First contact: Agents meet Haskell
Simulating epidemics using Functional Reactive Programming

An Agent-Based Approach

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Abstract

TODO: cite my own 1st paper from SSC2017: add it to citations
TODO: refine it: start with simulating epidemics and then go into ABS
Agent-Based Simulation (ABS) is a methodology in which a system is simulated in a bottom-up approach by modelling the micro interactions of its constituting parts, called agents, out of which the global macro system behaviour emerges. So far, the Haskell community hasn't been much in contact with the community of ABS due to the latter's primary focus on the object-oriented programming paradigm. This paper tries to bridge the gap between those two communities by introducing the Haskell community to the concepts of ABS. We do this by deriving an agent-based implementation for the simple SIR model from epidemiology. In our approach we leverage the basic concepts of ABS with functional reactive programming from Yampa and Dunai which results in a surprisingly fresh, powerful and convenient EDSL for programming ABS in Haskell.

Index Terms

Functional Reactive Programming, Agent-Based Simulation

I. INTRODUCTION

In this paper we derive a pure functional approach for agent-based simulation in Haskell. We start from a very simple solution running in the Random Monad, then making the transition to Yampa

The aim of this paper is to show how ABS can be done in Haskell and what the benefits and drawbacks are. We do this by introducing the SIR model of epidemiology and derive an agent-based implementation for it based on Functional Reactive Programming. By doing this we give the reader a good understanding of what ABS is, what the challenges are when implementing it and how we solved these in our approach. We then discuss details which must be paid attention to in our approach and its benefits and drawbacks. The contribution is a novel approach to implementing ABS with powerful time-semantics and more emphasis on specification and possibilities to reason about the correctness of the simulation.

II. DEFINING AGENT-BASED SIMULATION

Agent-Based Simulation (ABS) is a methodology to model and simulate a system where the global behaviour may be unknown but the behaviour and interactions of the parts making up the system is of knowledge. Those parts, called agents, are modelled and simulated out of which then the aggregate global behaviour of the whole system emerges. So the central aspect of ABS is the concept of an agent which can be understood as a metaphor for a pro-active unit, situated in an environment, able to spawn new agents and interacting with other agents in some neighbourhood by exchange of messages [?]. We informally assume the following about our agents TODO: need some references here, we cannot claim this without citation here (cite Peers book):

- They are uniquely addressable entities with some internal state over which they have full, exclusive control.
- They are pro-active which means they can initiate actions on their own e.g. change their internal state, send messages, create new agents, terminate themselves.
- They are situated in an environment and can interact with it.
- They can interact with other agents which are situated in the same environment by means of messaging.

Epstein [?] identifies ABS to be especially applicable for analysing *"spatially distributed systems of heterogeneous autonomous actors with bounded information and computing capacity"*. Thus in the line of the simulation types *Statistic* [†], *Markov* [‡], *System Dynamics* [§], *Discrete Event* [∓], ABS is the most powerful one as it allows to model the following:

- Linearity & Non-Linearity [†‡§∓] - the dynamics of the simulation can exhibit both linear and non-linear behaviour.
- Time [†‡§∓] - agents act over time, time is also the source of pro-activity.
- States [‡§∓] - agents encapsulate some state which can be accessed and changed during the simulation.

Fig. 1: Transitions in the SIR compartment model.

- Feedback-Loops [§∓] - because agents act continuously and their actions influence each other and themselves, feedback-loops are the norm in ABS.
- Heterogeneity [∓] - although agents can have same properties like height, sex,... the actual values can vary arbitrarily between agents.
- Interactions - agents can be modelled after interactions with an environment or other agents, making this a unique feature of ABS, not possible in the other simulation models.
- Spatiality & Networks - agents can be situated within e.g. a spatial (discrete 2d, continuous 3d,...) or network environment, making this also a unique feature of ABS, not possible in the other simulation models.

III. THE SIR MODEL

To explain the concepts of ABS and of our functional reactive approach to it, we introduce the SIR model as a motivating example. It is a very well studied and understood compartment model from epidemiology [?] which allows to simulate the dynamics of an infectious disease like influenza, tuberculosis, chicken pox, rubella and measles [?] spreading through a population. In this model, people in a population of size N can be in either one of three states *Susceptible*, *Infected* or *Recovered* at a particular time, where it is assumed that initially there is at least one infected person in the population. People interact with each other *on average* with a given rate β per time-unit and get infected with a given probability γ when interacting with an infected person. When infected, a person recovers *on average* after δ time-units and is then immune to further infections. An interaction between infected persons does not lead to re-infection, thus these interactions are ignored in this model. This definition gives rise to three compartments with the transitions as seen in Figure 1.

