NOTES

IDLE

Hello!

This note is mainly used to record some trivial but interesting problems that I encounter in daily life.

FIGURE 1. Night City

CONTENTS

1. 2023.12.22

1.1. **Identification of co-character groups with Lie algebras.** Settings: G complex connected reductive algebraic group, fix a splitting datum $(B, H, \{X_\alpha\})$.

There is an identification

$$
\begin{aligned}
I: \quad X_*(H) \otimes \mathbb{C} &\longrightarrow \mathfrak{h} \\
\phi &\longmapsto d\phi(1)
\end{aligned}
$$

The weight lattice for G is $P = \{\lambda \in X^*(H) \otimes \mathbb{C} \,|\, \langle \lambda, \check{\alpha} \rangle \in \mathbb{Z} \text{ for all } \alpha \in \Delta\}$
The co-weight lattice is $\check{P} = \{\check{\lambda} \in X_*(H) \otimes \mathbb{C} \,|\, \langle \alpha, \check{\lambda} \rangle \in \mathbb{Z} \text{ for all } \check{\alpha} \in \check{\Delta}\}$
Under this identification we have

$$
\begin{aligned}
&\{\check{\lambda} \in \mathfrak{h} \,|\, exp(\check{\lambda}) = 0\} \\
&= \{\check{\lambda} \in X_*(H) \otimes \mathbb{C} \,|\, \phi(exp(d\check{\lambda}(1))) = 1 \text{ for all } \phi \in X^*(H)\} \\
&= \{\check{\lambda} \in X_*(H) \otimes \mathbb{C} \,|\, exp(\langle \phi, \check{\lambda} \rangle) = 1 \text{ for all } \phi \in X^*(H)\} \\
&= \{\check{\lambda} \in X_*(H) \otimes \mathbb{C} \,|\, \langle \phi, \check{\lambda} \rangle \in 2\pi i \mathbb{Z} \text{ for all } \phi \in X^*(H)\} \\
&= 2\pi i X_*(H).
\end{aligned}
$$

And similarly

$$
\check{P} = \{\check{\lambda} \in \mathfrak{h} \,|\, exp(2\pi i \check{\lambda}) \in Z(G)\}
$$

There is also an identification

$$
\begin{aligned}
J: \quad X^*(H) \otimes \mathbb{C} &\longrightarrow \mathfrak{h}^* \\
\varphi &\longmapsto d\varphi
\end{aligned}
$$

Under this identification ... I forget what I want to say.

1.2. **Pinnings of algebraic groups.**

Theorem 1.1 (see AdC18)**.** *Inner automorphism group* Inn(G) *of G is equal to* G/Z(G), Inn(G) *acts freely and transitively on the set of pinnings.*

I know that there is an analogy in compact connected groups, and it has been proved.

Theorem 1.2. *If* G *is a compact connected group, then its inner automorphism group* Inn(G) = G/Z(G) *acts freely and transitively on the set of pinnings. (where pinnings for it is a bit different from the complex case, that $(T, B_\mathbb{C}, \mathcal{X})$ is a pinning if T is a maximal torus, $B_\mathbb{C}$ a Borel of $G_\mathbb{C}$ containing T, $\mathcal{X}$ is a set of real rays in simple root space.)*

Proof. Here's proof from mathoverflow Lspise's answer:

To prove this, I'll use a few pieces of structure theory:

1. All maximal tori in G are G°-conjugate.

2. All Borel subgroups of $G_{\mathbb{C}}$ are $G_{\mathbb{C}}^\circ$-conjugate.

3. For every maximal torus T in G, the map $\mathbf{W}(G^\circ, T) \to \mathbf{W}(G_{\mathbb{C}}^\circ, T_{\mathbb{C}})$ is an isomorphism.

4. If G_{sc} and $(G_{\mathbb{C}})_{\mathrm{sc}}$ are the simply connected covers of the derived groups of G° and $G_{\mathbb{C}}^\circ$, then $(G_{\mathrm{sc}})_{\mathbb{C}}$ equals $(G_{\mathbb{C}})_{\mathrm{sc}}$.

