

THE AGENTS NEW CLOTHS? TOWARDS PURE FUNCTIONAL PROGRAMMING IN ABS

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ABSTRACT

The established, traditional approach to implement and engineer Agent-Based Simulations (ABS) so far has primarily been object-oriented with Python and Java being the most popular languages. In this paper we explore an orthogonal route to this problem and investigate the pure functional programming paradigm, using the language Haskell, for implementing ABS. We give a short, high level introduction into core pure functional programming paradigm features and how they can be made of use to implement ABS. To put our approach to a practical test we present as a case-study a *full and verified* implementation of the seminal Sugarscape model. With this case-study we are able to show that pure functional programming as in Haskell has a valid place in engineering clean, robust and maintainable ABS implementations. Further we show that we can directly leverage the benefits of pure functional programming to ABS: we have very few potential bugs at runtime, can easily exploit data-parallelism, concurrency is much less painful to add, code-testing in general is very convenient and powerful and with property-based testing in particular we present a new code-testing technique to ABS which hasn't been discussed before. The main drawback of using the pure functional programming paradigm we identified its lack of performance, which is clearly behind object-oriented approaches.

Keywords: Agent-Based Simulation, Functional Programming, Haskell, Concurrency, Parallelism, Property-Based Testing, Validation & Verification.

1 INTRODUCTION

The traditional approach to Agent-Based Simulation (ABS) has so far always been object-oriented techniques, due to the influence of the seminal work of Epstein et al (Epstein and Axtell 1996) in which the authors claim "[...] *object-oriented programming to be a particularly natural development environment for Sugarscape specifically and artificial societies generally* [...]" (p. 179). This work established the metaphor in the ABS community, that *agents map naturally to objects* (North and Macal 2007) which still holds up today.

In this paper we challenge this metaphor and present ways of implementing ABS with the functional programming paradigm using the language Haskell (Hudak, Hughes, Peyton Jones, and Wadler 2007). We

claim that functional programming has its place in ABS because of ABS' *scientific computing* nature where results need to be reproducible and correct while simulations should be able to exploit parallelism and concurrency as well. The established object-oriented approaches need considerably high effort and might even fail to deliver these objectives due to its conceptually different approach to programming. In contrast, we claim that by using functional programming for implementing ABS it is less difficult to add parallelism and correct concurrency, the resulting simulations are easier to test and verify, guaranteed to be reproducible already at compile-time, have fewer potential sources of bugs and are ultimately more likely to be correct.

To substantiate our claims we present fundamental concepts and advanced features of functional programming and how they can be used to engineer clean, maintainable and reusable ABS implementations. Further we discuss how the well known benefits of functional programming in general are applicable in ABS. To put our claims to test we conducted a practical case-study in which we implemented the *full* SugarScape model (Epstein and Axtell 1996) in Haskell. In this case study:

- We developed techniques for engineering a clean, maintainable, general and reusable *sequential* implementation in Haskell. Those techniques are directly applicable to other ABS implementations as well due to the complex nature of the Sugarscape model.
- We explored ways of exploiting data-parallelism to speed up the execution in our sequential implementation.
- We explored techniques for implementing a *concurrent* implementation using Software Transactional Memory (STM).
- We conducted performance measurements of our sequential and concurrent implementation.
- We explored ways of code-testing our implementation. We show how to use property-based testing for ensuring the correctness of individual agent parts, and unit-testing of the whole simulation which serves both as regression test and to check model hypotheses.
- Our Sugarscape implementation is fully validated against the dynamics reported in the book (Epstein and Axtell 1996) and an already existing NetLogo replication (Weaver). Due to lack of space we discuss the validation process in Appendix A.

We present the challenges encountered in this case-study and discuss benefits and drawbacks. As will become apparent, our results support our claims that pure functional programming has indeed its place in ABS and that it has the mentioned benefits. On the other hand, due to its heavier approach in terms of engineering, its approach pays only off in cases of high-impact and large-scale simulations which results might have far-reaching consequences e.g. influence policy decisions. Because its only those models which require high confidence in correctness and stability of the software this heavier approach is almost never necessary for quick prototyping and small case-studies of ABS models for which NetLogo and the established object-oriented approaches using Python, Java, C++ are perfectly suitable. Still, we hope to distil the developed techniques into a general purpose ABS Haskell library so implementing models becomes much easier and quicker and makes using Haskell attractive for prototyping models as well.

The aim and contribution of this paper is to introduce the functional programming paradigm using Haskell to ABS on a *conceptual* level, identifying benefits, difficulties and drawbacks. This is done by presenting the above mentioned case-study which introduces general implementation techniques applicable to ABS. To the best of our knowledge, we are the first to do so.

The structure of the paper is as follows. In Section 2 we present related work on functional programming in ABS. In Section 3 we introduce the functional programming paradigm, establish key features, motivate why we chose Haskell, discuss its type system, side-effects, parallelism and concurrency and how ABS can be implemented on top of them building on Monads, Continuations and FRP. In Section 4 we present our Sugarscape case-study and with its challenges encountered and benefits and drawbacks. In Section

5 we discuss our initial claims in the light of the case-study. In Section 6 we conclude and point out further research. We added an Appendix A in which we give a deeper insight into our process of validating our Sugarscape model against the book and (Epstein and Axtell 1996) and an already existing NetLogo replication (Weaver).