The dynamics of this model over time can be formalized using the System Dynamics (SD) approach [?] which models a system through differential equations. For the SIR model we get the following equations:

$$\frac{\mathrm{d}S}{\mathrm{d}t} = -infectionRate \tag{1}$$

$$\frac{\mathrm{d}I}{\mathrm{d}t} = infectionRate - recoveryRate \tag{2}$$

$$\frac{\mathrm{d}R}{\mathrm{d}t} = recoveryRate \tag{3}$$

$$infectionRate = \frac{I\beta S\gamma}{N} \tag{4}$$

$$recoveryRate = \frac{I}{\delta} \tag{5}$$

Solving these equations is then done by integrating over time. In the SD terminology, the integrals are called *Stocks* and the values over which is integrated over time are called *Flows*. The $1+$ in $I(t)$ amounts to the initially infected agent - if there wouldn't be a single infected one, the system would immediately reach equilibrium.

$$S(t) = N + \int_0^t -infectionRate \, \mathrm{d}t \tag{6}$$

$$I(t) = 1 + \int_0^t infectionRate - recoveryRate \, \mathrm{d}t \tag{7}$$

$$R(t) = \int_0^t recoveryRate \, \mathrm{d}t \tag{8}$$

There exist a huge number of software packages which allow to conveniently express SD models using a visual approach like in Figure 2.

Running the SD simulation over time results in the dynamics as shown in Figure 3 with the given variables.
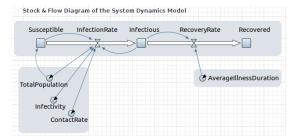
Fig. 2: A visual representation of the SD stocks and flows of the SIR compartment model. Picture taken using AnyLogic Personal Learning Edition 8.1.0.
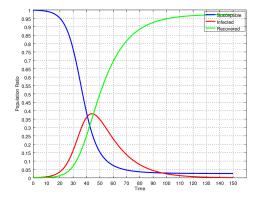

Fig. 3: Dynamics of the SIR compartment model using the System Dynamics approach. Population Size $N = 1,000$, contact rate $\beta = \frac{1}{5}$, infection probability $\gamma = 0.05$, illness duration $\delta = 15$ with initially 1 infected agent. Simulation run for 150 time-steps.

An Agent-Based approach

The SD approach is inherently top-down because the emergent property of the system is formalized in differential equations. The question is if such a top-down behaviour can be emulated using ABS, which is inherently bottom-up. Also the question is if there are fundamental drawbacks and benefits when doing so using ABS. Such questions were asked before and modelling the SIR model using an agent-based approach is indeed possible. It is important to note that SD can be seen as operating on averages thus treating the population completely continuous which results in non-discrete values of stocks e.g. 3.1415 infected persons. Thus the fundamental approach to map the SIR model to an ABS is to discretize the population and model each person in the population as an individual agent. The transition between the states are no longer happening according to continuous differential equations but due to discrete events caused both by interactions amongst the agents and time-outs.

TODO: this is already a too technical explanation which fixes the implementation details already on messaging / data-flow - this is too early and in deriving our approach we will implement 4 different approaches (feedback of all agent-states, data-flow, environment, transactions) TODO: the main point is that we are implementing a state-chart with the transitions are the main thing to consider

- Every agent makes *on average* contact with β random other agents per time unit. In ABS we can only contact discrete agents thus we model this by generating a random event on average every β time units. Note that we need to sample from an exponential CDF because the rate is proportional to the size of the population as [**?**] pointed out.
- An agent does not know the other agents' state when making contact with it, thus we need a mechanism in which agents reveal their state in which they are in *at the moment of making contact*. Obviously the already mentioned messaging which allows agents to interact is perfectly suited to do this.
  - *Susceptibles*: These agents make contact with other random agents (excluding themselves) with a "Susceptible" message. They can be seen to be the drivers of the dynamics.
  - *Infected*: These agents only reply to incoming "Susceptible" messages with an "Infected" message to the sender. Note that they themselves do *not* make contact pro-actively but only react to incoming one.
  - *Recovered*: These agents do not need to send messages because contacting it or being contacted by it has no influence on the state.

- Transition of susceptible to infected state - a susceptible agent needs to have made contact with an infected agent which happens when it receives an "Infected" message. If this happens an infection occurs with a probability of γ. The infection can be calculated by drawing p from a uniform random-distribution between 0 and 1 - infection occurs in case of $\gamma >= p$. Note that this needs to be done for *every* received "Infected" message.
- Transition of infected to recovered - a person recovers *on average* after δ time unites. This is implemented by drawing the duration from an exponential distribution [**?**] with $\lambda = \frac{1}{\delta}$ and making the transition after this duration.

For a more in-depth introduction of how to approximate an SD model by ABS see [**?**] who discusses a general approach and how to compare dynamics and [**?**] which explain the need to draw the illness-duration from an exponential-distribution. For comparing the dynamics of the SD and ABS approach to real-world epidemics see [**?**].

IV. DERIVING A FUNCTIONAL APPROACH

In this section we will derive a functional approach for implementing an agent-based simulation of the SIR model. We will start out with a very naive approach and show its limitations which can be overcome by bringing in FRP. Then in three steps we will add more concepts and generalisations, ending up at the final approach which utilises monadic stream functions (MSF) [**?**], a generalisation of FRP. Although we presented a high-level agent-based approach to the SIR model in the previous section, which focused only on the states and the transitions, we haven't talked about technical implementation details on how to actually implement such a state-machine. In these steps we will ultimately present four different approaches on how to implement these states and transitions. Although all result *on average* in the same dynamics, not all of them are equally expressive and testable.