5. Every compact Lie group has a finite subgroup that meets every component.

I only need (4) to prove that, for every maximal torus T in G, the map from T to the set of conjugation-fixed elements of $T/\mathbf{Z}(G^\circ)$ is surjective. This is probably a well known fact in its own right for real-group theorists.

Now consider triples $(T, B_{\mathbb{C}}, \mathcal{X})$ as follows: T is a maximal torus in G; $B_{\mathbb{C}}$ is a Borel subgroup of $G_{\mathbb{C}}^\circ$ containing $T_{\mathbb{C}}$, with a resulting set of simple roots $\Delta(B_{\mathbb{C}}, T_{\mathbb{C}})$; and $\mathcal{X}$ is a set consisting of a real ray in each complex simple root space (i.e., the set of positive real multiples of some fixed non-0 vector). (Sorry about the pair of modifiers "complex simple".) I will call these 'pinnings', although it doesn't agree with the usual terminology (where we pick individual root vectors, not rays). I claim that $G^\circ/\mathbf{Z}(G^\circ)$ acts simply transitively on the set of pinnings.

Once we have transitivity, freeness is clear: if $g \in G^\circ$ stabilises some pair $(T, B_{\mathbb{C}})$, then it lies in T, and so stabilises every complex root space; but then, for it to stabilise some choice of rays $\mathcal{X}$, it has to have the property that $\alpha(g)$ is positive and real for each simple root α; but also $\alpha(g)$ is a norm-1 complex number, hence trivial, for each simple root α, hence for each root α, so that g is central.

For transitivity, since (1) all maximal tori in G are G°-conjugate, so (2) for every maximal torus T in G, the Weyl group $\mathbf{W}(G^\circ_{\mathbb{C}}, T_{\mathbb{C}})$ acts transitively on the Borel subgroups of $G^\circ_{\mathbb{C}}$ containing $T_{\mathbb{C}}$, and (3) $\mathbf{W}(G^\circ, T) \to \mathbf{W}(G^\circ_{\mathbb{C}}, T_{\mathbb{C}})$ is an isomorphism, it suffices to show that all possible sets $\mathcal{X}$ are conjugate. Here's the argument that I came up with to show that they are even T-conjugate; I think it can probably be made much less awkward. Fix a simple root α, and two non-0 elements X_α and X'_α of the corresponding root space. Then there are a positive real number r and a norm-1 complex number z such that $X'_\alpha = rzX_\alpha$. Choose a norm-1 complex number w such that $w^2 = z$. There is then a unique element s_{ad} of $T_{\mathbb{C}} / \mathbf{Z}(G^\circ_{\mathbb{C}})$ such that $\alpha(s_{\mathrm{ad}}) = w$, and $\beta(s_{\mathrm{ad}}) = 1$ for all simple roots $\beta \neq \alpha$. By (4), we can choose a lift s_{sc} of s_{ad} to $(G_{\mathrm{sc}})_{\mathbb{C}} = (G_{\mathbb{C}})_{\mathrm{sc}}$, which necessarily lies in the preimage $(T_{\mathbb{C}})_{\mathrm{sc}}$ of (the intersection with the derived subgroup of) T, and put $t_{\mathrm{sc}} = s_{\mathrm{sc}} \cdot \overline{s_{\mathrm{sc}}}$. Then

$$\alpha(t_{\mathrm{sc}}) = \alpha(s_{\mathrm{sc}})\overline{\alpha(s_{\mathrm{sc}})} = \alpha(s_{\mathrm{sc}})\overline{\alpha(s_{\mathrm{sc}})}^{-1} = w \cdot \overline{w}^{-1} = z,$$

and, similarly, $\beta(t_{\mathrm{sc}}) = 1$ for all simple roots $\beta \neq \alpha$. Now the image t of t_{sc} in $G^\circ_{\mathbb{C}}$ lies in $T_{\mathbb{C}}$ and is fixed by conjugation, hence lies in T; and $\mathrm{Ad}(t)X_\alpha = zX_\alpha$ lies on the ray through X'_α.

