Deep Learning - 236606 - HW2

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1. **DNN architecture:**

   See mydnn.py for the DNN class as well as all relevant routines to run the class. For each experiment see relevant .py file which uses the above class.

2. **Experiments**

   a. **Batch size:**

   In this set of experiments, we used a network consisting of a single hidden layer with 128 neurons and using ReLU and softmax activations in aim to reveal the relationship between the batch size (i.e. how many samples are being calculated before each backpropagation step) to the learning performance.

   We ran the network with batch sizes 128, 1024, 10K and 20K and for each one accounted for the total runtime and overall accuracy and loss on the test set.

   In these sets of experiments, we calculated the validation set accuracy and loss **for every iteration** as ascribed in the assignment hence run-times are extremely longer.

   The other parameters aside from batch size were kept the same (no regularization was used at this stage) and learning rate was left the same to see the effects properly. That being said, the epoch number changed as so to allow convergence (but not overkill run-time). Below are the accuracy and loss plot for the test and validation sets for the different batch sizes

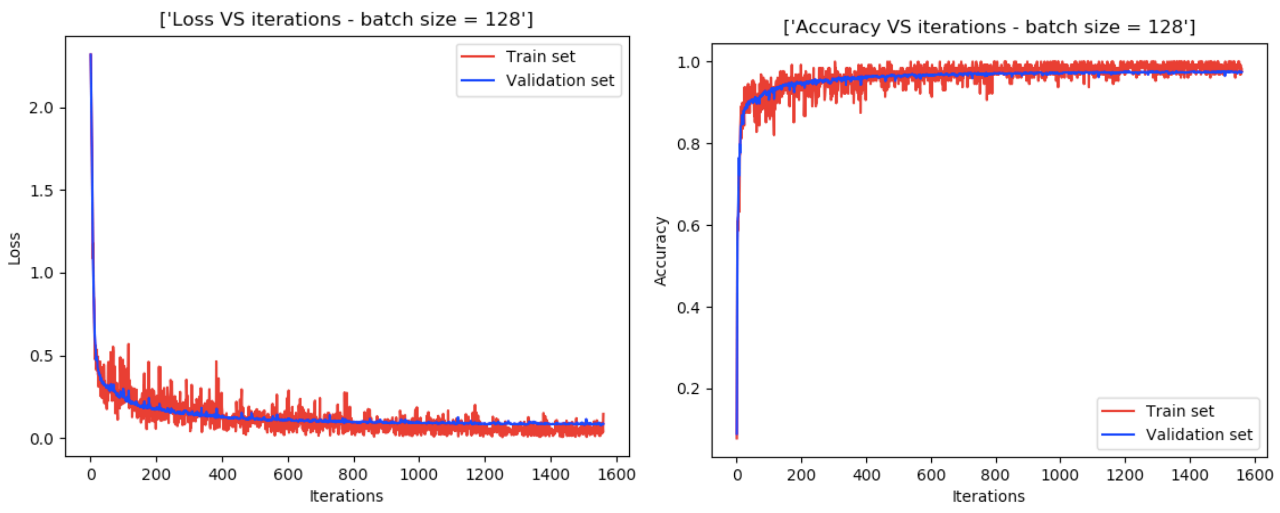


Figure 1 - Loss and accuracy VS iterations for 128 batch size

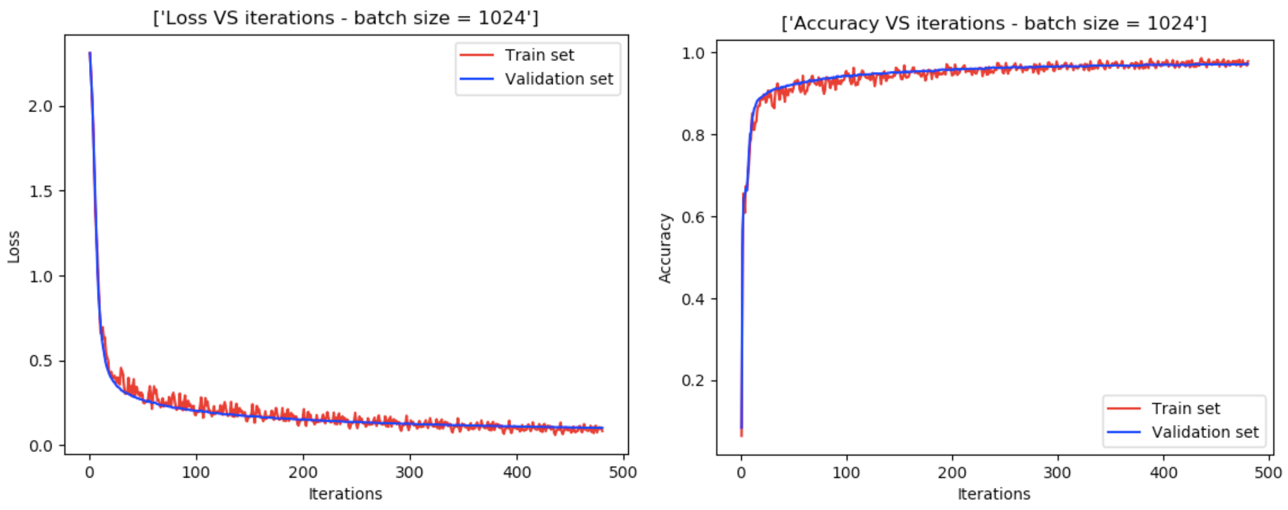