A. Step I: Naive beginnings

In our first step we start with modelling the states of the agents for which we simply use an Algebraic Data Type (ADT):

Also agents are ill for some duration meaning we need to keep track when a potentially infected agent recovers. Also as previously mentioned, a simulation is stepped in discrete or continuous time-steps thus we introduce a notion of *time* and Δt by defining:

Now we can represent every agent simply as a tuple of its SIR-state and its potential recovery time. We hold all our agents simply in a list and define helper functions:

Next we need to think about how to actually step our simulation. For this we define a function which simply steps our simulation with a fixed Δt until a given time t where in each step the agents are processed and the output is fed back into the next step. TODO: need a much better explanation and maybe split up into more steps? As already mentioned in previous sections, the agent-based implementation of the SIR model is inherently stochastic which means we need access to a random-number generator. We decided to use the Rand Monad at this point as threading a generator through the simulation and the agents is very cumbersome. Thus our simulation stepping runs in the Rand Monad:

Now we can implement the behaviour of an individual agent. First we need to distinguish between the agents SIR-states:

An agent gets fed all the agents states so it can draw random contacts. Note that this includes also the agent itself thus we would need to omit the agent itself to prevent making contact with itself. We decided against that as it complicates the solution and for larger numbers of agent population the probability for an agent to make contact with itself is so small that it can be neglected.

From our implementation it becomes apparent that only the behaviour of a susceptible agent involves randomness and that a recovered agent is simply a sink: it does nothing - its state stays constant.

Lets look how we can implement the behaviour of a susceptible agent. It simply makes contact on average with a number of other agents and gets infected with a given probability if an agent it has contact with is infected. When the agent gets infected it calculates also its time of recovery by drawing a random number from the exponential distribution meaning it is ill on average for illnessDuration.

The infected agent is trivial. It simply recovers after the given illness duration which is implemented as follows:

1) Results: We run the simulation for $t = 150$ time-units with a fixed $\Delta t = 1.0$. With increasing number of agents the dynamics approach the one of the SD simulation. TODO: add pictures

Reflecting on our first naive approach we can conclude that it already introduced the most fundamental concepts of ABS

- Time - the simulation occurs over virtual time which is modelled explicitly divided into *fixed* Δt where at each the agents are executed.
- Agents - we implement each agent as an individual behaviour which depends on the agents state.
- Feedback - the output state of the agent in the current time-step t is the input state for the next time-step $t + 1$.
- Environment - as environment we implicitly assume a fully-connected network where every agent 'knows' every other agents, including itself and thus can make contact with every other agent (including itself).
- Stochasticity - its an inherently stochastic simulation, which is indicated by the Rand Monadic type.

- Deterministic - repeated runs with the same initial random-number generator result in same dynamics. This may not come as a surprise but in Haskell we can guarantee that property statically already at compile time because our simulation runs in the Rand monad and NOT in the IO Monad. This guarantees that no external, uncontrollable sources of randomness can interfere with the simulation.
- Dynamics - it works as expected: with increasing number of agents our solution approaches the SD dynamics

Nonentheless our approach has also weaknesses and dangers:

1) Δt is passed explicitly as argument to the agent and needs to be dealt with explicitly. It seems to be not very elegant and a potential source of errors - can we do better and find a more elegant solution?
2) The way our agents are represented is not very elegant: the state of the agent is explicitly encoded in an ADT and when processing the agent the function needs always first distinguish between the states. Can we express it in a more implicit, functional way?
3) The states of all agents of the current step are fed back into every agent in the next step so that an agent can pick its contacts. Although agents cannot change the states, this reveals too much information e.g. the illness duration is of no interest to the other agents. Although we could just feed in the SIRStates without the illness duration, the problem is more of conceptual nature: it should be the agent which decides to whom it reveals which information.

We move now to the next step in which we will address points 1 and 2, point 3 will be solved in step 3.

B. Step II: Adding FRP

As shown in the first step, the need to handle Δt explicitly can be quite messy, is inelegant and a potential source of errors, also the explicit handling of the state of an agent and its behavioural function is not very functional. We can solve both these weaknesses by switching to the functional reactive programming (FRP) paradigm, because it allows to express systems with discrete and continuous time-semantics. TODO: need more introduction. In this step we are focusing on arrowized FRP using the library Yampa. In it, time is handled implicit and cannot be messed with and the whole system is built on the concept of signal-functions (SF). A signal-function is basically a continuation which allows then to capture state using closures. Both these fundamental features allow us to tackle the weaknesses of our first step and push our approach further towards a truly functional approach.

We start by defining our agents now as a signal-function which receives the states of all agents as input and outputs the state of the agent:

Now we can re-define the behaviour of an agent to be the following:

Depending on the initial state we return one of three functions. Most notably is the difference that we are now passing a random-number generator instead of running in the random-monad because signal-functions as implemented in Yampa are not capable of being monadic. We see that the recovered agent ignores the random-number generator which is in accordance with the implementation of step 1 where it acts as a sink which returns constantly the same state:

The implementation of a susceptible agent in FRP is a bit more involved but much more expressive and elegant:

The implementation works as follows: the agent behaves as susceptible until it becomes infected, then it behaves as an infected agent by switching into the *infectedAgent* SF. Instead of randomly drawing the number of contacts to make we now follow a fundamentally different approach by using the occasionally function. It generates on average an event after the given time - the important difference is that in each time-step we generate either a single event or no event. This requires a fundamental different approach in selecting the right Δt and sampling the system as will be shown in results. When occasionally generates an event we then draw a random element from the other agents state and if we picked an infected agent we need to draw a random boolean which results in True with a uniform probability of 0.05 given in *infectivity*. If the boolean is True this means the agent got infected thus returning the Infected state and an Event with unit which signals that the agent will switch now into the infected behaviour - the state and behaviour is now implicit and much more expressive.