Since G also acts on the set of pinnings, we have a well defined map $p : G \to G^\circ / \mathbf{Z}(G^\circ)$ that restricts to the natural projection on G°. Now $\ker(p)$ meets every component, but it contains $\mathbf{Z}(G^\circ)$, so it need not be finite. Applying (5) to the Lie group $\ker(p)$ yields the desired subgroup H. Note that, as requested in your improved classification, conjugation by any element of H fixes a pinning, hence, if inner, must be trivial.

$\square$

Remark 1.3. *The pinnings show that split reductive algebraic groups look like butterflies!*

According to Milne, Grothendieck used the following analogy to talk about the structure theory of algebraic groups. An algebraic group is like a butterfly. The body of the butterfly is the maximal torus. The wings are two opposite Borel subgroups. And the pins are the pinning maps.

FIGURE 2. Pinning of butterfly

1.3. **Coherent continuation representations.** Until the end of today, G denotes a real reductive group in Harish-Chandra class.

Let's recall some basic definitions of coherent continuation representations, in order for me to do calculations in the case of real classical groups.

Fix a connected reductive complex Lie group $G_{\mathbb{C}}$, together with a Lie group homomorphism $\iota : G \to G_{\mathbb{C}}$ such that its differential $d\iota : \mathrm{Lie}(G) \to \mathrm{Lie}(G_{\mathbb{C}})$ has the following two properties:

(1) the kernel of $d\iota$ is contained in the center of $\mathrm{Lie}(G)$;
(2) the image of $d\iota$ is a real form of $\mathrm{Lie}(G)$.

The analytical weight lattice of $G_{\mathbb{C}}$ is identified with a subgroup of ${}^a\mathfrak{h}^*$ via $d\iota$. We write $Q_\iota \subset {}^a\mathfrak{h}^*$ for this subgroup. Denote root lattice by $Q_{\mathfrak{g}}$, weight lattice by $Q^{\mathfrak{g}}$, then

$$Q_{\mathfrak{g}} \subset Q_\iota \subset Q^{\mathfrak{g}}.$$

They are all W stable subgroups. And we have some categories:

(1) $\mathrm{Rep}(\mathfrak{g}, Q_\iota)$ the category of all finite-dimensional representations of $\mathfrak{g}$ whose weight are contained in $Q_{\mathfrak{g}}$, it can be identified with $\mathrm{Rep}(\mathrm{Lie}(G_{\mathbb{C}}), Q_\iota)$ and hence $\mathrm{Rep}_{\mathrm{finite}}(G_{\mathbb{C}})$;
(2) $\mathrm{Rep}(G)$ the category of all Casselman-Wallach representations of G, and its full subcategories $\mathrm{Rep}_\mu(G)$, $\mathrm{Rep}_S(G)$.

To denote their Grothendieck groups, we replace Rep by $\mathcal{K}$.

Fix a Q_ι-coset $\Lambda = \lambda + Q_\iota \subset {}^a\mathfrak{h}^*$.

Definition 1.4 (coherent family). *A $\mathcal{K}(G)$-valued Λ coherent family is a map*

$$\Phi : \Lambda \to \mathcal{K}(G)$$

satisfies the following two conditions:

(1) *for all $\nu \in \Lambda$, $\Phi(\nu) \in \mathcal{K}_\nu(G)$;*
(2) *for all representations $F \in \mathrm{Rep}(\mathfrak{g}, Q_\iota)$ and all $\nu \in \Lambda$,*

$$F \cdot (\Phi(\nu)) = \sum_\mu \Phi(\nu + \mu),$$

where μ runs over all weights of F, counted with multiplicities.