Figure 2 - Loss and accuracyt for the 10K batch size

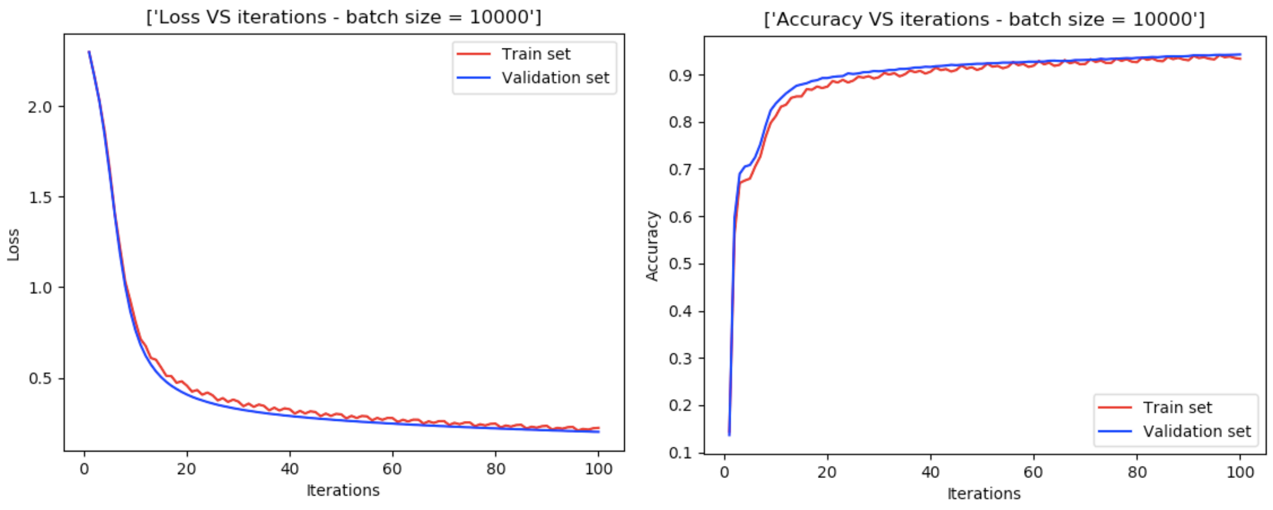


Figure 3 - Loss and accuracy for 10K samples batch

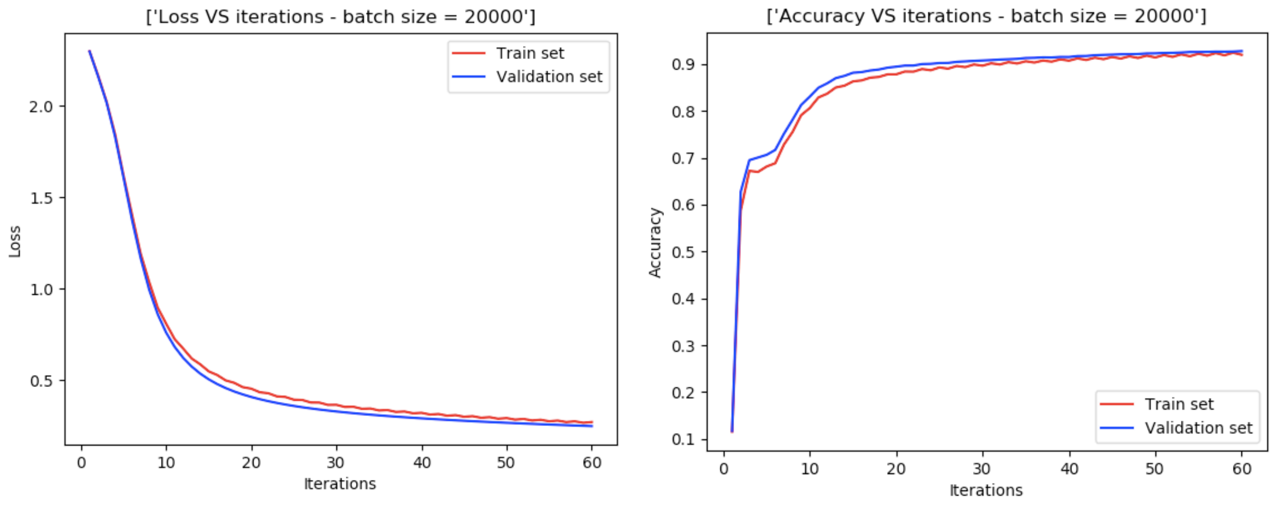


Figure 4 - Loss and accuracy to 20K batch size

Below is a table concluding the results from the different trials:

Batch size	epochs	Total iterations	Run-time	Loss train set	Accuracy train set	Loss test set	Accuracy test set
128	4	1560	1280 sec	0.0840874	0.9754	0.0831195	0.9747
1024	10	480	400 sec	0.0829664	0.978515	0.1041465	0.9691
10K	30	150	270 sec	0.2006139	0.9425	0.1775361	0.9482
20K	30	60	270 sec	0.2700259	0.9195	0.2555614	0.9261

First, we can observe the obvious that the more iterations of back propagation (i.e. using smaller batch sizes) results in longer run-time yet increased accuracy. That said, we also saw that when using larger batch sizes, we are consequently decreasing the learning rate hence preferably a greater one should be used when using large batches (as stated before, we kept it constant in the results for the sake of uniformity).

b. Regularization:

In these set of tests, we used the same architecture as in section 2.a (batch size of 1024 samples for 10 epochs) only now we are looking for the regularization effects on generalization when using different norms (L1, L2 and none). As part of that, we also ran a brief optimization on the weight decay parameter for each norm and used it when comparing the results.