We deal with randomness different now and implement signal-functions built on the noiseR function provided by Yampa. This function takes a range of values and the random-number generator as input and returns the *next* value in the range. This is another example of the statefulness of a signal-function as it needs to keep track of the changed random-number generator internally through the use of continuations and closures. Here we provide the implementation of *randomBoolSF*, drawRandomElemSF function works similar but takes the list as input:

Implementing the infected agent in FRP is also a bit more involved but much more expressive too:

The infected agent behaves as infected until it recovers on average after the illness duration after which it behaves then as a recovered agent by switching into *recoveredAgent*. As in the susceptible agent we use the occasionally function to generate the event when the agent recovers. Note that the infected agent ignores the states of the other agents as its behaviour is completely independent of them.

Running and stepping the simulation works now a bit different:

Yampa provides the function *embed* which allows to run a signal-function for a given number of steps where in each step one provides the Δt and an optional input. We run the signal-function *stepSimulation*:

This function takes all the signal-functions and current states of all agents and returns a signal-function which has the unit-type as input and returns a list of agent-states. What we need to do is to run all agents signal-functions in parallel where all the agent-states are passed as inputs and collect the output of all signal-functions into a list. Fortunately Yampa provides the function *dpSwitch* for this task TODO: explain the d in p, and why we use it. Its first argument is the pairing-function which pairs up the input to the signal-functions, the second argument is the collection of signal-functions, the third argument is a signal-function generating the switching event and the last argument is a function which generates the continuation after the switching event has occurred. pSwitch then returns a new signal-function which runs all the signal-functions in parallel (thus the p) and switching into the continuation when the switching event occurs. The continuation-generation function gets passed the signal-functions after they were run in parallel and the data of the switching event which in combination allows us to recursively switch back into the stepSimulation function. In every step we generate a switching event which passes the final agent-states to the continuation-generation which in turn simply returns stepSimulation recursively but now with the new signal-functions and the new agent-states. Note that we delay the switching event always by one step because the continuations are evaluated always upon switching which would result in an infinite switching loop if not delayed using *notYet*.

1) Results: As already mentioned, because we are now using the occasionally function we need a different approach of sampling the system. In the infected agent occasionally determines the time when an agent recovers so its a discrete transition but in a susceptible agent it determines the average number of contacts per time unit: it is a rate with an exponential distribution. This requires us to sample the system with small enough Δt to arrive at the correct solution - if we choose a too large Δt, we loose events which will result in dynamics which do not approach the SD dynamics. We investigated the behaviour of occasionally under varying Δt, which we added as Appendix TODO add appendix. From this it becomes apparent that we need to step our simulation with a $\Delta t <= 0.1$ to arrive at a sufficiently close approximation of the SD dynamics. TODO: show results

TODO: ALARM ALARM !!!! our dynamics do not approach the ones of SD yet, Step 1 solution is much better, probably because we need supersampling! when reducing time-deltas to 0.001 we arrive at good solutions. TODO: compare the results to random-monad solution 100 agents. with smaller and smaller time-delta we need only 1 infected agent and arrive already at a good approximation. this is not the case in random-monad where because of 1.0 time-delta it results in too non fine-grained resolution

By moving on to FRP using Yampa we made a huge improvement in clarity, expressivity and robustness of our implementation. Also by using explicit time-semantics with *occasionally* we can achieve extremely fine grained stochastics: as opposed to draw random number of events we create only a single event or not. This requires then to sample the system with a much smaller Δt: we are treating it as a truly continuous system. Still we are not too happy about our approach as we feed back all agents states into every agent, something we want to omit in an agent-based simulation. We now move on to step 3 in which we introduce a more general and much more controlled mechanism for feeding back agent-states.

C. Step III: Adding data-flow

In this step we will introduce a data-flow mechanism between agents which makes the feedback explicit. As already mentioned in the previous step, by revealing the state of every agent to all other agents makes the interactions implicit and deprives the agent of its control over which other agent sees its data. As a remedy we introduce data-flows which allow an agent to send arbitrary data to other agents. The data will be collected from the sending agents and distributed to the receivers after each step, which means that we have a delay of one Δt and a round-trip takes $2\Delta t$ - which is exactly the behaviour we had before: feedback. This change requires then a different approach of how the agents interact with each other: a susceptible agent then sends to a random agent a data-flow indicating a contact. Only infected agents need to reply to such contact requests by revealing that they are infected. The susceptible agents then need to check for incoming replies which means they were in contact with an infected agent.

