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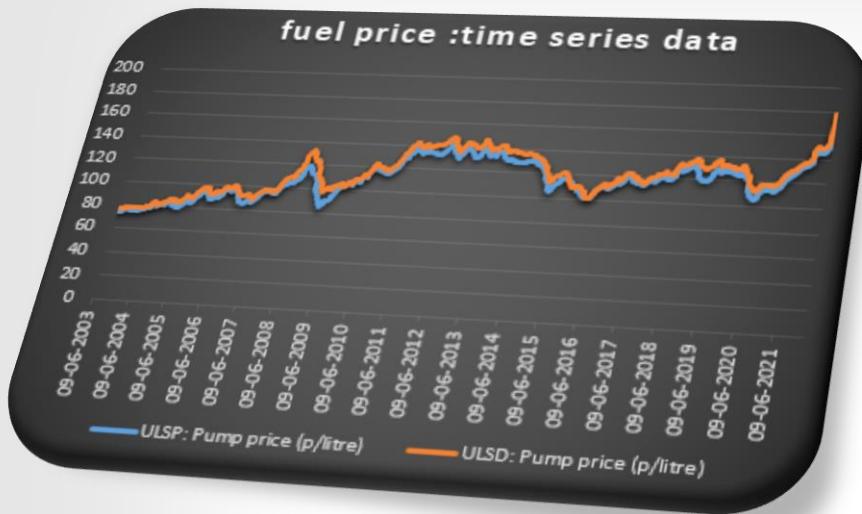
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# PRICE OF PETROL & DIESEL IN UK

- a Time Series analysis

This is a project work on time series ,for UG sem 6 (STS-A-DSE-B-6-2-P),based on BEIS' weekly road fuel price statistics  
 9/6/2003 – 28/3/2022 where we intend to decompose various components of the data, forecast next few values based on data and compare forecasted values with observed values if available

Date: 14july, 2022



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## ABSTRACT:

Here in this project, we have collected BEIS' weekly road fuel price statistics providing average UK retail 'pump' prices on a weekly basis. We have this weekly observations for both petrol price and diesel price, for the time period 9june,2003 to 28march,2022. Throughout the project work I intend to .....

- Obtain the common descriptive measures such as 'mean', 'median', 'quartile', 'range', 'variance' of the data
- Check if any linear or non-linear relationship between petrol and diesel price
- Plot the time series data for easy visualization
- Decompose various components of time series such as
  - Fitting different 'trend line' to the data by 'Method of Least Square', computing their 'coefficient of determination', constructing corresponding 'ANOVA table'. Comparing such lines and selecting the most appropriate one.
  - Check whether there is 'seasonal' and/or 'cyclical' component. If any then analyse them.
  - Examine the 'irregular component' with the help of 'residual plot'.
- Based on the observed time series data forecasting immediate future values.

## INTRODUCTION:

Millions of years ago, algae and plants lived in shallow seas. After dying and sinking to the seafloor, the organic material mixed with other sediments and was buried. Over millions of years under high pressure and high temperature, the remains of these organisms transformed into what we know today as fossil fuels. Coal, natural gas, and petroleum are all fossil fuels that formed under similar conditions. Today, petroleum is found in vast underground reservoirs where ancient seas were located.

Energy is an inseparable component of any economic development. Oil has become the world's most important source of energy since the mid-1950s in such a way that it is named as 'lifeblood of the nations'. In general, low oil prices are considered good for importers of oil because it not only improves consumer spending but also improves the trade balance of a country. Therefore an increase in oil prices has a significant negative impact on the GDP growth in all oil importing countries. On the other side, decrease in Oil price is bad for oil exporters as it could put a depression in revenues of oil exporting countries where oil exports play an enormously important role in supporting economic growth and government finances.

When we think of oil, we tend to think of fuel for our cars, trucks, and planes, and heating oil. However, there are a myriad of uses for crude oil in petro-chemical sector that affects our lives – which means that reducing our dependence on oil may not be as easy as it seems, even after knowing its environmental impacts and non-renewability very well. The European Commission has established a gradual transfer from petroleum-based economy to a more carbohydrate-based economy for 2030, where atleast 20% of transportation fuel should be produced from biomass. The United Kingdom (UK) [the country, not to be confused with Great Britain, its largest island whose name is also loosely applied to the whole country] is a sovereign country in Europe, has the world's sixth-largest economy by nominal gross domestic product (GDP), and the eighth-largest by purchasing power parity (PPP). Having a high-income economy and a very high human development index rating, the United Kingdom ranks 15th in the world for oil consumption, accounting for about 1.6% of the world's total consumption. UK Oil Consumption data is updated yearly, averaging 1,731.920 Barrel/Day from Dec 1965 to 2020. The data reached an all-time high of 2,228.136 Barrel/Day in 1973 and a record low of 1,191.787 Barrel/Day in 2020. The United Kingdom holds 2,754,685,000 barrels of proven oil reserves as of 2016, ranking 30th in the world and accounting for about only 0.2% of the world's total oil reserves of 1,650,585,140,000 barrels. It produces every year an amount equivalent to 14.4% of its total proven reserves (as of 2016), and imports 11% of its oil consumption (177,443 barrels per day in 2016). Through its extensive supply chain, the oil & gas industry employs hundreds of thousands of people and makes a major contribution to the UK economy in terms of tax revenues, technologies and exports. For the oil importer countries, oil price increase and economic growth are negatively correlated while all things being equal.

So, keeping track of oil price in the United Kingdom is essential, what we will try through our project work.

## COLLECTED DATA :

BEIS' weekly road fuel price statistics provide average UK retail ('pump') prices on a weekly basis. The data is used to monitor road fuel prices in the UK, and to compare UK road fuel prices with other EU countries. Weekly price data is published on the BEIS [website](#) the day after collection.

Here we extract the weekly tabulated data on the time period **9june,2003 to 28march,2022** as follows...

| Date       | ULSP  | ULSD  |
|------------|-------|-------|
| 09-06-2003 | 74.59 | 76.77 |
| 16-06-2003 | 74.47 | 76.69 |
| 23-06-2003 | 74.42 | 76.62 |
| 30-06-2003 | 74.35 | 76.51 |
| 07-07-2003 | 74.28 | 76.46 |
| 14-07-2003 | 74.21 | 76.41 |
| 21-07-2003 | 75.07 | 76.9  |
| 28-07-2003 | 75.1  | 76.86 |
| 04-08-2003 | 75.12 | 76.81 |
| 11-08-2003 | 75.44 | 77.08 |
| 18-08-2003 | 75.81 | 77.44 |
| 26-08-2003 | 76.05 | 77.68 |
| 01-09-2003 | 76.13 | 77.59 |
| 08-09-2003 | 76.23 | 77.67 |
| 15-09-2003 | 76.2  | 77.63 |
| 22-09-2003 | 76.15 | 77.55 |
| 29-09-2003 | 76.08 | 77.53 |
| 06-10-2003 | 76.43 | 77.66 |
| 13-10-2003 | 75.9  | 77.43 |

:

For the data and throughout the project hereafter .....

- Unit of price = pence/liter
- ULSP =( Ultra low sulphur unleaded) petrol
- ULSD = (Ultra low sulphur unleaded )diesel

We will also denote petrol price and diesel price by 'p' or ' $p_t$ ' and 'd' or ' $d_t$ ' respectively.

BEIS stands for Department for Business, Energy & Industrial Strategy a department of the government of the United Kingdom .

|            |        |        |
|------------|--------|--------|
| 07-02-2022 | 146.33 | 150.3  |
| 14-02-2022 | 146.95 | 151.1  |
| 21-02-2022 | 147.77 | 151.95 |
| 28-02-2022 | 149.22 | 153.36 |
| 07-03-2022 | 152.95 | 158.56 |
| 14-03-2022 | 159.96 | 169.48 |
| 21-03-2022 | 165.37 | 177.47 |
| 28-03-2022 | 162.65 | 176.44 |

*<remaining entire data set is attached in appendix section ,at the end of this project>*

# METHODOLOGY:

## DESCRIPTIVE MEASURES :

### ■ 5NUMBER SUMMARY & BOX PLOT

The five-number summary is a set of descriptive statistics that provides information about a dataset. It consists of the five most important sample percentiles

---- the sample minimum (smallest observation)

---- the lower quartile or Q1 (the value under which 25% of data points are found when arranged in increasing order )

---- the median or Q2 (the middle value)

---- the upper quartile or Q3 (the value under which 75% of data points are found when arranged in increasing order )

---- the sample maximum (largest observation)

The five-number summary gives information about the location [median], spread [midspread or interquartile range IQR : Q3-Q1] ,range [sample maximum – minimum] and skewness [ comparing distance between Q1toQ2 & Q2toQ3 ] of the observations. To readily visualize all such information we draw Boxplot. In addition to the box representing midrange and a midline at median on a box plot, there are lines, called whiskers, extending from the box indicating variability outside the upper and lower quartiles. Conventionally, from above( /below) the upper( /lower) quartile , a distance of 1.5 times the IQR is measured out and a whisker is drawn up( /down) to the largest( /smallest) observed data point from the dataset that falls within this distance. Because the whiskers must end at an observed data point, the whisker lengths can look unequal, even though 1.5 IQR is the same for both sides. All other observed data points outside the boundary of the whiskers are plotted as a dot, a small circle, a star, etc. are called outliers.

It is possible to quickly compare several sets of observations by comparing their five-number summaries and boxplots.

### ■ SAMPLE MEAN & VARIANCE

The sample mean or average is the most commonly used ‘measure of location’ for a dataset, which is the sum of those values divided by the number of values. Using mathematical notation, if a sample of  $n$  observations on variable  $X$  is taken, the sample mean is

$$\bar{x} = \sum_i x_i / n$$

it is easy to interpret but is widely affected in presence of extreme observations.

Sample variance can be defined as the average of the squared difference of data points from the mean of the data set. It is an ‘absolute measure of dispersion’ and is used to check the deviation of data points with respect to the data's average.The formula is given by

$$s_x^2 = \frac{\sum_i (x_i - \bar{x})^2}{n - 1}$$

### ■ SAMPLE COVARIANCE

Suppose we have  $n$  pairs of observations  $(x_i, y_i)$ ;  $i=1,2,\dots,n$  of two variables  $x, y$  and we

want to know how well or in which direction they vary together. Such a measure is sample covariance , given by

$$cov(x, y) = \frac{\sum_i (x_i - \bar{x})(y_i - \bar{y})}{n - 1}$$

this measure remains unaffected by origin change, but is not unit free. Note that in particular sample variance of  $x$  is  $s_x^2 = cov(x, x)$

## ■ . SAMPLE CORRELATION COEFFICIENT & LINEAR REGRESSION|

The degree or extent to which two variables are linearly related or associated is called the correlation between the variables. It is measured by product moment correlation coefficient due to K. Pearson

$$\begin{aligned} r_{xy} &= \frac{\sum_i (x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_i (x_i - \bar{x})^2} \sqrt{\sum_i (y_i - \bar{y})^2}} \\ &= \frac{cov(x, y)}{s_x \cdot s_y} \quad \text{where } |r_{xy}| \leq 1 \end{aligned}$$

Now based on the observations we can fit a straight line such that our model is

$y_i = (a + bx_i) + e_i$  ; $i=1,2,\dots,n$  where  $e_i$  's are random errors satisfying assumption  $E(e_i) = 0$  &  $cov(e_i, e_j) = \begin{cases} \sigma^2, & i=j \\ 0, & i \neq j \end{cases}$ . The unknown parameters  $a, b$  are

estimated in such a way that error sum of squares  $L = \sum_i e_i^2$  is minimized. The normal equations are given by

$$\begin{cases} \frac{\partial L}{\partial a} = 0 \\ \frac{\partial L}{\partial b} = 0 \end{cases} \quad \text{i.e.} \quad \begin{aligned} \sum_i y_i &= na + b \sum_i x_i \\ \sum_i y_i x_i &= a \sum_i x_i + b \sum_i x_i^2 \end{aligned} \quad \dots\dots\dots \text{solving the equations we get}$$

$$\begin{cases} \hat{b} = r_{xy} \frac{s_y}{s_x} \\ \hat{a} = \bar{y} - \hat{b}\bar{x} \end{cases} \quad \text{and } MSE = \frac{\sum_i (y_i - \hat{a} - \hat{b}x_i)^2}{n-2} \text{ gives an estimate of } \sigma^2$$

Note that  $r_{xy} = \begin{cases} +1 : \text{when all } (x_i, y_i) \text{ exactly lies on a straight line with +ve slope} \\ 0 : \text{when } x_i, y_i \text{ 's are uncorrelated, i.e there is no linear relationship} \\ -1 : \text{when all } (x_i, y_i) \text{ exactly lies on a straight line with -ve slope} \end{cases}$

in general the quantity  $r_{xy}^2$  is termed as 'coefficient of determination' ,which measures how much proportion of the total variability of  $y$  is explained by the least square linear regression equation  $y_x = (\hat{a} + \hat{b}x)$  ,means how well the straight line is fitted to the data.