Below are the accuracy and loss VS weight decay size for the three options:

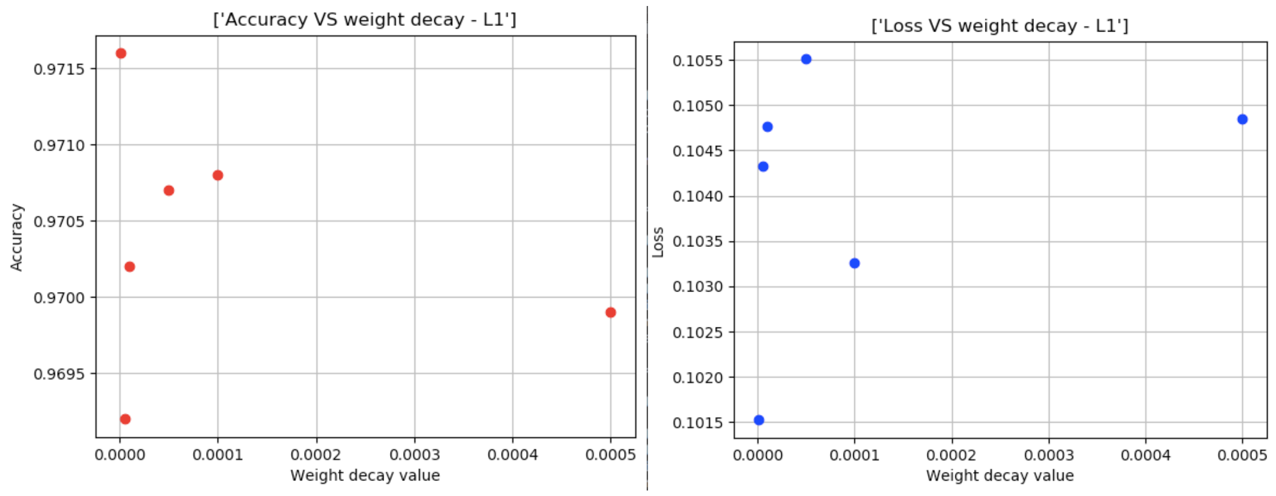


Figure 5 - Loss and accuracy VS weight decay under L1 norm

The weight decay with the best accuracy is 5e-07, giving a loss of 0.101523 and accuracy of 0.9716 (on the test set)

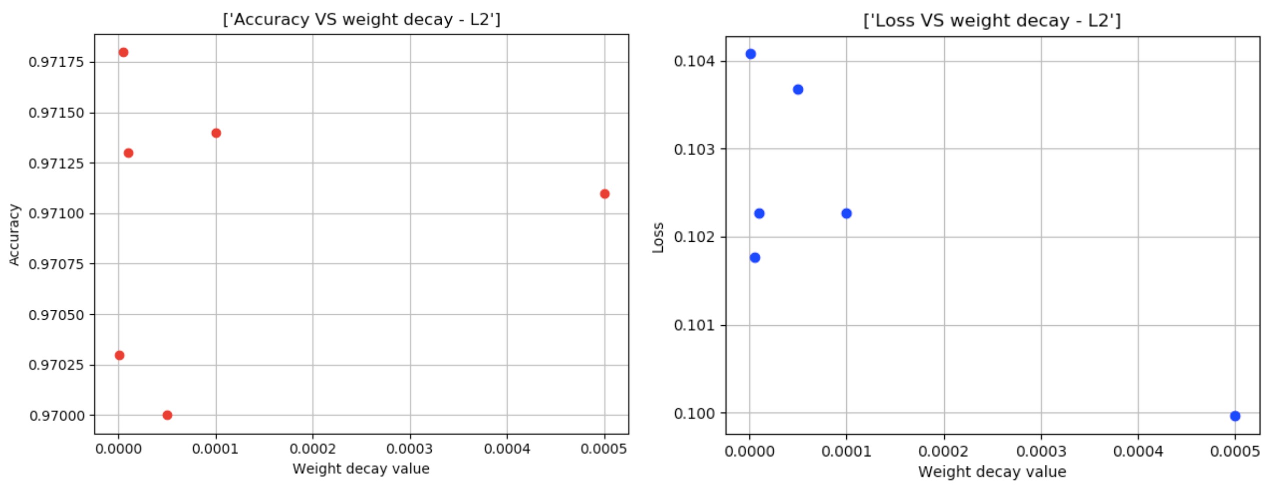


Figure 6 - Loss and accuracy VS weight decay under L2 norm

The weight decay with the max accuracy is 5e-06 giving a loss of 0.10176 and accuracy of 0. 0.9718 (again on the test set)