First we need a way of addressing agents, which we do by introducing unique agent ids. Also we need a data-package which identifies the receiver and carries the data:

Next we need more general input and output types of our agents signal-functions. We introduce a new input type which holds both the agent-id of the agent and the incoming data-flows from other agents:

We also introduce a new output type which holds both the outgoing data-flows to other agents and the observable state the agent wants to reveal to the outside world:

Note that by making the observable state explicit in the types we give the agent further control of what it can reveal to the outside world which allows an even stronger separation between the agents internal state / data and what the agent wants the world to see.

Now we can then generalise the agents signal-functions to the following type:

For our SIR implementation we need concrete types, so we need to define what the type parameters *o* and *d* are. For *d* we simply define an ADT which defines a contact-message, and for *o* which defines the type of the observable state, we use the existing SIR-state. Now we can define the type synonyms for our SIR implementation:

Obviously the existing implementation from step 2 needs to be adjusted. Lets look at the initial agent-behaviour:

It still takes a random-number generator, the initial sir-state and returns the corresponding signal-function depending on the state now in addition it now takes a list of agent ids. When using data-flow we need to know the ids of the agents we are communicating with - we need to know our neighbourhood, or seen differently: we need to have access to the environment we are situated in. In our case our environment is a fully connected read-only network in which all agents know all other agents. The easiest way of representing a fully connected network is simply using a list. Again when we look at the functions which are returned we see that recovered agent is still the same: it is a sink which ignores the environment and the random-number generator.

The implementation is nearly the same as in step 2 but instead of returning only the sir-state now the output of an agents signal-function is of type *AgentOut*:

The behaviour of the infected agent now explicitly ignores the environment which was not apparent in step 2 on this level:

The implementation of the infected agent now basically works the same as in step 2 but it additionally needs to reply to incoming contact data-flows with an "Infected" reply. This makes the difference to step 2 very explicit: in the data-flow approach agents now make explicit contact with each other which means that the susceptible agent sends out contact data-flows to which only infected agents need to reply. Note that at the moment of recovery the agent can still infect others because it will still reply with Infected. The response mechanism is implemented in *respondToContactWith*:

Note that the order of data packages in a data-flow is not specified and must not matter as it happens virtually at the same time, thus we always append at the front of the outgoing data-flow list.

Lets look at the susceptible agent behaviour. As already mentioned before, the feedback interaction between agents works now very explicit due to the data-flow but needs a different approach in our implementation:

Again the implementation is very similar to step 2 with the fundamental difference how contacts are made and how infections occur. First the agent checks if it got infected. This happens if an infected agent replies to the susceptible agents contact AND the susceptible agent got infected with the given probability. Note that *gotInfected* runs in the Random-Monad which we run using *runRand* and the random-number generator. To update our random-number generator to the changed one, we use the *rec* keyword of Arrows, which allows us to refer to a variable after it is defined. In combination with iPre we introduced a local state - the random-number generator - which changes in every step. If the agent got infected, it simply returns an AgentOut with Infected as observable state and a switching event which indicates the switch to infected behaviour. If the agent is not infected it draws from occasionally to determine if it should make contact with a random agent. In case it should make contact it simply sends a data-package with the contact susceptible data to the receiver - only an infected agent will reply.

Stepping the simulation now works a little bit different as the input/output types have changed and we need to collect and distribute the data-flow amongst the agents:

The distribution of the data-flows happens in the *switchingEvt* signal-function and is then passed on to the continuation-generation function as in step 2. The difference is that it creates now a list of AgentIn for the next step instead of a list of all the agents sir-states of the previous step. Again the continuation-generation function recursively returns the stepSimulation signal-function. The pairing function of pSwitch is now slightly more straightforward as it just pairs up the AgentIn with its corresponding signal-function.

1) Reflection: It seems that by introducing the data-flow mechanism we have complicated things but this is not so. Data-flows make the feedback between agents explicit and gives the agents full control over the data which is revealed to other agents. This also makes the fact even more explicit, that we cannot fix the connections between the agents already at compile time e.g. by connecting SFs which is done in many Yampa applications (TODO: cite Henrik papers) because agents interact with each other randomly. One can look at the data-flow mechanism as a kind of messaging but there are fundamental differences: messaging almost always comes up as an approach to managing concurrency and involves stateful message-boxes which can be checked an emptied by the receivers - this is not the case with the data-flow mechanism which behaves indeed as a flow where data is not stored in a messagebox but is only present in the current simulation-step and if ignored by the agent will be gone in the next step. Also by distinguishing by the internal and the observable state of the agent, we give the agent even more control of what is visible to the outside world. So far we have a pretty decent implementation of an agent-based SIR approach. The next three steps focus - as this 3rd one - on introducing more concepts and generalising our implementation so far. What we are lacking at the moment is a general treatment of the environment and of synchronised transactional behaviour between agents. To be able to conveniently introduce both we want to make use of monads which is not possible using Yampa. In the next step we make the transition to Monadic Stream Functions (MSF) as introduced in Dunai [**?**]. The authors of Dunai implement BearRiver which is a re-implementation of Yampa on top of MSF which should allow us to easily replace Yampa with MSFs in our implementation of Step 3.