## ■ TIME SERIES :

Time series is set of data collected and arranged in accordance of time. A time series is the result of a number of movements which are caused by numerous economic, political, natural and other factors. The analysis of time series means decomposing the past data into components and then projecting them forward. A time series typically has four components, though on occasions only one or two of these may eclipse the others....

- **TREND :-** Over a long period, a time series will have an overall tendency either to move

upwards or downwards or sometimes a blend thereof, though the actual movement will not be regular. This general, smooth, long term ,average tendency of the time series data is known as the ‘secular trend’. For example, at a glance the sales of a popular soft drink manufacturer is likely to reveal an increasing trend.

It is to be noted that, the term ‘long period’ has no exact definition but relative to the context of discussion. Trend is the easiest component in a time series data to capture and then the most well discussed one.

- **SEASONAL VARIATION** :- The fluctuations in a time series due to the rhythmic forces which operate in a regular ,periodic manner over a span of less than a year and have the same or almost same pattern year after year are called seasonal variations. For example, for a soft drink manufacturer, while the yearly sales may be on the increase, the sales are likely to be high every summer and low every winter. Clearly, any time series data recorded annually can not have seasonality.

If seasonality is present in a time series data it can be extracted easily as ratio or residual of observed values to trend values.

- **CYCCLICAL MOVEMENTS** :- These are oscillatory movements—a series of repeated sequences---superimposed on the original data. These movements are of longer duration than a year. The sales of a company, for example, may be high because the level of economic activity may be high. Similarly, the sales may be low due to overall subdued economic activity.

Although these variations may be recurrent, these are seldom found to be of similar pattern having same period and amplitude of oscillations. Also all cyclic components whose wavelength exceeds the length of the observed data is have to be accumulated in Trend component. A satisfactory method for the direct measurement of cyclical variations is not available. In most of the cases these variations are so intermixed with random variations that it is very difficult, if not impossible, to separate them.

- **IRREGULAR COMPONENT** :- Apart from the three systematic components, almost all the time series contain another factor called the ‘random’ or ‘irregular’ or ‘residual fluctuations’ .These fluctuations are purely random ,erratic ,unforeseen, unpredictable and are due to numerous non-recurring and irregular circumstances which are beyond the control of human-hand but obviously a part of our system ,such as earthquakes, wars, floods, strike etc. Irregular component may or may not be significant. As the name suggests, it is not possible to isolate irregular component and study them exclusively, nor one can forecast or estimate them precisely.

- Forecasting time series data implies that predictions about the future values are made only from the past data, and other factors, no matter how important they might potentially be, are not considered.

## ■ **POLYNOMIAL CURVE FITTING:-**

As trend line is a smooth curve , an obvious choice is fitting a polynomial curve( since polynomial functions are continuous and differentiable everywhere).

Here we assume that our suitable trend equation is a polynomial of degree ‘ $p$ ’ in time element ‘ $t$ ’. That is our mathematical model is

$$y_t = (a_0 + a_1 t + a_2 t^2 + \dots + a_p t^p) + e_t$$

so here are  $p$  covariates  $t, t^2, \dots, t^p$ ; response variable  $y_t$  ;( $p+1$ ) parameters  $a_0, a_1, a_2, \dots, a_p$  .

$e_t$  's are random errors satisfying assumption  $E(e_t) = 0$  &  $\text{cov}(e_t, e_{t'}) = \begin{cases} \sigma^2, & t = t' \\ 0, & t \neq t' \end{cases}$ . let

there are  $n$  observations of the pair  $(t, y_t)$ . The parameters are estimated from the data by 'method of least square', which consists of minimizing  $L = \sum_t e_t^2$

$$= \sum_t (y_t - a_0 + a_1 t + a_2 t^2 + \dots + a_p t^p)^2$$

the  $(p+1)$  normal equations are given by  $\frac{\partial L}{\partial a_i} = 0 \quad \forall i = 0, 1, 2, \dots, p$

$$\text{i.e. } \begin{cases} \sum_t y_t = n a_0 + a_1 \sum_t t + a_2 \sum_t t^2 + \dots + a_p \sum_t t^p \\ \sum_t t y_t = a_0 \sum_t t + a_1 \sum_t t^2 + a_2 \sum_t t^3 + \dots + a_p \sum_t t^{p+1} \\ \vdots \\ \sum_t t^p y_t = a_0 \sum_t t^{p+1} + a_1 \sum_t t^{p+2} + a_2 \sum_t t^{p+3} + \dots + a_p \sum_t t^{2p} \end{cases}$$

solving these equations we get estimated value of  $a_0, a_1, a_2, \dots, a_p$ . Then fitted value at time  $t$  is

$$\hat{y}_t = \hat{a}_0 + \hat{a}_1 t + \hat{a}_2 t^2 + \dots + \hat{a}_p t^p$$

**EXAMPLE :** when  $p=2$ , and origin of  $t$  is so chosen that  $t = 0, \pm 1, \pm 2, \dots, \pm k$ ;  $n = 2k + 1$  we have

$$\hat{a}_2 = \frac{n \sum_t y_t t^2 - \sum_t y_t \sum_t t^2}{n \sum_t t^4 - (\sum_t t^2)^2}; \quad \hat{a}_1 = \frac{\sum_t y_t t}{\sum_t t^2}; \quad \hat{a}_0 = \frac{\sum_t y_t - \hat{a}_2 \sum_t t^2}{n}$$

#### ● MERITS :

- i. It is possible to compute trend values for all the periods and predict the value for a period lying outside the observed data.
- ii. The functional form is easy to comprehend for layman. Also using suitable transformation, various other kind of equation can be fitted by this method.
- iii. The results of the method of least squares are most satisfactory because the fitted trend satisfies the two most important properties,
  - $\sum_t (y_t - \hat{y}_t) = 0$  : the position of fitted trend equation is such that the sum of deviations of observations above and below this equal to zero.
  - $\sum_t (y_t - \hat{y}_t)^2 = 0$  : the sums of squares of deviations of observations, about the trend equations, are minimum among all polynomials of same degree

#### ● DEMERITS :

- i. It is not flexible like if some observations are added, then the entire calculations are to be done once again.
- ii. The computation of trend values, on the basis of this method, doesn't take into account the other components of a time series and hence forecasting may not always be reliable except immediate future.
- iii. Since the choice of a particular trend is arbitrary, the method is not, strictly, objective.

### ■ TESTING OF SIGNIFICANCE:-

Based on our model, mentioned earlier—  $y_t = (a_0 + a_1 t + a_2 t^2 + \dots + a_p t^p) + e_t$

we want to test whether the covariate has any significant effect on response variable or not. So,

our testing problem will be,  $\mathbf{H}_0 : a_i = 0 , \forall i = 1, 2, \dots, p$  vs  $\mathbf{H}_1 : a_i \neq 0 , \exists \text{ atleast one } i \text{ in } 1, 2, \dots, p$   
Now, consider the following table

| Source of variation | sum of squares<br>or SS   | d.f.  | mean squares<br>or MS   | F statistic                |
|---------------------|---|-------|-------------------------|----------------------------|
| regression          | $SS_{reg} = \sum_t (\hat{a}_0 + \hat{a}_1 t + \hat{a}_2 t^2 + \dots + \hat{a}_p t^p - \bar{y}_t)^2$ | p     | $MS_{reg} = SS_{reg}/p$ | $F = \frac{MS_{reg}}{MSE}$ |
| error               | $SSE = \sum_t (y_t - \hat{a}_0 - \hat{a}_1 t - \hat{a}_2 t^2 - \dots - \hat{a}_p t^p)^2$            | n-p-1 | $MSE = SSE/(n-p-1)$     |                            |
| total               | $SS_{total} = \sum_t (y_t - \bar{y}_t)^2$   | n-1   | ----                    |                            |

where  $SS_{total} = SS_{reg} + SSE$  is an orthogonal splitting, i.e  $SS_{reg}$  and  $SSE$  independent.

Under  $H_0 : a_i = 0 , \forall i = 1, 2, \dots, p$ ; the model reduces to  $y_t = a_0 + e_t$

$$\Rightarrow \hat{a}_0 = \bar{y}_t \text{ and so } SSE_{H_0} = SS_{total}$$

Hence a large positive value of  $(SSE_{H_0} - SSE) = SS_{reg}$  indicates deviation from  $H_0$ . It can be shown that  $F = \frac{MS_{reg}}{MSE} \sim F$  distribution with df (p, n-p-1). We reject  $H_0$  in favour of  $H_1$  at level  $\alpha$  iff  $F_{observed} > F_{\alpha;p,n-p-1}$  [upper  $\alpha$  point of F distribution with df (p, n-p-1)]

### • $R^2$ :-

The ratio of 'sum square due to regression' to the 'total variation' is an obvious measure of how well our model is fitted to the data. Then the most general definition of the 'coefficient of determination' is

$$R^2 = \frac{SS_{reg}}{SS_{total}} = 1 - \frac{SSE}{SS_{total}}$$

taking value between 0 and 1. More the value of  $R^2$  close to 1 indicates a better fit of model.

However each time we add a new covariate to the model the R-squared is guaranteed to increase, with a decrease in error df, even if the predictor variable isn't useful. So, penalizing with error df, we define our modified measure

$$\text{adjusted } R^2 = 1 - \frac{MSE}{MS_{total}} = 1 - (1 - R^2) \frac{n-1}{n-p-1}$$

it may be possible that in some case by adding new covariate  $R^2$  is increased but adjusted  $R^2$  is decreased. Same as  $R^2$ , the value of adjusted  $R^2$  lies between 0 and 1. More the value of adjusted  $R^2$  close to 1 indicates that all the predictor variables in model have better significant effects.

➤ It is to be mentioned that, in time-series analysis when fitting trend line, our goal is not to search for a curve that almost exactly passes through all data points (over-fitting), but to capture the overall long-term tendency of the time series.

## ■ FORECAST ACCURACY :-

It is to be noted that ,size of residuals are not a reliable indication of how large the forecasts error are likely to be, it can only be determined by considering how well a model performs on new data that were not used while fitting. If our training data is given by {  $y_1, y_2, \dots, y_T$  } and test data is given by {  $y_{T+1}, y_{T+2}, \dots$  }, then ‘forecast error’ is defined as their difference

$$e_T = y_{T+h} - \hat{y}_{T+h|T}$$

Some useful measures of forecast accuracy are given by

- **Root Mean Squared Error** :  $RMSE = \sqrt{\text{mean}(e_t^2)}$  ---gives an idea about average distance between observed and forecasted value.
- **Mean absolute percentage error** :  $MAPE = \text{mean}\left(\left|\frac{e_t}{y_t} \times 100\right|\right)$  ---- which measures the average magnitude of proportions of forecast error to observed value.

higher these measures ,better the accuracy

# ANALYSIS:

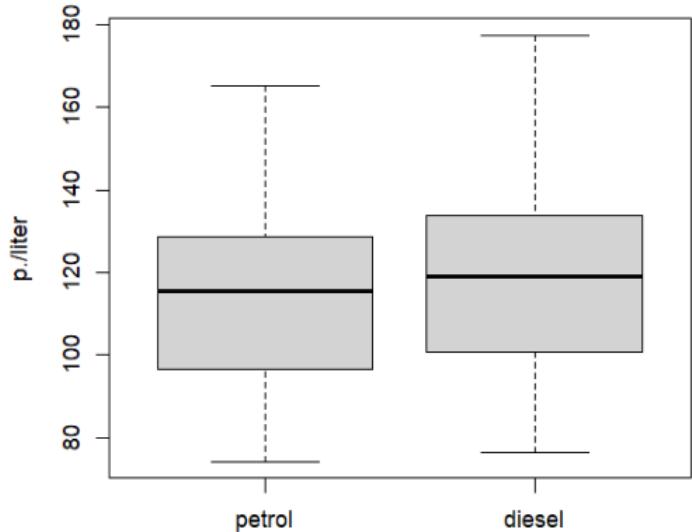
## ► DESCRIPTIVE ANALYSIS :

After getting the data , the first thing we intend to study is the '5point' summary of the data, and to visualize easily we add corresponding box-plots.

