbols in translational motion and rotational motion,

Translational Motion	Rotational Motion
1. Linear displacement=s	1. Angular displacement=θ
2. Linear velocity, $v = \frac{ds}{dt}$	2. Angular velocity, $\omega = \frac{d\theta}{dt}$
3. Linear acceleration, $a = \frac{dv}{dt} = \frac{d^2s}{dt^2}$	3. Angular acceleration, $\alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$
4. Mass, m	4. Moment of inertia, I
5. Linear momentum, $p = mv$	5. Angular momentum, $L = I\omega$
6. Force, $F = \frac{dp}{dt} = ma$	6. Torque, $\tau = \frac{dL}{dt} = I\alpha$
7. Work done by force, $W = Fs$	7. Work done by torque, $W = \tau\theta$
8. Translational kinetic energy, $E_k = \frac{1}{2}mv^2$	8. Rotational Kinetic energy, $E_k = \frac{1}{2}I\omega^2$
9. Equations of translational motion: i. $s = ut$ ii. $v = u + at$ iii. $s = ut + \frac{1}{2}at^2$ iv. $v^2 = u^2 + 2as$	9. Equations of rotational motion: i. $\theta = \omega_0 t$ ii. $\omega = \omega_0 + \alpha t$ iii. $\theta = \omega_0 t + \frac{1}{2}\alpha t^2$ iv. $\omega^2 = \omega_0^2 + 2\alpha\theta$

2. The moment of inertia, $I = \sum_{i=1}^{n} m_i r_i^2$
3. Radius of gyration, $K = \sqrt{\frac{r_1^2 + r_2^2 + \ldots + r_n^2}{n}}$
4. The total kinetic energy of a rolling body, K.E. $= \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$
5. Conservation of angular momentum, $I_1\omega_1 = I_2\omega_2$
6. The acceleration of a cylinder rolling down an inclined plane, $a = \frac{g\sin\theta}{1 + \frac{I}{Mr^2}}$

FORMULAE

1. Restoring force, $F = -ky$, where, k is the restoring force constant
2. The wave equation for S.H.M., $y = A\sin(\omega t + \phi_0)$
3. Energy in SHM: Kinetic energy, $E_k = \frac{1}{2}k(A^2 - x^2) = \frac{1}{2}m\omega^2(A^2 - x^2)$

   and potential energy, $E_p = \frac{1}{2}kx^2 = \frac{1}{2}m\omega^2x^2$

   Total energy, $E = \frac{1}{2}m\omega^2A^2$
4. The displacement, velocity, acceleration, time period and frequency of SHM:

S. N.	Physical Quantity	Formula
i.	Displacement	$y = A\sin(\omega t + \phi)$
ii.	Velocity	$v = \omega\sqrt{A^2 - y^2}$, At mean position, $y = 0$, $v = A\omega = v_{max}$ At extreme position, $y = A$, $v = 0 = v_{min}$
iii.	Acceleration	$a = -\omega^2 y$ At mean position, $y = 0$, $a = 0 = a_{min}$ At extreme position, $y = A$, $a = -\omega^2 A = a_{max}$
iv.	Time period	$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{\text{Displacement}}{\text{Acceleration}}}$ In oscillation of spring, $T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{\text{Inertia factor}}{\text{Spring factor}}}$
v.	Frequency	$f = \frac{1}{T} = \frac{\omega}{2\pi}$

5. Oscillation of simple pendulum: (i) Time period, $T = 2\pi\sqrt{\frac{l}{g}}$ (ii) frequency, $f = \frac{1}{2\pi}\sqrt{\frac{g}{l}}$
6. Oscillation of loaded spring: $T = 2\pi\sqrt{\frac{m}{k}}$

C. VISCOSITY

FORMULAE

1. The viscous force, $F = -\eta A\frac{dv}{dx}$, where η is called the coefficient of viscosity
2. The equation of continuity is Av = constant i.e., $A_1v_1 = A_2v_2$
3. The Bernoulli's formula, $P + \rho gh + \frac{1}{2}\rho v^2$ = constant, for horizontal flow of liquid, $h = 0$
4. Poiseuille's formula, $Q = \frac{\pi Pr^4}{8\eta l}$
5. Stoke's formula, $F = 6\pi\eta r v_t$
6. Terminal velocity, $v_t = \frac{2}{9}\frac{r^2}{\eta}(\rho - \sigma)g$