D. Step IV: Generalising to Monadic Stream Functions

TODO: write a bit introductory words for this subsection

1) Identity Monad: We start by making the transition to BearRiver by simply replacing Yampas signal-function by BearRivers which is the same but takes an additional type-parameter *m* which indicates the monad. If we replace this type-parameter with the identity monad we should be able to keep the code exactly the same, except from a few type-declarations, because BearRiver re-implements all necessary functions we are using from Yampa [1]. We start by re-defining our general agent signal-function, introducing the monad (stack) our SIR implementation runs in and the sir-agents signal-function:

We also have to add the *SIRMonad* to the existing *stepSimulation* type-declarations and we are nearly done. The function *embed* for running the simulation is not provided by BearRiver but by Dunai which has important implications. Dunai does not know and care about time in MSFs, which is exactly what BearRiver builds on top of MSFs. It does so by adding a *ReaderT Double* which carries the Δt. This means that *embed* returns a computation in the *ReaderT Double* Monad which we need to run explicitly using *runReaderT*. This then results in an identity computation which we simply peel away using *runIdentity*. Here is the complete code of *runSimulation*:

Note that embed does not take a list of Δt any more but simply a list of inputs for each step to the top level signal-function.

2) Random Monad: Using the Identity Monad does not gain us anything but it was a first step towards a more general solution. Our next step is to replace the Identity Monad by the Random Monad which will allow us to get rid of the RandomGen arguments to our functions and run the whole simulation within the RandomMonad *again* just as we started but now with the full features functional reactive programming! We start by re-defining the SIRMonad and SIRAgent:

Note that we parametrise the Random Monad with a RandomGen g thus this requires to add the RandomGen type-class to all functions where it was not yet added. We also simply remove all RandomGen arguments to all functions except *runSimulation*. The question is now how to access this random monad functionality within the MSF context. For the function *occasionally*, there exists a monadic pendant *occasionallyM* which requires a MonadRandom type-class. Because we are now running within a MonadRandom instance we simply replace *occasionally* with *occasionallyM*. Running *gotInfected* is now much easier. Using the function *arrM* of Dunai allows us to run a monadic action in the stack as an arrow. We then directly run gotInfected by lifting it into the random-monad. This can be seen in the susceptible agent running in the random monad SF:

Note also that *drawRandomElemSF* doesn't take a random number generator as well as it has been reimplemented to make full use of the MonadRandom in the stack:

Instead of *noiseR* which requires a RandomGen, it makes use of Dunai *getRandomRS* stream function which simply runs *getRandomR* in the MonadRandom.

Finally because our innermost monad is now the Random Monad instead of the Identity in *runSimulation* we need to replace *runIdentity* by *evalRand*:

3) Reflections: By making the transition to MSFs we can now stack arbitrary number of Monads. As an example we could add a StateT monad on the type of AgentOut which would allow to conveniently manipulate the AgentOut e.g. in case where one sends more than one message or the construction of the final AgentOut is spread across multiple functions which allows easy composition. When implementing this one needs to replace the dpSwitch with an individual implementation in which one runs the state monad isolated for each agent. We could even add the IO monad if our agents require arbitrary IO e.g. reading/writing from files or communicating over TCP/IP. Although one could run in the IO monad, one should not do so as we would loose all guarantees about the reproducibility of our simulation. In ABS we need deterministic behaviour under all circumstances where repeated runs with the same initial conditions, including the random-number generator, should result in the same dynamics. If we allow IO we loose the ability to guarantee the reproducibility at compile-time even if the agents never use IO facilities and just run in the IO for printing debug messages. So far making the transition to MSFs does not seem as compelling as making the move from the RandomMonad in step 1 to FRP in step 2. Running in the RandomMonad within FRP is convenient but we could achieve the same with passing RandomGen around as we showed in Step 3. In the next step we introduce the concept of a read/write environment which we realise using a StateT monad. This will show the real benefit of the transition to MSFs as without it, implementing a general Environment access would be quite cumbersome.

E. Step V: Adding an environment

In this step we will add an environment in which the agents exist and through which they interact with each other. This is a fundamental different approach to agent-agent interaction but is as valid as the interactions in the previous steps. In ABS agents are often situated within a discrete 2d environment TODO: cite sugarscape, schelling, agentzero, which is simply a finite nxm grid with either a moore or neumann neighbourhood. Agents are either static or can move freely around with cells allowing either single or multiple occupants. We can directly transfer the SIR model to a discrete 2d environment by placing the agents on a corresponding 2d grid with an unrestricted neighbourhood. The behaviour of the agents is the same but they select their neighbours directly from the environment using the provided neighbourhood. Also instead of using data-flow to communicate, agents now communicate through the environment by revealing their current state to their neighbours by placing

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[1]This was not quite true at the time we wrote this paper, where *occasionally*, *noiseR* and *dpSwitch* were missing. We simply implemented these functions and created a pull request using git.

it on their cell. Agents can read (actually they could write too) then the states of all their neighbours which tells them if a neighbour is infected or not. This allows us to implement the infection mechanism close to step 1 and 2.