box-plot for petrol and diesel price data

```
> summary(pricedata[,-1])
```

|                            |                            |
|----------------------------|----------------------------|
| ULSP..Pump.price..p.litre. | ULSD..Pump.price..p.litre. |
| Min. : 74.21               | Min. : 76.41               |
| 1st Qu.: 96.57             | 1st Qu.: 100.93            |
| Median :115.56             | Median :119.14             |
| Mean :112.71               | Mean :117.22               |
| 3rd Qu.:128.59             | 3rd Qu.:133.71             |
| Max. :165.37               | Max. :177.47               |



the number of observations for both petrol and diesel price is

```
> n  
[1] 982 .There is no outlier.
```

for diesel average value of data is slightly larger than petrol ,

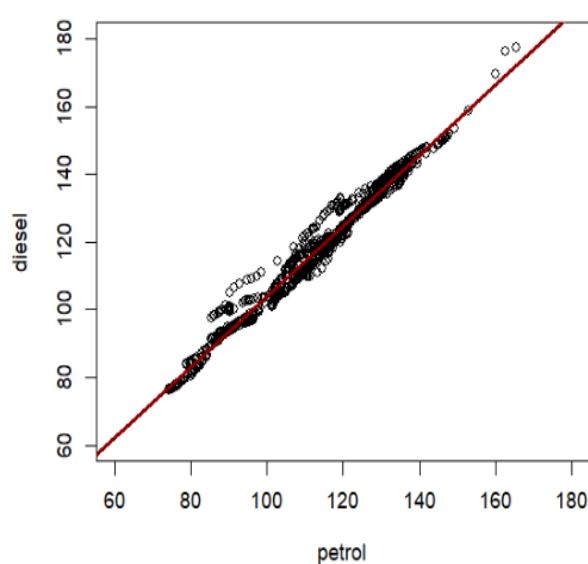
in both case mean<median , also 1<sup>st</sup> quartile is little more apart from median compared to median & 3<sup>rd</sup> quartile —implies the data has slightly -ve skewness for both petrol and diesel.

Using sample variance as measure of dispersion for the data

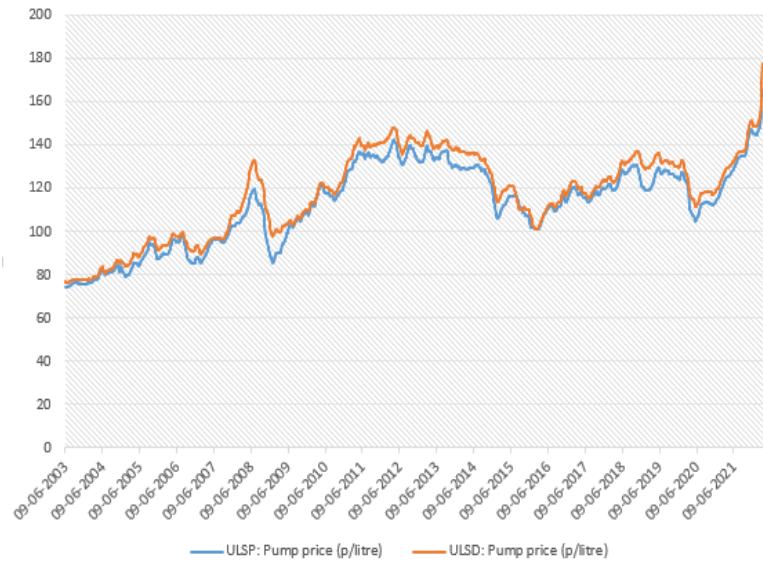
```
> var(p)      > var(d)      > cor(p,d)  
[1] 355.0034  [1] 393.7688  and [1] 0.9904448
```

see that petrol and diesel price is very highly +ve correlated. If we plot the pair (petrol price, diesel price) for various week , we get an almost straight line. Plotting the prices over time ,we also notice that petrol price and diesel price data looks very similar in nature.

petrol vs diesel price(p./liter) for fixed time



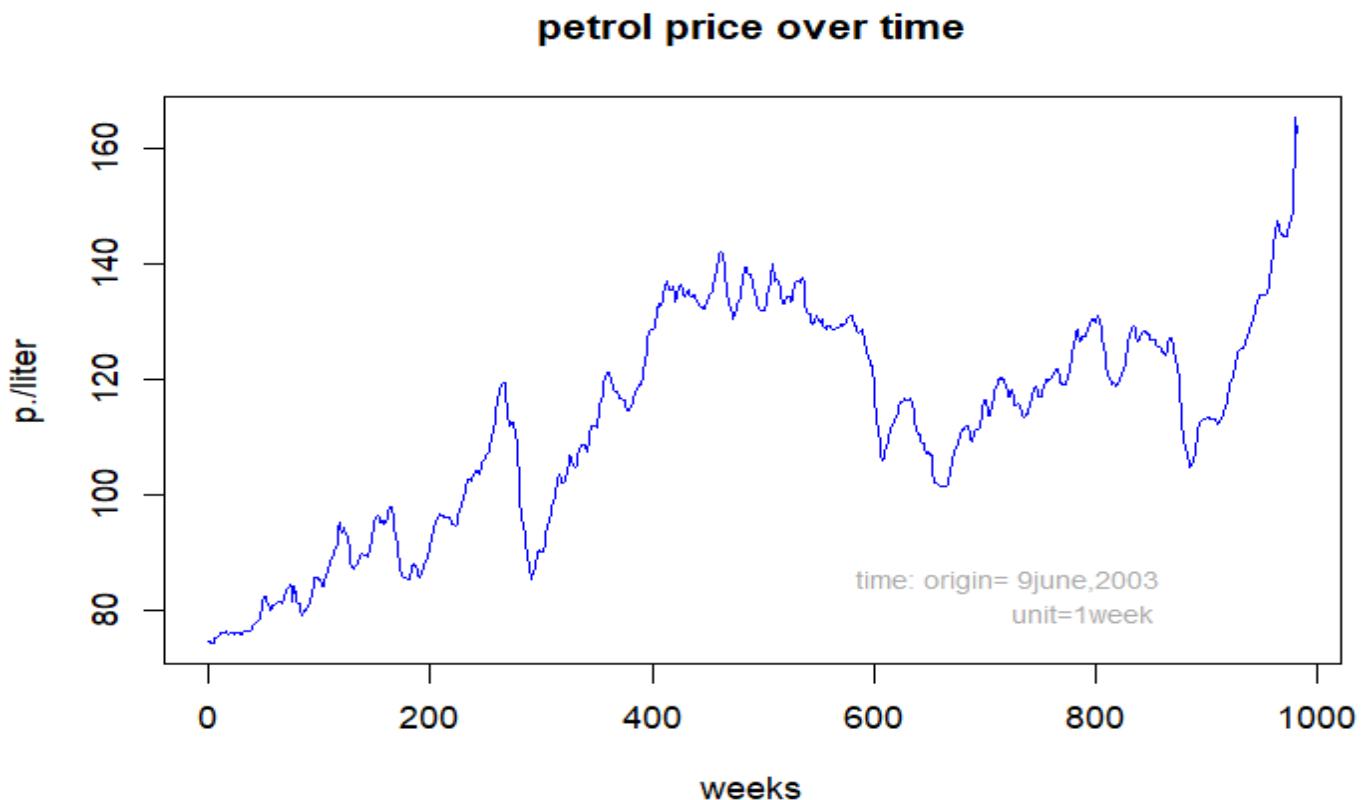
fuel price over time



- So from now on, we first obtain time series analysis and forecasting of one data , without loss of generality ,say petrol price data, and later regress the other i.e diesel price based on petrol price

## ✚ TIME SERIES ANALYSIS :

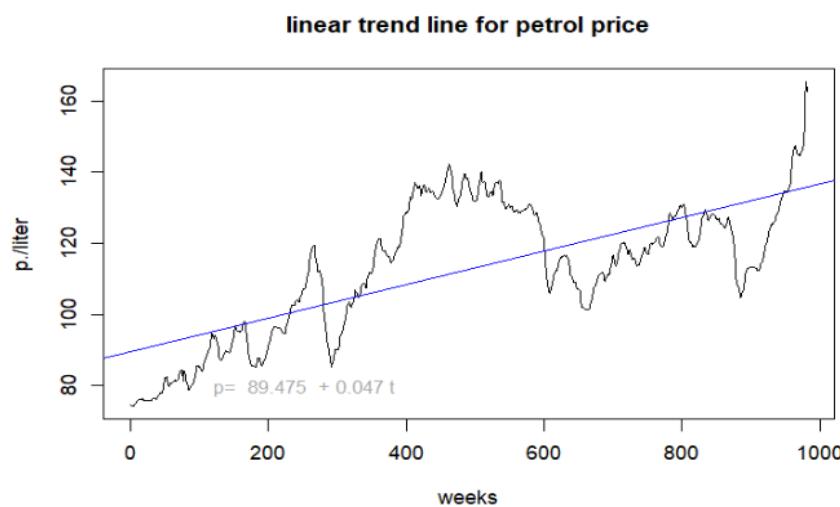
As the first step of time series analysis , plot the weekly petrol price data over time, taking 9june 2003 as origin and unit as 1week



- **TREND** : Lets start with linear trend, the best fitted straight line for this data is given by  
 > lm(p~t)

```
call:  
lm(formula = p ~ t)
```

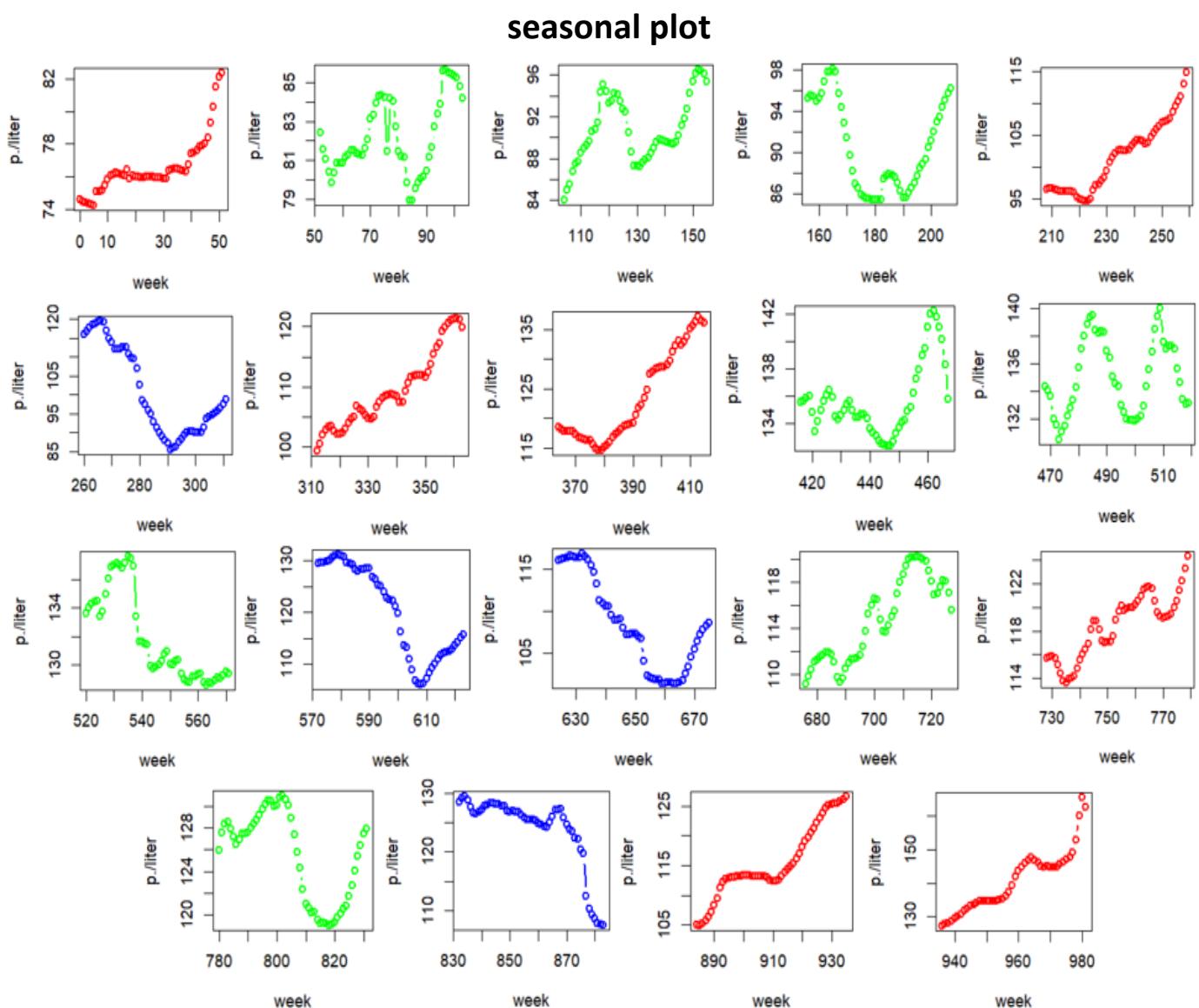
```
Coefficients:  
(Intercept)          t  
89.47510           0.04736  
Residual standard error: 13.22 on 980 degrees of freedom  
Multiple R-squared:  0.5083,   Adjusted R-squared:  0.5078  
F-statistic: 1013 on 1 and 980 DF,  p-value: < 2.2e-16
```



Here ANOVA suggests that corresponding F statistic is significant , which simply means that we have enough evidence to reject the null hypothesis “there is no trend” in favour of alternate hypothesis “there is linear trend” .But from the value of  $R^2$  it is evident that only 50.83% of total variation of the data is explained by linear trend equation ,which is not so satisfying. We can improve our trend equation, but for time being let skip it, only gaining knowledge from the slope of the linear trend that there is an overall increasing trend in the data.