B. SURFACE TENSION

Values of Physical Constants

Surface tension of water = 7.0×10^{-2} Nm^{-1}

FORMULAE

1. Surface tension, $T = \frac{\text{Force}}{\text{Length}} = \frac{F}{L}$
2. Work done, W = surface tension × increase in area of the liquid surface i.e. $W = T \times A$
3. Height of liquid in capillary tube, $h = \frac{2T\cos\theta}{r\rho g}$, where θ is angle of contact

A. FLUID STATIC

Values of Physical Constants

Specific gravity of steel = 7.8

Density of water = 1000 Kg/m^3

FORMULAE

1. Pascal's law, $P = \frac{F_1}{A_1} = \frac{F_2}{A_2}$
2. According to Archimedes' principle,
   Loss in weight of a body in a liquid
   = weight of liquid displaced
   = volume × density × acceleration due to gravity
3. Apparent weight of solid in a liquid
   = True weight – weight of liquid displaced
   $= mg - V'\rho' g$
4. When a body just floats,
   Weight of the body = weight of liquid displaced
   $V\rho g = V'\rho' g$
   or, $\frac{V'}{V} = \frac{\rho'}{\rho}$
   i.e., $\frac{\text{Volume of immersed portion}}{\text{Total volume of the solid}} = \frac{\text{Density of solid}}{\text{Density of liquid}}$
5. Specific gravity $= \frac{\text{Density of substance at t°C}}{\text{Density of water at 4°C}}$

ELECTRICITY & MAGNETISM

14. ELECTRICAL CIRCUITS

FORMULAE

1. Kirchhoff's first (current) law: Sum of incoming currents – sum of out going currents = 0, i.e. $\Sigma I = 0$
2. Kirchoff's second (Voltage) law: $\Sigma E - \Sigma IR = 0$
3. Wheat stone bridge circuit Principle: In balanced conditions, $\frac{P}{Q} = \frac{X}{R}$
4. Meterbridge: In balancing condition $\frac{R_1}{R_2} = \frac{l_1}{l_2}$
5. Potentiometer:
   i. Principle, $V \propto l$ (for I, A and ρ constant)
   ii. Comparison of emf: $\frac{E_1}{E_2} = \frac{l_1}{l_2}$
   iii. Determination of internal resistance:
   $r = \left(\frac{E}{V} - 1\right) R = \left(\frac{l_1}{l_2} - 1\right) R$
6. Conversion of galvanometer into ammeter
   i. Internal resistance of ammeter is very small, $R_a = \frac{GS}{G + S}$
   ii. Value of shunt is, $S = \frac{I_g G}{I - I_g}$
7. Conversion of galvanometer into voltmeter:
   i. Internal resistance is very large; $R_V = R + G$
   ii. Value of multiplier, $R = \frac{V}{I_g} - G$

18. ELECTROMAGNETIC INDUCTION

FORMULAE

1. Magnetic Flux: $\phi = \vec{B} \cdot \vec{A} = BA\cos\theta$. (where θ is the angle between $\vec{B}$ and $\vec{A}$)
2. Formulae about induced emf, current, charge and power:
   i. Induced emf, $E = -N\frac{d\phi}{dt}$
   ii. Induced current, $I = \frac{E}{R} = -\frac{N}{R} \cdot \frac{d\phi}{dt}$
   iii. Induced charge, $q = Idt = -\frac{E}{R} d\phi$
   iv. Induced power, $P = IE = \frac{E^2}{R} = N^2 \left(\frac{d\phi}{dt}\right)^2 \cdot \frac{1}{R}$
3. Magnitude of induced emf:
   i. For a conducting rod moving in a uniform magnetic field, $E = Blv \sin\theta$,
   ii. For a conducting rod rotating with angular velocity ω in a uniform magnetic field, $E = BAf$
   iii. For a disc of radius r rotating in uniform magnetic field, $E = BAf$.
   iv. For a rectangular coil rotating in a uniform magnetic field, $E = BA\omega N \sin\omega t = E_0 \sin\omega t$
4. Self induction:
   i. Magnetic flux, $\phi = LI$
   ii. Induced emf in the coil, $E = -L\frac{dI}{dt}$
   iii. For a solenoid, $L = \frac{\mu_0 N^2 A}{l}$
   iv. For a plane circular coil, $(L) = \frac{\mu_0 N^2 A}{2r}$
   v. The energy stored by a inductor, $U = \frac{1}{2} LI^2$
5. Mutual induction
   i. The emf induced by neighbouring coil, $E = -M\frac{dI}{dt}$
   ii. The mutual inductance of two long coaxial solenoids each of length l, area of cross-section A wound in air is,
   $M = \frac{\mu_0 N_1 N_2 \pi r_1^2}{2r_2}$, r_2 is radius of bigger coil or $r_2 > r_1$
6. Transformer:
   i. Relation of alternating voltage, alternating current and number of turns in two coils. $\frac{V_s}{V_p} = \frac{N_s}{N_p} = \frac{I_p}{I_s}$
   ii. Efficiency, $\eta = \frac{\text{output power}}{\text{intput power}} \times 100\%$

