

Functional Reactive Agent-Based Simulation

Jonathan Thaler
School of Computer Science
University of Nottingham
jonathan.thaler@nottingham.ac.uk

Thorsten Altenkirch
School of Computer Science
University of Nottingham
thorsten.altenkirch@nottingham.ac.uk

Abstract

Agent-Based Simulation (ABS) is a methodology in which a system is simulated in a bottom-up approach by modelling the micro interactions of its constituting parts, called agents, out of which the global macro system behaviour emerges. So far, the Haskell community hasn't been much in contact with the community of ABS due to the latter's primary focus on the object-oriented programming paradigm. This paper tries to bridge the gap between those two communities by introducing the Haskell community to the concepts of ABS using the simple SIR model from epidemiology. Further we present our library *FrABS* which allows to implement ABS the first time in Haskell. In this library we leverage the basic concepts of ABS with functional reactive programming using Yampa which results in a surprisingly powerful and convenient EDSL for formulating ABS.

Index Terms

Functional Reactive Programming, Agent-Based Simulation

I. INTRODUCTION

In Agent-Based Simulation (ABS) one models and simulates a system by modeling and implementing the pro-active constituting parts of the system, called *Agents* and their local interactions. From these local interactions then the emergent property of the system emerges. ABS is still a young field, having emerged in the early-to-mid 90s primarily in the fields of social simulation and computational economics.

The aim of this paper is to show how ABS can be done in Haskell and what the benefits and drawbacks are. We do this by introducing the SIR model of epidemiology and derive an agent-based implementation for it which is based on our *FrABS* library. By doing this we give the reader a good understanding of what ABS is, what the challenges are when implementing it and how we solved these in our library. We then discuss details which must be paid attention to in our approach and its benefits and drawbacks. The contribution is a novel approach to implementing ABS with powerful time-semantics and more emphasis on specification and possibilities to reason about the correctness of the simulation.

II. BACKGROUND

A. Agent Based Simulation

ABS is a methodology to model and simulate a system where the global behaviour may be unknown but the behaviour and interactions of the parts making up the system is of knowledge. Those parts, called agents, are modelled and simulated out of which then the aggregate global behaviour of the whole system emerges. So the central aspect of ABS is the concept of an agent which can be understood as a metaphor for a pro-active unit, situated in an environment, able to spawn new agents and interacting with other agents in some neighbourhood by exchange of messages [1]. We informally assume the following about our agents:

- They are uniquely addressable entities with some internal state over which they have full, exclusive control.
- They are pro-active which means they can initiate actions on their own e.g. change their internal state, send messages, create new agents, terminate themselves.
- They are situated in an environment and can interact with it.
- They can interact with other agents which are situated in the same environment by means of messaging.

Epstein [2] identifies ABS to be especially applicable for analysing "*spatially distributed systems of heterogeneous autonomous actors with bounded information and computing capacity*". Thus in the line of the simulation types *Statistic*[†], *Markov*[‡], *System Dynamics*[§], *Discrete Event*[⊖], ABS is the most powerful one as it allows to model the following:

- Linearity & Non-Linearity^{†‡⊖} - the dynamics of the simulation can exhibit both linear and non-linear behaviour.
- Time^{†‡⊖} - agents act over time, time is also the source of pro-activity.
- States^{‡⊖} - agents encapsulate some state which can be accessed and changed during the simulation.
- Feedback-Loops^{§⊖} - because agents act continuously and their actions influence each other and themselves, feedback-loops are the norm in ABS.
- Heterogeneity[⊖] - although agents can have same properties like height, sex,... the actual values can vary arbitrarily between agents.



Fig. 1: Transitions in the SIR compartment model.

- Interactions - agents can be modelled after interactions with an environment or other agents, making this a unique feature of ABS, not possible in the other simulation models.
- Spatiality & Networks - agents can be situated within e.g. a spatial (discrete 2d, continuous 3d,...) or network environment, making this also a unique feature of ABS, not possible in the other simulation models.

B. Functional Reactive Programming

TODO: short introduction, following all the other papers which is basically useless because it only scratches the surface... For a good introduction into FRP using Yampa we refer to the papers of [3], [4] and [5].

III. THE SIR MODEL

To explain the concepts of ABS and of our functional reactive approach to it, we introduce the SIR model as a motivating example. It is a very well studied and understood compartment model from epidemiology which allows to simulate the dynamics of an infectious disease spreading through a population. In this model, people in a population of size N can be in either one of three states *Susceptible*, *Infected* or *Recovered* at a particular time, where it is assumed that initially there is at least one infected person in the population. People interact with each other *on average* with a given rate β per time-unit and get infected with a given probability γ when interacting with an infected person. When infected, a person recovers *on average* after δ time-units and is then immune to further infections. An interaction between infected persons does not lead to re-infection, thus these interactions are ignored in this model. This definition gives rise to three compartments with the transitions as seen in Figure 1.

The dynamics of this model over time can be formalized using the System Dynamics (SD) approach which models a system through differential equations. For the SIR model we get the following equations:

$$\begin{aligned}\frac{dS}{dt} &= -infectionRate \\ \frac{dI}{dt} &= infectionRate - recoveryRate \\ \frac{dR}{dt} &= recoveryRate\end{aligned}$$

$$\begin{aligned}infectionRate &= \frac{I\beta S\gamma}{N} \\ recoveryRate &= \frac{I}{\delta}\end{aligned}$$

Solving these equations is then done by integrating over time. In the SD terminology, the integrals are called *Stocks* and the values over which is integrated over time are called *Flows*¹.

$$\begin{aligned}S(t) &= N + \int_0^t -infectionRate \, dt \\ I(t) &= 1 + \int_0^t infectionRate - recoveryRate \, dt \\ R(t) &= \int_0^t recoveryRate \, dt\end{aligned}$$

There exist a huge number of software packages which allow to conveniently express SD models using a visual approach like in Figure 2.

Running the SD simulation over time results in the dynamics as shown in Figure 3 with the given variables.

¹The 1+ in $I(t)$ amounts to the initially infected agent - if there wouldn't be a single infected one, the system would immediately reach equilibrium.



Fig. 2: A visual representation of the SD stocks and flows of the SIR compartment model. Picture taken using AnyLogic Personal Learning Edition 8.1.0.

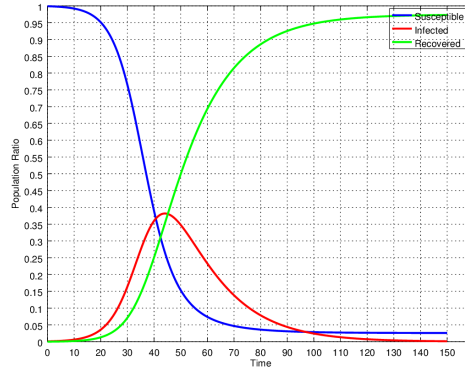


Fig. 3: Dynamics of the SIR compartment model using the System Dynamics approach. Population Size $N = 1000$, contact rate $\beta = \frac{1}{5}$, infection probability $\gamma = 0.05$, illness duration $\delta = 15$ with initially 1 infected agent. Simulation run for 150 time-steps.

An Agent-Based approach

The SD approach is inherently top-down because the emergent property of the system is formalized in differential equations. The question is if such a top-down behaviour can be emulated using ABS, which is inherently bottom-up. Also the question is if there are fundamental drawbacks and benefits when doing so using ABS. Such questions were asked before and modelling the SIR model using an agent-based approach is indeed possible. It is important to note that SD treats the population completely continuous which results in non-discrete values of stocks e.g. 3.1415 infected persons. Thus the fundamental approach to map the SIR model to an ABS is to discretize the population and model each person in the population as an individual agent. The transition between the states are no longer happening according to continuous differential equations but due to discrete events caused both by interactions amongst the agents and time-outs.

- Every agent makes *on average* contact with β random other agents per time unit. In ABS we can only contact discrete agents thus we model this by generating a random event on average every β time units. Note that we need to sample from an exponential CDF because the rate is proportional to the size of the population as [6] pointed out.
- An agent does not know the other agents' state when making contact with it, thus we need a mechanism in which agents reveal their state in which they are in *at the moment of making contact*. Obviously the already mentioned messaging which allows agents to interact is perfectly suited to do this.
 - *Susceptibles*: These agents make contact with other random agents (excluding themselves) with a "Susceptible" message. They can be seen to be the drivers of the dynamics.
 - *Infected*: These agents only reply to incoming "Susceptible" messages with an "Infected" message to the sender. Note that they themselves do *not* make contact pro-actively but only react to incoming one.
 - *Recovered*: These agents do not need to send messages because contacting it or being contacted by it has no influence on the state.
- Transition of susceptible to infected state - a susceptible agent needs to have made contact with an infected agent which happens when it receives an "Infected" message. If this happens an infection occurs with a probability of γ . The infection can be calculated by drawing p from a uniform random-distribution between 0 and 1 - infection occurs in case of $\gamma \geq p$. Note that this needs to be done for *every* received "Infected" message.