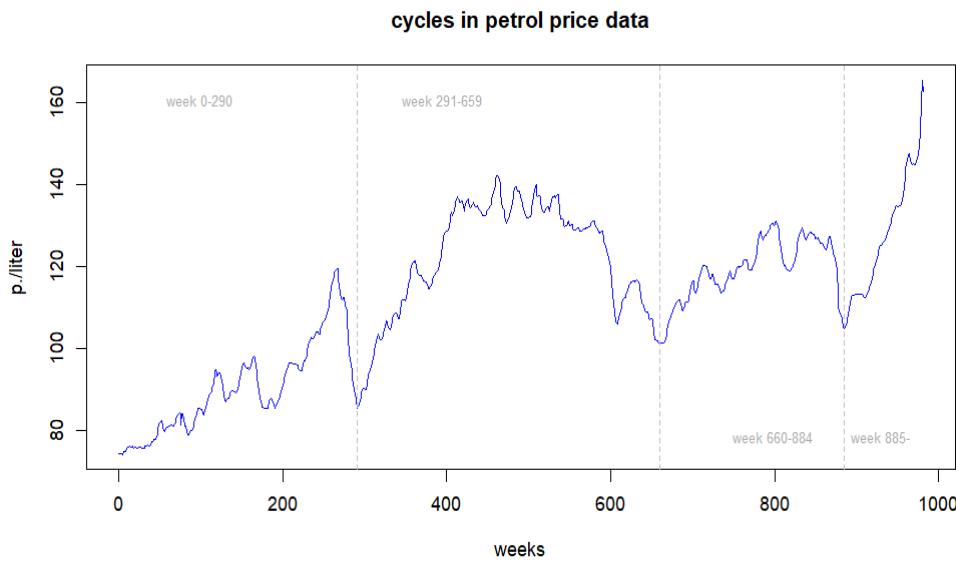
- **SEASONALITY :** If there is seasonality in a time series data, then plot of the data from one year should look alike plot from another year.

Here after plotting the data for every 52 weeks(1year) ,we see increasing pattern in some case as well as decreasing or fluctuating pattern.



So it is appealing that our data have no seasonality.

- **CYCICAL COMPONENT :** From the time-plot we notice that the data have three complete cycle of unequal length and one incomplete cycle at the end.



- we can bypass the hardship of separating ‘trend’ and ‘cyclic’ component by capturing so called ‘trend-cycle’ [The trend-cycle is the component that represents variations of low frequency in a time series, the high frequency fluctuations having been filtered out], and dumping everything others in residual part .

## ○ TREND-CYCLE DECOMPOSITION:

In the absence of seasonality here, trend-cycle is nothing but the entire systematic part of the time series. The rises and falls in the time-plot suggests that this systematic part can be modelled by either a polynomial curve or by a series of sums of sine and cosine curve (harmonic analysis). For sake of computational simplicity we stick to polynomial model.

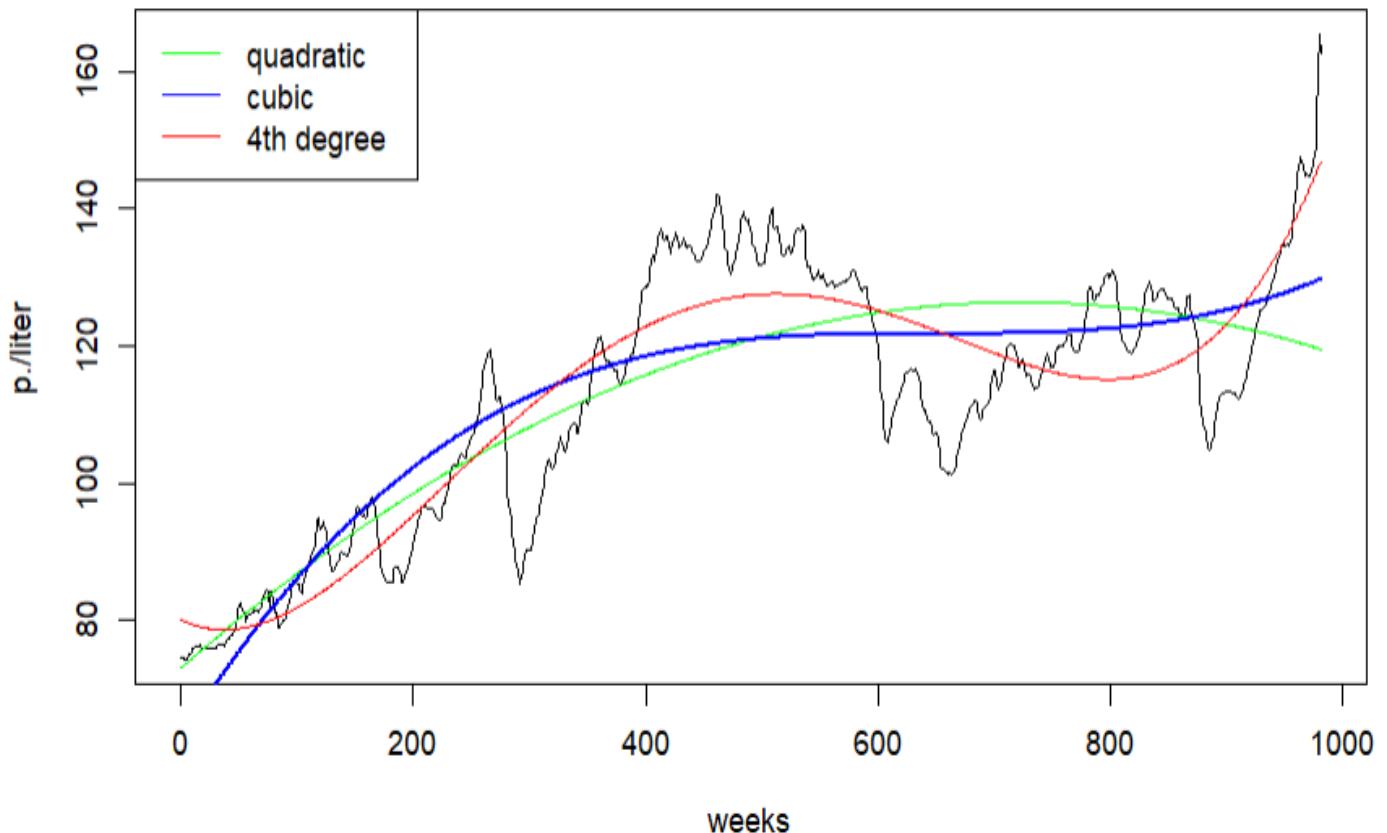
Let fit polynomials with degree  $p=2,3,\dots$  to the petrol price data by ‘method of least square and compare them ( remember straight line already have found inefficient ).....

| Trend-cycle eqn. | Intercept  | coeff. Of t | $t^2$      | $t^3$     | $t^4$    | R <sup>2</sup> | ANOVA   |
|------------------|------------|-------------|------------|-----------|----------|----------------|---|
| quadratic        | 73.0826611 | 0.1477221   | -0.0001023 |           |          | 0.6607         | Residual standard error: 10.99 on 979 degrees of freedom<br>Multiple R-squared: 0.6607, Adjusted R-squared: 0.66<br>F-statistic: 953.3 on 2 and 979 DF, p-value: < 2.2e-16  |
| cubic            | 6.29E+01   | 2.73E-01    | -4.21E-04  | 2.16E-07  |          | 0.703          | Residual standard error: 10.28 on 978 degrees of freedom<br>Multiple R-squared: 0.703, Adjusted R-squared: 0.7021<br>F-statistic: 771.8 on 3 and 978 DF, p-value: < 2.2e-16 |
| 4th degree       | 8.00E+01   | -7.85E-02   | 1.19E-03   | -2.34E-06 | 1.30E-09 | 0.797          | Residual standard error: 8.506 on 977 degrees of freedom<br>Multiple R-squared: 0.797, Adjusted R-squared: 0.7962<br>F-statistic: 959 on 4 and 977 DF, p-value: < 2.2e-16   |

As the degree of the polynomial , the residual variability decreases, R<sup>2</sup> value increases that means the polynomial better fits the data. Quadratic model with 66% explained variability is not a good one. But note that , while the cubic model averages out rises and falls of the cycle , the 4<sup>th</sup> degree model try to mimic them. So to avoid ‘over-fitting’ we

choose the cubic equation as our suitable trend-cycle equation, though it has smaller  $R^2$  value than the 4<sup>th</sup> degree one.

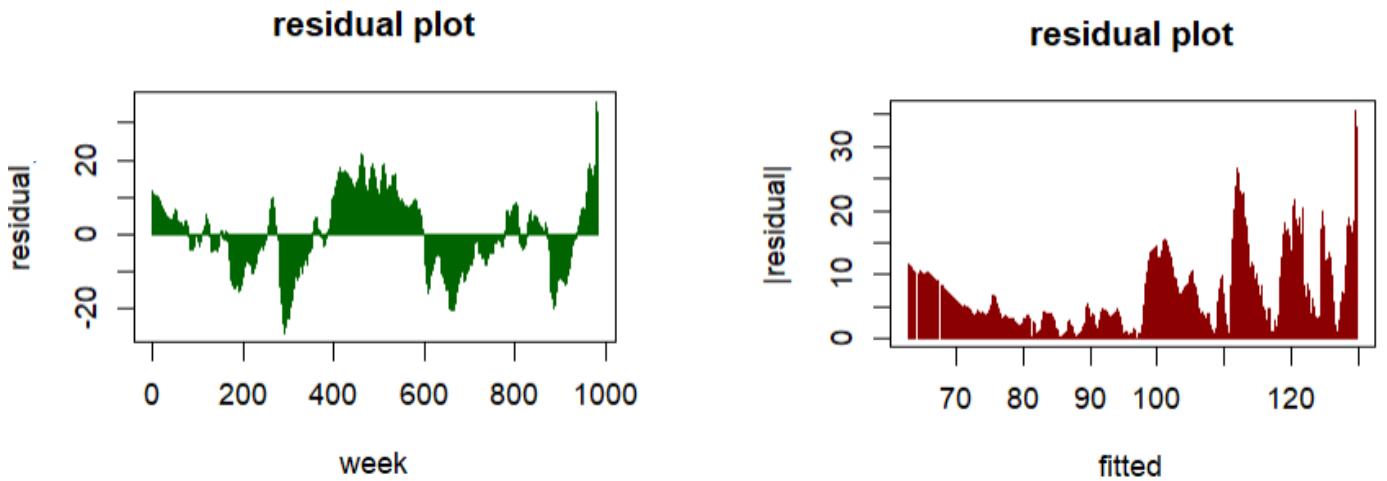
### various trend-cycle line for petrol price



For time  $t$ , the estimated trend-cycle component of petrol price is thus given by

$\hat{\tau}_t = 6.29 * 10 + 2.73 * 10^{-1}t - 4.21 * 10^{-4}t^2 + 2.16 * 10^{-7}t^3$ , which accounts for explaining 70.3% variation in petrol price data.

- **RESIDUALS :** Here the magnitude of deviation of observed & fitted values is not proportional to fitted values, so we can use additive residuals, i.e. for time  $t$  observed value of petrol price,  $p_t = \hat{\tau}_t + z_t$ ; where  $z_t$ 's are residuals.
- Plotting the residuals over time, we see that the pattern is not entirely random. Residuals



of same sign have a tendency to occur side by side, i.e residuals are supposed to be highly autocorrelated for initial few lags. The ‘fitted vs |residual| plot’ justifies the additive model.