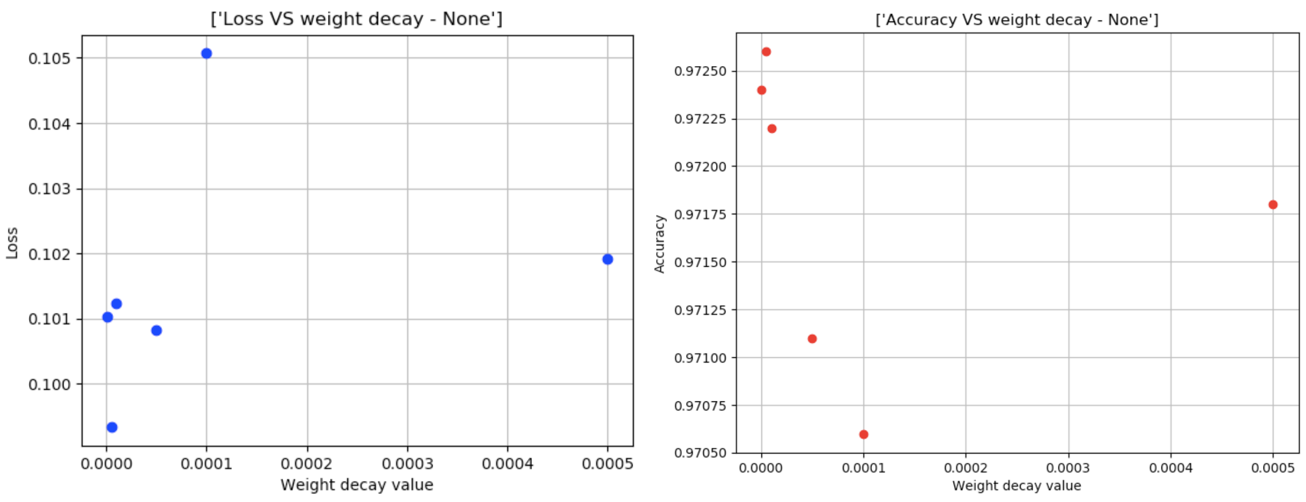


Figure 7 - Loss and accuracy VS weight decay without regularization

The weight decay with the max accuracy = 0.0001, where the loss is 0.1016795774770573, and accuracy = 0.9717

After optimizing the weight decay parameter, we now train the net with each regularization norm in accordance to the results we got (**changing only the norm type**) and specifically observing the regularization effects, hence the generalization error (over-fitting)

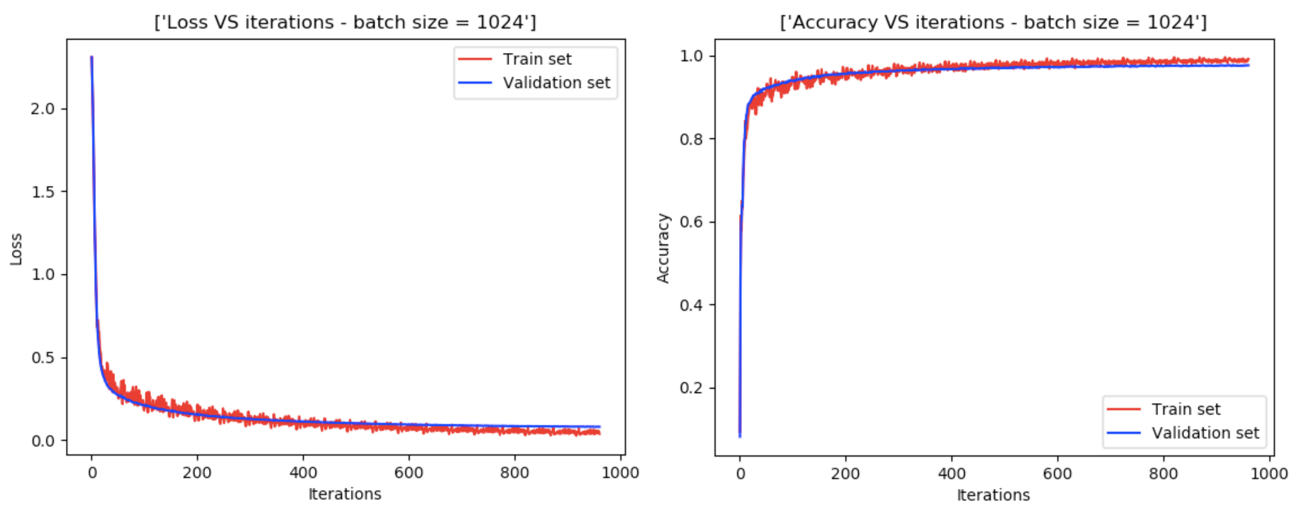


Figure 8 - Loss and accuracy plots for batch size 1024 with L1 regularization

And zooming in we can see the generalization error is starting to grow as we run more iterations:

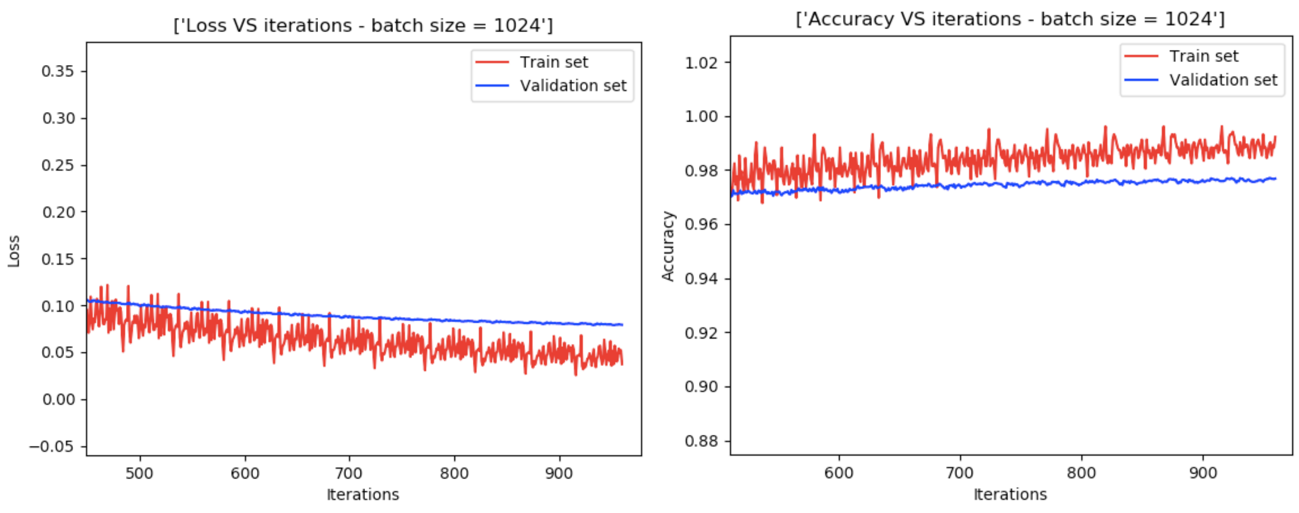


Figure 9 - Zooming in on figure 8 we can see the generalization error starting to develop (while the train set error is decreasing the validation set isn't effected – hence we are starting to overfit)

And running it over the test set we obtain: Loss = 0.08187, accuracy= 0.9749

For L2 regularization

Looking at the zoom in of the accuracy and loss curve we can see the generalization error development:

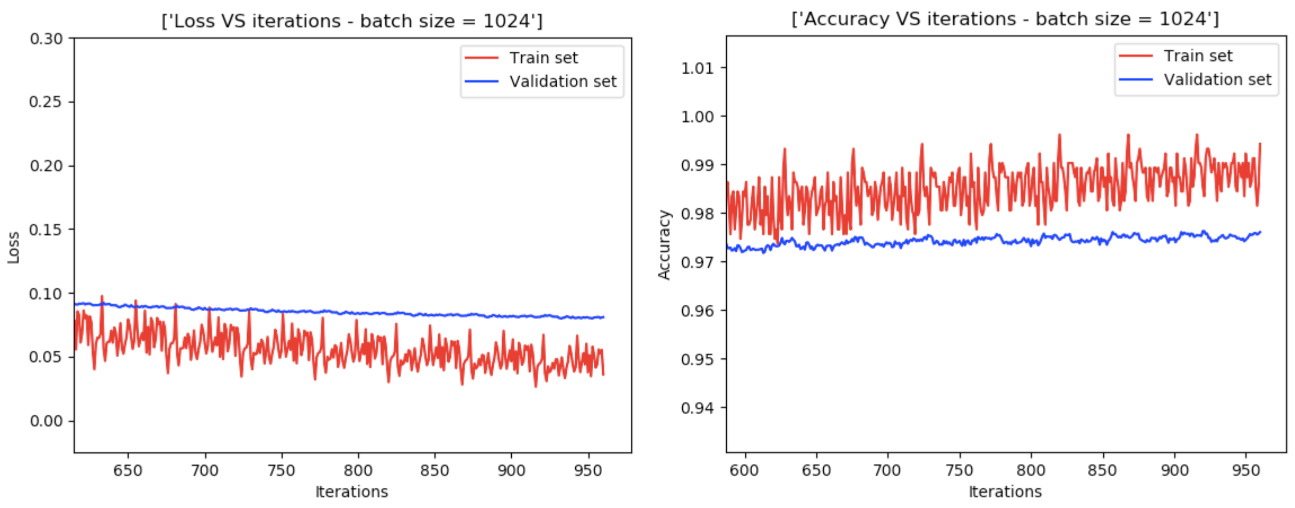


Figure 10 - Accuracy and loss curves for L2 regularization

And running it over the test set we obtain: Loss =0.081762, accuracy= 0.9737 – slightly better than the L1 regularization