The set of coherent families is denoted by $\mathrm{Coh}_\Lambda(\mathcal{K}(G))$

We have the integral Weyl group $W_\Lambda = \{w \in W \mid w\lambda - \lambda \in Q_\iota\}$ there is a representation of W_Λ on $\mathrm{Coh}_\Lambda(\mathcal{K}(G))$ by

$$(w \cdot \Phi)(\nu) = \Phi(w^{-1}\nu).$$

This is called a coherent continuation representation.

2. 2023.12.23

2.1. **Review of regular parameter.** Following yesterday's discussion of coherent continuation representations, we define two basis for it, use Langland classification.

Suppose H is a Cartan subgroup of G, by which we mean a subgroup that is the centralizer of a Cartan subalgebra of $\mathrm{Lie}(G)$ (In the case of linear groups, it's just a real algebraic torus.). H has a unique maximal compact subgroup (since G is in Harish-Chandra's class, adjoint representation of H is trivial, H_0 lies in the center of H, as we know, all maximal subgroups of Lie groups are conjugate to each other, use Cartan decomposition of H, we win. In fact, H has a unique maximal compact torus and a unique maximal split torus) We denote by $\Delta_{\mathfrak{h}} \subset \mathfrak{h}^*$ the root system of $\mathfrak{g}$. A root is called imaginary if $\check{\alpha} \in \mathfrak{t}$ (i.e. roots which take pure imaginary values on $\mathrm{Lie}(H)$), an imaginary root α is called compact if $\mathfrak{g}_\alpha$ and $\mathfrak{g}_{-\alpha}$ is contained in the complexified Lie algebra of a common compact subgroup of G.(this is equivalent to $\mathfrak{g}_\alpha$ belong to a Lie subalgebra of a compact subgroup, since $\mathfrak{g}_\alpha$ lies in a Lie subalgebra of maximal compact subgroup is equivalent to $\mathfrak{g}_{-\alpha}$ Lies in the same subgroup)

There are two important facts about the representation theory of H:

(1) Every Casselman-Wallach representation of H is finite-dimensional, it follows directly from that H is an extension of an abelian group and a finite group;

(2) For every $\Gamma \in \mathrm{Irr}(\mathrm{H})$ differential of Γ is a direct sum of one-dimensional representations attached to a unique $\mathrm{d} \in \mathfrak{h}^*$. (it is true because H is in Harish-Chandra's class).

For every Borel subalgebra $\mathfrak{b}$ of $\mathfrak{g}$ containing $\mathfrak{h}$, write

$$\xi_{\mathfrak{b}} : \mathfrak{h} \to {}^a\mathfrak{h},$$

for the linear isomorphism attached to $\mathfrak{b}$ defined by

$$\mathfrak{h} \hookrightarrow \mathfrak{b} \to \mathfrak{b}/[\mathfrak{b}, \mathfrak{b}] = {}^a\mathfrak{h}.$$

The transpose inverse of this map is still denoted by $\xi_{\mathfrak{b}} : {}^a\mathfrak{h}^* \to \mathfrak{h}^*$.

Write

$$W({}^a\mathfrak{h}^*, \mathfrak{h}^*) := \{\xi_{\mathfrak{b}} : {}^a\mathfrak{h}^* \to \mathfrak{h}^* \,|\, \mathfrak{b} \text{ is a Borel subalgebra containing } \mathfrak{h}\}.$$

Definition 2.1. *For every element $\xi \in W({}^a\mathfrak{h}^*, \mathfrak{h}^*)$, put*

$$\delta(\xi) := \frac{1}{2} \cdot \sum_{\alpha \text{ is an imaginary root in } \xi\Delta^+} \alpha - \sum_{\beta \text{ is an compact imaginary root in } \xi\Delta^+} \beta \in \mathfrak{h}^*.$$

Write $\mathcal{P}_\Lambda(G)$ for the set of all triples $\gamma = (H, \xi, \Gamma)$, where H is a Cartan subgroup of G, $\xi \in W({}^a\mathfrak{h}^*, \mathfrak{h}^*)$, and

$$\Gamma : \quad \Lambda \quad \longrightarrow \quad \mathrm{Irr}(\mathrm{H})$$
$$\nu \quad \longmapsto \quad \Gamma_\nu$$

is a map with the following properties:

