An English Mathematician, **George Boole** gave the concept of Boolean Algebra or Algebra of Logic in his two revolutionary papers titled **The Mathematical Analysis** **of Logic** and **An Investigation of the Laws of Thought**. **Claude Shanon** applied Boolean Algebra to design electrical circuits. Further development of Boolean Algebra led to the birth of Digital Electronics.

## **Boolean Constant: False(0)** or **True(1)** values are known as Boolean Constants or Logical Constants or Truth Values.

## **Boolean Statement:** A statement is said to be a Boolean or Logical Statement if it has a definite value, which is either **False(0)** or **True(1)**.

# Examples:

|  |  |
| --- | --- |
| **Logical Statement** | **Non Logical Statement** |
| It is raining outside. | What is your name? |
| The door is open. | Who is Ms. Jameson? |
| New Delhi is the capital of India. | What is the capital of India |

**Boolean operators:** Operators used in Boolean algebra are known as Boolean or logical operators. Basic logical operators and their notations are shown below:

|  |  |  |
| --- | --- | --- |
| **Operator** | **Symbol** | **Example** |
| NOT | ' | X' |
| AND | . | X.Y or XY |
| OR | + | X+Y |

**Boolean Variable:** A variable, which holds False(0) or True(1) value, is known as Boolean variable (X, Y, Z …).

**Boolean Expression:** A meaningful combination of Boolean operators (AND/OR/NOT), Boolean operand/variable (X, Y, Z etc.) and Boolean constant (0 or 1) is known as Boolean Expression (Logical Expression). Single letter will be used to represent a Boolean variable. Examples of Boolean expressions are given below:

i) X+Y.Z

ii) A(B+C)+BC'

iii) U OR V AND NOT Z

Boolean algebra is an algebraic structure on a set {0, 1} together with Boolean operators '(NOT), **.** (AND) & +(OR) with the following postulates satisfied.

* **Closure Property**

If X & Y are two Boolean variables then:

i) X+Y ∈ {0, 1} ∀ X, Y ∈ {0, 1}

ii) X.Y ∈ {0, 1} ∀ X, Y ∈ {0, 1}

iii) X' ∈ {0, 1} ∀ X ∈ {0, 1}

* **Commutative Property**

If X & Y are two Boolean variables then:

i) X+Y = Y+X ∈ {0, 1} ∀ X, Y ∈ {0, 1}

ii) X.Y = Y.X ∈ {0, 1} ∀ X, Y ∈ {0, 1}

* **Associative Property**

If X & Y are two Boolean variables then:

i) X+(Y+Z) = (X+Y)+Z = X+Y+Z ∈ {0, 1} ∀ X, Y, Z ∈ {0, 1}

ii) X.(Y.Z) = (X.Y).Z = X.Y.Z ∈ {0, 1} ∀ X, Y, Z ∈ {0, 1}

* **Existence of Identity**

For every X ∈ {0, 1} such that

i) X + 0 = X

ii) X . 1 = X

* **Law of Inverse**

For every X ∈ {0, 1} ∃ X' ∈ {0, 1} such that

i) X + X' = 1

ii) X . X' = 0

* **Idempotent Law**

For every X ∈ {0, 1} such that

i) X + X = X

ii) X . X = X

* **Law of Involution or Complementation** **Law**

For every X ∈ {0, 1} ∃ X' ∈ {0, 1} such that

(X')' = X

* **Distributive Law**

For every X, Y, Z ∈ {0, 1} such that

i) X.(Y + Z) = X.Y + X.Z

ii) (X + Y).(X + Z) = X + Y.Z

* **Absorption Law**

For every X, Y, Z ∈ {0, 1} such that

i) X + X.Y = X

ii) X.(X + Y) = X

* **Demorgan’s Law**

For every X, Y ∈ {0, 1} such that

i) (X + Y)' = X' . Y'

ii) (X . Y)' = X' + Y'

* **Other Important Laws**

a) X + 1 = 1 ∀ X ∈ {0, 1}

b) X . 0 = 0 ∀ X ∈ {0, 1}

c) X + X'.Y = X + Y ∀ X, Y ∈ {0, 1}

d) X' + X.Y = X' + Y ∀ X, Y ∈ {0, 1}

**Boolean Function:** A Function with logical variable, logical constant & logical operator representing a truth value (0 or 1) is known as Boolean Function. Examples are given below:

i) F = X + Y.Z

ii) F(X,Y) = X' + Y' + Y.X

iii) F(X,Y) = Σ(0, 2)

The function has same meaning as F(X,Y) = X'.Y' + X.Y'

iv) F(X,Y,Z) = Π (1, 2, 4 )

The function has same meaning as F(X,Y,Z)=(X+Y+Z').(X+Y'+Z).(X'+Y+Z)

**Duality Principal** states that every algebraic expression deducible from the postulates of Boolean algebra remains valid if the operators (+ and .) and identity elements (0 and 1) are interchanged.