2 RELATED WORK

The amount of research on using pure functional programming with Haskell in the field of ABS has been moderate so far. Most of the papers are related to the field of Multi Agent Systems and look into how agents can be specified using the belief-desire-intention paradigm (De Jong 2014, Sulzmann and Lam 2007, Jankovic and Such 2007).

A multi-method simulation library in Haskell called *Aivika 3* is described in the technical report (Sorokin 2015). It supports implementing Discrete Event Simulations (DES), System Dynamics and comes with basic features for event-driven ABS which is realised using DES under the hood. Further it provides functionality for adding GPSS to models and supports parallel and distributed simulations. It runs in IO for realising parallel and distributed simulation but also discusses generalising their approach to avoid running in IO.

In his masterthesis (Bezirgiannis 2013) the author investigated Haskell's parallel and concurrency features to implement (amongst others) *HLogo*, a Haskell clone of the NetLogo simulation package, focusing on using Software Transactional Memory for a limited form of agent-interactions. *HLogo* is basically a re-implementation of NetLogos API in Haskell where agents run within IO and thus can also make use of STM functionality.

There exists some research (Di Stefano and Santoro 2005, Varela, Abalde, Castro, and Gulías 2004, Sher 2013) of using the functional programming language Erlang (Armstrong 2010) to implement ABS. The language is inspired by the actor model (Agha 1986) and was created in the 1986 by Joe Armstrong for Eriksson for developing distributed high reliability software in telecommunications. The actor model can be seen as quite influential to the development of the concept of agents in ABS which borrowed it from Multi Agent Systems (Wooldridge 2009). It emphasises message-passing concurrency with share-nothing semantics (no shared state between agents) which maps nicely to functional programming concepts. The mentioned papers investigate how the actor model can be used to close the conceptual gap between agent-specifications which focus on message-passing and their implementation. Further they also showed that using this kind of concurrency allows to overcome some problems of low level concurrent programming as well.

Using functional programming for DES was discussed in (Jankovic and Such 2007) where the authors explicitly mention the paradigm of FRP to be very suitable to DES.

A domain-specific language for developing functional reactive agent-based simulations was presented in (Schneider, Dutchyn, and Osgood 2012, Vendrov, Dutchyn, and Osgood 2014). This language called FRAB-JOUS is human readable and easily understandable by domain-experts. It is not directly implemented in FRP/Haskell but is compiled to Haskell code which they claim is also readable. This supports that FRP is a suitable approach to implement ABS in Haskell. Unfortunately, the authors do not discuss their mapping of ABS to FRP on a technical level, which would be of most interest to functional programmers.

3 FUNCTIONAL PROGRAMMING

Functional programming (FP) is called *functional* because it makes functions the main concept of programming, promoting them to first-class citizens: functions can be assigned to variables, they can be passed as arguments to other functions and they can be returned as values from functions. The roots of FP lie in the

Lambda Calculus which was first described by Alonzo Church (Church 1936). This is a fundamentally different approach to computing than imperative programming (which includes established object-orientation) which roots lie in the Turing Machine (Turing 1937). Rather than describing *how* something is computed as in the more operational approach of the Turing Machine, due to the more *declarative* nature of the Lambda Calculus, code in functional programming describes *what* is computed.

In our research we are using the *pure* functional programming language Haskell. The paper of (Hudak, Hughes, Peyton Jones, and Wadler 2007) gives a comprehensive overview over the history of the language, how it developed and its features and is very interesting to read and get accustomed to the background of the language. The main points why we decided to go for Haskell are:

- Rich Feature-Set - it has all fundamental concepts of the pure functional programming paradigm of which we explain the most important below. Further, Haskell has influenced a large number of languages, underlining its importance and influence in programming language design.
- Real-World applications - the strength of Haskell has been proven through a vast amount of highly diverse real-world applications (Hudak, Hughes, Peyton Jones, and Wadler 2007), is applicable to a number of real-world problems (O'Sullivan, Goerzen, and Stewart 2008) and has a large number of libraries available ¹.
- Modern - Haskell is constantly evolving through its community and adapting to keep up with the fast changing field of computer science. Further, the community is the main source of high-quality libraries.

3.1 Fundamentals

To explain the central concepts of functional programming, we give an implementation of the factorial function in Haskell:

```
factorial :: Integer -> Integer
factorial 0 = 1
factorial n = n * factorial (n-1)
```

When looking at this function we can identify the following:

1. Declarative - we describe *what* the factorial function is rather than how to compute it. This is supported by *pattern matching* which allows to give multiple equations for the same function, matching on its input.
2. Immutable data - in functional programming we don't have mutable variables - after a variable is assigned, it cannot change its contents. This also means that there is no destructive assignment operator which can re-assign values to a variable. To change values, we employ recursion.
3. Recursion - the function calls itself with a smaller argument and will eventually reach the case of 0. Recursion is the very meat of functional programming because they are the only way to implement loops in this paradigm due to immutable data.
4. Static Types - the first line indicates the name and the type of the function. In this case the function takes one Integer as input and returns an Integer as output. Types are static in Haskell which means that there can be no type-errors at run-time e.g. when one tries to cast one type into another because this is not supported by this kind of type-system.