16. MAGNETIC FIELD

FORMULAE

1. Lorentz Force:
   i. The magnitude of magnetic Lorentz force experienced by moving charge in magnetic field,
   $F = q\,|\vec{v} \times \vec{B}| = Bqv \sin\theta$
   ii. The magnitude of magnetic Lorentz force experienced by current carrying conductor in magnetic field B,
   $F = I\,(\vec{l} \times \vec{B}) = BIl \sin\theta$.
2. Magnetic torque and moving coil meters:
   i. The magnetic torque in a rectangular coil $\tau = BINA\cos\theta$, where θ is the direction of the current with respect to uniform magnetic field.
   ii. The magnetic moment of a current loop, $\mu = IA$, I = current, A = Area of loop.
   iii. The angular displacement of needle in moving coil meter, $\theta = \left(\frac{BNA}{k}\right) I$
   Where k is restoring torque per units twist.
   iv. The current sensitivity of coil meter, $\frac{\theta}{I} = \frac{BNA}{k}$
   v. The voltage sensitivity of coil meter, $\frac{V}{I} = \frac{BNA}{kR}$
3. Biot and Savart law:
   i. The magnitude of magnetic field dB due to a current element (Idl) is,
   $dB = \frac{\mu_0}{4\pi} \frac{Idl \sin\theta}{r^2}$
   ii. Magnetic Field at centre of circular current carrying coil, $B = \frac{\mu_0 NI}{2r}$
   iii. Magnetic Field at axis of circular current carrying coil, $B = \frac{\mu_0 NIR^2}{2(R^2 + x^2)}$
   iv. Magnetic Field due to an infinite long straight current carrying conductor, $B = \frac{\mu_0 I}{2\pi r}$.
   v. Magnetic Field due to an infinite long current carrying solenoid,
   $B = \mu_0 nI$ (at the center) and $B = \frac{\mu_0 nI}{2}$ (at the end).
4. Magnetic force per unit length, due to two current carrying conductor, $\frac{F}{l} = \frac{\mu_0 I_1 I_2}{2\pi r}$
5. Hall effect:
   i. Hall voltage, $V_H = \frac{BI}{net}$
   ii. Hall coefficient $H_C = \frac{1}{ne} = \frac{E_H}{BJ}$
   iii. Hall resistance, $R_H = \frac{BH_C}{t} = \frac{BH_C d}{A}$

15. THERMOELECTRIC EFFECTS

FORMULAE

1. The neutral temperature, $\theta_n = \frac{\theta_c + \theta_i}{2}$, where θ_c is the temperature of cold junction and θ_i is the temperature of inversion.
2. The thermo emf, $E = \alpha\theta + \frac{1}{2}\beta\theta^2$, where, α and β are constants.