But if we consider the 1<sup>st</sup> order differences

$$\Delta z_t = z_{t+1} - z_t$$

and plot them over  $t$

, we get a more or less random pattern.

the average value of observed  $\Delta z_t$ 's is

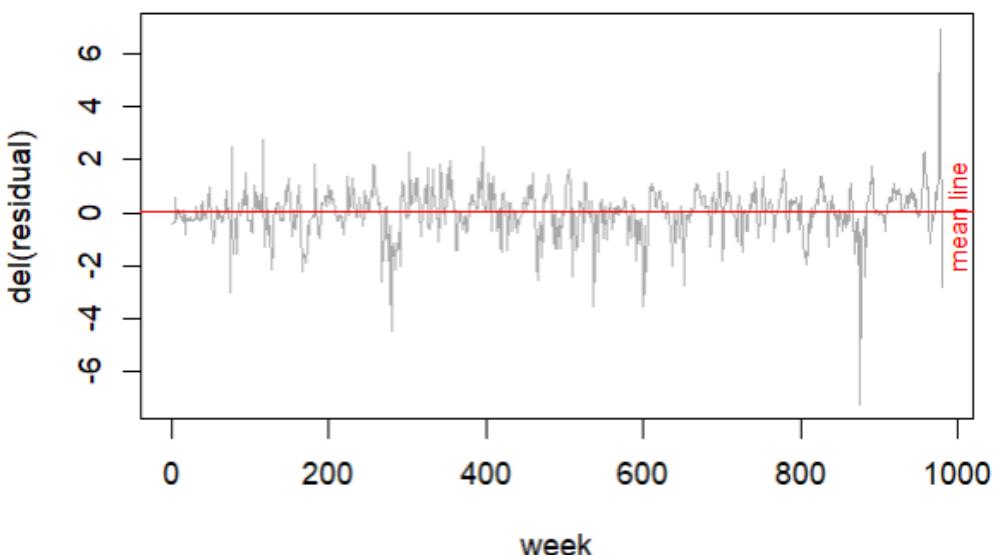
> `mean(diff(z))`

[1] 0.02163614 ,

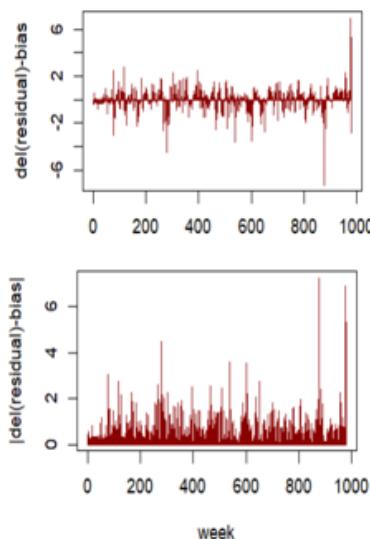
is an estimate of

Expectation( $\Delta z_t$ ).

### first difference of residuals over time



### adjusted first diff. of residuals over time



So, let us assume ,

$$\Delta z_t = \text{a random component with expectation } E(\Delta z_t)$$

Or,

$$(z_{t+1} - z_t) = \text{bias} + \\ \text{some random component with expectation 0}$$

we can thus predict the non random part of the residual as

$$\widehat{z_{t+1}} = 0.0216 + z_t, \\ \text{provided } z_t = p_t - \widehat{\tau}_t \text{ is known.}$$

## MODELLING PETROL PRICE :

Based on our analysis , we have obtained the following model:

$$p_t = \tau_t + z_t$$

price of petrol at time  $t$       trend cycle      residual : a non random part + random error

Hence ,

we can forecast the petrol price at time  $t$

$$\text{as } \widehat{p}_t = \widehat{\tau}_t + \widehat{z}_t$$

where the equation is explicitly given by

$$\widehat{\tau}_t = 6.29 * 10 + 2.73 * 10^{-1}t - 4.21 * 10^{-4}t^2 + 2.16 * 10^{-7}t^3$$

$$\widehat{z}_t = 0.0216 + z_{t-1}; \text{ if } z_{t-1} \text{ unknown, we can use } \widehat{z}_t = 2 * 0.0216 + z_{t-2}$$

Or so on.. with some reduced accuracy.

## PETROL vs DIESEL PRICE :

As we have seen earlier, for observations on same time point (petrol price , diesel price) is extremely highly correlated.

```
> cor(p,d)
```

```
[1] 0.9904448
```

So , we can predict diesel price based on petrol price using 'least square linear regression'. The best fitted linear regression equation is given by

```
> lm(d~p)
```

```
call:
lm(formula = d ~ p)
```

```
Coefficients:
(Intercept)          p
-0.3467        1.0431
```

i.e. diesel price at time  $t$ ,

$$d_t = -0.3467 + 1.0431p_t$$

Residual standard error: 2.738 on 980 degrees of freedom

Multiple R-squared: 0.981, Adjusted R-squared: 0.981

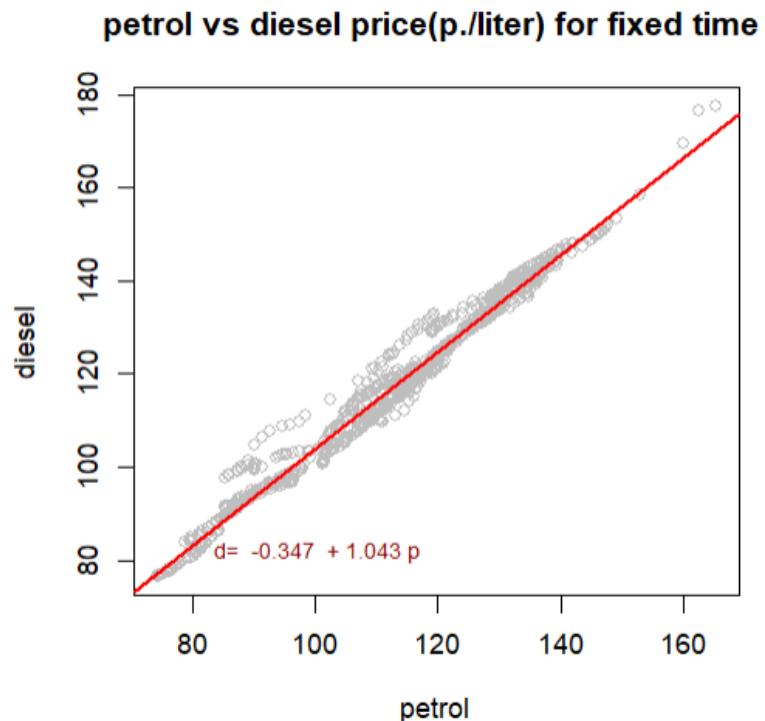
F-statistic: 5.055e+04 on 1 and 980 DF, p-value: < 2.2e-16

such regression equation can explain

98.1% variation of the response variable diesel price data.

Thus, we can model the diesel price at time  $t$

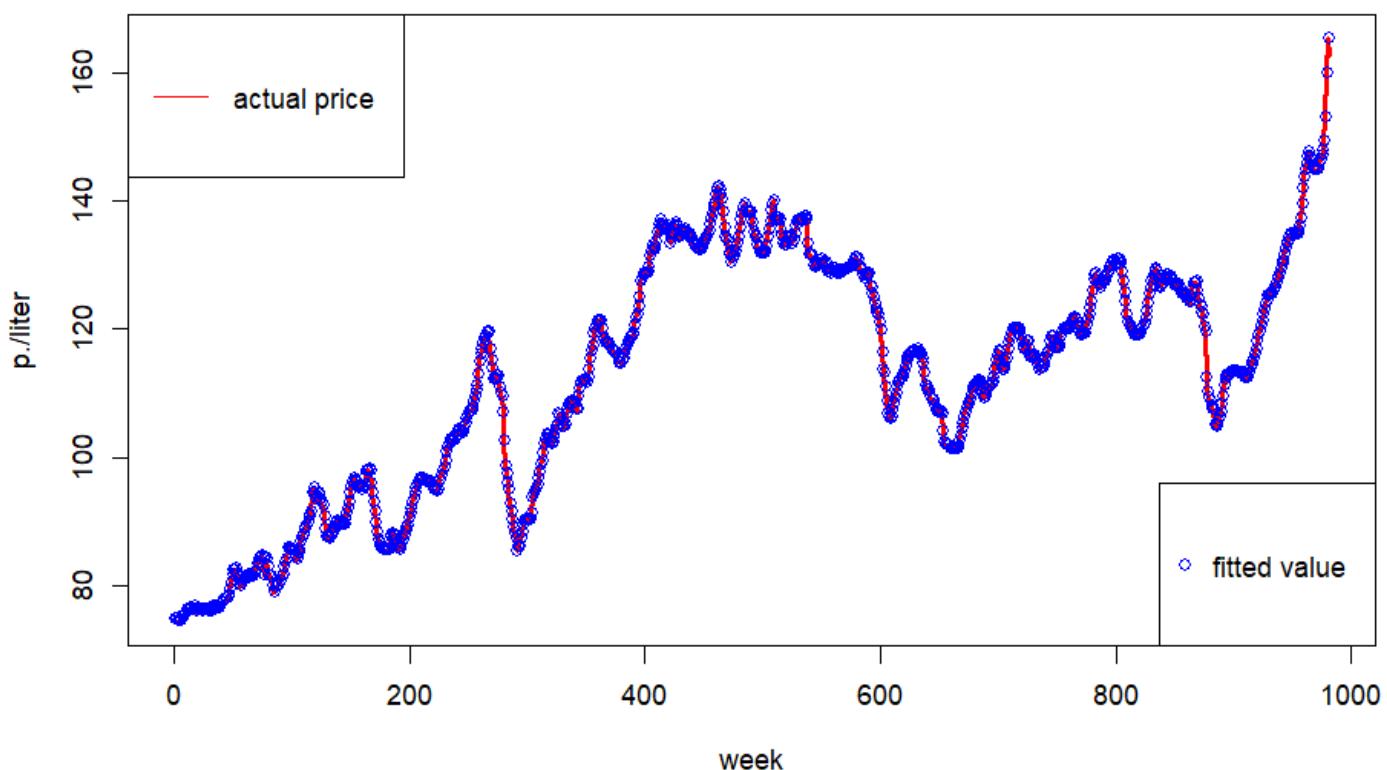
$$\text{as } \hat{d}_t = -0.3467 + 1.0431\hat{p}_t$$



## ✓ comparing observed & fitted values:

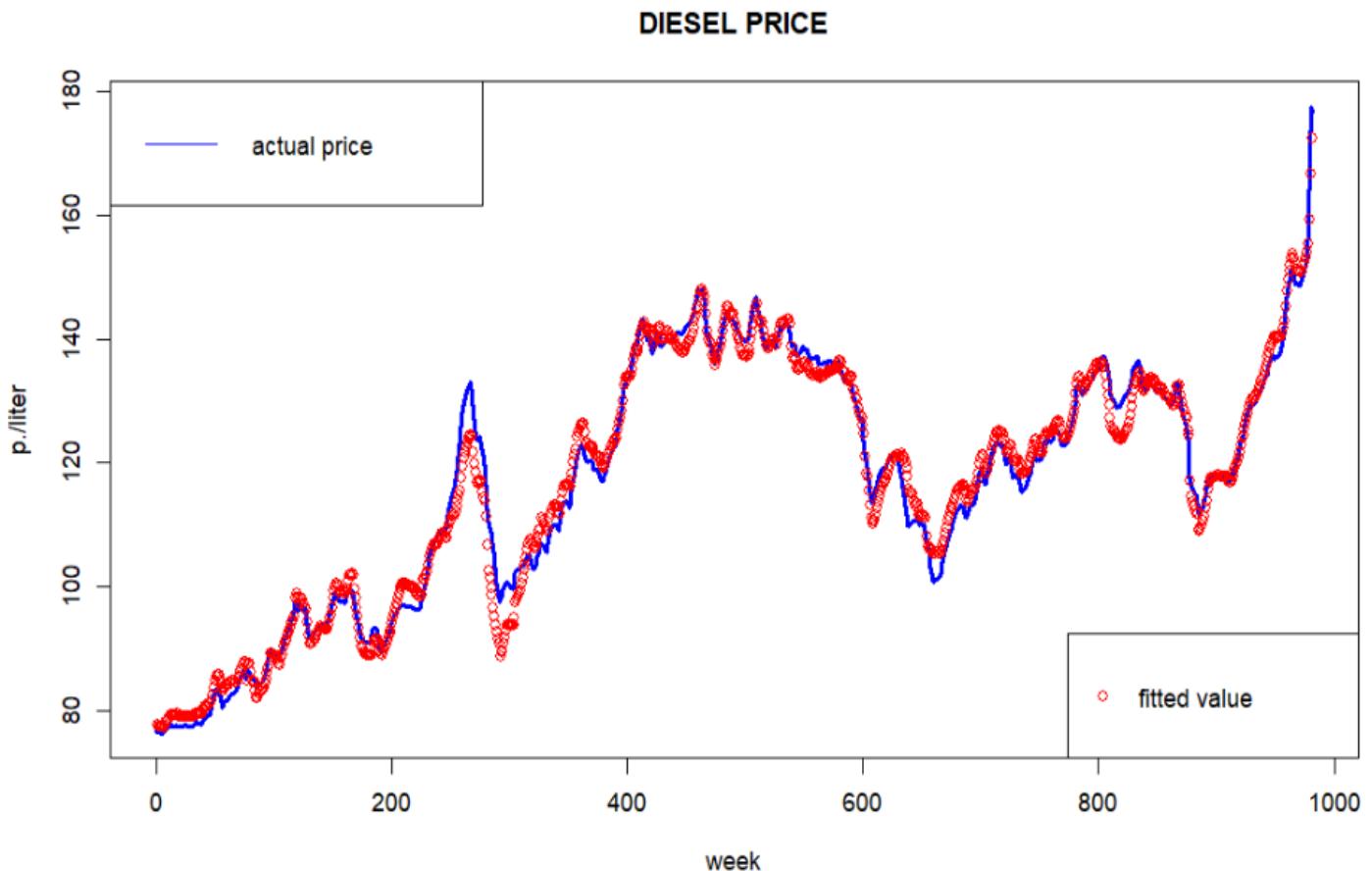
For petrol, we can see that fitted values almost overlap the observed values throughout