¹https://wiki.haskell.org/Applications_and_libraries

5. Explicit input and output - all data which are required and produced by the function have to be explicitly passed in and out of it. There exists no global mutable data whatsoever and data-flow is always explicit.
6. Referential transparency - calling this function with the same argument will *always* lead to the same result, meaning one can replace this function by its value. This means that when implementing this function one can not read from a file or open a connection to a server. This is also known as *purity* and is indicated in Haskell in the types which means that it is also guaranteed by the compiler.

It may seem that one runs into efficiency-problems in Haskell when using algorithms which are implemented in imperative languages through mutable data which allows in-place update of memory. The seminal work of (Okasaki 1999) showed that when approaching this problem with a functional mind-set this does not necessarily be the case. The author presents functional data structures which are asymptotically as efficient as the best imperative implementations and discusses the estimation of the complexity of lazy programs.

For an excellent and widely used introduction to programming in Haskell we refer to (Hutton 2016). Other, more exhaustive books on learning Haskell are (Lipovaca 2011, Allen and Moronuki 2016). For an introduction to programming with the Lambda-Calculus we refer to (Michaelson 2011). For more general discussion of functional programming we refer to (Hughes 1989, MacLennan 1990, Hudak, Hughes, Peyton Jones, and Wadler 2007).

3.2 Side-Effects

One of the fundamental strengths of Haskell is its way of dealing with side-effects in functions. A function with side-effects has observable interactions with some state outside of its explicit scope. This means that its behaviour depends on history and that it loses its referential transparency character, which makes understanding and debugging much harder. Examples for side-effects are (amongst others): modifying a global variable, await an input from the keyboard, read or write to a file, open a connection to a server, drawing random-numbers,...

Obviously, to write real-world programs which interact with the outside-world we need side-effects. Haskell allows to indicate in the *type* of a function that it does or does *not* have side-effects. Further there are a broad range of different effect types available, to restrict the possible effects a function can have to only the required type. This is then ensured by the compiler which means that a program in which one tries to e.g. read a file in a function which only allows drawing random-numbers will fail to compile. Haskell also provides mechanisms to combine multiple effects e.g. one can define a function which can draw random-numbers and modify some global data. The most common side-effect types are: *IO* allows all kind of I/O related side-effects: reading/writing a file, creating threads, write to the standard output, read from the keyboard, opening network-connections, mutable references; *Rand* allows drawing random-numbers; *Reader / Writer / State* allows to read / write / both from / to an environment.

A function without any side-effect type is called *pure*, and the *factorial* function is indeed pure. Below we give an example of a function which is not pure. The *queryUser* function *constructs* a computation which, when executed asks the user for its user-name and compares it with a given user-configuration. In case the user-name matches it returns True, and False otherwise after printing a corresponding message.

```
queryUser :: String -> IO Bool
queryUser username = do
    -- print text to console
    putStr "Type in user-name: "
```

```

-- wait for user-input
str <- getLine
-- check if input matches user-name
if str == username
then do
  putStrLn "Welcome!"
  return True
else do
  putStrLn "Wrong user-name!"
  return False

```

The *IO* in the first line indicates that the function runs in the IO effect and can thus (amongst others) print to the console and read input from it. What seems striking is that this looks very much like imperative code - this is no accident and intended. When we are dealing with side-effects, ordering becomes important, thus Haskell introduced the so-called *do*-notation which emulates an imperative style of programming. Whereas in imperative programming languages like C, commands are chained or composed together using the `;` operator, in functional programming this is done using function composition: feeding the output of a function directly into the next function. The machinery behind the *do*-notation does exactly this and desugars this imperative-style code basically into function compositions which run custom code between each line, depending on the type of effect the computation runs in (IO in this case). This approach of function composition with custom code in between each function allows to emulate a broad range of imperative-style effects, including the above mentioned ones. For a technical, in-depth discussion of the concept of side-effects and how they are implemented in Haskell using Monads, we refer to the following papers: (Moggi 1989, Wadler 1992, Wadler 1995, Wadler 1997, Jones 2002).

Although it might seem very restrictive at first, we get a number of benefits from making the type of effects we can use in the function explicit. First we can restrict the side-effects a function can have to a very specific type which is guaranteed at compile time. This means we can have much stronger guarantees about our program and the absence of potential errors already at compile-time which implies that we don't need test them with e.g. unit-tests. Second, because effect-runners are themselves *pure*, we can execute effectful functions in a very controlled way by making the effect-context explicit in the parameters to the effect-runner. This allows a much easier approach to isolated testing because the history of the system is made explicit.

Further, this type system allows Haskell to make a very clear distinction between parallelism and concurrency. Parallelism is always deterministic and thus pure without side-effects because although parallel code runs concurrently, it does by definition not interact with data of other threads. This can be indicated through types: we can run pure functions in parallel because for them it doesn't matter in which order they are executed, the result will always be the same due to the concept of referential transparency. Concurrency is potentially non-deterministic because of non-deterministic interactions of concurrently running threads through shared data. For a technical, in-depth discussion on Parallelism and Concurrency in Haskell we refer to the following books and papers: (Marlow 2013, O'Sullivan, Goerzen, and Stewart 2008, Harris, Marlow, Peyton-Jones, and Herlihy 2005, Marlow, Peyton Jones, and Singh 2009).

