

K-Map of 2 variables

$\begin{array}{c c} & A \\ \hline B & \end{array}$	A'	A
B'	0	2
B	1	3

K-Map of 3 variables

$\begin{array}{c c} & AB \\ \hline C & \end{array}$	A'B'	A'B	AB	AB'
C'	0	2	6	4
C	1	3	7	5

K-Map of 4 variables for SOP function

$\begin{array}{c c} & ab \\ \hline cd & \end{array}$	a+b	a+b'	a'+b'	a'+b
c+d	0	4	12	8
c+d'	1	5	13	9
c'+d'	3	7	15	11
c+d'	2	6	14	10

K-Map of 4 variables for POS function

$\begin{array}{c c} & ab \\ \hline cd & \end{array}$	a+b	a+b'	a'+b'	a'+b
c+d	0	4	12	8
c+d'	1	5	13	9
c'+d'	3	7	15	11
c+d'	2	6	14	10

1. Minimize using K-Map $F(A, B, C) = \sum(1, 2, 3, 6, 7)$

$\begin{array}{c c} & AB \\ \hline C & \end{array}$	A'B'	A'B	AB	AB'
C'		1	1	
C	1	1	1	

An overlapping pair and a quad. So the minimal solution is

$$F(A, B, C) = A'C + B$$

2. Minimize using K-Map $F(A, B, C, D) = \sum(4, 5, 6, 7, 12, 13, 14)$

AB \ CD	A'B'	A'B	AB	AB'
C'D'		1	1	
C'D		1	1	
CD		1		
CD'		1	1	

$$F(A, B, C, D) = A'B + BC' + BD'$$

3. Minimize using K-Map $F(A, B, C, D) = \prod(0, 1, 2, 3, 4, 5, 8, 9, 10, 11, 14)$

AB \ CD	A+B	A+B'	A'+B	A'+B'
C+D	0	0		0
C+D'	0	0		0
C'+D'	0			0
C'+D	0		0	0

$$F(A, B, C, D) = B(A+C)(A'+C'+D)$$

1. Minimise the following SOP functions using K-Map:

- $F(A, B, C, D) = \sum(0, 1, 2, 3, 4, 5, 8, 9, 12, 13)$
- $F(A, B, C, D) = \sum(0, 1, 2, 3, 5, 7, 8, 9, 12, 13, 14, 15)$
- $F(A, B, C, D) = \sum(0, 1, 2, 3, 5, 7, 8, 9, 10, 11, 13, 15)$
- $F(A, B, C, D) = \sum(0, 2, 4, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15)$
- $F(A, B, C, D) = \sum(0, 1, 2, 3, 4, 8, 9, 10, 11, 12)$
- $F(A, B, C, D) = \sum(1, 3, 4, 5, 7, 9, 11, 12, 13, 14)$

2. Minimise the following POS functions using K-Map:

- $F(A, B, C, D) = \prod(0, 2, 6, 7, 8, 9, 10, 11, 13, 15)$
- $F(A, B, C, D) = \prod(0, 4, 5, 9, 11, 12, 13, 15)$
- $F(A, B, C, D) = \prod(0, 4, 5, 7, 8, 9, 12, 13, 15)$
- $F(A, B, C, D) = \prod(0, 2, 4, 6, 7, 8, 9, 10, 12, 13, 14, 15)$
- $F(A, B, C, D) = \prod(0, 2, 4, 5, 6, 7, 10, 14)$