(1) $\Gamma_{\nu+\beta} = \Gamma_\nu \otimes \xi(\beta)$ for all $\beta \in Q_\iota$ and $\nu \in \Lambda$;

(2) $\mathrm{d}_\nu = \xi(\nu) + \delta(\xi)$ for all $\nu \in \Lambda$.(note that Cartan subgroups not always have irr repn with this differential!)

Here $\xi(\beta)$ is naturally viewed as a character of H by using the homomorphism $\iota : H \to H_{\mathbb{C}}$, and $H_{\mathbb{C}}$ is a Cartan subgroup of $G_{\mathbb{C}}$ containing $\iota(H)$.

2.2. **Outer automorphism of algebraic groups.**

Lemma 2.2 (a simple discover of abstract algebra). *G an abstract group, H a normal subgroup of G, if G acts transitively on a set X such that the restriction of this action to H is free and transitive, then the short exact sequence of groups*

$$1 \to H \to G \to G/H \to 1$$

Proof. Actually choose an element $x \in X$, we have

$$G \simeq \mathrm{Stab}_G(x) \ltimes H.$$

$\square$

Using this lemma, we can see that for a complex connected reductive algebraic group G, $\mathrm{Aut}(G)$ acts on the set of pinnings transitively, with its normal subgroup the $\mathrm{Inn}(G)$ acts freely and transitively, so every pinning gives us a split of the short exact sequence

$$1 \to \mathrm{Inn}(G) \to \mathrm{Aut}(G) \to \mathrm{Out}(G) \to 1.$$

Some facts about split connected reductive algebraic groups over arbitrary field k:

(1) {split connected reductive algebraic groups}$/_\backsim$ = {root datums}$/_\backsim$;
(2) For any two automorphisms of G, if they induce the same automorphism of root datum, then they differ by an inner automorphism;
(3) Fix a pinning, we get an isomorphism $\mathrm{Out}(G) \simeq \mathrm{Aut}(D_b)$.

3. 2023.12.26

Theorem 3.1 (Harish-Chandra's parameter of discrete series repn). *G be a connected semisimple Lie group (maybe there is an analogy for real reductive group), K a maximal torus of G, suppose that $\mathrm{rank}(K) = \mathrm{rank}(G)$, fix a compact maximal torus $T \in K$ and positive system Δ^+. Then for any analytically integral weight $\lambda + \rho$, there exist a discrete series representation π_λ, such that:*

    (1) *The infinitesimal character of π_λ is λ;*
    (2) *$\nu = \lambda + \rho - 2\rho_c$ is the highest weight of a minimal K type of π_λ;*
    (3) *If μ is any highest weight of irreducible K representation exist in π_λ, then*

$$\mu = \nu + \sum_{\alpha \in \Delta^+} n_\alpha \alpha$$

.

Moreover, $\pi_\lambda \simeq \pi_{\lambda'}$ if and only if λ and λ' lies in the same W_c orbit.

In the above theorem, $\rho - 2\rho_c$ is canonically realized as a continuous character of double cover of T, called $\tilde{T}$, which can be realized as a pull-back diagram:

$$\begin{array}{ccc} \tilde{T} & \longrightarrow & \mathbb{C}^\times \\ \downarrow & & \downarrow{\scriptstyle z \mapsto z^2} \\ T & \xrightarrow{2\rho - 4\rho_c} & \mathbb{C}^\times \end{array}$$

Similarly, λ is also a character of a double cover of T, with respect to -2λ, but the difference of -2λ and $2\rho - 4\rho_c$ is a square of a character of T, the two double covers are canonically isomorphic.