19. ALTERNATING CURRENTS

FORMULAE

1. Alternating current and voltage:

i. Instantaneous value of alternating current is, $I = I_0 \sin \omega t$

ii. Instantaneous value of alternating voltages, $V = V_0 \sin\omega t$

iii. $I_{rms} = \frac{I_0}{\sqrt{2}} = 0.707\, I_o$, I_o = peak value of current

iv. $V_{rms} = \frac{V_0}{\sqrt{2}} = 0.707\, V_o$, V_o = peak value of Voltage

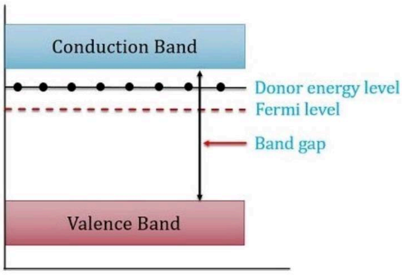
2. Parameters of Various a.c. circuits

S.N.	Parameter	R circuit	L circuit	C circuit	RL circuit	RC circuit	LCR circuit
1.	Alternating voltage and current	$V = V_o \sin\omega t$ $I = I_o \sin\omega t$	$V = V_o \sin\omega t$ $I = I_o \sin\left(\omega t - \frac{\pi}{2}\right)$	$V = V_o \sin\omega t$ $I = I_o \sin\left(\omega t + \frac{\pi}{2}\right)$	$V = V_o \sin\omega t$ $I = I_o \sin(\omega t - \phi)$	$V = V_o \sin\omega t$ $I = I_o \sin(\omega t + \phi)$	$V = V_o \sin\omega t$ $I = I_o \sin(\omega t \pm \phi)$
2.	Phase difference of current w.r.t. voltage	zero	lags by $\frac{\pi}{2}$	Leads by $\frac{\pi}{2}$	Lags by ϕ, $\tan \phi = \frac{X_L}{R}$	Leads by ϕ, $\tan \phi = \frac{X_C}{R}$	Leads by ϕ, $\tan\phi = \frac{X_L - X_C}{R}$ or, $\frac{X_C - X_L}{R}$
3.	Reactance	Zero	$X_L = \omega L$	$X_C = \frac{1}{\omega C}$	$X_L = \omega L$	$X_C = \frac{1}{\omega C}$	$X_L - X_C$ or, $X_C - X_L$
4.	Impedance	$Z = R$	$Z = X_L$	$Z = X_C$	$Z = \sqrt{R^2 + X_L^2}$	$Z = \sqrt{R^2 + X_C^2}$	$Z = \sqrt{R^2 + (X_L - X_C)^2}$
5.	Power factor ($\cos\phi$)	1	Zero	Zero	$\frac{R}{\sqrt{R^2 + X_L^2}}$	$\frac{R}{\sqrt{R^2 + X_C^2}}$	$\frac{R}{\sqrt{R^2 + (X_L - X_C)^2}}$
6.	Average Power	$V_{rms} I_{rms}$	Zero	Zero	$V_{rms} I_{rms} \cos\phi$	$V_{rms} I_{rms} \cos\phi$	$V_{rms} I_{rms} \cos\phi$

3. Some other formula regarding a.c.

i. Resonant frequency, $f_o = \frac{1}{2\pi\sqrt{LC}}$ for L-C-R series circuit.

ii. Quality factor, $Q = \frac{\omega_o L}{R} = \frac{1}{\omega_o CR}$ for L-C-R series circuit.



17. MAGNETIC PROPERTIES OF MATERIALS

FORMULAE

1. $\tan \delta = \frac{B_V}{B_H}$

2. Apparent dip, $\tan \delta' = \frac{\tan \delta}{\cos \theta}$

3. True dip from two apparent dips, $\cot^2\delta = \cot^2\delta_1 + \cot^2\delta_2$

4. $B_V^2 + B_H^2 = B^2$

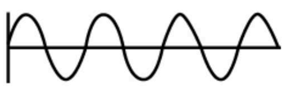
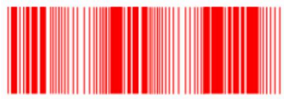
5. $\mu_r = 1 + \chi$

6. $\mu = \mu_0 \mu_r = \frac{B}{H} = \mu_0 (1 + \chi)$

7. $I = \frac{M}{V} = \frac{m}{A}$

8. $H = \frac{B_0}{\mu_0}$

9. $\chi_m = \frac{I}{H}$

Transverse waves	Longitudinal waves
1. 	1. 
2. Particles vibrate in a direction perpendicular to the direction of propagation of the wave.	2. Particles vibrate in a direction parallel to the direction of propagation of the wave.
3. Crests and troughs are formed	3. Compression and rarefa-cations are formed.
4. Formed on the surface of solids and liquids	4. Formed in solids,liquids and gas
5. May be elastic waves or non elastic waves eg.light wave,radio wave	5. Only elastic waves (mechanical)eg.sound wave,seismic wave.
6. Do not create pressure difference in the medium	6. Creates pressure difference in the medium