3. Algebraically prove the identities:

- $A'B'C' + A'BC' + ABC' + AB'C' + A'B'C + A'BC = A' + C'$

$$\text{LHS} = A'B'C' + A'BC' + ABC' + AB'C' + A'B'C + A'BC$$

Pairing 1st & 2nd term, 3rd & 4th term and 5th & 6th term

$$= A'C'(B' + B) + AC'(B + B') + A'C(B' + B)$$

$$= A'C' + AC' + A'C \quad \text{Since } B + B' = 1$$

$$= C'(A' + A) + A'C$$

$$= C' + A'C \quad \text{Since } A + A' = 1$$

$$= (C' + A')(C' + C) \quad \text{Applying Distributive Law: } (X + Y)(X + Z) = X + YZ$$

$$= A' + C' \quad \text{Since } C + C' = 1$$

$$= \text{RHS}$$

- b) $A'BC' + ABC' + AB'C' + A'BC + ABC + A'B'C' = B + C'$
 LHS = $A'BC' + ABC' + AB'C' + A'B'C' + A'BC + ABC$
 Pairing 1st & 2nd term, 3rd & 6th term and 4th & 5th term
 $= BC'(A' + A) + B'C'(A + A') + BC(A' + A)$
 $= BC' + B'C' + BC$ Since $A + A' = 1$
 $= C'(B + B') + BC$
 $= C' + BC$ Since $B + B' = 1$
 $= (C' + B)(C' + C)$ Applying Distributive Law $(X + Y)(X + Z) = X + YZ$
 $= B + C'$ Since $C + C' = 1$
 $= \text{RHS}$
- c) $A'B'C + A'B'C' + ABC + A'BC + A'BC' + ABC' = A' + B$
 d) $A'B'C' + A'B'C + ABC' + ABC + AB'C' + AB'C = A + B'$
 e) $A'B'C' + A'B'C + A'BC + ABC + AB'C' + AB'C = B' + C$
 f) $ABC' + A'B'C + A'BC + ABC + AB'C' + AB'C = A + C$

4. Algebraically prove the identities:

- a) $(A + B + C)(A + B + C')(A + B' + C')(A' + B' + C')(A' + B + C')(A' + B + C) = BC'$
 LHS = $(A + B + C)(A + B + C')(A + B' + C')(A' + B' + C')(A' + B + C')(A' + B + C)$
 Pairing 1st & 2nd term, 3rd & 4th term and 5th & 6th term
 $= (A + B + CC')(AA' + B' + C')(A' + B + C'C)$
 Applying Distributive Law $(X + Y)(X + Z) = X + YZ$
 $= (A + B)(B' + C')(A' + B)$ Since $AA' = 0$ and $CC' = 0$
 $= (AA' + B)(B' + C')$
 Pairing 1st & 3rd terms, applying Distributive Law $(X + Y)(X + Z) = X + YZ$
 $= B(B' + C')$ Since $AA' = 0$
 $= BB' + BC'$
 $= BC'$ Since $BB' = 0$
 $= \text{RHS}$
- b) $(A + B + C)(A + B + C')(A + B' + C')(A' + B' + C')(A' + B + C')(A + B' + C) = AC'$
 LHS = $(A + B + C)(A + B + C')(A + B' + C')(A' + B + C')(A + B' + C)(A' + B' + C')$
 Pairing 1st & 2nd term, 3rd & 6th term and 4th & 5th term
 $= (A + B + CC')(A + B'B + C')(A + B' + CC')$
 Applying Distributive Law $(X + Y)(X + Z) = X + YZ$
 $= (A + B)(A + C')(A + B')$ Since $B'B = 0$ and $CC' = 0$
 $= (A + BB')(A + C')$
 Pairing 1st & 3rd terms, applying Distributive Law $(X + Y)(X + Z) = X + YZ$
 $= A(A + C')$ Since $BB' = 0$
 $= AA' + AC'$
 $= AC'$ Since $AA' = 0$
 $= \text{RHS}$
- c) $(A' + B' + C)(A + B + C')(A + B' + C')(A' + B' + C')(A' + B + C')(A + B' + C) = B'C'$
 d) $(A + B + C)(A + B' + C)(A' + B' + C)(A' + B + C)(A + B + C')(A + B' + C') = AC$
 e) $(A + B + C)(A + B' + C)(A' + B' + C)(A' + B + C)(A' + B' + C')(A + B' + C') = B'C$
 f) $(A + B + C)(A + B' + C)(A' + B' + C)(A' + B + C)(A' + B' + C')(A' + B + C') = A'C$