### PETROL PRICE



the time. The error variance for the model  $\hat{p}_t = \hat{\tau}_t + \hat{z}_t$  is 0.799 unit<sup>2</sup>. The magnitude of error is on average 0.54% of the actual petrol price data. So, our derived model fits very well for analysing petrol price data.

```
> summary(p[-1]-estm_p)
   Min. 1st Qu. Median Mean 3rd Qu. Max.
-7.24547 -0.31837 0.02939 0.00000 0.48735 6.91751
> var(p[-1]-estm_p)
[1] 0.7992405
> 100*mean(abs(p[-1]-estm_p)/p[-1])
[1] 0.5361433
```



But for diesel price ,we have more error variance as compared to petrol, while petrol and diesel price variation almost same. A model generating error

```
> summary(d[-1]-estm_d)
   Min. 1st Qu. Median Mean 3rd Qu. Max.
-8.146695 -1.762154 -0.494200 0.000703 1.156723 10.862132
> var(d[-1]-estm_d)
[1] 7.683009
> 100*mean(abs(d[-1]-estm_d)/d[-1])
[1] 1.81379
```

1.81% of the actual diesel price on average, is not a bad one though.

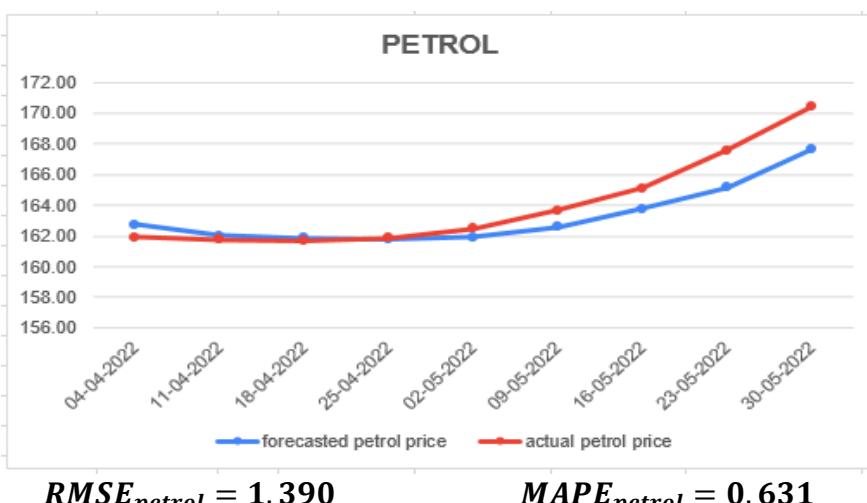
# ✓cross-checking:

*"A model that fits well, not necessarily forecasts well."*

We started our project work based on data till 28march,22. But in the meantime of our work being done , few more weekly observations are updated in the original website. So we can now compare these observed values with the predicted values from our model for these weeks.....

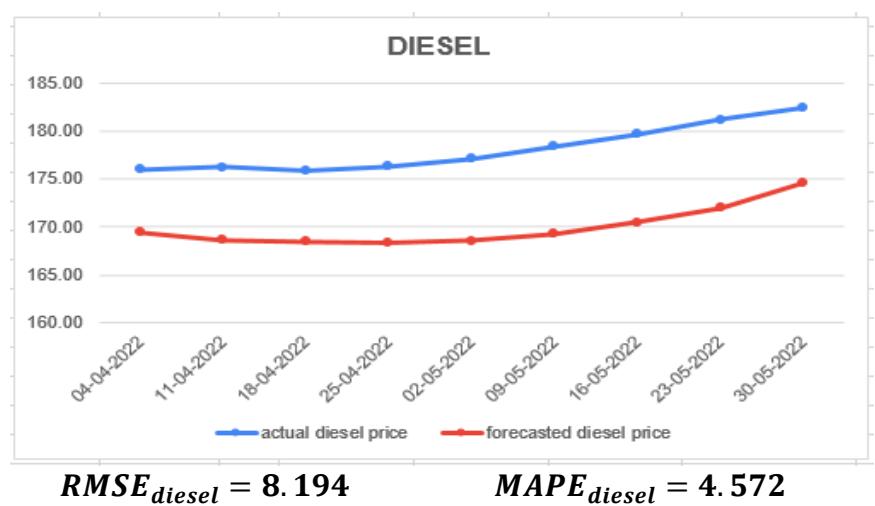
| date       | forecasted petrol price | actual petrol price | forecasted diesel price | actual diesel price |
|------------|-------------------------|---------------------|-------------------------|---------------------|
| 04-04-2022 | 162.74                  | 161.91              | 169.41                  | 176.00              |
| 11-04-2022 | 162.00                  | 161.78              | 168.64                  | 176.22              |
| 18-04-2022 | 161.87                  | 161.67              | 168.51                  | 175.93              |
| 25-04-2022 | 161.77                  | 161.84              | 168.39                  | 176.33              |
| 02-05-2022 | 161.94                  | 162.48              | 168.57                  | 177.06              |
| 09-05-2022 | 162.58                  | 163.68              | 169.24                  | 178.39              |
| 16-05-2022 | 163.78                  | 165.09              | 170.49                  | 179.67              |
| 23-05-2022 | 165.19                  | 167.59              | 171.96                  | 181.16              |
| 30-05-2022 | 167.69                  | 170.44              | 174.57                  | 182.37              |

---all prices are in same unit (pence/liter)



As we see, the forecasted prices for petrol match quite well with actual prices. MAPE value tells us that on average the error magnitude in forecasting is only 0.63% of the actual price, and from RMSE we say average distance of forecasted and observed values is 1.39unit.

But as we see, this is not so good for diesel price forecasting. MAPE value tells us that on average the error magnitude in forecasting is 4.57% of the actual diesel price, and from RMSE we say average distance of forecasted and observed values is 8.19unit.



We can also make projection of the average oil price for next few weeks....

| <i>date</i> | <i>forecasted petrol price</i> | <i>forecasted diesel price</i> |
|-------------|--------------------------------|--------------------------------|
| 6june,2022  | 170.54                         | 177.54                         |
| 13june,2022 | 170.64                         | 177.65                         |
| 20june,2022 | 170.73                         | 177.75                         |
| 27june,2022 | 170.83                         | 177.85                         |
| 4july,2022  | 170.93                         | 177.96                         |
| 11july,2022 | 171.03                         | 178.06                         |
| 18july,2022 | 171.13                         | 178.17                         |
| 25july,2022 | 171.23                         | 178.27                         |

---all prices are in same unit (pence/liter)

## CONCLUSION:

All we can say ,

- ⇒ In immediate future there is no tendency to reduce in fuel cost for UK peoples, as the time-plot is increasing in nature.
- ⇒ In absence of seasonality , we can not say in advance when price will increase or decrease significantly.
- ⇒ About the correctness of our model, it is obvious that any statistical techniques are based on the assumption that.....

*existing patterns will continue into the future*

----this assumption is more likely to be correct over the short term than it is over the long term, and for this reason these techniques that provide us with reasonably accurate forecasts for the immediate future, but the same may perform quite poorly further into the future (unless the data patterns are extraordinarily stable).

- ⇒ And lastly, we can't predict randomness that is why it is named so. We can't model it in advance, if the increasing curve faces a sudden dip with no systematic reason at all.

## REMARKS :-

- Note that for petrol price our forecasts are based on time series analysis. But for diesel instead of performing time series analysis once again, we noticed that there is a strong linear relationship between both price and choose the easy wayout of regressing upon the forecasted petrol price as forecasted diesel price of that same time. Though it saved our effort or time , some additional error introduced in this step. So, it is justified that forecast of diesel price would not be as good as that of petrol price.
- Here high correlation between petrol & diesel price is because they both are affected by some economic and other factors in a similar way, not due to any cause-effect relationship.

## ■ References:-

<https://www.gov.uk/government/statistics/weekly-road-fuel-prices>  
<https://www.ukogplc.com/page.php?pID=72>  
<https://otexts.com/fpp2/>  
<https://medium.com/vitrox-publication/what-is-a-time-series-forecasting-d020d657f11a>  
<https://www.statcan.gc.ca/en/dai/btd/tce-faq>  
<https://medium.com/Analytics-vidhya/time-series-decomposition-part-i-trend-cycle-computation-29fac227896a>

## ■ Books:-

*THE ANALYSIS OF TIME SERIES, AN INTRODUCTION* –Chris Chatfield  
*FUNDAMENTALS OF APPLIED STATISTICS* – S.C. Gupta & V.K. Kapoor

## ■ Softwares used:-

1. RStudio (R version 4.1.3 )
2. Microsoft Excel (2013)
3. Microsoft Word (2013)

## ■ Useful calculations:-

<https://docs.google.com/spreadsheets/d/1g18HLJNEJOJqcdXzaWP1NFyBFZaJBTcl/edit?usp=sharing&ouid=108980753175037445540&rtpof=true&sd=true>

## ■ Codes:-

<https://drive.google.com/file/d/1gSDbDmIv40ajwGHbfP1PJKMrRSDMETmg/view?usp=sharing>  
[https://drive.google.com/file/d/1Ahnmzd6Q3nxnwGDj16\\_3zR6ZDAmV2YQY/view?usp=sharing](https://drive.google.com/file/d/1Ahnmzd6Q3nxnwGDj16_3zR6ZDAmV2YQY/view?usp=sharing)  
<https://drive.google.com/file/d/19EiyuJn9uw0ixkik2FwXJ1haRxYB7t3t/view?usp=sharing>  
[https://drive.google.com/file/d/1\\_b5M3aCbJsH5\\_RDr6Y1AIXSiP1QAkHbu/view?usp=sharing](https://drive.google.com/file/d/1_b5M3aCbJsH5_RDr6Y1AIXSiP1QAkHbu/view?usp=sharing)  
<https://drive.google.com/file/d/1vXYsewhtMhvDz34B-ex0VIa-nE84eeiN/view?usp=sharing>

### **-:ACKNOWLEDGEMENT:-**

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### **-:DECLARATION:-**

**I, Ramit Nandi**, a student of B.Sc. Sem-VI, Statistics Honours, of University of Calcutta, Registration No. 012-1111-0558-19; Roll No. 193012-21-0381 hereby declare that I have done this piece of project work entitled as "Price of petrol & diesel in UK - a time-series analysis" under the supervision of **Prof. Oindrila Bose**, Department of Statistics, Asutosh College, as a part of B.Sc. Sem-VI examination according to the syllabus paper **DSE-B2**. I further declare that the piece of project has not been published elsewhere for any degree or diploma or taken from any published project.