Without regularization

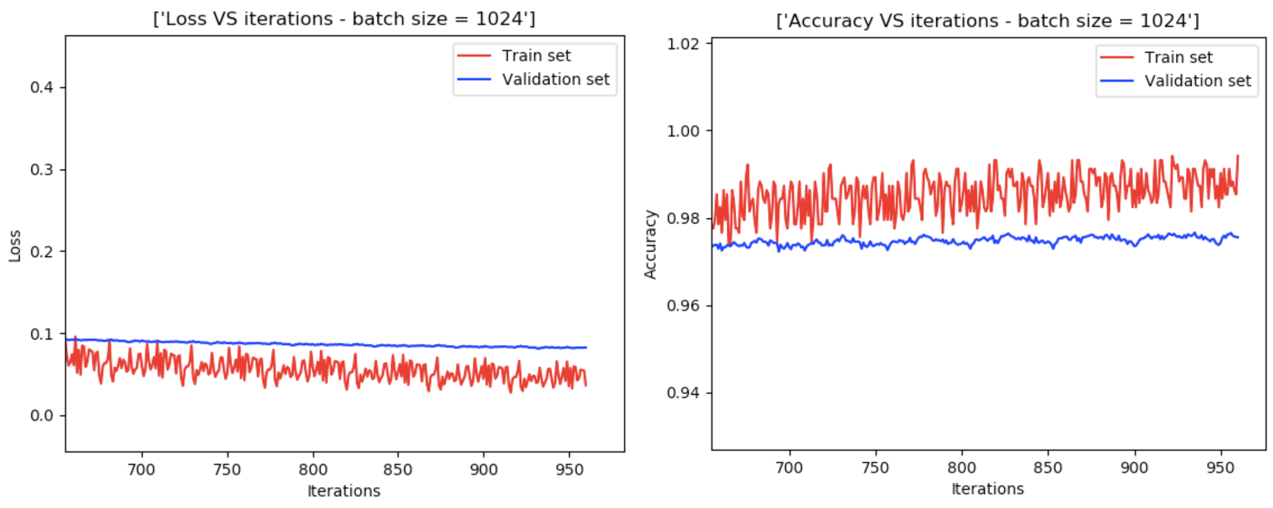


Figure 11 - Loss and accuracy without using regularization

Running it over the test set we obtain: Loss = 0.08237 and accuracy= 0.9741

Hence the best accuracy on the test set was using L2 regularization – that said, we also could have tried running for more training cycles and according to theory the generalization error would have increased when not using generalization (or using a weaker one).

c. **Architecture**:

In the following sets of experiments, we ran different configurations of architectures ranging from depth 2-3 and width up to 512 neurons aiming to investigate the affects on the accuracy of the test and train sets. We chose different configurations as to test both extremes of the overall weights as well as various configurations which add up to the same overall number of weights (give or take) only with different architectures (i.e. varying each layer opposite to the other).

i. **Configurations table and results:**

#	Depth	Width	Overall weights	Runtime	Loss train set	Accuracy train set	Loss test set	Accuracy test set
1	2	64	50,816	22.428	0.048452	0.9892	0.0856	0.9729
2	2	128	101,632	36.619	0.03774	0.99218	0.07867	0.9766
3	2	256	203,264	68.513	0.032775	0.991210	0.07744	0.9752
4	2	512	406,528	126.412	0.03132	0.99414	0.071216	0.9782
5	3	[64, 64]	54,912	27.664	0.06569	0.98535	0.097230	0.9697
6	3	[512, 512]	1,342,578,688	229.146	0.07086	0.981445	0.099679	0.9706
7	3	[128, 64]	109,184	43.134	0.053017	0.98730	0.092121	0.9707
8	3	[80, 512]	108,800	60.797	0.09717	0.96679	0.1155	0.9637
9	3	100, 200	100,400	48.432	0.051773	0.9882	0.09499	0.9702
10	3	512,80	443,168	136.785	0.045381	0.9892	0.083578	0.9749

ii. **Conclusions**

1. There seems to be a minimal number of weights needed to obtain better performance, which is logical under the input dimension (784) while using too much can also decrease accuracy, possibly because the more weights you use the more learning samples and iterations you need (as so they all converge and not to any local minimal).
2. As for generalization (or over-fitting), it seems using one layer we obtain greater generalization errors - possibly due to the regularization effects or its lack of expressiveness.

3. Balancing run-time and performance, a single hidden layer, with 256 weights seemed to perform well while also consume less time and in that sense, might be a good option to keep optimizing other parameters on.

d. Regression

In this section, we generated a synthetic data set from the function $f(x_1, x_2) = x_1 \exp(-x_1^2 - x_2^2)$, $where\ x_1, x_2 \in [-2, 2]$ comprising of 100 and 1000 samples (each containing a uniformly sampled x1 and x2 from the above interval). The data was then used to train a network as to optimize the loss for the two sample set sizes.

We tested multiple architectures for both cases, where the general guideline was to test regularization for the smaller data set (as to allow generalization) and try and use less weights (partially to allow quicker runtime and more trials)

We first tested different weight decays using the architecture:

{"input": 2, "output": 128, "nonlinear": "relu", "regularization": "l2"},

{"input": 128, "output": 64, "nonlinear": "sigmoid", "regularization": "l2"},

{"input": 64, "output": 1, "nonlinear": "none", "regularization": "l2"}

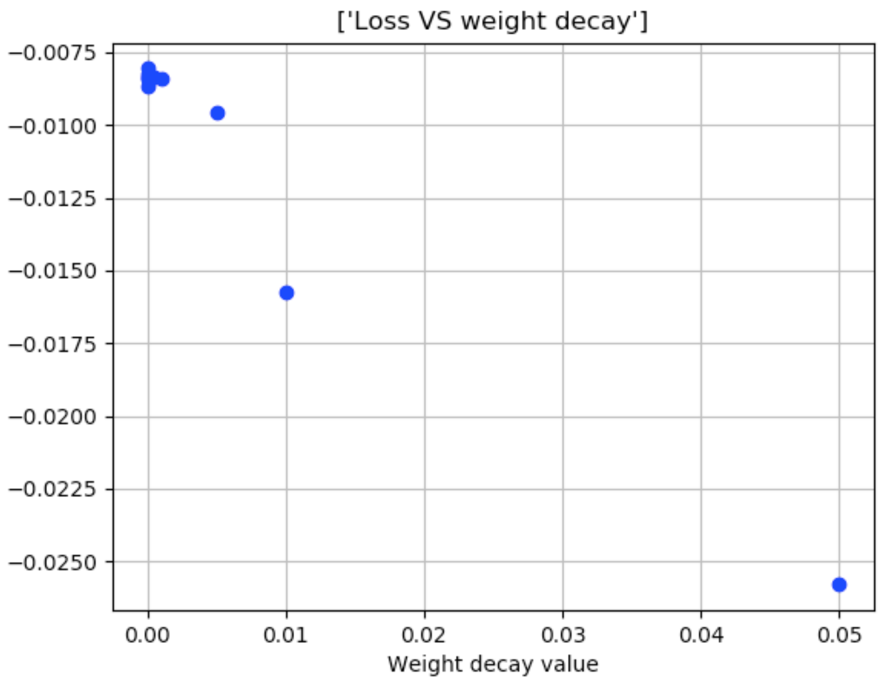


Figure 12 -Test set loss VS Weight decay for 100 samples. Seeing the above results, we chose to work weight minimal weight decay factor