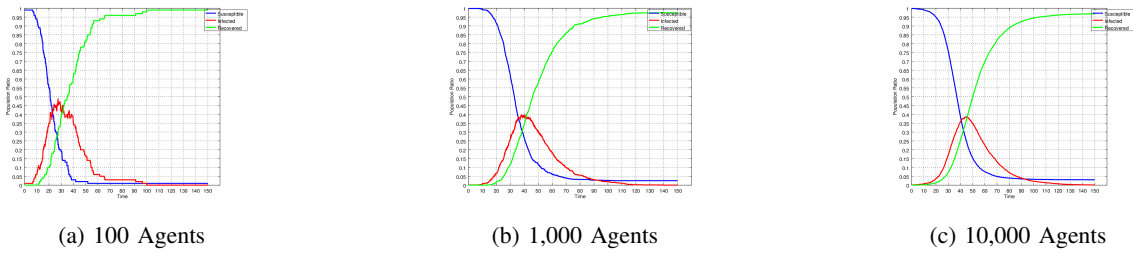


Fig. 4: Approximating the continuous dynamics of the system dynamics simulation using the agent-based approach. Model-parameters are the same ($\beta = \frac{1}{5}$, $\gamma = 0.05$, $\delta = 15$ with initially 1 infected agent) except population size. All simulations run for 150 time-steps.

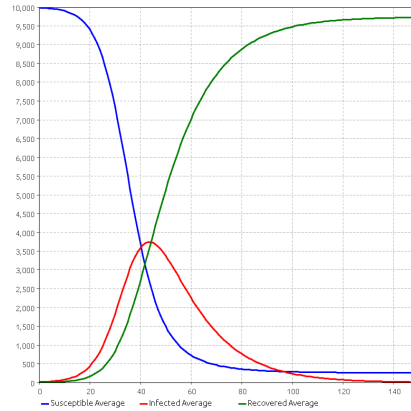


Fig. 5: TODO: replace by my own picture. Dynamics of the SIR compartment model using the agent-based approach with same parameters as in SD ($\beta = 1/5$, $\gamma = 0.05$, $\delta = 15$) but with a population of 10,000 where 10 are initially infected and the dynamics averaged over 4 replications.

- Transition of infected to recovered - a person recovers *on average* after δ time unites. This is implemented by drawing the duration from an exponential distribution [6] with $\lambda = \frac{1}{\delta}$ and making the transition after this duration.

In Figure 4 we give the dynamics simulating the SIR model with the agent-based approach.

As previously mentioned the agent-based approach is a discrete one which means that with increasing number of agents, the discrete dynamics approximate the continuous dynamics of the SD simulation. Still the dynamics of 10,000 Agents do not match the dynamics of the SD simulation perfectly. This is because as opposed to the SD simulation the agent-based approach is inherently a stochastic one as we continuously draw from random-distributions which drive our state-transitions. What we see in Figure 4 is then just a single run where the dynamics would result in slightly different shapes when run with a different random-number generator seed. The agent-based approach thus generates a distribution of dynamics over which ones needs to average to arrive at the correct solution. This can be done using replications in which the simulation is run with the exact same parameters multiple times but each with a different random-number generator see. The resulting dynamics are then averaged and the result is then regarded as the correct dynamics. We have done this as can be seen in Figure 5, using 4 replications, which matches the SD dynamics to a very satisfactory level ².

TODO: replace by my own pictures TODO: run replications with 1000 agents for 100 repls TODO: run replications with 10,000 agents for 10 repls

For a more in-depth introduction of how to approximate an SD model by ABS see [7] who discusses a general approach and how to compare dynamics and [6] which explain the need to draw the illness-duration from an exponential-distribution.

²Note that in the replications we are using 10 initially infected agents to ensure that no simulation run will terminate too early (meaning that the disease gets extinct after a few time steps) which would offset the dynamics completely. This happens due to "unlucky" random distributions which can be repaired by introducing more initially infected agents which increases the probability of spreading the disease in the very early stage of the simulation drastically. We found that when using 10 initially infected agents in a population of 10,000 (which amounts to 0.1%) is enough to never result in an early terminating simulation. This is also a fundamental difference between SD and ABS: the dynamics of the agent-based approach can result in a wide range of scenarios which includes also the one in which the disease gets extinct in the early stages (a lucky coincidence for mankind) - this is simply not possible in the SD approach. So we can argue that ABS is much closer to reality than SD as it allows to explore alternate futures in the dynamics.

We will derive the implementation of this approach in the next section, with the complete code provided in Appendix A. As will be seen our approach allows us express this behaviour very explicitly and is looking very much like a formal ABS specification of the problem.

IV. FUNCTIONAL REACTIVE ABS

TODO: cross reference to the code in the appendix by giving line-numbers e.g. the full implementation of a susceptible agent can be seen in

The challenges one faces when implementing an ABS plain, without support from a library are manifold. In the paper (TODO: cite my own paper on update-strategies) the authors discuss already the fundamental things to consider in a programming language agnostic way. Generally one faces the following challenges:

- Agent Representation - how do we represent an agent? The ABS community implements agents as objects (as in Java, Python or C++) as they claim that the mapping of an agent on an object is natural. The question is how to represent an agent in Haskell?
- Agent-Agent Interaction - how can agents interact with other agents? In object-orientation we have method-calls which allows to call other objects and mutate their state. Also this is not available in Haskell, so how do we solve this problem without resorting to the IO monad?
- Environment representation - how can we represent an environment? Also an environment must have the ability to update itself e.g. regrow some resources.
- Agent-Environment interaction - how can agents interact (read / write) with the environment they are situated in?
- Agent Updating - how is the set of agents organised, how are they updated and how is it managed (deleting, adding during simulation)? In object-oriented implementations due to side-effects and mutable data in-order updates are easily done but this is not available in Haskell without resorting to the IO monad.

In the next subsections we will discuss each point by deriving a functional reactive implementation of the agent-based SIR model. For us it is absolutely paramount that the simulation should be pure and never run in the IO Monad (except of course the surrounding Yampa loop which allows rendering and output). The complete source-code can be seen in Appendix TODO: ref.

A. Agent Representation

An agent can be seen as a tuple $\langle id, s, m, e, b \rangle$.

- **id** - the unique identifier of the agent
- **s** - the generic state of the agent
- **m** - the set of messages the agent understands
- **e** - the type of the environment the agent is situated in and can interact with
- **b** - the behaviour of the agent

The id is simply represented as an Integer and must be unique for all currently existing agents in the system as it is used for message-delivery. A stronger requirement would be that the id of an agent is unique for the whole simulation-run and will never be reused - this would support replications and operations requiring unique agent-ids.

Each agent may have a generic state which could be any data type or compound data. A SIR agent's state can be represented using the an AGDT as follows:

```
data SIRState = Susceptible | Infected | Recovered
```

The behaviour of the agent is a signal-function which maps a tuple of an AgentIn and the environment to an AgentOut and the environment. It has the following signature:

```
type AgentBehaviour s m e = SF (AgentIn s m e, e) (AgentOut s m e, e)
```

AgentIn provides the necessary data to the agent-behaviour: its id, incoming messages, the current state s and a random-number generator.

AgentOut allows the agent to communicate changes: kill itself, create new agents, sending messages, an updated state s and a changed random-number generator.

The behaviour also gets the environment passed in, which the agent can read and also write by changing it and returning it along side the AgentOut. It is important to note, that the environment is completely generic and we do not induce any type-bounds on it.

Obviously AgentIn is read-only ³ whereas AgentOut is both read- and write-able. The first thing an agent-behaviour does is creating the default AgentOut from the existing AgentIn using the function.

```
agentOutFromIn :: AgentIn s m e -> AgentOut s m e
```

³Of course one could change it, but the changes are never propagated out of the function thus ultimately making them never happen due to lazy evaluation.

This will copy the relevant fields over to `AgentOut` on which one now primarily acts. The read-only and read/write character of both types is also reflected in the EDSL where most of the functions implemented also work that way: they may read the `AgentIn` and read/write an `AgentOut`.

Relevant functions for working on the agent-definition are:

```
kill :: AgentOut s m e -> AgentOut s m e
isDead :: AgentOut s m e -> Bool
onStart :: (AgentOut s m e -> AgentOut s m e) -> AgentIn s m e -> AgentOut s m e -> AgentOut s m e

setDomainState :: s -> AgentOut s m e -> AgentOut s m e
updateDomainState :: (s -> s) -> AgentOut s m e -> AgentOut s m e

createAgent :: AgentDef s m e -> AgentOut s m e -> AgentOut s m e
```

The function *kill* marks an agent for removal after the current iteration. The function *isDead* checks if the agent is marked for removal. The function *onStart* allows to change the `AgentOut` in the case of the start-event which happens on the very first time the agent runs. The function *setDomainState* allows to change the state of the agent by overriding it and *updateDomainState* allows to change it by keeping parts of it. The function *createAgent* allows to add an agent-definition⁴ to the `AgentOut` which results in creating a new agent from the given definition which will be active in the next iteration of the simulation.

Having these functions we build some reactive primitives into our EDSL meaning that they return signal-functions themselves. We start with the following functions:

```
doOnce :: (AgentOut s m e -> AgentOut s m e) -> SF (AgentOut s m e) (AgentOut s m e)
doOnceR :: AgentBehaviour s m e -> AgentBehaviour s m e
doNothing :: AgentBehaviour s m e

setDomainStateR :: s -> AgentBehaviour s m e
updateDomainStateR :: (s -> s) -> AgentBehaviour s m e
```

The *doOnce* function may seem strange at first but allows conveniently make actions (which are changing the agent-out) only once e.g. when making the transition from Susceptible to Infected changing the state to Infected just once. A more striking example would be to send a message just once after a transition. The *doOnceR* function is the reactive version, which allows to do a complete agent-behaviour only once. The function *doNothing* provides a convenient way of an agent-sink which is basically an agent which does literally nothing - the resulting agent-behaviour just transforms the agent-in to agent-out using the previously mentioned function *agentOutFromIn*.