# APPENDIX

## Data-sets:

| Date       | ULSP  | ULSD  | 08-03-2004 | 76.73 | 78.25 | 13-12-2004 | 82.73 | 85.98 | 19-09-2005 | 94.41 | 97.32 | 26-06-2006 | 94.97 | 97.36 |
|------------|-------|-------|------------|-------|-------|------------|-------|-------|------------|-------|-------|------------|-------|-------|
| 09-06-2003 | 74.59 | 76.77 | 15-03-2004 | 77.39 | 78.8  | 20-12-2004 | 81.46 | 85.17 | 26-09-2005 | 93.33 | 96.27 | 03-07-2006 | 95.27 | 97.6  |
| 16-06-2003 | 74.47 | 76.89 | 22-03-2004 | 77.44 | 78.92 | 27-12-2004 | 81.2  | 85.14 | 03-10-2005 | 93.51 | 96.61 | 10-07-2006 | 95.89 | 97.99 |
| 23-06-2003 | 74.42 | 76.82 | 29-03-2004 | 77.59 | 79.01 | 03-01-2005 | 81.17 | 85.12 | 10-10-2005 | 94.25 | 97.05 | 17-07-2006 | 96.88 | 98.74 |
| 30-06-2003 | 74.35 | 76.51 | 05-04-2004 | 77.86 | 79.23 | 10-01-2005 | 79.84 | 84.87 | 17-10-2005 | 94.14 | 97    | 24-07-2006 | 97.78 | 99.41 |
| 07-07-2003 | 74.28 | 76.46 | 12-04-2004 | 77.88 | 79.27 | 17-01-2005 | 78.93 | 83.94 | 24-10-2005 | 93.47 | 96.8  | 31-07-2006 | 97.83 | 99.41 |
| 14-07-2003 | 74.21 | 76.41 | 19-04-2004 | 78.04 | 79.43 | 24-01-2005 | 78.93 | 83.87 | 31-10-2005 | 92.79 | 96.57 | 07-08-2006 | 98.05 | 99.57 |
| 21-07-2003 | 75.07 | 78.9  | 26-04-2004 | 78.4  | 79.75 | 31-01-2005 | 79.54 | 84.23 | 07-11-2005 | 92.47 | 96.59 | 14-08-2006 | 97.82 | 99.45 |
| 28-07-2003 | 75.1  | 78.86 | 04-05-2004 | 79.28 | 80.84 | 07-02-2005 | 79.84 | 84.27 | 14-11-2005 | 90.53 | 94.92 | 21-08-2006 | 95.74 | 97.93 |
| 04-08-2003 | 75.12 | 78.81 | 10-05-2004 | 80.3  | 81.88 | 14-02-2005 | 80.03 | 84.32 | 21-11-2005 | 88.61 | 93.22 | 28-08-2006 | 94.4  | 96.67 |
| 11-08-2003 | 75.44 | 77.08 | 17-05-2004 | 81.49 | 82.83 | 21-02-2005 | 80.15 | 84.36 | 28-11-2005 | 87.35 | 91.8  | 04-09-2006 | 92.88 | 95.02 |
| 18-08-2003 | 75.81 | 77.44 | 24-05-2004 | 82.13 | 83.48 | 28-02-2005 | 80.45 | 84.65 | 05-12-2005 | 87.35 | 91.8  | 11-09-2006 | 91.48 | 95.34 |
| 25-08-2003 | 78.05 | 77.88 | 31-05-2004 | 82.35 | 83.81 | 07-03-2005 | 81.16 | 85.64 | 12-12-2005 | 87.2  | 91.48 | 18-09-2006 | 89.78 | 94.95 |
| 01-09-2003 | 78.13 | 77.59 | 07-06-2004 | 82.44 | 83.85 | 14-03-2005 | 81.84 | 86.11 | 19-12-2005 | 87.56 | 91.83 | 25-09-2006 | 88.2  | 93.51 |
| 08-09-2003 | 78.23 | 77.87 | 14-06-2004 | 81.54 | 82.8  | 21-03-2005 | 82.7  | 87.06 | 26-12-2005 | 87.88 | 92.12 | 02-10-2006 | 88.95 | 92.13 |
| 15-09-2003 | 78.2  | 77.83 | 21-06-2004 | 81.04 | 82.09 | 28-03-2005 | 83.4  | 87.88 | 02-01-2006 | 88.03 | 92.27 | 09-10-2006 | 86.6  | 91.74 |
| 22-09-2003 | 78.15 | 77.55 | 28-06-2004 | 80.41 | 81.04 | 04-04-2005 | 83.91 | 88.3  | 09-01-2006 | 88.56 | 92.94 | 16-10-2006 | 85.91 | 91.59 |
| 29-09-2003 | 78.08 | 77.53 | 05-07-2004 | 79.87 | 80.51 | 11-04-2005 | 85.61 | 89.82 | 16-01-2006 | 89.01 | 93.19 | 23-10-2006 | 85.74 | 91.22 |
| 06-10-2003 | 78.43 | 77.86 | 12-07-2004 | 80.34 | 81.05 | 18-04-2005 | 85.63 | 89.8  | 23-01-2006 | 89.55 | 93.62 | 30-10-2006 | 85.6  | 91.17 |
| 13-10-2003 | 75.9  | 77.43 | 19-07-2004 | 80.84 | 81.88 | 25-04-2005 | 85.51 | 89.62 | 30-01-2006 | 89.88 | 93.93 | 06-11-2006 | 85.54 | 91.12 |

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|            |       |       |            |       |       |            |       |       |            |       |       |            |       |       |
|------------|-------|-------|------------|-------|-------|------------|-------|-------|------------|-------|-------|------------|-------|-------|
| 20-10-2003 | 76.05 | 77.54 | 26-07-2004 | 80.87 | 81.7  | 02-05-2005 | 85.44 | 89.3  | 06-02-2006 | 89.73 | 93.85 | 13-11-2006 | 85.49 | 91.15 |
| 27-10-2003 | 78    | 77.51 | 02-08-2004 | 80.86 | 81.79 | 09-05-2005 | 85.36 | 89.46 | 13-02-2006 | 89.65 | 93.77 | 20-11-2006 | 85.45 | 91.17 |
| 03-11-2003 | 78    | 77.52 | 09-08-2004 | 81.18 | 82.14 | 16-05-2005 | 85.27 | 89.4  | 20-02-2006 | 89.6  | 93.75 | 27-11-2006 | 85.47 | 91.17 |
| 10-11-2003 | 75.94 | 77.47 | 16-08-2004 | 81.25 | 82.33 | 23-05-2005 | 84.77 | 89    | 27-02-2006 | 89.51 | 93.69 | 04-12-2006 | 85.46 | 91.5  |
| 17-11-2003 | 75.93 | 77.52 | 23-08-2004 | 81.51 | 82.82 | 30-05-2005 | 84.18 | 88.39 | 06-03-2006 | 89.37 | 93.63 | 11-12-2006 | 87.44 | 92.96 |
| 24-11-2003 | 76.02 | 77.67 | 31-08-2004 | 81.53 | 82.89 | 06-06-2005 | 84.01 | 88.23 | 13-03-2006 | 89.55 | 93.85 | 18-12-2006 | 87.7  | 93.26 |
| 01-12-2003 | 78    | 77.66 | 06-09-2004 | 81.37 | 82.92 | 13-06-2005 | 84.99 | 88.96 | 20-03-2006 | 90.19 | 94.42 | 25-12-2006 | 87.9  | 93.41 |
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| 26-01-2004 | 76.44 | 78.08 | 01-11-2004 | 84.27 | 86.42 | 08-08-2005 | 89.67 | 93.63 | 15-05-2006 | 96.39 | 98.74 | 19-02-2007 | 86.6  | 90.55 |
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| 16-02-2004 | 76.44 | 78    | 22-11-2004 | 81.46 | 85.17 | 29-08-2005 | 91.42 | 95.33 | 05-06-2006 | 95.24 | 97.55 | 12-03-2007 | 88.48 | 92.17 |
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| 20-08-2007 | 95.31 | 96.59 | 04-08-2008 | 115.06 | 128.19 | 20-07-2009 | 102.22 | 102.99 | 05-07-2010 | 117.91 | 120.44 | 20-06-2011 | 136.03 | 140.05 |
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| 17-09-2007 | 94.65 | 96.45 | 11-09-2008 | 112.2  | 123.89 | 17-08-2009 | 103.96 | 104.45 | 02-08-2010 | 116.51 | 119.03 | 17-08-2011 | 134.97 | 139.36 |

11 10-09-2008 13 17-08-2009 15 02-08-2010 17 18-07-2011 19

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| 07-01-2008 | 103.37 | 108.29 | 22-12-2008 | 88.01  | 99.94  | 07-12-2009 | 108.64 | 109.87 | 22-11-2010 | 118.99 | 122.74 | 07-11-2011 | 134.41 | 140.42 |
| 14-01-2008 | 103.82 | 108.77 | 29-12-2008 | 87.34  | 99.17  | 14-12-2009 | 108.5  | 109.57 | 29-11-2010 | 119.21 | 123.2  | 14-11-2011 | 133.68 | 140.53 |
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12 03-01-2009 14 08-02-2010 16 18-01-2011 18 24-01-2011 20

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| 13-01-2014 | 130.36 | 138.34 | 29-12-2014 | 113.16 | 120.36 | 14-12-2015 | 104    | 108.07 | 28-11-2016 | 113.72 | 116.57 | 13-11-2017 | 118.91 | 122.69 |
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| 03-03-2014 | 129.36 | 137.15 | 16-02-2015 | 107.26 | 114.73 | 01-02-2016 | 101.36 | 100.84 | 16-01-2017 | 118.63 | 122.1  | 01-01-2018 | 120.19 | 123.51 |
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| 24-03-2014 | 128.57 | 136    | 09-03-2015 | 110.07 | 117.51 | 22-02-2016 | 101.39 | 101.13 | 06-02-2017 | 120.18 | 123.25 | 22-01-2018 | 121.5  | 125.07 |
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| 18-08-2014 | 129.29 | 133.74 | 03-08-2015 | 116.12 | 115.17 | 18-07-2016 | 111.92 | 113    | 03-07-2017 | 113.74 | 115.83 | 18-06-2018 | 127.92 | 131.98 |
| 25-08-2014 | 128.34 | 133.16 | 10-08-2015 | 115.48 | 114.2  | 25-07-2016 | 111.69 | 112.89 | 10-07-2017 | 113.58 | 115.29 | 25-06-2018 | 127.18 | 131.27 |
| 01-09-2014 | 128.01 | 132.77 | 17-08-2015 | 114.58 | 112.05 | 01-08-2016 | 111.05 | 112.43 | 17-07-2017 | 113.89 | 115.58 | 02-07-2018 | 126.54 | 130.83 |
| 08-09-2014 | 128.34 | 133.09 | 24-08-2015 | 113.23 | 111.05 | 08-08-2016 | 109.75 | 111.4  | 24-07-2017 | 113.99 | 115.73 | 09-07-2018 | 126.97 | 131.53 |
| 15-09-2014 | 128.43 | 133.13 | 31-08-2015 | 111.21 | 109.92 | 15-08-2016 | 109.28 | 111.02 | 31-07-2017 | 114.16 | 115.99 | 16-07-2018 | 127.47 | 131.85 |
| 22-09-2014 | 128.62 | 133.32 | 07-09-2015 | 110.9  | 109.78 | 22-08-2016 | 109.63 | 111.71 | 07-08-2017 | 114.78 | 116.64 | 23-07-2018 | 127.5  | 131.9  |
| 29-09-2014 | 128.59 | 133.23 | 14-09-2015 | 110.6  | 109.95 | 29-08-2016 | 110.43 | 112.55 | 14-08-2017 | 115.53 | 117.43 | 30-07-2018 | 127.54 | 131.94 |
| 06-10-2014 | 126.78 | 131.43 | 21-09-2015 | 110.61 | 110.52 | 05-05-2016 | 110.97 | 113.1  | 21-08-2017 | 116.04 | 118.05 | 06-08-2018 | 128.02 | 132.39 |
| 13-10-2014 | 126.5  | 131.08 | 28-09-2015 | 109.47 | 110.45 | 12-05-2016 | 111.31 | 113.42 | 28-08-2017 | 116.4  | 118.24 | 13-08-2018 | 128.39 | 132.56 |
| 20-10-2014 | 125.4  | 130    | 05-10-2015 | 108.94 | 110.57 | 15-05-2016 | 111.29 | 113.23 | 04-05-2017 | 116.95 | 118.77 | 20-08-2018 | 128.79 | 132.97 |
| 27-10-2014 | 125.11 | 129.72 | 12-10-2015 | 108.97 | 110.78 | 26-05-2016 | 111.36 | 113.38 | 11-05-2017 | 118.14 | 119.94 | 27-08-2018 | 129.18 | 133.2  |
| 03-11-2014 | 123.94 | 128.56 | 19-10-2015 | 109.01 | 111.17 | 03-10-2016 | 111.65 | 113.71 | 18-05-2017 | 118.85 | 120.59 | 03-05-2018 | 129.76 | 133.67 |
| 10-11-2014 | 122.94 | 127.59 | 26-10-2015 | 108.04 | 110.61 | 10-10-2016 | 112.35 | 114.4  | 25-05-2017 | 118.87 | 120.92 | 10-05-2018 | 130.24 | 134.31 |
| 17-11-2014 | 122.5  | 127.31 | 02-11-2015 | 107.21 | 110.14 | 17-10-2016 | 113.72 | 116    | 02-10-2017 | 118.13 | 120.37 | 17-05-2018 | 130.59 | 134.57 |
| 24-11-2014 | 122.29 | 127.17 | 09-11-2015 | 107.14 | 110.08 | 24-10-2016 | 115.2  | 117.7  | 09-10-2017 | 117.16 | 120.51 | 24-05-2018 | 130.59 | 134.77 |
| 27         | 29     | 31     | 33         | 35     |        |            |        |        |            |        |        |            |        |        |
| 01-10-2018 | 129.98 | 134.86 | 18-02-2019 | 119.05 | 129.23 | 08-07-2019 | 126.86 | 131.68 | 25-11-2019 | 125.32 | 130.08 | 13-04-2020 | 109.27 | 116.15 |
| 08-10-2018 | 130.15 | 135.53 | 25-02-2019 | 119.22 | 129.66 | 15-07-2019 | 127.13 | 131.86 | 02-12-2019 | 124.81 | 129.79 | 20-04-2020 | 108.63 | 115.73 |
| 15-10-2018 | 130.81 | 136.63 | 04-03-2019 | 119.72 | 130.25 | 22-07-2019 | 127.81 | 132.21 | 09-12-2019 | 124.75 | 129.79 | 27-04-2020 | 107.88 | 115.22 |
| 22-10-2018 | 130.98 | 136.93 | 11-03-2019 | 120.1  | 130.59 | 29-07-2019 | 128.03 | 132.6  | 16-12-2019 | 124.33 | 129.56 | 04-05-2020 | 107.56 | 114.94 |
| 29-10-2018 | 130.64 | 136.99 | 18-03-2019 | 120.48 | 130.85 | 05-08-2019 | 128.37 | 132.61 | 23-12-2019 | 124.16 | 129.81 | 11-05-2020 | 107.45 | 114.83 |
| 05-11-2018 | 130.11 | 137.03 | 25-03-2019 | 120.83 | 131.15 | 12-08-2019 | 128.36 | 132.59 | 30-12-2019 | 124.96 | 130.54 | 18-05-2020 | 105.09 | 112.22 |
| 12-11-2018 | 128.94 | 137.08 | 01-04-2019 | 121.7  | 131.48 | 19-08-2019 | 128.17 | 13     |            |        |        |            |        |        |