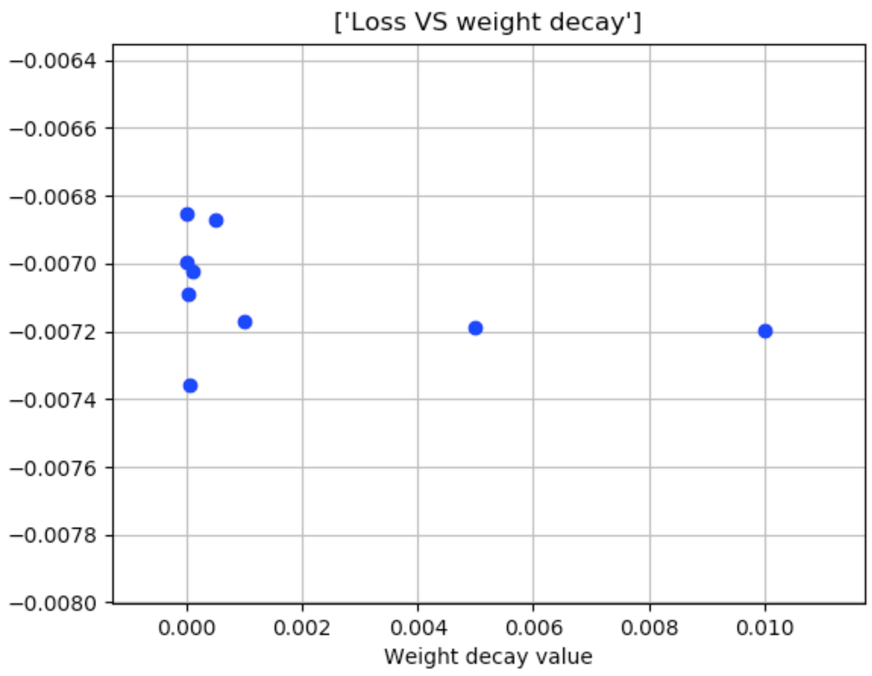


Figure 13 - Test set loss VS weight decay for 1000 training samples. Here also we chose to work with minimal weight decay to obtain minimal loss

We then ran the network for a few architectures and recorded the test set loss;

For depth 2 we used ReLU activation and L2 regularization, and for depth 3 we used ReLU and sigmoid (in this order) and L2 regularization as well.

For both train sample sizes, we ran for 200 epochs with batch size 10 for 100 train samples and 100 for 1000 train samples.

#	Depth	Width	Loss train set	Loss test set	Loss train set	Loss test set
			100 train samples		1000 train samples	
1	2	64	-0.00359728	-0.01040215	-0.00806548	-0.00835385
2	2	128	-0.01758234	-0.01108232	-0.00676666	-0.00834461
3	2	256	-0.01116733	-0.01198914	-0.00708678	-0.00870745
4	2	400	-0.00277952	-0.00914418	-0.0049995	-0.00661973
5	3	[64, 64]	-0.00419778	-0.00678353	-0.00682882	-0.00707502
6	3	[128, 64]	-0.00516225	-0.00743214	-0.00367851	-0.00429218
7	3	[64, 128]	-0.00732223	-0.00769308	-0.00682058	-0.00662957
8	3	[100, 200]	-0.01928708	-0.03556572	-1.59984514	-1.39835986

Following the rough optimization on architecture, we settled for the architecture marked orange for the 100 train-sample-set and the one marked yellow for the 1000 train samples. Generally speaking, the more weights we induce the more samples are needed to properly train it, yet too few cause a generalization error as seen in the shallow architectures above.

Below are the training and validation loss curves for the chosen schemes:

For the 100-sample train set:

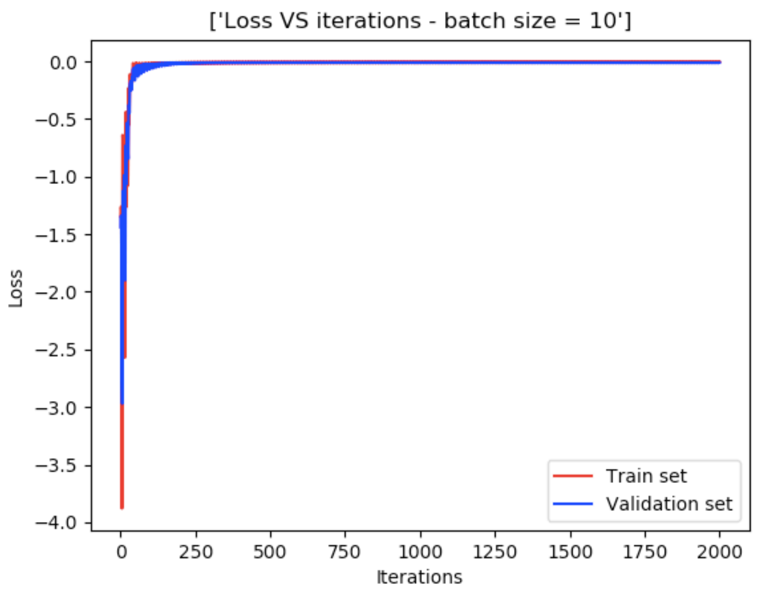


Figure 14 - Loss VS iterations for test and validation sets

And for the 1000 sample train set:

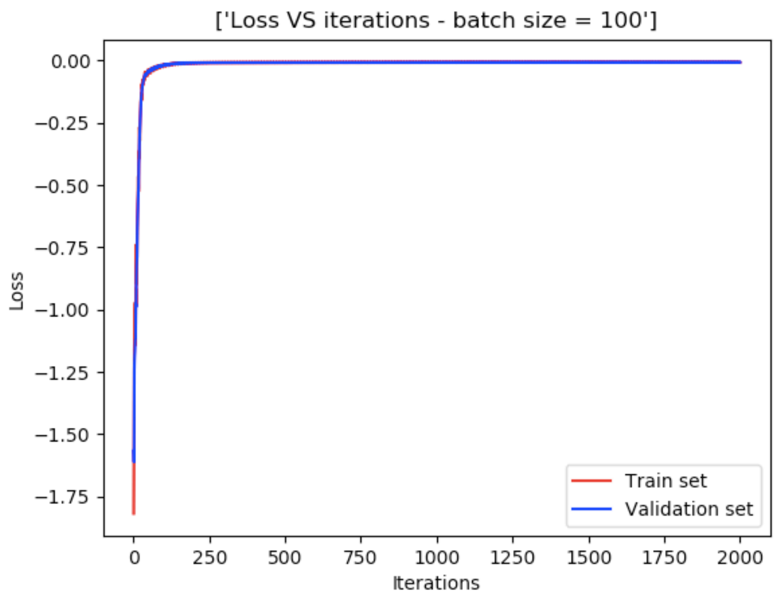
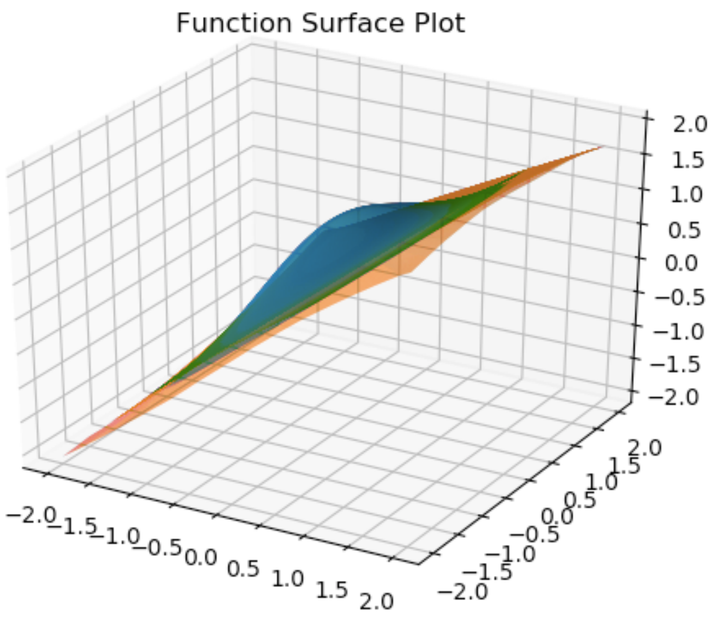
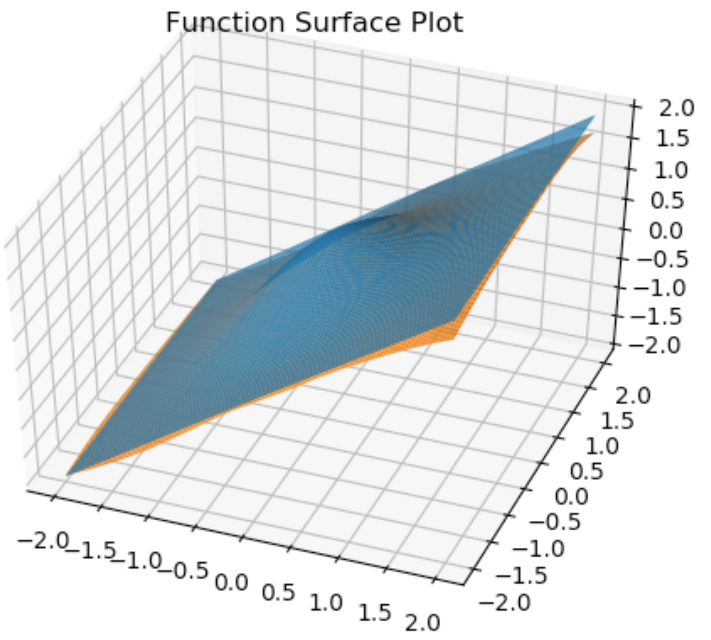


Figure 15 - Loss VS iterations for the 1000 sample train set

To conclude, we plot the function over the [-2, 2] grid with the true and estimated values (test set and network output):
For the 1000-sample set:



And for the 100-sample set:



Naturally, the 1000-sample test set derived a more accurate fit, especially noticeable on the out-skirts of the grid.