Often we want some more reactive behaviour e.g. making a transition from one behaviour to another on a given event. For this we provide the following:

```
type EventSource s m e = SF (AgentIn s m e, AgentOut s m e) (AgentOut s m e, Event ())
transitionOnEvent :: EventSource s m e -> AgentBehaviour s m e -> AgentBehaviour s m e -> AgentBehaviour s m e
```

The function *transitionOnEvent* takes an event-source which creates the event, an agent-behaviour which is run until the event hits and an agent-behaviour which is run at the event and after. The event-source is a signal-function itself to allow maximum of flexibility and gets both agent-in and agent-out and returns a (potentially changed) agent-out and the event upon to switch. The workings of this functionality can be seen for implementing the susceptible agent where we use a *transitionOnEvent* and a specific event-source which generates an event when the susceptible agent got infected.

Sometimes we need our transition event to rely on time-semantics e.g. in SIR where an infected agent recovers *on average* after δ time-units. For this we provide the following function

```
transitionAfterExp :: RandomGen g => g
                    -> Double
                    -> AgentBehaviour s m e
                    -> AgentBehaviour s m e
                    -> AgentBehaviour s m e
```

It takes a random-number generator, the *average* time-out, the behaviour to run before the time-out and the behaviour to run after the time-out where the function will return then the according behaviour. For implementing this behaviour we initially used Yampas *after* function which generates an event after given time-units but this would not result in the correct dynamics as we rather need to create a random-distribution of time-outs than a deterministic time-out which occurs always after the same time. For this we implemented our own function, called *afterExp*, which now takes a random-number generator a time-out and some value of type `b` and creates a signal-function which ignores its input and creates an event *on average* after `DTime`.

```
afterExp :: RandomGen g => g -> DTime -> b -> SF a (Event b)
```

B. Agent-Agent Interaction

Agent-agent interaction is the means of an agent to directly address another agent and vice versa. Inspired by the actor model we implement a *messaging* with share nothing semantics. In this case the agent send messages which will arrive in the

⁴AgentDef simply contains the initial state, behaviour and id of the agent (amongst others).

next step of the simulation at the receiver thus being kind of asynchronous - a round-trip would always take at least two steps, independent of the sampling time. Depending on the semantics of the model we sometimes need synchronous interactions e.g. when only one agent can change the environment or decisions need to be made within one step - this wouldn't be possible with the asynchronous messaging. For this we introduced the concept of *conversations* which allow two agents to interact with each other for an arbitrary number of requests and replies without the simulation being advanced - time is halted and only the two agents are active until they finish their conversation.

1) *Messaging*: Each Agent can send a message to an other agent through AgentOut-Signal where incoming messages are queued in the AgentIn-Signal and can be processed when the agent is active the next time. The agent is free to ignore the messages and if it does not process them they will simply be lost. This is in fundamental contrast to the actor model where messages stay in the message-box of the receiving actor until the actor has processed them. We chose a different approach as time has a different meaning in ABS than in a system of actors where there is basically no global notion of time. Note that due to the fact we don't have method-calls in FP, messaging will always take some time, which depends on the sampling interval of the system. This was not obviously clear when implementing ABS in an object-oriented way because there we can communicate through method calls which are a way of interaction which takes no simulation-time. We need a set of messages *m* the agents understand. In the case of the SIR model we simply use the following:

```
data SIRMsg = Contact SIRState
```

In addition we provide the following functions in our EDSL to support messaging.

```
type AgentMessage m = (AgentId, m)
```

```
sendMessage :: AgentMessage m -> AgentOut s m e -> AgentOut s m e
sendMessageTo :: AgentId -> m -> AgentOut s m e -> AgentOut s m e
sendMessages :: [AgentMessage m] -> AgentOut s m e -> AgentOut s m e
broadcastMessage :: m -> [AgentId] -> AgentOut s m e -> AgentOut s m e
```

```
hasMessage :: (Eq m) => m -> AgentIn s m e -> Bool
onMessage :: (AgentMessage m -> acc -> acc) -> AgentIn s m e -> acc -> acc
onMessageFrom :: AgentId -> (AgentMessage m -> acc -> acc) -> AgentIn s m e -> acc -> acc
onMessageType :: (Eq m) => m -> (AgentMessage m -> acc -> acc) -> AgentIn s m e -> acc -> acc
```

Most of the functions are pretty self-explanatory, we will shortly explain the *onMessage**. The function *onMessage* provides a way to react to incoming messages by using a callback function which manipulates an accumulator, thus resembling the workings of fold. The functions *onMessageFrom* and *onMessageType* provide the same functionality but filter the messages accordingly. We can now write a function which allows an agent to reply to an incoming Contact message with another contact message of a given SIRstate. TODO: refer to the appendix and give line-numbers instead of inserting it here.

Sometimes we also need discrete semantics like changing the behaviour of an agent on reception of a specific message. For this we provide the function *transitionOnMessage* which works the same way as *transitionOnEvent* but now on a message instead.

```
transitionOnMessage :: (Eq m) => m -> AgentBehaviour s m e -> AgentBehaviour s m e -> AgentBehaviour s m e
```

Of course messaging sometimes may have specific time-semantics as in our SIR model. There susceptible agents make contact with *n* other agents on average *per time unit*. To implement this we randomly need to generate messages with a given frequency within some time-interval by drawing from the exponential random-distribution. This is already supported by Yampa using *occasionally* and we have built on it a the following:

```
type MessageSource s m e = e -> AgentOut s m e -> (AgentOut s m e, AgentMessage m)
```

```
sendMessageOccasionallySrc :: RandomGen g => g
-> Double
-> MessageSource s m e
-> SF (AgentOut s m e, e) (AgentOut s m e)
```

```
constMsgReceiverSource :: m -> AgentId -> MessageSource s m e
randomNeighbourNodeMsgSource :: m -> MessageSource s m (Network l)
randomNeighbourCellMsgSource :: (s -> Discrete2dCoord) -> m -> Bool -> MessageSource s m (Discrete2d AgentId)
randomAgentIdMsgSource :: m -> Bool -> MessageSource s m [AgentId]
```

The function *sendMessageOccasionallySrc* takes a random-number generator, the frequency of messages to generate *on average per time-unit* a message-source and returns a signal-function which takes a tuple of an agent-out and environment and returns an agent-out. This signal-function which performs the actual generating of the messages needs to be fed in the tuple but only returns the changed agent-out but not the environment - this guarantees statically at compile-time that the environment cannot be changed in this process. This is also directly reflected in the type of *MessageSource* which takes an environment and agent-out and returns a tuple with a changed agent-out and a message. We provide pre-defined messages-sources like *constMsgReceiverSource* which always generates the same message, *randomNeighbourNodeMsgSource* which picks a random neighbour from a network-environment (see below), *randomNeighbourCellMsgSource* which picks a random neighbour from a discrete 2d-grid environment (see below) and *randomAgentIdMsgSource* which randomly picks an element from an environment

which is a list of AgentIds (omitting the sender True/False). As can be seen in TODO: refer to appendix the susceptible agent makes use of this function.

2) *Conversations*: The messaging as implemented above works well for one-directional, virtual asynchronous interaction where we don't need a reply at the same time. A perfect use-case for messaging is making contact with neighbours in the SIRS-model: the agent sends the contact message but does not need any response from the receiver, the receiver handles the message and may get infected but does not need to communicate this back to the sender. A different case is when agents need to transact in the time-step one or multiple times: agent A interacts with agent B where the semantics of the model (and thus messaging) need an immediate response from agent B - which can lead to further interactions initiated by agent A. The Sugarscape model has three use-cases for this: sex, warfare and trading amongst agents all need an immediate response (e.g. wanna mate with me?, I just killed you, wanna trade for this price?). The reason is that we need to transact now as all of the actions only work on a 1:1 relationship and could violate resource-constraints. For this we introduce the concept of a conversation between agents. This allows an agent A to initiate a conversation with another agent B in which the simulation is virtually halted and both can exchange an arbitrary number of messages through calling and responding without time passing (something not possible without this concept because in each iteration the time advances). After either one agent has finished with the conversation it will terminate it and the simulation will continue with the updated agents (note the importance here: *both* agents can change their state in a conversation). The conversation-concept is implemented at the moment in the way that the initiating agent A has all the freedom in sending messages, starting a new conversation,... but that the receiving agent B is only able to change its state but is not allowed to send messages or start conversations in this process. Technically speaking: agent A can manipulate an AgentOut whereas agent B can only manipulate its next AgentIn. When looking at conversations they may look like an emulation of method-calls but they are more powerful: a receiver can be unavailable to conversations or simply refuse to handle this conversation. This follows the concept of an active actor which can decide what happens with the incoming interaction-request, instead of the passive object which cannot decide whether the method-call is really executed or not.

```

type AgentConversationReply s m e = Maybe (m, AgentIn s m e, e)

type AgentConversationReceiver s m e = (AgentIn s m e
    -> e
    -> AgentMessage m
    -> AgentConversationReply s m e)

type AgentConversationSender s m e = (AgentOut s m e
    -> e
    -> Maybe (AgentMessage m)
    -> (AgentOut s m e, e))

conversation :: AgentMessage m
    -> AgentConversationSender s m e
    -> AgentOut s m e
    -> AgentOut s m e

conversationEnd :: AgentOut s m e -> AgentOut s m e

```

The agent-conversations sender is the initiator of the conversation which can only be an agent which is just run and has started a conversation with a call to the function *conversation*. After the agent has run, the simulation-system will detect the request for a conversation and start processing it by looking up the receiver and calling the functions with passing the values back and forth. While a conversation is active, time does not advance and other agents do not act. Note that due to its nature, conversations are only available when using the *sequential* update-strategy (see below).