4 CASE-STUDY: PURE FUNCTIONAL SUGARSCAPE

To explore how to approach ABS based on pure functional programming concepts as introduced before, we did a *full and verified* implementation of the seminal Sugarscape model (Epstein and Axtell 1996). We chose the model because it is quite well known in the ABS community, it was highly influential in sparking the interest in ABS, it is quite complex with non-trivial agent-interactions and it used object-oriented techniques and explicitly advocates them as a good fit to ABS.

Our goal was first to develop techniques and concepts to show *how* to engineer a clean, maintainable and robust ABS in Haskell. The second step was then to identify benefits and drawbacks to identify *why* one would follow such an approach. In a third step we pushed the benefits of pure functional programming further and tried to find a remedy for the drawbacks. Absolutely paramount in our research was, that we are being *pure*, which avoids the IO effect type under all circumstances because we would practically lose all strong compile time guarantees².

TODO: (Macal 2016) sugarscape is level 4, chapter 2 is level 3 or 2?

4.1 A Functional View

Due to the fundamentally different approaches of functional programming (FP) an ABS needs to be implemented fundamentally different as well compared to established object-oriented (OO) approaches. We face the following challenges:

1. How can we represent an Agent, its local state and its interface?
2. How can we implement direct agent-to-agent interactions?
3. How can we implement an environment and agent-to-environment interactions?

The fundamental building blocks to solve these problems are *recursion* and *continuations*. In recursion a function is defined in terms of itself: in the process of computing the output it *might* call itself with changed input data. Continuations in turn are functions which allow to encapsulate the execution state of a program by capturing local variables and pick up computation from that point later on by returning a new function.

Thus we implement an agent as a continuation: this lets us encapsulate arbitrary complex agent-state which is only visible and accessible from within the continuation - the agent has exclusive access to it. This allows also to switch behaviour dynamically e.g. switching from one mode of behaviour to another like in a state-machine, simply by returning new functions which encapsulate the new behaviour. If no change in behaviour should occur, the continuation simply recursively returns itself with the new state captured.

The fact that we design an agent as a function, raises the question of the interface of it: what are the inputs and the output? Note that the type of the function has to stay the same although we might switch into different continuations - our interface needs to capture all possible cases of behaviour. The way we define the interface is strongly determined by the direct agent-agent interaction. In case of Sugarscape agents need to be able to conduct two types of direct agent-agent interaction: 1. one-directional, where agent A sends a message to agent B without requiring agent B to synchronously reply to that message e.g. repaying a loan or inheriting money to children; 2. bi-directional, where two agents negotiate over multiple steps e.g. accepting a trade, mating or lending. Thus it seems reasonable to define as input type an algebraic data-type which defines all possible incoming messages and agent handles. The agents continuation is then called every time the agent receives a message and can process it, update its local state and might change its behaviour. As output we define a data-structure which allows the agent to communicate to the simulation kernel 1. whether it wants to be removed from the system or not, 2. a list of new agents this agent wants to spawn, 3. a list of messages the agent wants to send to other agents. Further because the data is completely local, the agent also returns a data-structure which holds all *observable* information, the agent wants to share with the outside world. Together with the continuation this guarantees that the agent is in full control over its local state, which no one can mutate or access from outside. This also implies that one can only get information out of

²The code is freely accessible from <https://github.com/thalerjonathan/phd/tree/master/public/towards/SugarScape>

the agent by running its function. It also means that the output type of the function has to cover all possible input cases - it cannot change or depend on the input.

The simulation kernel keeps track of the existing agents and the message-queue and processes the queue one element at a time. The newly messages of an agent are inserted *at the front* of the queue, ensuring that synchronous bi-directional messages are possible without violating resources constraints. The Sugarscape model specifies that in each tick or time-step all agents run in random order and do their thing. Thus to start the agent-behaviour in a new time-step, the core inserts a Tick message to each agent in random order which then results in them being executed and emitting new messages. The current time-step has finished when all messages in the queue have been processed.

TODO: give a short example of continuation agent

Obviously the agents in the Sugarscape are located in a discrete 2d environment where they move around and harvest resources, which means the need to read and write data of environment. This is conveniently implemented by adding a State side-effect type to the agent continuation function. Further we also add a Random effect type because dynamics in most ABS in general and Sugarscapes in particular are driven by random number streams, so our agent needs to have access to one as well. All of this low level continuation plumbing is not necessary to implement and there exists a high quality library called *dunai*, based on research on Functional Reactive Programming (Hudak, Courtney, Nilsson, and Peterson 2003) and Monadic Stream Functions (Perez, Baerenz, and Nilsson 2016, Perez 2017).

4.2 Memory

Haskell is notorious for its space-leaks due to lazy evaluation: data is only evaluated when required, which. Even for simple programs one can be hit by a serious space-leak where unevaluated code pieces (thunks) builds up in memory until they are needed, leading to dramatically increased memory usage for a problem which could be solved using a fraction.

It is no surprise that our highly complex Sugarscape implementation initially suffered severely from space-leaks, piling up about 40 MByte of memory per second. In simulation this is a big issue, threatening the value of the whole implementation despite its other benefits: because simulations might run for a (very) long time or conceptually forever, one must make absolutely sure that the memory usage stays constant.