**Example**:

* **Law of Inverse**

i) X+X' = 1

ii) X.X' = 0

* **Distributive Law**

i) X.(Y + Z) = X.Y + X.Z

ii) X + Y.Z = (X + Y).(X + Z)

For every example of Duality Principal, (i) is Dual of (ii) and vice versa.

* **Absorption Law**

i) X + X.Y = X

ii) X.(X + Y) = X

* **Demorgan’s Law**

i) (X + Y)' = X' . Y'

ii) (X . Y)' = X' + Y'

**Truth Table:** A truth table is a table showing all possible combinations of 0 and 1 that can be assigned to input variables present in an expression and the resultant output obtained from operations. If a Boolean expression contains **n** variables then there will **2n** outputs (**2n** rows in a truth table).

Rows for 2 variables Truth Table is give below:

|  |  |  |  |
| --- | --- | --- | --- |
| X | Y | Min Term | Max Term |
| 0 | 0 | X'.Y' | X+Y |
| 0 | 1 | X'.Y | X+Y' |
| 1 | 0 | X.Y' | X'+Y |
| 1 | 1 | X.Y | X'+Y' |

Rows for 3 variables Truth Table is give below:

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| X | Y | Z | Min Term | Max Term |
| 0 | 0 | 0 | X'.Y'.Z' | X+Y+Z |
| 0 | 0 | 1 | X'.Y'.Z | X+Y+Z' |
| 0 | 1 | 0 | X'.Y.Z' | X+Y'+Z |
| 0 | 1 | 1 | X'.Y.Z | X+Y'+Z' |
| 1 | 0 | 0 | X.Y'.Z' | X'+Y+Z |
| 1 | 0 | 1 | X.Y'.Z | X'+Y+Z' |
| 1 | 1 | 0 | X.Y.Z' | X'+Y'+Z |
| 1 | 1 | 1 | X.Y.Z | X'+Y'+Z' |

Rows for 4 variables Truth Table is give below:

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| X | Y | Z | W | Min Term | Max Term |
| 0 | 0 | 0 | 0 | X'.Y'.Z'.W' | X+Y+Z+W |
| 0 | 0 | 0 | 1 | X'.Y'.Z'.W | X+Y+Z+W' |
| 0 | 0 | 1 | 0 | X'.Y'.Z.W' | X+Y+Z'+W |
| 0 | 0 | 1 | 1 | X'.Y'.Z.W | X+Y+Z'+W' |
| 0 | 1 | 0 | 0 | X'.Y.Z'.W' | X+Y'+Z+W |
| 0 | 1 | 0 | 1 | X'.Y.Z'.W | X+Y'+Z+W' |
| 0 | 1 | 1 | 0 | X'.Y.Z.W' | X+Y'+Z'+W |
| 0 | 1 | 1 | 1 | X'.Y.Z.W | X+Y'+Z'+W' |
| 1 | 0 | 0 | 0 | X.Y'.Z'.W' | X'+Y+Z+W |
| 1 | 0 | 0 | 1 | X.Y'.Z'.W | X'+Y+Z+W' |
| 1 | 0 | 1 | 0 | X.Y'.Z.W' | X'+Y+Z'+W |
| 1 | 0 | 1 | 1 | X.Y'.Z.W | X'+Y+Z'+W' |
| 1 | 1 | 0 | 0 | X.Y.Z'.W' | X'+Y'+Z+W |
| 1 | 1 | 0 | 1 | X.Y.Z'.W | X'+Y'+Z+W' |
| 1 | 1 | 1 | 0 | X.Y.Z.W' | X'+Y'+Z'+W |
| 1 | 1 | 1 | 1 | X.Y.Z.W | X'+Y'+Z'+W' |

**Logic Gate:** An electronic gadget which can perform some logical operation like NOT/ AND/ OR operation on Boolean values.