C. Environment representation

Agents are situated within an environment which itself is *not* an agent - although it can have its own behaviour e.g. regrowing resources, it cannot send or receive messages from agents. Thus we treat the environment completely generic by allowing any given type captured in the type-variable *e*. Environment behaviour is optional but if required, implemented using a signal-function which simply maps *e* to *e*:

```

type EnvironmentBehaviour e = SF e e

```

By allowing a signal-function as the environment behaviour gives us the opportunity to implement reactive behaviour and time-semantics in environments as well. Again it is essential to note that throughout the whole simulation implementation we never put any bounds on the environment nor make assumptions about its type (except for providing utility-functions which allow agents to operate on existing environment implementations).

Although environments can be anything (even be of unit-type, if no environment is required at all), there exist a few standard environments in ABS which are provided in all ABS packages. We provide implementations for them and discuss them below. Note that we don't provide the APIs of the environments here as it is out-of-scope of this paper.

1) *Network*: A network environment gives agents access to a network, represented by a graph where the nodes are agent-ids and the edges represent neighbourhood information. We implemented fully-connected, Erdos-Renyi and Barabasi-Albert networks. In our case the networks are undirected and the labels can be labelled, carrying arbitrary data or being unlabelled, having unit-type. Agents can then perform the usual graph algorithms on these networks.

2) *Discrete 2D*: A discrete 2d environment gives agents access to a 2d-grid with dimensions of $N \times M \in \mathbb{N}$ cells. The cells are of a generic type c and can thus be anything from AgentIds to resource-sites with single or multiple occupants. Such an environment has a defined neighbourhood of either Moore (8 neighbours) or Von-Neumann (4 neighbours). Agents can then query the environment for cells using neighbourhoods, radius or specific positions, change the cells and update them in the environment.

3) *Continuous 2D*: A continuous 2d environment gives agents access to a continuous 2d-space with dimensions of $N \times M \in \mathbb{R}$. This space can contain an arbitrary number of objects of the generic type o where each of them has a coordinate within this space. Agents can query for objects within a given radius, add, remove and update them. Also we provide functions to move objects either in a given or random direction.

D. Agent-Environment interaction

In ABS agents almost always are situated within an environment. We follow a subtle different approach and implement it in a way that agents have access to a generic environment of type e as discussed above instead of being situated within. It is important to note the subtle difference of agents *having access* to the environments instead of *being situated* within them. This allows to free us making assumptions within an environment how agents use these environments and also allows us to stack multiple environments e.g. agents moving on a discrete 2d-grid but relying on neighbourhood from a network. Our SIR implementation uses a list of all AgentIds as the environment which means that every agent knows all the existing agents of the simulation and can address them. We could have used a Network environment using a fully-connected graph but the memory-consumptions of the library FGL we are using for graphs are unacceptable in case of fully-connected networks of a larger numbers of agents (10,000). Each agent gets the environment passed in through the AgentIn and can change it by passing a changed version of the environment out through AgentOut.

E. Agent Updating

For agents to be pro-active, they need to be able to perceive time. Also agents must have the opportunity to react to incoming messages and manipulate the environment. The work of (TODO: cite our own paper on update-strategies) identified four possible ways of doing this where we only implemented the *sequential*- and *parallel-strategy*⁵. Implementing these iteration-strategies using Haskell and FRP is not as straight-forward as in e.g. Java because one does not have mutable data which can be updated in-place. We implement both update-strategies basically by running all agents behaviour signal-functions every δt , so when running a simulation for a duration of t the number of steps is $\frac{t}{\delta t}$. It is important to realise that in our approach of a single behaviour function we merge both pro-activity, time-dependent behaviour and message-receiving behaviour. A different approach would be to have callbacks for messages in addition to the normal agent-behaviour but this would be quite cluttered and unelegant.

In both the sequential and parallel update-strategy after one iteration there is one single environment left. An environment can have an optional behaviour which allows the environment to update its cells. This is a regular SF thus having also the time of the simulation available. Note that the environment cannot send messages to agents because it is not an agent itself. An example of an environment behaviour would be to regrow some good on each cell according to some rate per time-unit (inspired by SugarScape regrowing of Sugar).

1) *Sequential*: In this strategy the agents are updated one after another where the changes (messages sent, environment changed,...) of one agent are visible to agents updated after. Basically this strategy is implemented as a variant of fold which allows to feed output of one agent (e.g. messages and the environment) forward to the other agents while iterating over the list of agents. For each agent the agent-behaviour signal-function is called with the current AgentIn as input to retrieve the according AgentOut. The messages of the AgentOut are then distributed to the receivers AgentIn. The environment of the agent, which is passed in through AgentIn and returned through AgentOut will then be passed forward to the next agent $i + 1$ AgentIn in the current iteration. The last environment is then the final environment in the current iteration and will be returned by the callback function together with the current AgentOuts. As previously mentioned, conversations are *only* possible within this update-strategy because only in this strategy agents act after another which is a fundamental requirement for conversations to make sense and work correctly.

⁵The other two strategies being the *concurrent*- and *actor-strategy*, both requiring to run within the STM-Monad, which is not possible with Yampa. The work of Ivan Perez in [8] implemented a library called Dunai, which is the same as Yampa but capable of running in an arbitrary Monad. We leave this for further research.

2) *Parallel*: The parallel strategy is *much* easier to implement than the sequential but is of course not applicable to all models because of its different semantics. Basically this strategy is implemented as a map over all agents which calls each agent-behaviour signal-function with the agents `AgentIn` to retrieve the new `AgentOut`. Then the messages are distributed amongst all agents. Each agent receives a copy of the environment upon which it can work and return a changed one. Thus after one iteration there are n versions of environments where n is equal to the number of agents. These environments must then be collapsed into a final one which is always domain-specific thus the model implementer needs to provide a corresponding function.

```
type EnvironmentCollapsing e = [e] -> e
```

Of course not all models have environments which can be changed and in the SIR model we indeed use a list of `AgentIds` which won't change during execution (meaning, the agents only read it). Because of this environment-collapsing functions are optional and when none is provided the environments returned by the agents are ignored and always the initial one is provided. To make this more explicit we introduce a wrapper which wraps a signal-function which is the same as agent-behaviour but omits the environment from the out tuples. When this wrapper is used one can guarantee statically at compile-time that the environment cannot be changed by the agent-behaviour. We also provide a function which completely ignores the environment, which allows to reason already at compile time that no environment access will happen ever.

```
type ReactiveBehaviourIgnoreEnv s m e = SF (AgentIn s m e) (AgentOut s m e)
type ReactiveBehaviourReadEnv s m e = SF (AgentIn s m e, e) (AgentOut s m e)
```

```
ignoreEnv :: ReactiveBehaviourIgnoreEnv s m e -> AgentBehaviour s m e
readEnv :: ReactiveBehaviourReadEnv s m e -> AgentBehaviour s m e
```

We use both functions in our SIR implementation TODO: refer to appendix.

F. Non-Reactive Features

Not all models have such explicit time-semantics as the SIR model. Such models just assume that the agents act in some order but don't rely on any time-outs, timed transitions or rates. These models are more of imperative nature and map therefore naturally to a monadic style of programming using the `do`-notation. Unfortunately we don't have this functionality available within a SF thus to support the programming of such imperative models, we implemented wrapper-functions which allow to provide both non-monadic and monadic functionality which runs within a wrapper-signal function.

1) Non-monadic (pure) wrapping:

```
agentPure :: (e -> Double -> AgentIn s m e -> AgentOut s m e -> (AgentOut s m e, e)) -> AgentBehaviour s m e
agentPureReadEnv :: (e -> Double -> AgentIn s m e -> AgentOut s m e -> AgentOut s m e) -> AgentBehaviour s m e
agentPureIgnoreEnv :: (Double -> AgentIn s m e -> AgentOut s m e -> AgentOut s m e) -> AgentBehaviour s m e
```

The function `agentPure` wraps a non-monadic (pure) function and wraps it in a SF. This pure function has the following arguments: the environment, the current local time of the agent, the `AgentIn`, a default `AgentOut` and must return the tuple of `AgentOut` and the environment.

The function `agentPureReadEnv` works the same as `agentPure` but omits the environment from the return-type thus ensuring statically at compile-time that an agent which implements its behaviour in this way can only read the environment but never change it.

The function `agentPureIgnoreEnv` is an even stricter version of `agentPureReadEnv` and omits environment from the agents behaviour altogether thus ensuring statically at compile-time that an agent which wraps its behaviour in this function will never access the environment.