As a remedy, Haskell allows to add so-called strictness pragmas to code-modules which forces strict evaluation of all data even if it is not used. Carefully adding this conservatively file-by file applying other techniques of forcing evaluation removed most of the memory leaks. Another memory leak was caused by selecting the wrong data-structure for our environment, for which we initially used an immutable array. The problem is that in the case of an update the whole array is copied, causing memory leaks AND a performance problem. We replaced it by an IntMap which uses integers as key (mapping 2d coordinates to unique integers is trivial) and is internally implemented as a radix-tree which allows for very fast lookups and inserts because whole subtrees can be re-used.

4.3 Testing

We implemented a number of tests for agent functions which don't cover a whole sub-part of an agents behaviour: checks whether an agent has died of age or starved to death, the metabolism, immunisation step, check if an agent is a potential borrower or fertile, lookout, trading transaction. What all these functions have

in common is that they are not pure computations like utility functions but require an agent-continuation which means they have access to the agent state, environment and random-number stream. This allows testing to capture the *complete* system state in one location, which allows the checking of much more invariants than in approaches which have implicit side-effects. What is particularly powerful is that one has complete control and insight over the changed state before and after e.g. a function was called on an agent: thus it is very easy to check if the function just tested has changed the agent-state itself or the environment or other data provided to the agent through a Monad: the new environment is returned after running the agent and can be checked for equality of the initial one - if the environments are not the same, one simply lets the test fail. This behaviour is very hard to emulate in OOP because one can not exclude side-effect at compile time, which means that some implicit data-change might slip away unnoticed. In FP we get this for free.

We tested these functions with an approach called *property-based* testing. Although it is now available in a wide range of programming languages and paradigms, property-based testing has its origins in Haskell (Claessen and Hughes 2002) and we argue that for that reason it really shines in pure functional programming. Property-based testing allows to formulate *functional specifications* in code which then the property-testing library (e.g. QuickCheck) tries to falsify by automatically generating random test-data covering as much cases as possible. When an input is found for which the property fails, the library then reduces it to the most simple one. For a technical, in-depth discussion on property-based testing in Haskell we refer to the following papers: (Claessen and Hughes 2002).

We implement custom data-generators for our agent state and environment and its cells and then let QuickCheck generate the random data and us running the agent with the provided data, checking for the properties. An example for such a property is that an agent has starved to death in case its sugar (or spice) level has dropped to 0. The corresponding property-test generates a random agent state and also a random sugar level which we set in the agent state. We then run the function which returns True in case the agent has starved to death. We can then check that this flag is true only if the initial random sugar level was less then or equal 0.

We found that property-based testing works surprisingly well in this context because properties seem to be quite abound here. Also, it is clear to see that this kind of testing is especially well suited to ABS, firstly due to ABS stochastic nature and second because we can formulate specifications, meaning we describe *what* to test instead of *how* to test (again the declarative nature of functional programming shines through). Also the deductive nature of falsification in property-based testing suits very well the constructive nature of ABS.

4.4 Concurrency and parallelism

To see how difficult it was to build a concurrent implementation we took the existing sequential implementation and added concurrency to it using Software Transactional Memory (STM). The main idea behind STM is that instead of locking and synchronising access to shared data, STM executes code-blocks as atomic transactions which either commit successfully in case no dirty-read happened or retries in case the value was changed since its last read. Although STM exists in other languages as well, Haskell's type-system guarantees that retries have no persisting side-effects, which is crucial for the retry-semantics of STM implementations. We have written a separate paper about using STM to implement concurrent ABS TODO cite my paper in TOMACS, thus we will not go into more detail here but refer to it instead.

5 DISCUSSION

Probably the biggest strength is that we can guarantee reproducibility at compile time: given identical initial conditions, repeated runs of the simulation will lead to same outputs. This is of fundamental importance in simulation and addressed in the Sugarscape model: *"... when the sequence of random numbers is specified ex ante the model is deterministic. Stated yet another way, model output is invariant from run to run when*

all aspects of the model are kept constant including the stream of random numbers." (page 28, footnote 16)
- we can guarantee that in our pure functional approach already at compile time.

Refactoring is very convenient and quickly becomes the norm: guided by types (change / refine them) and relying on compiler to point out problems, results in very effective and quick changes without danger of bugs showing up at run-time. This is not possible in Python because of its lack of compiler and types, and much less effective in Java due to its dynamic type-system although through IDE support one arrives also at a highly strong refactoring performance there.

Adding data-parallelism is easy and often requires simply swapping out a data-structure or library function against its parallel version. Concurrency, although still hard, is less painful to address and add in a pure functional setting due to immutable data and explicit side-effects. Further, the benefits of implementing concurrent ABS based on STM has been shown (TODO: cite my own TOMACS paper) which underlines the strength of Haskell for concurrent ABS due to its strong guarantees about retry-semantics.

Testing in general allows much more control and checking of invariants due to the explicit handling of effects - together with the strong static type system, nothing slips the testing-code as everything is explicit. Property-based testing in particular is a perfect match to testing ABS due to the random nature in both and because it supports convenient expressing of specifications. Thus we can conclude that in a pure functional setting, testing is very expressive and powerful and supports working towards an implementation which is very likely to be correct.

The main drawback of our approach is performance, which at the moment does not come close to object-oriented implementations. There are two main reasons for it: first, FP is known for being slower due to higher level of abstractions which are bought by slower code in general and second, updates are the main bottleneck due to immutable data which requires to copy the whole (or subparts) of a data structure in cases of a change. One way to address that problem is to add data-parallelism and concurrency. The first one is easily possible as already explained above, the use of concurrency we have addressed in our STM paper (TODO cite). Another potential improvement of this problem is the use of linear types (TODO: cite linear haskell paper), which allow to annotate a variable with how often it is used within a function. From this a compiler can derive aggressive optimisations, resulting in imperative-style performance but retaining the declarative nature of the code.