Truth Table for NOT NOT Gate (Inverter)

|  |  |
| --- | --- |
| X | X' |
| 0 | 1 |
| 1 | 0 |

NOT gate has **one** input and **one** output.

X

X'

Truth Table for 2-input AND 2-input AND Gate

X

Y

X.

Y

|  |  |  |
| --- | --- | --- |
| X | Y | X.Y |
| 0 | 0 | 0 |
| 0 | 1 | 0 |
| 1 | 0 | 0 |
| 1 | 1 | 1 |

Truth Table for 3-input AND 3-input AND Gate

|  |  |  |  |
| --- | --- | --- | --- |
| X | Y | Z | X.Y.Z |
| 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 |
| 0 | 1 | 0 | 0 |
| 0 | 1 | 1 | 0 |
| 1 | 0 | 0 | 0 |
| 1 | 0 | 1 | 0 |
| 1 | 1 | 0 | 0 |
| 1 | 1 | 1 | 1 |

AND gate has **two** or more inputs but only **one** output.

Truth Table for 2-input OR 2-input OR Gate

|  |  |  |
| --- | --- | --- |
| X | Y | X+Y |
| 0 | 0 | 0 |
| 0 | 1 | 1 |
| 1 | 0 | 1 |
| 1 | 1 | 1 |

Truth Table for 3-input OR 3-input OR Gate

|  |  |  |  |
| --- | --- | --- | --- |
| X | Y | Z | X+Y+Z |
| 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 1 |
| 0 | 1 | 0 | 1 |
| 0 | 1 | 1 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 0 | 1 | 1 |
| 1 | 1 | 0 | 1 |
| 1 | 1 | 1 | 1 |

  
OR gate has **two** or more inputs but only **one** output

Truth Table for 2-input XOR 2-input XOR Gate

|  |  |  |
| --- | --- | --- |
| X | Y | X⊕Y |
| 0 | 0 | 0 |
| 0 | 1 | 1 |
| 1 | 0 | 1 |
| 1 | 1 | 0 |

Truth Table for 3-input XOR 3-input XOR Gate

|  |  |  |  |
| --- | --- | --- | --- |
| X | Y | Z | X⊕Y⊕Z |
| 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 1 |
| 0 | 1 | 0 | 1 |
| 0 | 1 | 1 | 0 |
| 1 | 0 | 0 | 1 |
| 1 | 0 | 1 | 0 |
| 1 | 1 | 0 | 0 |
| 1 | 1 | 1 | 1 |

XOR gate has **two** or more inputs but only **one** output

X

Y

X



Y



Z

Z

Truth Table for 2-input NAND 2-input NAND Gate

|  |  |  |
| --- | --- | --- |
| X | Y | (X.Y)' |
| 0 | 0 | 1 |
| 0 | 1 | 1 |
| 1 | 0 | 1 |
| 1 | 1 | 0 |

Truth Table for 3-input NAND 3-input NAND Gate

|  |  |  |  |
| --- | --- | --- | --- |
| X | Y | Z | (X.Y.Z)' |
| 0 | 0 | 0 | 1 |
| 0 | 0 | 1 | 1 |
| 0 | 1 | 0 | 1 |
| 0 | 1 | 1 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 0 | 1 | 1 |
| 1 | 1 | 0 | 1 |
| 1 | 1 | 1 | 0 |

NAND gate has **two** or more inputs but only **one** output.

Truth Table for 2-input NOR 2-input NOR Gate

|  |  |  |
| --- | --- | --- |
| X | Y | (X+Y)' |
| 0 | 0 | 1 |
| 0 | 1 | 0 |
| 1 | 0 | 0 |
| 1 | 1 | 0 |

Truth Table for 3-input NOR 3-input NOR Gate

|  |  |  |  |
| --- | --- | --- | --- |
| X | Y | Z | (X+Y+Z)' |
| 0 | 0 | 0 | 1 |
| 0 | 0 | 1 | 0 |
| 0 | 1 | 0 | 0 |
| 0 | 1 | 1 | 0 |
| 1 | 0 | 0 | 0 |
| 1 | 0 | 1 | 0 |
| 1 | 1 | 0 | 0 |
| 1 | 1 | 1 | 0 |