2) *Monadic wrapping*: The monadic wrappers work basically the same way as the pure version, with the difference that they run within the state-monad with `AgentOut` being the state to pass around.

```
agentMonadic :: (e -> Double -> AgentIn s m e -> State (AgentOut s m e) e) -> AgentBehaviour s m e
agentMonadicReadEnv :: (e -> Double -> AgentIn s m e -> State (AgentOut s m e) ()) -> AgentBehaviour s m e
agentMonadicIgnoreEnv :: (Double -> AgentIn s m e -> State (AgentOut s m e) ()) -> AgentBehaviour s m e
```

We also provide monadic versions of the pure primitives of our EDSL which work on `AgentOut`, discussed above. They run in the state-monad where the state is the agent-out as this is the data which is manipulated step-by-step for the final output.

```
agentIdM :: State (AgentOut s m e) AgentId

sendMessageM :: AgentMessage m -> State (AgentOut s m e) ()
sendMessageToM :: AgentId -> m -> State (AgentOut s m e) ()
onMessageM :: (Monad mon) => (acc -> AgentMessage m -> mon acc) -> AgentIn s m e -> acc -> mon acc

createAgentM :: AgentDef s m e -> State (AgentOut s m e) ()

killM :: State (AgentOut s m e) ()
isDeadM :: State (AgentOut s m e) Bool

getDomainStateM :: State (AgentOut s m e) s
updateDomainStateM :: (s -> s) -> State (AgentOut s m e) ()
```

```

setDomainStateM :: s -> State (AgentOut s m e) ()
domainStateFieldM :: (s -> t) -> State (AgentOut s m e) t

For the environment-behaviour we provide a monadic wrapper as well.
type EnvironmentMonadicBehaviour e = (Double -> State e ())
environmentMonadic :: EnvironmentMonadicBehaviour e -> EnvironmentBehaviour e

```

G. Randomness

Most of the ABS are inherent stochastic processes and so is the agent-based SIR implementation. This means that agents behaviour depends on random-numbers. So far the drawing from these random-number was hidden behind functions but sometimes an agent needs to draw a random-number directly. For this we provide each agent with its own random-number generator which must be initialized when creating the agent-definition. The agent can then draw random-numbers in a pure way without having to resort to the IO Monad. Still it can become very cumbersome because the changed random-number generator needs to be updated - for this we provide a few convenience functions and monadic implementations.

TODO: discuss random functions for agents.

V. SAMPLING THE SYSTEM

When sampling the system, the correct time-delta must be selected which depends on the highest frequency which occurs in a time-reactive function in the whole system. For example in the FrSIR model we want infected agents to make on average contact with 5 other agents per time-unit, which means with a frequency of $\frac{1}{5}$. This functionality is built on Yampas function occasionally which behaviour we investigated under differing time-deltas with the above frequency. In this investigation we simply sampled occasionally with different time-deltas for a duration of 1000 time-units and the event-frequency of $\frac{1}{5}$. The results can be seen in Figure 6c and are quite striking. The plot clearly shows that occasionally needs quite high sampling frequency even for comparatively low event-frequency - this becomes of course worse for higher event-frequencies.

The other time-reactive function which occurs in the FrSIR model is the timed transition from infected to recovered which occurs on average with an exponential random-distribution after 15 time-units. This functionality is built on a custom implementation of Yampas after which creates an event after a time-out of the passed in time-duration drawn from an exponential random-distribution. Clearly this function has different semantics as although it also continuously emit events over time - NoEvent before the time was hit, and Event x after the time hit - the relevant point is that it switches to Event at some discrete point in time. This is implemented as simply adding up the time-deltas until the accumulator is GE than the previously drawn exponential time-out. We also investigated the behaviour of this function under varying time-deltas using a time-out of 15 (drawn from an exponential distribution within the function). Our approach was to sample the afterExp until an event occurs (this is one of the occasions where lazy evaluation really shines as one simply repeats the time-delta stream forever but then searches for the first occurrence of an event, which MUST occur at some point due to mathematical exponential distribution and our parameters to it, so it will always terminate) and then see when it has occurred. We run this with 10,000 replications with different random-number seeds and average the resulting times. The results can be seen in Figure 6a. The result is striking in another way: this function seems to be pretty invariant to the time-deltas, for obvious reasons: we are basically just interested in the "after"-condition of the whole semantics whereas in occasionally we are interested in the "repeatedly"-conditions. If we under the afterExp then we can be off by one time-delta. If we under sample occasionally we keep losing events, the close time-delta and event-frequency are, the more we lose. Of course afterExp can also be used for very short time-outs e.g. $\frac{1}{15}$. We have investigated the behaviour of this function for various time-deltas as well as seen in Figure ???. Here the result is much more striking and shows that afterExp is vulnerable to small time-outs as well as occasionally. To show that occasionally is not vulnerable to very low frequencies of e.g. one event every 5 time-steps we plotted the behaviour of this under varying time-steps in Figure 6d. The result shows that for low frequencies occasionally works fine with larger time-deltas

Super-Sampling

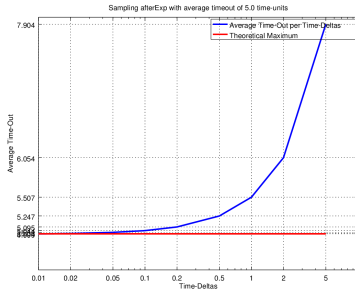
of course performance is a big issue and it decreases as time-deltas get smaller and smaller. if we could perform subsampling just for the given high-frequency function with the remaining system running in lower frequency then we could achieve substantial performance-increase. TODO: talk a little bit more about the costs of too small time-deltas: need to go through all agents all the time

formula of calculating-steps = steps per time-unit * time-to-run-the-simulation

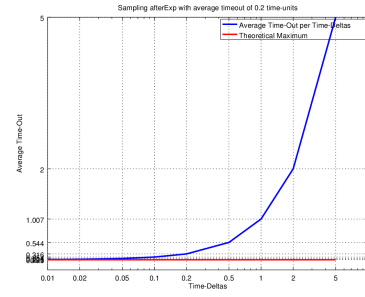
The problem is that *embed* does not really help because when running a sf with it, the signal-functions time does not advance. Thus we implemented a new signal-function which allows us to super-sample another signal-function.

```
superSampling :: Int -> SF a b -> SF a [b]
```

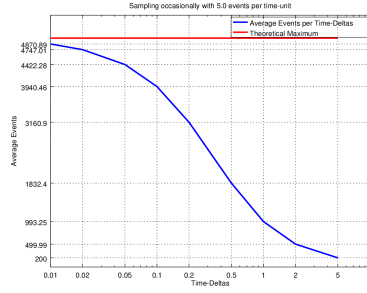
It takes the number of super-samples n and the signal-function sf to sample and returns a new signal-function which performs the super-sampling. It does this by evaluating sf for n times with dt of $\frac{dt}{n}$ and the same input argument a for all n evaluations. At time 0 no super-sampling is done and just a single output of sf is calculated. A list of b is returned with length of n containing the result of the n evaluations of sf . If 0 or less super samples are requested exactly one is calculated.



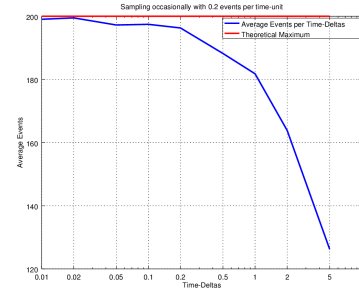
(a) Sampling *afterExp* with an average time-out of 5.



(b) Sampling *afterExp* with an average time-out of 0.2.



(c) Sampling *occasional* with a frequency of $\frac{1}{5}$ (average of 5 events per time-unit). The theoretical average is 5000 events within this time-frame.



(d) Sampling *occasional* with a frequency of 5 (average of 0.2 events per time-unit). The theoretical average is 200 events within this time-frame.

Fig. 6: Sampling the *afterExp* and *occasional* functions to visualize the influence of sampling frequencies on the occurrence of the respective events. Time-deltas are $[5, 2, 1, \frac{1}{2}, \frac{1}{5}, \frac{1}{10}, \frac{1}{20}, \frac{1}{50}, \frac{1}{100}]$. The experiments for *afterExp* used 10,000 replications. The experiments for *occasional* ran for 1000 time-units with 100 replications.

We ran tests super-sampling both *occasionally* (Figure 7a, Figure 7b) and *afterExp* (Figure). They work the same way as above except that now the time-delta is fixed to 1.0 but using increasing numbers of super-samples. The results are as expected: as the number of super-samples increase, so increases the accuracy.

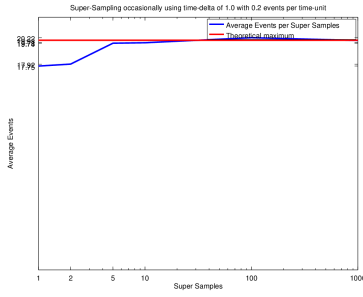
At first this might not seem to be a real win as we still need to calculate a big number of samples every time. The big win comes though when these super-sampled signal-functions are embedded in a larger system which could run on a comparatively low frequency of e.g. 1 dt. So we are then increasing the sampling-frequency just where we need it and keep the frequency low where it is not required.