6 CONCLUSIONS

Our results strongly hint that our claim that pure functional programming has indeed its place in ABS is valid but for now we conclude that this is only so in cases of high-impact and large-scale simulations which results might have far-reaching consequences e.g. influence policy decisions. The reason is that engineering a proper implementation of a non-trivial ABS model takes substantial effort in pure functional programming due to different techniques required. Still, we plan on distilling the developed techniques of the case-study into a general purpose ABS library. This should make implementing models much easier and quicker and using Haskell attractive for prototyping models as well.

6.1 Further Research

Often, ABS models build on an underlying DES core, especially when they follow an event-driven approach (Meyer 2014) where they need to schedule actions into the future. Due to the time-driven approach of the Sugarscape model, we didn't implement this in our case-study but our pure functional simulation core is conceptually already very close to a DES approach. We plan on extending it to allow true event-based ABS, which we want to feed into our general purpose ABS library as well.

Due to the recursive nature of functional programming we believe that it is also a natural fit to implement recursive simulations as the one discussed in (Gilmer and Sullivan 2000). In recursive ABS agents are able to halt time and 'play through' an arbitrary number of actions, compare their outcome and then to resume time and continue with a specifically chosen action e.g. the best performing or the one in which they haven't died. More precisely, an agent has the ability to run the simulation recursively a number of times where this number is not determined initially but can depend on the outcome of the recursive simulation. So recursive ABS gives each Agent the ability to run the simulation locally from its point of view to anticipate its actions in the future and change them in the present. Due to the lack of implicit side-effects and referential transparency, combined with the recursive nature of pure functional programming, we think that implementing a recursive simulation in such a setting should be straight-forward.

This research is only a first step towards an even more extensive use of type systems in ABS. The next step is to investigate the use of dependent types in ABS, using the pure functional language Idris. Dependent types allow to express even stronger guarantees at compile time, going as far as programs being valid proofs for the correctness of the implemented feature.

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A VALIDATING SUGARSCAPE IN HASKELL

Obviously we wanted our implementation to be correct, which means we validated it against the informal reports in the book. Also we use the work of (Weaver) which replicated Sugarscape in NetLogo and reported on it³.

In addition to the informal descriptions of the dynamics, we implemented tests which conceptually check the model for emergent properties with hypotheses shown and expressed in the book. Technically speaking we have implemented that with unit-tests where in general we run the whole simulation with a fixed scenario and test the output for statistical properties which, in some cases is straight forward e.g. in case of Trading the authors of the Sugarscape model explicitly state that the standard deviation is below 0.05 after 1000 ticks. Obviously one needs to run multiple replications of the same simulation, each with a different random-number generator and perform a statistical test depending on what one is checking: in case of an expected mean one utilises a t-test and in case of standard-deviations a chi-squared test.

A.1 Terracing

Our implementation reproduce the terracing phenomenon as described on page TODO in Animation and as can be seen in the NetLogo implementation as well. We implemented a property-test in which we measure the closeness of agents to the ridge: counting the number of same-level sugars cells around them and if there is at least one lower then they are at the edge. If a certain percentage is at the edge then we accept terracing. The question is just how much, this we estimated from tests and resulted in 45%. Also, in the terracing animation the agents actually never move which is because sugar immediately grows back thus there is no need for an agent to actually move after it has moved to the nearest largest cite in can see. Therefore we test that the coordinates of the agents after 50 steps are the same for the remaining steps.

A.2 Carrying Capacity

Our simulation reached a steady state (variance < 4 after 100 steps) with a mean around 182. Epstein reported a carrying capacity of 224 (page 30) and the NetLogo implementations (Weaver) carrying capacity fluctuates around 205 which both are significantly higher than ours. Something was definitely wrong - the carrying capacity has to be around 200 (we trust in this case the NetLogo implementation and deem 224 an outlier).

After inspection of the NetLogo model we realised that we implicitly assumed that the metabolism range is *continuously* uniformly randomized between 1 and 4 but this seemed not what the original authors intended: in the NetLogo model there were a few agents surviving on sugarlevel 1 which was never the case in ours as the probability of drawing a metabolism of exactly 1 is zero when drawing from a continuous range. We thus changed our implementation to draw a discrete value as the metabolism.

This partly solved the problem, the carrying capacity was now around 204 which is much better than 182 but still a far cry from 210 or even 224. After adjusting the order in which agents apply the Sugarscape rules, by looking at the code of the NetLogo implementation, we arrived at a comparable carrying capacity of the NetLogo implementation: agents first make their move and harvest sugar and only after this the agents metabolism is applied (and ageing in subsequent experiments).

For regression-tests we implemented a property-test which tests that the carrying capacity of 100 simulation runs lies within a 95% confidence interval of a 210 mean. These values are quite reasonable to assume, when

³Note that lending didn't properly work in their NetLogo code and that they didn't implement Combat

looking at NetLogo - again we deem the reported Carrying Capacity of 224 in the Book to be an outlier / part of other details we don't know.

One lesson learned is that even such seemingly minor things like continuous vs. discrete or order of actions an agent makes have substantial impact on the dynamics of a simulation.