In fact, there is a one to one correspondence! Which I finally get it:

$$\{\text{genuie characters of } \tilde{T}_{\rho_{im}}\} \simeq \{(\Gamma, \nu) \in X^*(T) \times \mathfrak{t}^* \,|\, d\Gamma = \nu + \rho_{im} - 2\rho_{im,c}\}.$$
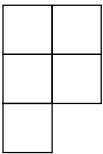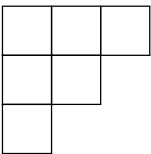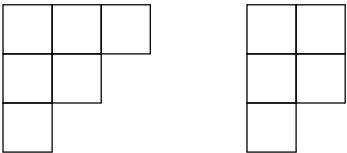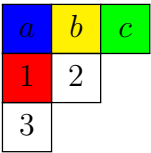
The correspondence is given by
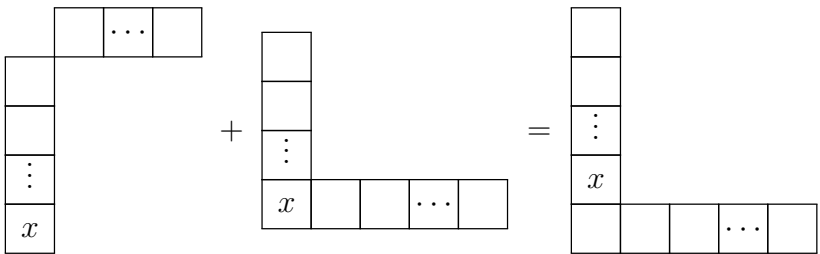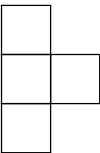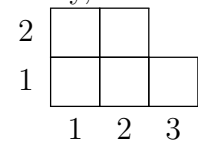
$$\chi \mapsto (\chi \otimes (\rho_{im} - 2\rho_{im,c}), d\chi)$$
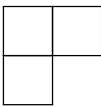$$(\Gamma, \nu) \mapsto \Gamma \otimes (2\rho_{im,c} - \rho_{im})$$

Definition 3.2 (Langlands parameter). *Let G be a linear real reductive group, the Langlands parameters for G is a triple $(H, \gamma, R_{i\mathbb{R}}^+)$. Such that*
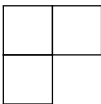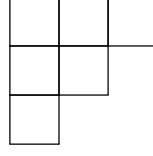
    (1) Hladky á

4. 2024.2.27

Today, I learned how to draw the Young tableau.

$$\begin{array}{c}
2 \\
1
\end{array}
\begin{array}{ccc}
\square & \square \\
\square & \square & \square
\end{array}$$
$$\begin{array}{ccc}
& 1 & 2 & 3
\end{array}$$

$$\begin{array}{c}
\square \\
\square & \square \\
\square
\end{array}$$

$$\boxed{\cdots} \;\; \vdots \;\; x \;\; + \;\; \boxed{\cdots} \;\; x \;\; = \;\; \vdots \;\; x \;\; \boxed{\cdots}$$

$$\begin{array}{ccc}
a & b & c \\
1 & 2 \\
3
\end{array}$$

$$a = \;\boxed{}\; + b$$

这是中文 $\square$ 这是英文
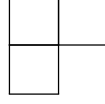
杨图 1:

杨图 2:

For a compact connected Lie group G, choose a maximal torus T, there is a canonical 1-1 correspondence:

$$X^*(T)/W \longleftrightarrow (X^*(T) + \rho)/W \longleftrightarrow \mathrm{Irr}(G)$$

Moreover, if we choose a positive system Ψ^+, and then ρ_{Ψ^+}, the above sets also canonically isomorphic to the set of dominant analytically integral weights:

$$\{\lambda \in X^*(T) \mid \langle \lambda, \alpha \rangle \geq 0, \text{for all } \alpha \in \Psi^+\}$$

This can be generalized to parameter of discrete series representations for connected semisimple Lie groups G with compact Cartan subgroup T :

$$(X^*(T) + \rho)^{reg}/W_c$$

If a discrete series representation V has minimal K type which has maximal weight ν, then it's parameter is $\lambda = \nu - \rho + 2\rho_c$.(not sure)

Note that for a fixed infinitesimal character, there are exactly $|W/W_c|$ different discrete series representations.