1) *Super Sampling in SIR*: We are using super-sampling in our SIR implementation to increase performance. We do this by keeping the time-delta at 1.0 and super-sampling the relevant functions with time-semantics which are *transitionAfterExp* and *sendMessageOccasionallySrc*. For both we provide in our EDSL versions which support super-sampling:

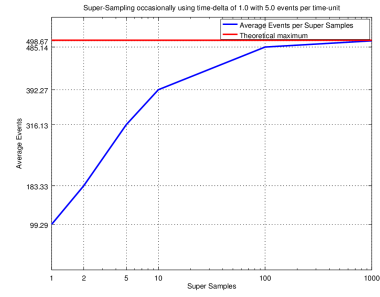
```
sendMessageOccasionallySrcSS :: RandomGen g => g
    -> Double
    -> Int
    -> MessageSource s m e
    -> SF (AgentOut s m e, e) (AgentOut s m e)

transitionAfterExpSS :: RandomGen g =>
    g
    -> Double
    -> Int
    -> AgentBehaviour s m e
    -> AgentBehaviour s m e
    -> AgentBehaviour s m e
```

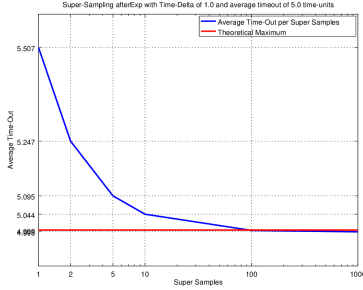
Both now take an additional parameter which determines the number of super-samples to be calculated. According to the above observations of the *occasionally* and *afterExp* functions which are the foundations of both of the functions we sample *sendMessageOccasionallySrcSS* with 20 super-samples and *transitionAfterExpSS* with 2. This will ensure that by using a time-delta of 1.0 we only calculate t steps when running a simulation for t time but that we sample our relevant functions with enough resolution to capture its frequencies. Optimally we should increase the number of super-samples for *sendMessageOccasionallySrcSS* to about 100 this will result in lower performance as *every* agent will perform this super-



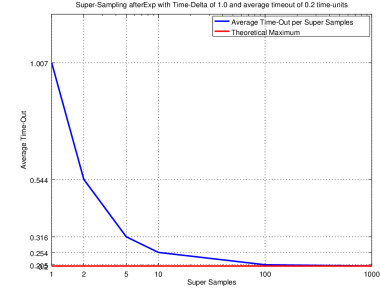
(a) Super-Sampling the *occasional* function with event-frequency of 5 (average of 0.2 events per time-unit). The theoretical average is 20 event within this time-frame.



(b) Super-Sampling the *occasional* function with event-frequency of $\frac{1}{5}$ (average of 5 events per time-unit). The theoretical average is 500 event within this time-frame.



(c) Super-Sampling the *afterExp* function with average time-out of 5.



(d) Super-Sampling the *afterExp* function with average time-out of 0.2.

Fig. 7: Super-Sampling the *afterExp* and *occasional* functions to visualize the influence of increasing number of super-samples on the average occurrence of the respective events. The time-delta is fixed to 1.0 in both cases with super-samples of [1, 2, 5, 10, 100, 1000]. The experiments for *afterExp* used 10,000 replications. The experiments for *occasional* ran for 100 time-units with 100 replications.

sampling, which will also result in a performance decrease. So in the end it is a struggle of performance vs. sufficiently close approximation.

VI. RESULTS AND DISCUSSION

A. Emulating System Dynamics

Due to the continuous time-semantics which can be expressed in our ABS approach we can also emulate system dynamics. Every stock and flow is then just an agent which exchange messages where the simulation is stepped using the *parallel* update-strategy. We add wrappers and type-definitions for convenience which increases the expressivity of the code, resembling system dynamics specifications. As a proof-of-concept we emulated the system dynamics of the SIR model, the code can be seen in Appendix B. Note that the code really looks like a system dynamics specification with the integrals of the mathematical specification directly showing up in the code - the implementation is correct per definition. Due to the internal implementation of the *integral* function of Yampa which uses the rectangle-rule to integrate, one must ensure to sample the system dynamics with small enough $t\Delta$ as can be seen in Figure 8.

B. Spatiality and Networks

When trying to emulate the dynamics of the SIR model using an agent-based approach the question arises what we ultimately gain from doing so when we could have generated the dynamics much quicker and smoother using the system-approach. The difference is that the agent-based approach is a stochastic one and can thus also generate "degenerated" dynamics e.g. in which the disease dies out after a few steps or even can't spread from patient zero - in this case ABS is clearly a benefit as it allows to investigate *alternative futures*, something not possible with system-dynamics in which the disease will never die out prematurely.

Another advantage of ABS over the system-dynamics approach is that agents can be heterogeneous and make use of spatial- and/or network-information defining the neighbourhood. We can thus simulate the spread of the disease throughout a population which is laid out on a 2d-grid or one can investigate spreading of the disease throughout a network of agents where some are

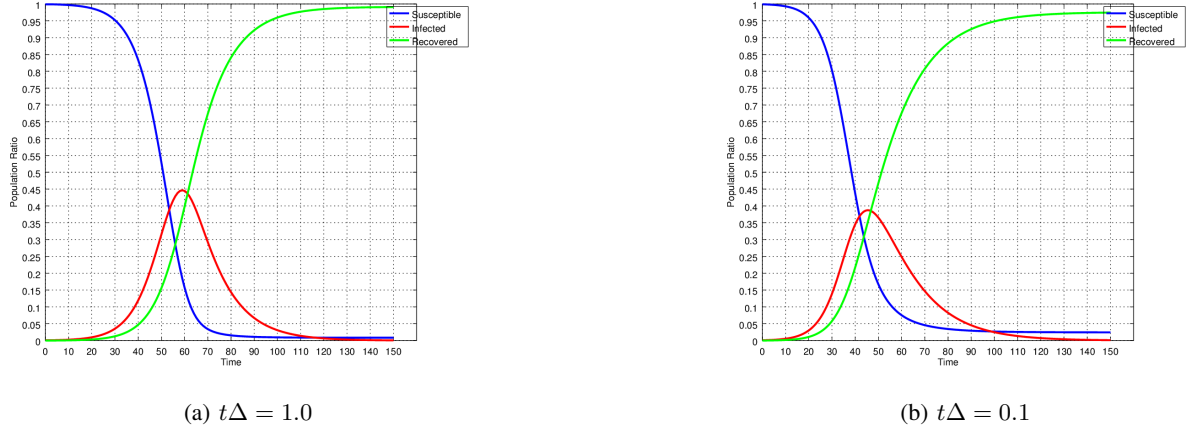


Fig. 8: Simulating the SIR model with our system dynamics emulation using different time-deltas. Note that although $t\Delta = 0.1$ might seem very close the system dynamic solution, there are still subtle differences to the initial Figure 3 which uses $t\Delta = 0.01$.

vaccinated and others not. We provide already suitable environments to simulate these cases and show an example of spreading the disease on a 2d-grid in Figure 9.

When using a 2d-grid or network one needs to set them up in the initialization code so there is a little more work to do there but the implementation of the agents differ just in one single line, which is where the neighbourhood is picked TODO: refer to appendix code. Instead of *randomAgentIdMsgSource* one uses either *randomNeighbourNodeMsgSource* in the case of a network or *randomNeighbourCellMsgSource* in case of a 2d-grid.

C. Agents as Signals

Due to the underlying nature and motivation of Functional Reactive Programming and Yampa, agents can be seen as signals which are generated and consumed by a Signal-Function which is the behaviour of an agent. If an agent does not change the output-signal is constant, if the agent changes e.g. by sending a message, changing its state,... it changes and a dead agent has no signal at all. The question is if the agents of our agent-based SIR implementation are true signals: do the dynamics change when we sample the system with $t\Delta = 0$? We hypothesize that our agents are true signals, thus they should not change when time does not change because they are completely time-dependent and rely completely on time-semantics. When actually running the simulation with $t\Delta = 0$ one gets the results as in 10.

As can be seen the dynamics are becoming constant *but* with a minor delay: infected increases a bit while susceptible decreases as can be seen in Figure 11. This is due to the delay of message delivery which takes one $t\Delta$, independent of its value - messages are also delivered when $t\Delta = 0$. Only message-generating functions which depend on non-zero $t\Delta$ to generate messages will then stop generate messages. Reactive functions which act on incoming messages can still create change as they do not rely on time-semantics but just on the discrete event of a message-arrival - which is the case in the transition from susceptible to infected.

Note that agents of non-time semantic models won't exhibit this behaviour - the dynamics will change even in case of a $t\Delta = 0$ - they just act on every update and don't care about the time-delta and just assume that every update occurs after a $t\Delta$ independent of the actual value of it. We implemented the function *doRepeatedlyEvery* which allows to transform a non-time semantic agent-behaviour into one. It is built on Yampas *repeatedly* function and has the following signature:

```
doRepeatedlyEvery :: Time -> AgentBehaviour s m e -> AgentBehaviour s m e
```

This function takes a time interval and an agent-behaviour signal-function and returns a new agent-behaviour signal-function which runs the argument signal-function every time-interval. Note that this function is subject to sampling-issues too e.g. when the time-interval is very small one needs to run the simulation with a $t\Delta \leq Time$ otherwise the dynamics would show delayed activation of the agent-behaviour.

