CS 240 Algorithm Design and Analysis (Spring 2021)
Midterm Exam

Instructions

- Time: 10:15-11:55am (100 minutes)
- This exam is closed-book, but you may bring an A4-size cheat sheet. Put all the study materials and electronic devices into your bag and put your bag in the front, back, or sides of the classroom.
- You can write your answers in either English or Chinese.
- Two blank pieces of paper are attached on the back, which you can use as scratch paper. Raise your hand if you need more paper.

1 (10 pt)

Each question has one or more correct answer(s). Select all the correct answer(s). For each question, you get 0 point if you select one or more wrong answers, but you get 1 point if you select a non-empty proper subset of the correct answers.

Please enter your answers in the following table.

1	2	3	4	5

1. Which of the following is/are correct?

   A. 100^{100} is $O(\log n)$

   B. $n \log n$ is $O(n^2)$

   C. $n^{\sqrt{n}}$ is $O((\log n)^n))$

   D. n^{100} is $O(1.01^n)$

   E. None of the above

2. Suppose $T(n) = 10n^3 + 8n^2 \log_2 n + 2^{\frac{5}{2}\log_2 n} + 2n \times (\frac{9}{2})^{1+\log_2 n}$. Which of the following is/are correct?

   A. $T(n)$ is $O(n^3)$

   B. $T(n)$ is $O(n^4)$

   C. $T(n)$ is $\Omega(n^3)$

   D. $T(n)$ is $\Omega(n^2)$

   E. $T(n)$ is $\Theta(n^3)$

3. Let X and Y be two decision problems. Suppose we know that X $\leq_p$ Y. Which of the following can we infer?

A. If Y is NP-complete, then so is X.

B. If Y is NP-complete and X is in NP, then X is NP-complete.

C. If X is NP-complete and Y is in NP, then Y is NP-complete.

D. X and Y can't both be NP-complete.

E. If Y is in P, then X is in P.

4. Which of the following is/are known to be correct?

   A. 2-coloring $\leq_P$ 3-coloring

   B. 3-coloring $\leq_P$ 2-coloring

   C. 3-coloring $\leq_P$ 4-coloring

   D. 4-coloring $\leq_P$ 3-coloring

   E. None of above

5. Which of the following is/are known to be correct?

   A. If P=NP, then 3-SAT $\in$ Co-NP

   B. If X is NP-complete, then X cannot be solved in polynomial time

   C. The brute-force algorithm for solving TSP is NP-complete

   D. If $X \in$ NP, then $\bar{X} \in$Co-NP, where $\bar{X}$ is the complement of X

   E. If $X \in$ EXP, then X does not have a poly-time certifier

2 (10 pt)

Assume you want to distribute n projects to m employees (one project can only be completed by one employee, and one employee can accomplish at most one project). However, each project has a certain degree of difficulty, and each employee has a certain degree of ability. Every employee can only complete a project that is no more difficult than his/her ability. For example, if the employees' abilities are $\{3, 2, 4\}$ and the project difficulties are $\{3, 5\}$, then only one project can be completed. Your goal is to complete as many projects as possible.

Please give an efficient algorithm and prove your algorithm is correct. Show the time complexity of your algorithm.

3 (10 pt)

We are given a color picture consisting of an $m \times n$ array A of pixels, where each pixel specifies a triple of red, green, and blue (RGB) intensities. Suppose that we wish to compress this picture slightly. Specifically, we wish to remove one pixel from each of the m rows, so that the whole picture becomes one pixel narrower. To avoid disturbing visual effects, however, we require that the pixels removed in two adjacent rows be in the same or adjacent columns. A "seam" is defined as the set of removed pixels.

(a) Show that the number of such possible seams grows at least exponentially in m, assuming that $n > 1$.

(b) Suppose that for each pixel $A[i, j]$, we have calculated a real-valued disruption measure $d[i, j]$, indicating how disruptive it would be to remove pixel $A[i, j]$. Intuitively, the lower a pixels disruption measure is, the more similar the pixel is to its neighbors. Suppose further that we define the disruption measure of a seam to be the sum of the disruption measures of its pixels. Find the best seam, i.e., the seam with the smallest disruption measure.

4 (10 pt)

Suppose you are the manager of a clothing shop with j different jackets and t different shirts. One day, c customers come to the shop. Each of them has his own (possibly multiple) favorite jackets and shirts but only wants to buy one jacket and one shirt of his favorite. We say that a deal is "successful" if you sell to a customer one jacket and one shirt and both of them are his favorite. Note that one jacket or shirt can only be sold to at most one customer. Design an efficient algorithm that maximizes the number of successful deals. (Hint: design a flow network with jacket, shirt and customer nodes.)

5 (10 pt)

5.1 Consider the following statement: if a problem in **NP** can be solved in polynomial time, then it is known that all problems in NP can be solved in polynomial time. Is this statement correct? Justify your answer.

5.2 Recall that the subset-sum problem is to decide whether any subset of S can sum up to a target positive integer t, where S is a given set of positive integers. Show that the following problem is NP-Complete by reduction from subset-sum.

MAX-SATURATED-FLOW: Consider a modified network flow problem in which each edge must either have zero flow or be completely saturated (i.e., the flow is equal to the capacity of the edge). A flow is called a saturated flow if it satisfies this additional constraint, as well as all the usual constraints of the max-flow problem. The problem is whether there is a saturated flow of value $\geq k$, where k is a given number.