5. 2024.2.28

Some facts about continuous complex characters of some common abelian topological groups:

field	character group	correspondence
$(\mathbb{R}_+^\times, \times)$	$\mathbb{C}$	$\xi \mapsto (t \mapsto t^\xi)$
$(\mathbb{C}^\times, \times)$	$\mathbb{C} \times \mathbb{Z}$	$(\xi, n) \mapsto (z \mapsto |z|^\xi (\frac{z}{|z|})^n)$
$(\mathbb{S}^1, \times)$	$\mathbb{Z}$	$n \mapsto (z \mapsto z^n)$
$(\mathbb{Q}_p, +)$	$\mathbb{Q}_p$	$\xi \mapsto (x \mapsto \exp(2\pi i \xi \{x\}_p))$

Remark 5.1. *Note that all continuous characters of $\mathbb{Q}_p$ are automatically locally constant (smooth), which is consistent with the case of real lie groups.*

6. 2024.10.22

Lemma 6.1. *The associated variety of an irreducible Casselman-Wallach representation of a real reductive group in Harish-Chandra class is the closure of a nilpotent orbit in $\mathfrak{g}^*$.*

Proof. By the work of Borho, Brylinski [BB], and Joseph [Jos], the associated variety of any primitive ideal (annihilator of an irreducible $\mathfrak{g}$-module) is the closure of a single nilpotent orbit. So we only need to prove that $\mathrm{gr}(\mathrm{Ann}_{\mathcal{U}(\mathfrak{g})}(\mathrm{V}))$ is a primitive ideal.

Since the K-finite part V_K is dense in V, $\mathrm{Ann}_{\mathcal{U}(\mathfrak{g})}(V) = \mathrm{Ann}_{\mathcal{U}(\mathfrak{g})}(V_K)$, we only need to consider the annihilator of the irreducible $(\mathfrak{g}, K)$-module V_K.

If G is connected, then K is also connected, so V_K is irreducible as $\mathcal{U}(\mathfrak{g})$-module, in this case, $\mathrm{Ann}_{\mathcal{U}(\mathfrak{g})}(V)$ is a primitive ideal.

If G is not connected, let G_0 be the identity component of G, $K_0 = G_0 \cap K$ is a maximal compact subgroup of G_0, then $V_K|_{G_0}$ is a finite length $(\mathfrak{g}, K_0)$-module, it has an irreducible quotient $V_K \to V_0$, then according to the theory of induction of $(\mathfrak{g}, K)$-modules [KV, Chapter 2] we have an injective morphism of $(\mathfrak{g}, K_0)$-modules:

$$(6.1) \qquad V_K|_{K_0} \hookrightarrow (\mathrm{induced}_{K_0}^K(V_0))|_{K_0} \cong \bigoplus_{\text{double cosets } K_0 k K_0} k V_0,$$

where $k V_0$ be the $(\mathfrak{g}, K_0)$-module whose underlying space is V_0, whose action by $X \in \mathfrak{g}$ is the usual action by $\mathrm{Ad}(k)X$, whose action by $g \in K_0$ is the usual action by $k^{-1}gk$, it has the same annihilator in $\mathcal{U}(\mathfrak{g})$ as V_0 since G is in Harish-Chandra's class, and hence the annihilator of V_K is the same as the annihilator of V_0, which is a primitive ideal.

$\square$

7. 2024.11.28

Any split connected reductive group over arbitrary field can be defined and split over prime subfield!

The split rational form is unique.

Classification of split reductive group is the same over different fields.

(They are work of Chevalley)

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[KV] W. Knapp and D. Vogan, *Cohomological Induction and Unitary Representations*, Princeton Mathematical Series, vol. 45, Princeton University Press, Princeton, NJ, 1995.