NOR gate has **two** or more inputs but only **one** output

**SOP Expression**: A Boolean expression containing only product term is called **S**um **O**f **P**roducts.

a) F(X,Y)= X'.Y' + X'.Y + X.Y

b) F(X,Y)= X' + X'.Y + X.Y' (X' is a product term since X'=X'.1)

c) F(X,Y,Z)= X'.Y'.Z', X'.Y'.Z, X'.Y.Z', X'.Y.Z

d) F(X,Y,Z)= Z + Y'.Z + X'.Y.Z' + X'.Y.Z + X.Y.Z

**POS Expression**: A Boolean expression containing only sum (factor) term is called **P**roduct **O**f **S**ums.

a) F(X,Y)= (X'+Y).(X+Y).(X+Y')

b) F(X,Y)= Y'.(X'+Y)(X+Y') (Y' is a sum term because Y'= Y'+0)

c) F(X,Y,Z)= (X'+Y'+Z').(X'+Y+Z').(X'+Y+Z).(X+Y'+Z')

d) F(X,Y,Z)= X.(X'+Y'+Z).(Y+Z').(X'+Y+Z).(X+Y+Z')

A **Minterm** of **n** variables (X1, X2,… Xn) is a product term where all the **n** variables (X1, X2, … Xn) appear either in direct form or in complemented form once only once.

i) Min terms of 2 variables (X,Y) are X'.Y', X'.Y, X.Y', X.Y

ii) Min terms of 3 variables (X,Y,Z) are X'.Y'.Z', X'.Y'.Z, X'.Y.Z', X'.Y.Z,…

A **Maxterm** of **n** variables (X1, X2,… Xn) is a sum term (factor term) where all the **n** variables (X1, X2, … Xn) appear either in direct form or in complemented form once only once.

i) Max terms of 2 variables (X,Y) are X'+Y', X'+Y, X+Y', X+Y

ii) Max terms of 3 variables (X,Y,Z) are X'+Y'+Z', X'+Y'+Z, X'+Y+Z', X'+Y+Z,…

**Canonical SOP**: If SOP expression contains only Min terms, then that SOP expression is called Canonical SOP (Normalized SOP).

i) F(X,Y)=X'.Y' + X'.Y + X.Y

ii) F(X,Y,Z)=X'.Y'.Z' + X'.Y'.Z + X'.Y.Z' + X'.Y.Z

**Canonical POS**: If POS expression contains only Max terms, then that POS expression is called Canonical POS (Normalized POS).

i) F(X,Y)=(X'+Y).(X+Y).(X+Y')

ii) F(X,Y,Z)=(X'+Y'+Z').(X'+Y+Z').(X'+Y+Z).(X+Y'+Z')

**To obtain Canonical SOP Expression form a Truth Table**:

* Look for rows with output 1
* While in input columns, 1 represent direct form and 0 represent complimented form.

**To obtain Canonical POS Expression form a Truth Table**

* Look for rows with output 0
* While in input columns, 0 represent direct form and 1 represent complimented form.

**Deriving SOP and POS from Truth Table given below**:

Row X Y Z F

0 0 0 0 0

1 0 0 1 0

2 0 1 0 1

3 0 1 1 0

4 1 0 0 1

5 1 0 1 1

6 1 1 0 0

7 1 1 1 1

a) **Derivation of SOP Expression**

3rd row, 5th row, 6th row and 8th row have output 1.

3rd row input is X is 0, Y is 1 and Z is 0 so the product term will X'.Y.Z' or 2

5th row input is X is 1, Y is 0 and Z is 0 so the product term will X.Y'.Z' or 4

6th row input is X is 1, Y is 0 and Z is 1 so the product term will X.Y'.Z or 5

8th row input is X is 1, Y is 1 and Z is 1 so the product term will X.Y.Z or 7

Canonical SOP of F(X, Y, Z)= X'.Y.Z'+X.Y'.Z'+X.Y'.Z+X.Y.Z

Short hand Canonical SOP of F(X, Y, Z)=Σ(2, 4, 5, 7)

b) **Derivation of POS Expression**

1st row, 2nd row, 4th row and 7th row have output 0.