F. Implementation

We start by defining our discrete 2d environment for which we use an indexed 2-dimensional array. In each cell the agents will store their current state, thus we use the *SIRState* as type for our array data:

Next we redefine our monad-stack and agent signal-function. We use a StateT transformer on top of our Random Monad from step 4 with the previously defined SIREnv as type for the state. Our agent signal-function now has only unit input and output type as we removed the data-flow mechanism for reasons of clarity. This also indicates through the types that the actions of the agents are only visible in side-effects through the monad stack they are running in.

Instead of having a unique AgentId an agent is now initialised through its Disc2dCoord and its initial state.

Again the recovered agent behaviour is the shortest one:

The implementation of a susceptible agent is now a bit different and a mix of step 1 and step 4. Instead of using data-flows the agent directly queries the environment for its neighbours and randomly selects one of them. The remaining behaviour is similar to the previous steps:

Note that the susceptible agent itself changes its state in the environment from Susceptible to Infected upon infection.

The behaviour of an infected agent is nearly the same with the difference that upon recovery the infected agent updates its state in the environment from Infected to Recovered.

Running the simulation is now slightly different as we have an initial environment and also need to peel away the StateT transformer:

As initial state we use the initial environment and instead of returning agent-states we simply return a list of environments, one for each step. The agent-states can then be extracted from each environment.

Due to the different approach of returning the SIREnv in every step, we implemented our own MSF:

G. Results

In addition to plotting the dynamics we implemented rendering functionality of the environments using the gloss library. This allows us to cycle arbitrarily through the steps and inspect the spreading of the disease over time visually.

When using an unrestricted neighbourhood we should arrive at the same dynamics of the SD approach. TODO: show dynamics AND 2d-grid renderings of unrestricted neighbourhood

In agent-based simulation the neighbourhood on a discrete neighbourhood is often restricted to either a moore or von neumann neighbourhood (TODO: draw small diagrams). This should result in different dynamics as agents have now a reduced subset of agents they can infect.

TODO: show dynamics AND 2d-grid renderings using a moore neighbourhood TODO: add analysis from the first draft paper

H. Reflection

At first the environment approach might seem a bit overcomplicated and one might ask what we have gained over using an unrestricted neighbourhood where all agents can contact all others we arrive at the same dynamics as in SD. The real win is that we can introduce arbitrary restrictions on the neighbourhood as shown using the moore or von neumann neighbourhood. Of course the environment is not restricted to a discrete 2d grid and can be anything from a continuous n-dimensional space to a complex network - one only needs to change the type of the StateT monad and provide corresponding neighbourhood querying functions. The ability to place the heterogeneous agents in a generic environment is also the fundamental advantage of an agent-based over a SD approach as it allows to simulate much more realistic scenarios.

Note that for reasons of clarity we have removed the data-flow approach from this implementation which results in the unit-types of input and output. Of course in a full blown agent-based simulation library we would combine both approaches and leave the AgentIn/AgentOut types with the data-flow functionality in place as implemented in step 4.

In step 3 and 4 we already have one variant of possible environment scenarios: the non-proactive read-only one. Implementing a pro-active read-only is easy: we use an agent as environment which broadcasts updates to all agents through data-flows. Implementing non pro-active read/write environments is very easy possible using MSFs and the StateT monad as shown in this step. If we want our environment to be pro-active e.g. regrowing some resources, then we simply add an environment-agent which acts upon the environment state. The convenient thing is that although conceptually all agents act at the same time, technically by using *mapM* in *stepSimulation* they are run after another which also serialises the environment access which gives every agent exclusive read/write access while it is active.

Attempting to introduce a non/pro-active read/write environment to the Yampa implementation is quite cumbersome. A possible solution would be to add another type-parameter *e* which captures the type of the environment and then pass it in through the input and allow it to be returned in the output of an agent signal-function. We would then end up with n copies

of the environment - one for each agent - which we need to fold back into a single environment. When we have a pro-active environment we could also add it as another agent but this results in even more problems when folding it back - the solution would be to run a separate environment signal-function which acts on the folded environment after all agents are run. All these problems are not an issue when using MSFs with a StateT which is a compelling example for making the transition to the more general MSFs.

In the last step we will introduce synchronised agent-transaction as the final agent-agent interaction mechanism. This is a quite sophisticated concept: synchronised agent-transactions which allow an arbitrary number of interactions between two agents without time lag. The use-case for this are price negotiations between multiple agents where the agents need to come to an agreement in the same time-step as described in TODO cite sugarscape.

I. Step VI: Adding agent transactions

Imagine two agents A and B want to engage in a bartering process where agent A, is the seller who wants to sell an asset to agent B who is the buyer. Agent A sends Agent B a sell offer depending on how much agent A values this asset. Agent B receives this sell offer, checks if the price satisfies its utility, if it has enough wealth to buy the asset and replies with either a refusal or its own price offer. Agent A then considers agent Bs offer and if it is happy it replies to agent B with an acceptance of the offer, removes the asset from its inventory and increases its wealth. Agent B receives this acceptance offer, puts the asset in its inventory and decreases its wealth (note that this process could involve a potentially arbitrary number of steps without loss of generality). We can see this behaviour as a kind of multi-step transactional behaviour because agents have to respect their budget constraints which means that they cannot spend more wealth or assets than they have. This implies that they have to 'lock' the asset and the amount of cash they are bartering about during the bartering process. If both come to an agreement they will swap the asset and the cash and if they refuse their offers they have to 'unlock' them. In classic OO implementations it is quite easy to implement this as normally only one agent is active at a time due to sequential (discrete event scheduling approach) scheduling of the simulation. This allows then agent A which is active, to directly interact with agent B through method calls. The sequential updating ensures that no other agent will touch the asset or cash and the direct method calls ensure a synchronous updating of the mutable state of both objects with no time passing between these updates.