A.3 Wealth Distribution

By visual comparison we validated that the wealth distribution (page 32-37) becomes strongly skewed with a histogram showing a fat tail, power-law distribution where very few agents are very rich and most of the agents are quite poor. We compute the skewness and kurtosis of the distribution which is around a skewness of 1.5, clearly indicating a right skewed distribution and a kurtosis which is around 2.0 which clearly indicates the 1st histogram of Animation II-3 on page 34. Also we compute the Gini coefficient and it varies between 0.47 and 0.5 - this is accordance with Animation II-4 on page 38 which shows a gini-coefficient which stabilises around 0.5 after. We implemented a regression-test testing skewness, kurtosis and gini-coefficients of 100 runs to be within a 95% confidence interval of a two-sided t-test using an expected skewness of 1.5, kurtosis of 2.0 and gini-coefficient of 0.48.

A.4 Migration

With the information provided by (Weaver) we could replicate the waves as visible in the NetLogo implementation as well. Also we propose that a vision of 10 is not enough yet and shall be increased to 15 which makes the waves very prominent and keeps them up for much longer - agent waves are travelling back and forth between both Sugarscape peaks. We haven't implemented a regression-test for this property as we couldn't come up with a reasonable straight forward approach to implement it.

A.5 Pollution and Diffusion

With the information provided by (Weaver) we could replicate the pollution behaviour as visible in the NetLogo implementation as well. We haven't implemented a regression-test for this property as we couldn't come up with a reasonable straight forward approach to implement it.

A.6 Mating

Initially we could not replicate Figure III-1 (TODO: page) - our dynamics first raised and then plunged to about 100 agents and go then on to recover and fluctuate around 300. This findings are in accordance with (Weaver), where they report similar findings - also when running their NetLogo code we find the dynamics to be qualitatively the same.

Also at first we weren't able to reproduce the cycles of population sizes. Then we realised that our agent-behaviour was not correct: agents which died from age or metabolism could still engage in mating before actually dying - fixing this to the behaviour that agents which died from age or metabolism won't engage in mating solved that and produces the same swings as in (Weaver). Although our bug might be obvious, the lack of specification of the order of the application of the rules is an issue in the SugarScape book.

A.7 Inheritance

We couldn't replicate the findings of the Sugarscape book regarding the Gini coefficient with inheritance. The authors report that they reach a gini coefficient of 0.7 and above in Animation III-4. Our Gini coefficient fluctuated around 0.35. Compared to the same configuration but without inheritance (Animation III-1) which reached a Gini coefficient of about 0.21, this is indeed a substantial increase - also with inheritance we reach a larger number of agents of around 1,000 as compared to around 300 without inheritance. The Sugarscape book compares this to chapter II, Animation II-4 for which they report a Gini coefficient of around 0.5 which we could reproduce as well. The question remains, why it is lower (lower inequality) with inheritance?

The baseline is that this shows that inheritance indeed has an influence on the inequality in a population. Thus we deemed that our results are qualitatively the same as the make the same point. Still there must be some mechanisms going on behind the scenes which are unspecified in the original Sugarscape.

A.8 Cultural Dynamics

We could replicate the cultural dynamics of AnimationIII-6 / Figure III-8: after 2700 steps either one culture (red / blue) dominates both hills or each hill is dominated by a different culture. We wrote a test for it in which we run the simulation for 2.700 steps and then check if either culture dominates with a ratio of 95% or if they are equal dominant with 45%. Because always a few agents stay stationary on sugarlevel 1 (they have a metabolism of 1 and cant see far enough to move towards the hills, thus stay always on same spot because no improvement and grow back to 1 after 1 step), there are a few agents which never participate in the cultural process and thus no complete convergence can happen. This is accordance with (Weaver).

A.9 Combat

Unfortunately (Weaver) didn't implement combat, so we couldn't compare it to their dynamics. Also, we weren't able to replicate the dynamics found in the Sugarscape book: the two tribes always formed a clear battlefront where some agents engage in combat e.g. when one single agent strays too far from its tribe and comes into vision of the other tribe it will be killed almost always immediately. This is because crossing the sugar valley is costly: this agent wont harvest as much as the agents staying on their hill thus will be less wealthy and thus easier killed off. Also retaliation is not possible without any of its own tribe anywhere near.

We didn't see a single run where an agent of an opposite tribe "invaded" the other tribes hill and ran havoc killing off the entire tribe. We don't see how this can happen: the two tribes start in opposite corners and quickly occupy the respective sugar hills. So both tribes are acting on average the same and also because of the number of agents no single agent can gather extreme amounts of wealth - the wealth should rise in both tribes equally on average. Thus it is very unlikely that a super-wealthy agent emerges, which makes the transition to the other side and starts killing off agents at large. First: a super-wealthy agent is unlikely to emerge, second making the transition to the other side is costly and also low probability, third the other tribe is quite wealthy as well having harvested for the same time the sugar hill, thus it might be that the agent might kill a few but the closer it gets to the center of the tribe the less like is a kill due to retaliation avoidance - the agent will simply get killed by others.

Also it is unclear in case of AnimationIII-11 if the R rule also applies to agents which get killed in combat. Nothing in the book makes this clear and we left it untouched so that agents who only die from age (original R rule) are replaced. This will lead to a near-extinction of the whole population quite quickly as agents kill

each other off until 1 single agent is left which will never get killed in combat because there are no other agents who could kill it - instead it will enter an infinite die and reborn cycle thanks to the R rule.