|            |        |        |            |        |        |            |        |        |            |        |        |
|------------|--------|--------|------------|--------|--------|------------|--------|--------|------------|--------|--------|
| 31-08-2020 | 113.29 | 118.18 | 14-12-2020 | 113.82 | 118.57 | 29-03-2021 | 125.13 | 129.32 | 12-07-2021 | 132.47 | 135.29 |
| 07-09-2020 | 113.37 | 118.22 | 21-12-2020 | 114.43 | 119.17 | 05-04-2021 | 125.24 | 129.35 | 19-07-2021 | 133.2  | 135.95 |
| 14-09-2020 | 113.32 | 118.18 | 28-12-2020 | 114.91 | 119.65 | 12-04-2021 | 125.4  | 129.39 | 26-07-2021 | 133.48 | 136.2  |
| 21-09-2020 | 113.31 | 118.16 | 04-01-2021 | 115.39 | 119.97 | 19-04-2021 | 125.48 | 129.49 | 02-08-2021 | 134.21 | 136.89 |
| 28-09-2020 | 113.3  | 118.14 | 11-01-2021 | 116.14 | 120.61 | 26-04-2021 | 125.8  | 129.77 | 09-08-2021 | 134.7  | 137.31 |
| 05-10-2020 | 113.26 | 118.11 | 18-01-2021 | 116.93 | 121.52 | 03-05-2021 | 126.09 | 130    | 16-08-2021 | 134.77 | 136.77 |
| 12-10-2020 | 113.19 | 118.05 | 25-01-2021 | 118.1  | 122.7  | 10-05-2021 | 126.53 | 130.43 | 23-08-2021 | 134.68 | 137.15 |
| 19-10-2020 | 113.18 | 118.08 | 01-02-2021 | 119.14 | 123.7  | 17-05-2021 | 127.19 | 131.03 | 30-08-2021 | 134.66 | 137.11 |
| 26-10-2020 | 113.14 | 118.08 | 08-02-2021 | 119.67 | 124.09 | 24-05-2021 | 127.89 | 131.52 | 06-09-2021 | 134.76 | 137.19 |
| 02-11-2020 | 113.11 | 118.06 | 15-02-2021 | 120.53 | 125.01 | 31-05-2021 | 128.15 | 131.82 | 13-09-2021 | 134.75 | 137.19 |
| 09-11-2020 | 112.5  | 117.28 | 22-02-2021 | 121.27 | 125.71 | 07-06-2021 | 128.69 | 132.38 | 20-09-2021 | 134.86 | 137.35 |
| 16-11-2020 | 112.35 | 117.07 | 01-03-2021 | 122.17 | 126.62 | 14-06-2021 | 129.47 | 133.17 | 27-09-2021 | 135.19 | 137.95 |
| 23-11-2020 | 112.42 | 117.13 | 08-03-2021 | 122.94 | 127.4  | 21-06-2021 | 130.1  | 133.69 | 04-10-2021 | 136.1  | 139.2  |
| 30-11-2020 | 112.61 | 117.41 | 15-03-2021 | 123.9  | 128.24 | 28-06-2021 | 130.73 | 134.27 | 11-10-2021 | 137.17 | 140.66 |
| 07-12-2020 | 113.17 | 117.99 | 22-03-2021 | 124.57 | 129.02 | 05-07-2021 | 131.7  | 134.13 | 18-10-2021 | 139.46 | 143.19 |

41 42 43 44

|            |        |        |            |        |        |
|------------|--------|--------|------------|--------|--------|
| 25-10-2021 | 141.81 | 145.9  | 10-01-2022 | 144.82 | 148.65 |
| 01-11-2021 | 143.7  | 147.48 | 17-01-2022 | 144.8  | 148.7  |
| 08-11-2021 | 144.9  | 148.84 | 24-01-2022 | 144.87 | 148.81 |
| 15-11-2021 | 145.87 | 149.76 | 31-01-2022 | 145.74 | 149.68 |
| 22-11-2021 | 146.89 | 150.73 | 07-02-2022 | 146.33 | 150.3  |
| 29-11-2021 | 147.53 | 151.31 | 14-02-2022 | 146.95 | 151.1  |
| 06-12-2021 | 146.89 | 150.61 | 21-02-2022 | 147.77 | 151.95 |
| 13-12-2021 | 146.22 | 149.88 | 28-02-2022 | 149.22 | 153.36 |
| 20-12-2021 | 145.16 | 148.89 | 07-03-2022 | 152.95 | 158.56 |
| 27-12-2021 | 144.92 | 148.82 | 14-03-2022 | 159.96 | 169.48 |
| 03-01-2022 | 145.04 | 148.85 | 21-03-2022 | 165.37 | 177.47 |
|            | ....   | ....   | 28-03-2022 | 162.65 | 176.44 |

45 46

## descriptive.R

```
library("readxl")
setwd("F:\\PROJECT WORK _OB") #change directory as per convenience
pricedata=data.frame(read_excel("fuel prices time series.xlsx"))

summary(pricedata[,-1])
boxplot(pricedata[,-1],names=c("petrol","diesel"),ylab="p./liter",main="box-plot for petrol and diesel price data")
n=length(pricedata[[1]]) #no. of obs
t=0:(n-1)
p=pricedata[[2]] #petrol price
d=pricedata[[3]] #diesel price
var(p)
var(d)
cor(p,d)

plot(t,p,type="l",col="blue",xlab="weeks",ylab="p./liter",main="petrol price over time")
lines(t,d,col="red")
mtext("time: origin= 9june,2003 ",side=1,line=-3,adj=.8,col="darkgrey",cex=.8)
```

```

mtext("unit=1week ", side=1, line=-2, adj=.8, col="darkgrey", cex=.8)
legend("topleft", c("petrol", "diesel"), col=c("blue", "red"), lwd=1)

trend.R

modelp1=lm(p~t) #linear trend
summary(modelp1)
modelp2=lm(p~poly(t, 2, raw=T)) #quadratic trend
summary(modelp2)
modelp3=lm(p~poly(t, 3, raw=T)) #cubic trend
summary(modelp3)
modelp4=lm(p~poly(t, 4, raw=T)) #4th degree polynomial trend
summary(modelp4)

plot(t,p,type="l",xlab="weeks",ylab="p./liter",main="linear trend line for
petrol price")
abline(modelp1,col="blue")
mtext(paste("p= ",round(coef(modelp1)[1],3),"",
+round(coef(modelp1)[2],3),"t"),side=1,line=-2,adj=.2,col="darkgrey",cex=.95)

plot(t,p,type="l",xlab="weeks",ylab="p./liter",main="various trend-cycle
line for petrol price")
lines(t,fitted(modelp2),col="green")
lines(t,fitted(modelp3),col="blue",lwd=2)
lines(t,fitted(modelp4),col="red")
legend("topleft", c("quadratic", "cubic", "4th
degree"), col=c("green", "blue", "red"), lwd=1)

plot(t,residuals(modelp3),type="h",xlab="week",ylab="residuals",
      main="residual plot of cubic trend",col="darkgreen")
plot(fitted(modelp3),abs(residuals(modelp3)),type="h",xlab="fitted",ylab="|residual|",
      main="fitted vs residual",col="darkgreen")

```

## **residual.R**

```

z=residuals(modelp3)
plot(t,z,type='h',col="darkgreen",xlab="week",ylab="|residual|",main="resid
ual plot")
plot(fitted(modelp3),abs(z),type='h',col="darkred",xlab="fitted",ylab="|res
idual|",main="residual plot")

plot(t[-n],diff(z)-
>dz,type='l',lwd=.8,col="darkgrey",,xlab="week",ylab="del(residual)",main="first
difference of residuals over time")
abline(h=mean(dz),col="red")
mtext("mean line",side=4,line=-1,cex=.85,col="red")

plot(t[-n],((dz-
mean(dz))),type='h',col="darkred",,xlab="week",ylab="del(residual)-
bias",main="adjusted first diff. of residuals over time")

```

```

plot(t[-n],abs((dz-
mean(dz))),type='h',col="darkred",,xlab="week",ylab="|del(residual)-
bias|",main="adjusted first diff. of residuals over time")

estm_z= (z+mean(dz))[-n] # estimated z1 is based on z0 ... ,zn is based on
zn-1

```

## PvsD.R

```

plot(p,d,col="grey",xlab="petrol",ylab="diesel",main="petrol vs diesel
price(p./liter) for fixed time ")
abline(lm(d~p)->donp,col="red",lwd=2)
mtext(paste("d= ",round(coef(lm(d~p))[1],3)," +
",round(coef(lm(d~p))[2],3),"p"),side=1,line=-2,adj=.2,col="darkred",cex=.8)

```

## comparison.R

```

estm_p= (fitted(modelp3))[-1] +estm_z
estm_d= coefficients(donp)[1] + estm_p*coefficients(donp)[2]

plot(t,p,main="PETROL PRICE",xlab="week",ylab="p./liter",
      col="red",type="l",lwd=3)
points(t[-1],estm_p,col="blue",lwd=.05)
legend("topleft","actual price",col="red",lwd=1.5)
legend("bottomright","fitted value",col="blue",pch=1)
summary(p[-1]-estm_p)
var(p[-1]-estm_p)
100*mean(abs(p[-1]-estm_p)/p[-1])

summary(d[-1]-estm_d)
var(d[-1]-estm_d)
100*mean(abs(d[-1]-estm_d)/d[-1])
plot(t,d,main="DIESEL PRICE",xlab="week",ylab="p./liter",
      col="blue",type="l",lwd=3)
points(t[-1],estm_d,col="red",lwd=.05)
legend("topleft","actual price",col="blue",lwd=1.5)
legend("bottomright","fitted value",col="red",pch=1)

```