1st row input is X is 0, Y is 0 and Z is 0 so the product term will X+Y+Z or 0

2nd row input is X is 0, Y is 0 and Z is 1 so the product term will X+Y+Z' or 1

4th row input is X is 0, Y is 1 and Z is 1 so the product term will X+Y'+Z' or 3

7th row input is X is 1, Y is 1 and Z is 0 so the product term will X'+Y'+Z or 6

Canonical POS of F(X, Y, Z)=(X+Y+Z).(X+Y+Z').(X+Y'+Z').(X'+Y'+Z)

Short hand Canonical POS of F(X, Y, Z)=Π(0, 1, 3, 6)

|  |
| --- |
| **Missing terms from Short hand SOP forms the Short hand POS and vice versa.** |

**Karnaugh Map:** A Karnaugh Map is a graphical representation of a Boolean function in either Canonical SOP form. K-Map method reduces a Boolean function to its minimal form without any algebraic manipulation. K-map method ensures that no further minimization is possible. Kindly note each number inside the cell represent Min Term.

**K-Map of 2 variables**

**K-Map of 3 variables**

**K-Map of 4 variables**

****1. Minimize using K-Map F(A, B, C)=∑(1,2,3,6,7)

An overlapping pair and a quad. So the minimal solution is

F(A,B,C)=A'C + B

2. Minimize using K-Map F(A,B,C,D)=∑(4,5,6,7,12,13,14)

F(A,B,C,D)=A'B+BC'+BD'

3. Minimize using K-Map F(A,B,C,D)=∏(0,1,2,3,4,5,8,9,10,11,14)

A+B

A+B'

A'+B'

A'+B

C+D

0

0

0

C+D'

0

0

0

C'+D'

0

0

C'+D

0

0

0

**AB**

**CD**



F(A,B,C,D)=B(A+C)(A'+C'+D)

1. Algebraically prove the identities:

1. A'B'C'+A'BC'+ABC'+AB'C'+A'B'C+A'BC=A'+C'
2. A'BC'+ABC'+AB'C'+A'BC+ABC+A'B'C'=B+C'
3. A'B'C+A'B'C'+ABC+A'BC+A'BC'+ABC'=A'+B
4. A'B'C'+A'B'C+ABC'+ABC+AB'C'+AB'C=A+B'
5. A'B'C'+A'B'C+A'BC+ABC+AB'C'+AB'C=B'+C
6. ABC'+A'B'C+A'BC+ABC+AB'C'+AB'C=A+C

2. Algebraically prove the identities:

1. (A+B+C)(A+B+C')(A+B'+C')(A'+B'+C')(A'+B+C')(A'+B+C)=BC'
2. (A+B+C)(A+B+C')(A+B'+C')(A'+B'+C')(A'+B+C')(A+B'+C)=AC'
3. (A'+B'+C)(A+B+C')(A+B'+C')(A'+B'+C')(A'+B+C')(A+B'+C)=B'C'
4. (A+B+C)(A+B'+C)(A'+B'+C)(A'+B+C)(A+B+C')(A+B'+C')=AC
5. (A+B+C)(A+B'+C)(A'+B'+C)(A'+B+C)(A'+B'+C')(A+B'+C')=B'C
6. (A+B+C)(A+B'+C)(A'+B'+C)(A'+B+C)(A'+B'+C')(A'+B+C')=A'C

Obtain the other Canonical form for the following Boolean expressions given below:

1. F(A,B,C)=∑(0,2,4,6)
2. F(A,B,C)=∑(1,2,4,7)
3. F(A,B,C,D)=∑(0,2,6,11,13,14)
4. F(A,B,C)=∏(1,2,4,7)
5. F(A,B,C,D)=∏(0,1,2,4,5,8,10,12)

Obtained the minimal of form for the K-Maps of SOP function given below:

1. F(A,B,C,D)=∑(0,1,2,3,4,5,8,9,12,13)

2. F(A,B,C,D)=∑(0,1,2,3,5,7,8,12,13,14,15)

3. F(A,B,C,D)=∑(0,1,2,3,5,7,8,9,10,11,13,15)

4. F(A,B,C,D)=∑(0,2,4,6,7,8,9,10,11,12,13,14,15)

5. F(A,B,C,D)=∑(0,1,2,34,8,9,10,11,12)

Obtained the minimal of form for the K-Maps of POS function given below:

1. F(A,B,C,D)=∏(0,2,6,7,8,9,10,11,13,15)

2. F(A,B,C,D)=∏(0,4,5,9,11,12,13,15)

3. F(A,B,C,D)=∏(0,4,5,7,8,9,12,13,15)

4. F(A,B,C,D)=∏(0,2,4,6,7,8,9,10,12,13,14,15)

5. F(A,B,C,D)=∏(0,2,4,5,6,7,10,14)