HEAT AND THERMODYNAMICS

4. FIRST LAW OF THERMODYNAMICS

Values of Physical Constants

Universal gas constant = 8.314 $Jmol^{-1} K^{-1}$

Ratio of Specific heat capacity of gases γ = 1.42

Ratio of Specific heat capacities of a monoatomic ideal gas = 5/3

FORMULAE

1. First law of thermodynamics, $dQ = dU + dW$
2. The relationship of molar heat capacities is, $C_p - C_v = R$
3. The specific heat capacity ratio, $\gamma = \frac{C_p}{C_v} = \frac{c_p}{c_v}$, The value of γ depends on atomicity of gas
4. The equation for isothermal process, PV = constant or $P_1V_1 = P_2V_2$
5. The equations of adiabatic process are,

   i. $P_1V_1^{\gamma} = P_2V_2^{\gamma}$ ii. $T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$

   iii. $\frac{P_1^{\gamma-1}}{T_1^{\gamma}} = \frac{P_2^{\gamma-1}}{T_2^{\gamma}}$
6. Work done in isothermal expansion,

   i. $W = nRT \log_e \frac{V_2}{V_1}$ ii. $W = nRT \log_e \frac{P_1}{P_2}$

7. Work done in adiabatic expansion,

   i. $W = \frac{nR}{\gamma - 1}(T_1 - T_2)$ ii. $W = \frac{1}{\gamma - 1}(P_1V_1 - P_2V_2)$

5. SECOND LAW OF THERMODYNAMICS

FORMULAE

1. Efficiency of heat engine,

$$\eta = \frac{\text{Work done}}{\text{Heat input}} = \frac{W}{Q_1} = \frac{Q_1 - Q_2}{Q_1} = 1 - \frac{Q_2}{Q_1}$$

2. Efficiency of Carnot's engine,

$$\eta = 1 - \frac{Q_2}{Q_1} = 1 - \frac{T_2}{T_1}$$

3. The efficiency of Petrol engine and Diesel engine,

$$\eta = \frac{\text{Work done}}{\text{Heat input}} = \frac{W}{Q_1} = 1 - \left(\frac{1}{\rho}\right)^{\gamma - 1},$$

where

$\rho = \frac{V_2}{V_1}$ is called compression ratio and

$\gamma = \frac{c_p}{c_v}$ = specific heat ratio.

4. The coefficient of performance of a refrigerator,

$$\beta = \frac{Q_2}{W} = \frac{Q_1}{Q_2 - Q_1} = \frac{T_2}{T_1 - T_2}$$

MODERN PHYSICS

20. ELECTRONS

FORMULAE

1. Motion of electron in uniform electric field:
i. The charge of an electron, $e = 1.6 \times 10^{-19}$ C, negative nature and its mass, $m_e = 9.1 \times 10^{-31}$ kg.
Speed of light, $C = 3 \times 10^8$ m/sec
Planck's constant, $h = 6.62 \times 10^{-34}$ Js
ii. Its specific charge $\left(\frac{e}{m}\right) = 1.76 \times 10^{11}$ C / kg.
iii. Its path in electric field is parabolic in nature, and the equation of path, $y = \left(\frac{eV}{2mdu^2}\right) x^2$.
iv. Acceleration produced is, $a = \frac{F}{m} = \frac{eE}{m}$.
2. Motion of electron in uniform magnetic field:
i. Force experienced by an electron moving in a magnetic field, $F = Bev \sin\theta$
ii. The radius of circular path, $r = \frac{mv}{Be} = \frac{p}{Be} = \frac{\sqrt{2mE_k}}{Be}$
iii. The time period of revolution, $T = \frac{2\pi m}{Be}$
iv. The frequency of revolution, $f = \frac{1}{T} = \frac{Be}{2\pi m}$
v. The angular frequency, $\omega = 2\pi f = \frac{Be}{m}$
vi. The radius of a circle in helical path, $r = \frac{mv \sin\theta}{Be}$
vii. The pitch of helical path,
$= v\cos\theta \times T = \frac{2\pi mv\cos\theta}{Be} = \frac{2\pi r}{\tan\theta}$
3. The speed of electron in cross field, $v = \frac{E}{B}$
4. Millikan's oil drop experiment:
i. Radius of oil drop, $r = \sqrt{\frac{9\eta v_1}{2(\rho - \sigma)g}}$
ii. The charge of electron is determined from,
$q = \frac{6\pi\eta d}{V}\sqrt{\frac{9\eta v_1}{2(\rho - \sigma)g}}(v_1 + v_2)$ and $q = ne$