D. Randomness

It is important to note that if we disable super-sampling and run the simulation for a given time t but with e.g. two different $t\Delta$ we would end up with two different results, even if the $t\Delta$ are small enough to sample the time-dependent functions sufficiently. Also if we use super-sampling with $t\Delta = 1.0$ and create the exact same number of samples as when using no super-sampling but smaller $t\Delta$, then we also end up with different results. The reason for this behaviour is that all these time-dependent functions ultimately build upon drawing from random-distributions. With different $t\Delta$ we are generating random-samples to a

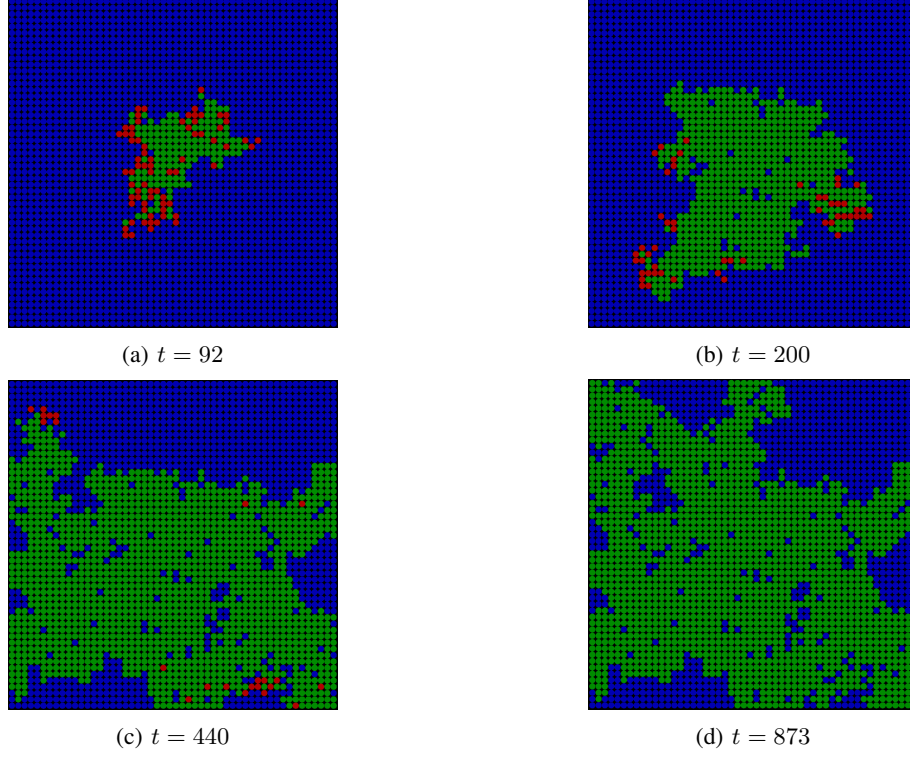


Fig. 9: Simulating SIR on a 52x52 grid with Moore neighbourhood using $t\Delta = 1$. Blue are susceptible, red are infected, green are recovered. The green areas act as protection as infected cannot cross the recovered border: this is particularly visible in the lower right corner of 9c where the disease has been contained in the blue island and has no means to escape. It may seem that the few remaining infected agents in the top left corner of 9c will die out soon but still it needs more than the already running simulation-time until the disease actually dies out with the last patient recovering at center top of 9d at $t = 873$.

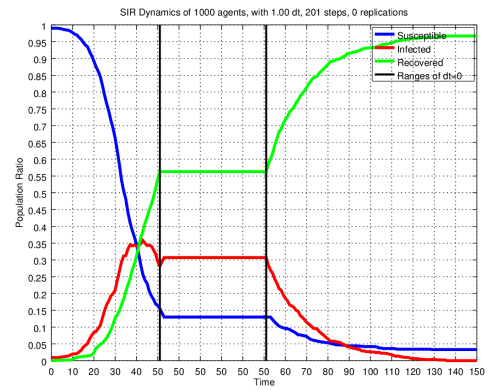
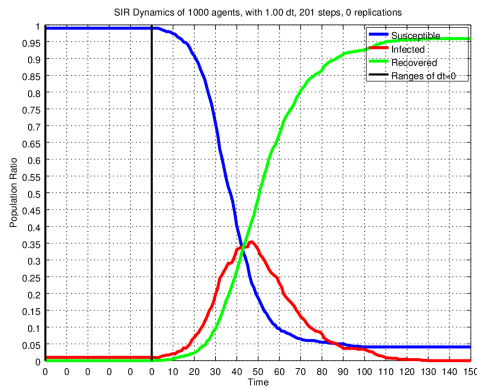


Fig. 10: Dynamics of agent-based SIR implementation of 1000 agents running with $t\Delta = 1$ with ranges of $t\Delta = 0$ marked with two vertical black lines.

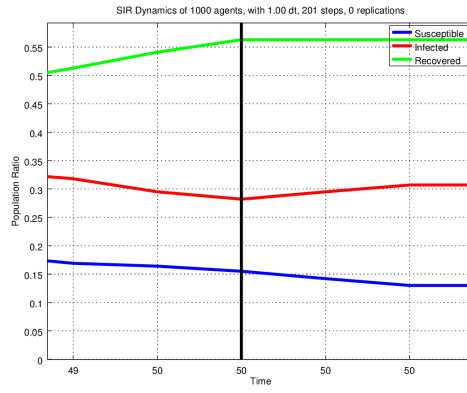
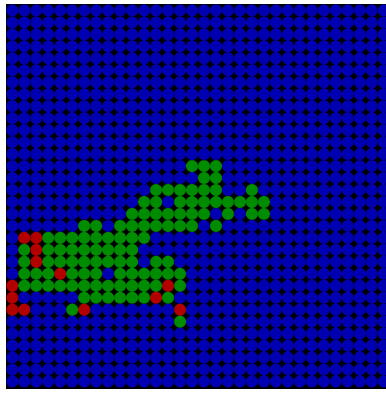
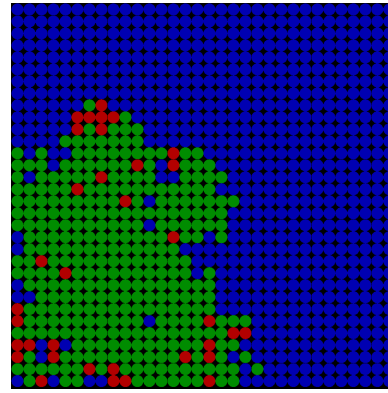


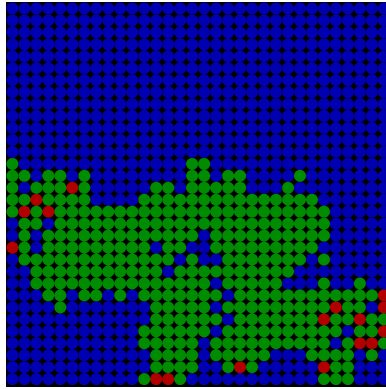
Fig. 11: Zoom-in to step 51, marked with the black line from where on $t\Delta = 0$ for the next 50 steps. The recovered ratio stays constant but a few agents get infected even *after* having switched to $t\Delta = 0$ which happens due to the message-delivery lag. After all messages have been delivered, the signal stays constant until non-zero $t\Delta$ are turned on again.



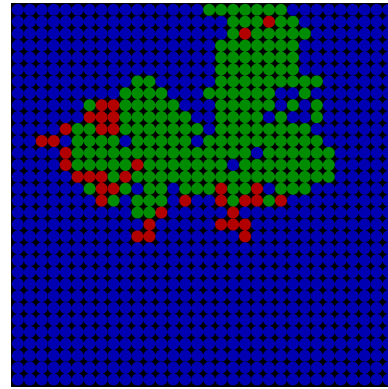
(a) $t\Delta = 0.1$, no super-sampling



(b) $t\Delta = 0.01$, no super-sampling



(c) $t\Delta = 1.0$ with 10 super-samples



(d) $t\Delta = 1.0$ with 100 super-samples

Fig. 12: Comparing results on 32x32 grid after $t = 200$ but with different $t\Delta$ and different number of super-samples. Note that a $t\Delta = 0.1$ is clearly not enough, whereas the others result in much more recovered agents.

greater or lesser extent, which would result in different random-number sequences which in turn ultimately leads to slightly different dynamics. When generating a plot of the dynamics this is not as visible, also this is the reason why one generates multiple replications, but this behaviour becomes strikingly apparent when simulating the SIR model on a 2d-grid as can be seen in Figure ??.

E. Message Round-trip Time

When reflecting on the messaging mechanism it becomes apparent that a round-trip from sender to receiver and back takes $2t\Delta$. A round-trip happens in our agent-based SIR approach to implement the transition from infected to susceptible - susceptible agents send Contact Susceptible messages to random agents (except itself) where only infected agents reply with a Contact Infected message. This means that it takes $2t\Delta$ until a susceptible agent might get infected. This becomes an issue if we want to match the dynamics of our agent-based approach to the system-dynamics one in which no time-delay happens - the agents act instantaneous with each other during one time-step. We have two solutions for this problem: either we resort to *conversations* or we increase the global sampling frequency of the system which matches the *message frequency* of messages which are subject to round-trips. Implementing conversations is only available in the *sequential* update-strategy and is much more cumbersome, so we followed the approach of increasing the frequency. We tried different frequencies and found that $t\Delta = 0.1$ results in a sufficiently good approximation to the system dynamics solution as can be seen in Figure TODO.