A.10 Spice

The book specifies for AnimationIV-1 vision between 1-10 and a metabolism between 1-5. The last one seems to be quite strange because the maximum sugar / spice an agent can find is 4 which means that agents with metabolism of either 5 will die no matter what they do because they can never harvest enough to satisfy their metabolism. When running our implementation with this configuration the number of agents quickly drops from 400 to 105 and continues to slowly degrade below 90 after around 1000 steps. The implementation of (Weaver) used a slightly different configuration for AnimationIV-1, where they set vision to 1-6 and metabolism to 1-4. Their dynamics stabilise to 97 agents after around 500+ steps. When we use the same configuration as theirs, we produce the same dynamics. Also it is worth noting that our visual output is strikingly similar to both the book AnimationIV-1 and (Weaver).

A.11 Trading

For trading we had a look at the NetLogo implementation of (Weaver): there an agent engages in trading with its neighbours *over multiple rounds* until MRSs cross over and no trade has happened anymore. Because (Weaver) were able to exactly replicate the dynamics of the trading time-series we assume that their implementation is correct. Unfortunately we think that the fact that an agent interacts with its neighbours over multiple rounds is made not very clear in the book. The only hint is found on page 102: *"This process is repeated until no further gains from trades are possible."* which is not very clear and does not specify exactly what is going on: does the agent engage with all neighbours again? is the ordering random? Another hint is found on page 105 where trading is to be stopped after MRS cross-over to prevent infinite loop. Unfortunately this is missing in the Agent trade rule T on page 105. Additional information on this is found in footnote 23 on page 107. Further on page 107: *"If exchange of the commodities will not cause the agents' MRSs to cross over then the transaction occurs, the agents recompute their MRSs, and bargaining begins anew."* This is probably the clearest hint that trading could occur over multiple rounds.

We still managed to exactly replicate the trading-dynamics as shown in the book in Figure IV-3, Figure IV-4 and Figure IV-5. The book is also pretty specific on the dynamics of the trading-prices standard-deviation: on page 109 the authors specify that at $t=1000$ the standard deviation will have always fallen below 0.05 (Figure IV-5), thus we implemented a property test which tests for exactly that property and the test passed. Unfortunately we didn't reach the same magnitude of the trading volume where ours is much lower around 50 but it is equally erratic, so we attribute these differences to other missing specifications or different measurements because the price-dynamics match that well already so we can safely assume that our trading implementation is correct.

According to the book, Carrying Capacity (Animation II-2) is increased by Trade (page 111/112). To check this it is important to compare it not against AnimationII-2 but a variation of the configuration for it where spice is enabled, otherwise the results are not comparable because carrying capacity changes substantially when spice is on the environment and trade turned off. We could replicate the findings of the book: the carrying capacity increases slightly when trading is turned on. Also does the average vision decrease and the average metabolism increase. This makes perfect sense: trading allows genetically weaker agents to survive which results in a slightly higher carrying capacity but shows a weaker genetic performance of the population.

According to the book, increasing the agent vision leads to a faster convergence towards the (near) equilibrium price (page 117/118/119, Figure IV-8 and Figure IV-9). We could replicate this behaviour as well.

According to the book, when enabling R rule and giving agents a finite life span between 60 and 100 this will lead to price dispersion: the trading prices won't converge around the equilibrium and the standard deviation will fluctuate wildly (page 120, Figure IV-10 and Figure IV-11). We could replicate this behaviour as well.

The Gini coefficient should be higher when trading is enabled (page 122, Figure IV-13) - We could replicate this behaviour.

Finite lives with sexual reproduction lead to prices which don't converge (page 123, Figure IV-14). We could reproduce this as well but it was important to re-set the parameters to reasonable values: increasing number of agents from 200 to 400, metabolism to 1-4 and vision to 1-6, most important the initial endowments back to 5-25 (both sugar and spice) otherwise hardly any mating would happen because the agents need too much wealth to engage (only fertile when have gathered more than initial endowment). What was kind of interesting is that in this scenario the trading volume of sugar is substantially higher than the spice volume - about 3 times as high.

From this part, we didn't implement: Effect of Culturally Varying Preferences, page 124 - 126, Externalities and Price Disequilibrium: The effect of Pollution, page 126 - 118, On The Evolution of Foresight page 129 / 130.

A.12 Diseases

We were able to exactly replicate the behaviour of Animation V-1 and Animation V-2: in the first case the population rids itself of all diseases (maximum 10) which happens pretty quickly, in less than 100 ticks. In the second case the population fails to do so because of the much larger number of diseases (25) in circulation. We used the same parameters as in the book. The authors of (Weaver) could only replicate the first animation exactly and the second was only deemed "good". Their implementation differs slightly from ours: In their case a disease can be passed to an agent who is immune to it - this is not possible in ours. In their case if an agent has already the disease, the transmitting agent selects a new disease, the other agent has not yet - this is not the case in our implementation and we think this is unreasonable to follow: it would require too much information and is also unrealistic. We wrote regression tests which check for animation V-1 that after 100 ticks there are no more infected agents and for animation V-2 that after 1000 ticks there are still infected agents left and they dominate: there are more infected than recovered agents.