1) Implementation: We start with the implementation of step 4 with the Random Monad and remove the data-flows from AgentIn and AgentOut. We then add a field in AgentOut which allows the agent to indicate that it wants to start a transaction with another agent with an initial data-package. Also we add a field in AgentIn which indicates an incoming transaction request from another agent with the given data-package. In addition we need another field in AgentOut which allows the agent to indicate that it accepts the incoming request:

We run the transactions in the specialised agent-transaction signal-functions *AgentTX* with different input and output types. This allows us to restrict the possible actions of an agent within a transaction:

The input *AgentTXIn* to an agent-transaction holds optional data and flags which indicate that the other agent has either committed or aborted the transaction.

The output *AgentTXOut* of an agent-transaction hold optional data a flag to abort the transaction and optional commit data which is Just in case the agent wants to commit. When committing the agent has to provide a potentially changed AgentOut and optionally a new agent behaviour signal-function. If the agent provides a signal-function when committing, the behaviour of the agent after the transaction will be this signal-function. If no signal-function is provided then the original one will be used.

We also provide type aliases for our SIR implementation:

Stepping the simulation is now slightly more complex as in every step we need to run the transactions. Fortunately it is easy to provide customised implementations of MSFs in dunai, which is a bit more tricky in Yampa and requires to expose internals.

The implementation of *runTransactions* is quite involved and omitted here because it would require too much space [2], but we will give a short informal description. All agents are iterated in an unspecified sequence and if an agent requests a transaction the other agent is looked up and the transaction-pair is run. This is done recursively until there are no transaction requests any-more (note that through the AgentOut of a committed transaction, an agent can request a new transaction within the same time-step). Running a transaction-pair works as follows: The target agents signal-function is run again (resulting in a second, or third,... execution, depending on how many transactions have this agent as target) but now with a $\Delta t = 0$. The target agent can then accept the incoming transaction or simply ignore it. If it is ignored the transaction will never start. The fact that the target agent signal-function is run more than once within a simulation step but with a $\Delta t = 0$ requires agents to make their actions time-dependent *but* they must listen to incoming transactions independent of time. The implementation of the infected agent below will make this more clear. When the transaction is accepted the system switches to running the transaction signal-functions after another with passing the data forward and backward between the two agents. It is most important to

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[2]The full code of all steps is freely available under https://github.com/thalerjonathan/phd/tree/master/public/HaskellABSTutorial/code

note that again the signal functions are run with $\Delta t = 0$ because conceptually transactions happen *instantaneously* without time advancing. This has important implications, and means that we cannot use any time-accumulating function e.g. integral or after within a transaction - simply because it makes no sense as no time passes. If *both* agents commit the transaction their new AgentOuts will replace the ones for the current simulation-step. If either one agent aborts the transaction the current AgentOuts of the current simulation-step will be used.

We provide a sequence diagram of data-flow in a multi-step negotiation as described in the introduction for a visual explanation of the complex protocol which is going on in a transaction.

Now it is time to look at the new agent implementations which use now the agent-transaction mechanism. The recovered agent is exactly the same but the susceptible and infected agent behaviour are very different now. Lets first look at the susceptible agent:

Instead of using a switch the susceptible agent behaves completely time-dependent and occasionally starts a new agent-transaction with a random agent. The function *susceptibleTx* handles the reply of the other agent. Note that we only commit with a continuation in case the agent becomes infected.

The infected agent is slightly less complex and still uses the switch mechanism:

The agent acts time-dependent which in this case is the transition from infected to recovered - if occasionallyM is run with a dt of 0 then no Event can happen (todo: is this really true??). The agent checks on every function call of infected for incoming transactions and accepts them all, independent of the state - only susceptible agents request transactions anyway. The agent simply replies with a Contact Infected and immediately commits the transaction in the transaction signal-function but does not switch into a new continuation.

2) Reflection: Note that the transactions run in the same monad as the normal agent behaviour signal-function which allows to add an environment as in step 5. In this case care must be taken when one has changed the environment but aborts the transaction as a roll back of the environment won't happen automatically. A different approach would allow to run the TX in a different monad and bring in e.g. the transactional state monad Control.Monad.Tx which supports rolling back of changes to the state.

The concept of agent-transactions is not explicitly known in the agent-based community and a novel development of this paper. The reason for this is that agent-transactions are already implicitly available in traditional OO implementations in which agents can call each others methods and change their state. By implementing this necessary and important concept in a pure functional approach we arrived at agent-transactions which make these synchronous, instantaneous, one-to-one interactions explicit.

V. Related Research

TODO

VI. Conclusions and Further Research

TODO: dependent types in ABS for further research

Acknowledgments