21. PHOTONS

FORMULAE

1. Energy of a photon, $E = hf = \frac{hc}{\lambda}$
2. It has zero rest mass, $m_o = m\sqrt{1 - \frac{v^2}{c^2}}$, for $v = c$, $m_o = 0$
3. From Einstein's mass-energy equivalence,
$E = hf = \frac{hc}{\lambda} = mc^2$
So, $m = \frac{hf}{c^2} = \frac{h}{c\lambda}$
4. Momentum of photon, $p = mv = mc = \frac{E}{c} = \frac{h}{\lambda}$
5. Work function $(\phi_0) = hf_0 = \frac{hc}{\lambda_0}$
6. Einstein's photoelectric equation, $hf = \phi_0 + E_k$ and $(E_k = \frac{1}{2}mv_{max}^2)$
7. Stopping potential (V_s), $eV_s = hf - \phi_0 = \frac{1}{2}mv_{max}^2$

23. QUANTIZATION OF ENERGY

FORMULAE

1. Bohr's theory:
i. Mathematical form of basic postulates
a. $\frac{mv^2}{r} = \frac{1}{4\pi\epsilon_0}\frac{Ze^2}{r^2}$ b. $mvr = \frac{nh}{2\pi}$
c. $\Delta E = hf = E_i - E_f$
ii. Radius of orbit of hydrogen like atom, $r_n = \frac{\epsilon_0 n^2 h^2}{\pi m Z e^2}$,
$\therefore r_n \propto n^2$
iii. Velocity of electron in an orbit, $v_n = \frac{Ze^2}{2\epsilon_0 nh}$, $v_n \propto \frac{1}{n}$
As, $r_n \propto n^2$, $v_n \propto \frac{1}{\sqrt{r_n}}$
iv. Energy of electron in an orbit of hydrogen like atom,
$E = \frac{me^4 Z^2}{8\epsilon_0^2 n^2 h^2} = -13.6\frac{Z^2}{n^2}$ eV
v. Rydberg constant, $R = \frac{me^4}{8\epsilon_0^2 n^2 ch^3} = 1.097 \times 10^7 m^{-1}$.
vi. The wavelength of radiation that is emitted in electron transition, $\frac{1}{\lambda} = RZ^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$
2. Dual nature of radiation:
de Broglie wavelength, $\lambda = \frac{h}{p} = \frac{h}{mv} = \frac{h}{\sqrt{2mE_k}} = \frac{h}{\sqrt{3mk_BT}}$
3. Heisenberg's uncertainty principle:
i. Position momentum uncertainty, $\Delta x \times \Delta p \geq \frac{h}{2\pi}$
ii. Energy time uncertainty, $\Delta E \times \Delta t \geq \frac{h}{2\pi}$
iii. Angular momentum and angular displacement uncertainly, $\Delta L \times \Delta\theta \geq \frac{h}{2\pi}$
4. X-Rays:
i. The minimum value of wavelength of X-rays is,
$\lambda = \frac{hc}{eV}$, i.e. $\lambda \,\alpha\, \frac{1}{V}$
ii. The energy of electron is $\frac{1}{2}mv_{max}^2 = eV$.
iii. Bragg's law of X-rays difference, $2d\sin\theta = n\lambda$, where $n = 1, 2, 3, \ldots$
5. Some constants; electric charge, $e = 1.16 \times 10^{-19}$ c,
Mass of electron, $m_e = 9.1 \times 10^{-31}$ kg,
Speed of light, $c = 3 \times 10^8$ m/s
Planck's constant, $h = 6.62 \times 10^{-34}$ Js

24. RADIOACTIVITY AND NUCLEAR REACTION

FORMULAE

1. The activity of a radioactive nucleus, $dN/dt = -\lambda N$, where $N = N_0 e^{-\lambda t}$, Where N_0 is the number of undecayed nuclei at time $t = 0$.
3. The half-life $T_{1/2}$ and the decay constant λ are related by the equation, $T_{1/2} = 0.693/\lambda$.
4. The relation of mean life and decay constant is $T_m = \frac{1}{\lambda}$