F. Recursive ABS

Due to the inherent recursive nature of functional programming we came up with the idea of *recursive* ABS in which agents can recursively run the simulation within the simulation which would allow them to project their own actions into the future. So far it only exists as a proof-of-concept and we are currently only aware of a single model [9] in the field of ABS which does recursive simulation. The implementation of recursive ABS is very natural due to the explicit data-flow and lack of side-effects which eases the task very much. Unfortunately we cannot go into detail of our approach as it is beyond the scope of the paper.

G. Advantages

We now look at a number of advantages this functional reactive approach to ABS has over to the traditional imperative, object-oriented implementation approaches done in Python, Java, C++.

1) *Continuous Time*: It seems that in our approach we combine the benefits of SD and ABS: we have continuous time-semantics but with individual, heterogeneous agents.

2) *Code close to specification*: As becomes directly evident when looking at the code of the agent-based implementation in Appendix A and system dynamics implementation in Appendix B, it looks very much like a specification. By creating this EDSL which allows to express powerful time-semantics it is possible to now create an ABS in a declarative style in Haskell where agent-implementation looks like a model-specification thus being correct by definition. arrowized programming is optional and only required when utilizing yampas time-semantics. if the model does not rely on time-semantics, it can use monadic-programming by building on the existing monadic functions in the EDSL which allow to run in the State-Monad which simplifies things very much - when to use yampas arrowized programming: time-semantics, simple state-chart agents - when not using yampas facilities: in all the other cases e.g. SugarScape is such a case as it proceeds in unit time-steps and all agents act in every time-step

3) *Being pure*: Because no part of the simulation runs in the IO monad ⁶ and we do not use `unsafePerformIO` ⁷ we can rule out a serious class of bugs caused by implicit data-dependencies and side-effects which can occur in traditional imperative implementations. Also we can statically guarantee the reproducibility of the simulation because of that. There are no side effects possible within the agents which would result in differences between same runs (e.g. file access, networking, threading). Every agent has access to its own random-number generator, allowing randomness to occur in the simulation but the random-generator seed is fixed in the beginning and can never be changed within an agent to come from e.g. the current system time, which would require to run within the IO Monad. This means that after initialising the agents, which *could* run in the IO Monad, the simulation itself runs completely deterministic. We provide functionality to render the output to a window (using the library Gloss) or writing to a text-file, meaning, parts of the simulation would run in the IO Monad. Here we rely on Yampas *reactimate* function which provides a well-defined way of communicating with the world in such a system. This function provides the $t\Delta$ for the next step, which *could* come from IO Monad but we keep the $t\Delta$ always fixed, thus removing another source of non-reproducibility where e.g. $t\Delta$ is influenced by system-dependent non-deterministic rendering-performance every step as happens in games or described by [10] in the context of FRP.

4) *Robust time handling*: cant mess around with time because delta-time is hidden from you (intentional design-decision by Yampa). this would be only very difficult and cumbersome to achieve in an object-oriented approach. TODO: experiment with it in Java - how could we actually implement this? I think it is impossible: may only achieve this through complicated application of patterns and inheritance but then has the problem of how to update the dt and more important how to deal with functions like integral which accumulates a value through closures and continuations. We could do this in OO by having a general base-class e.g. ContinuousTime which provides functions like `updateDt` and `integrate`, but we could only accumulate

⁶Except when using graphical rendering, but then only the rendering happens in the IO Monad - the simulation step is always a pure computation

⁷We actually do use it for generating unique agent-ids for newly created agents. This is necessary because need to guarantee that two invocations will result in two different ids, which would be difficult / impossible when running the simulation in the *parallel* update-strategy. This is not a problem as long as an agent does not rely on the absolute value of an agent-id but just uses it as an opaque identifier for sending/receiving.

a single integral value. TODO: this is also the case in e.g. RePast symphony in which one never deals with time-deltas but interacts with the scheduler through annotations

5) *Replications*: get replications for free without having to worry about side-effects and can even run them in parallel without headaches. TODO: need a little bit of discussion here as it is not that easy: we need to swap random-number generators out for new one in every replication, both in agents and environments

H. Disadvantages

We identify two main disadvantages.

1) *Performance*: Performance is currently nowhere near imperative object-oriented implementations. The reason for this is that we don't have in-place updates of data-structures and make no use of references. This results in lots of copying which is simply not necessary in the imperative languages with implicit effects. Also it is much more difficult to reason about time and space in our approach. Thus we see performance clearly as the main drawback of the functional approach and the only real advantage of the imperative approach over our.

2) *Steep learning curve*: Our approach is quite advanced in three ways. First it builds on the already quite involved FRP paradigm. Second it forces one to think properly of time-semantics of the model, how to sample it, how small $t\Delta$ should be and whether one needs super-sampling or not and if yes how many samples one should take. Third it requires to think about agent-interaction and update-strategies, whether one should use conversations which forces one to run sequentially or if one can run agents in parallel and use normal messaging which but incurs a time-delay which in turn would need to be considered when setting the $t\Delta$.

VII. RELATED RESEARCH

The amount of research on using the pure functional paradigm using Haskell in the field of ABS has been moderate so far. Most of the papers look into how agents can be specified using the belief-desire-intention paradigm [11], [12], [13]. A library for Discrete Event Simulation (DES) and System Dynamics (SD) in Haskell called *Aivika 3* is described in [14]. It comes with very basic features for ABS but only allows to specify simple state-based agents with timed transitions. [13] which discuss using functional programming for DES mention the paradigm of functional reactive programming (FRP) to be very suitable to DES. Tim Sweeney, CTO of Epic Games gave an invited talk in which he talked about programming languages in the development of game-engines and scripting of game-logic [15]. Although the fields of games and ABS seem to be very different, in the end they have also very important similarities: both are simulations which perform numerical computations and update objects in a loop either concurrently or sequential⁸. In games these objects are called *game-objects* and in ABS they are called *agents* but they are conceptually the same thing. The two main points Sweeney made were that dependent types could solve most of the run-time failures and that parallelism is the future for performance improvement in games. He distinguishes between pure functional algorithms which can be parallelized easily in a pure functional language and updating game-objects concurrently using software transactional memory (STM).

The thesis of [17] constructs two frameworks: an agent-modelling framework and a DES framework, both written in Haskell. They put special emphasis on parallel and concurrency in their work. The author develops two programs with strong emphasis on parallelism: HLogo which is a clone of the NetLogo agent-modelling framework and HDES, a framework for discrete event simulation.

[18] and [19] present a domain-specific language for developing functional reactive agent-based simulations. This language called FRABJOUS is human readable and easily understandable by domain-experts. It is not directly implemented in FRP/Haskell but is compiled to Yampa code - a FRP library for Haskell - which they claim is also readable. It seems that FRP is a promising approach to ABS in Haskell, an important hint we will follow in the section below.

VIII. CONCLUSION AND FUTURE RESEARCH

In this paper we presented a novel approach to implement agent-based models by Functional Reactive Programming (FRP) in Haskell building on the library Yampa which we termed Functional Reactive Agent-Based Simulation⁹.

In general, FRP tries to shift the direction of data-flow, from message passing onto data dependency. This helps reason about what things are over time, as opposed to how changes propagate TODO: cite ivan perez phd thesis. This of course raises the question whether FRP is *really* the right approach, because in our approach to ABS, message-passing is an essential concept. It is important to emphasize that agent-relations in interactions are never fixed in advance and are completely dynamic, forming a network. Maybe one has to look at message passing in a different way in FRP and to view and model it as a data-dependency but it is not clear how this can be done. The question is whether there is a mechanism in which we have explicit data-dependency but which is dynamic like message-passing.

⁸Gregory [16] defines computer-games as *soft real-time interactive agent-based computer simulations*

⁹For this research implemented a prototype library called *FrABS* which is available under <https://github.com/thalerjonathan/phd/tree/master/coding/libraries/frABS>. We plan on releasing it on Hackage in the future.

In the paper, TODO: cite my own art of iterating paper, the authors discuss four update-strategies of which we implemented only the *sequential* and *parallel* ones. The other two strategies are inherent concurrent ones, thus to implement them would require the agents to use concurrency features like STM or run in the IO Monad. Unfortunately both require monadic code which is not possible in Yampa. In [8] the authors provide a generalisation to FRP which also allows to run monads in FRP, which would make running the agents in the STM Monad possible. Using this approach we could then implement true concurrent agents allowing to implement the other two update-strategies. It would be interesting to see the benefits and drawbacks of our approach in these two strategies, especially when treating agents like signals as in the case of our SIR implementation. We leave this for further research.

Functional Programming is particularly strong for reasoning about an implementation and verification of an implemented model. How we can test, reason about and verify implemented models of ABS in our approach, we leave for further research in an accompanying paper which will be written for the agent-based simulation audience.

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APPENDIX A
FUNCTIONAL-REACTIVE CODE OF THE AGENT-BASED SIR MODEL

```
1 data SIRState = Susceptible | Infected | Recovered deriving (Eq)
2 data SIRMMsg = Contact SIRState deriving (Eq)
3
4 type SIRAgentState = SIRState
5
6 type SIREnvironment = [AgentId]
7
8 type SIRAgentDef = AgentDef SIRAgentState SIRMMsg SIREnvironment
9 type SIRAgentBehaviour = AgentBehaviour SIRAgentState SIRMMsg SIREnvironment
10 type SIRAgentBehaviourReadEnv = ReactiveBehaviourReadEnv SIRAgentState SIRMMsg SIREnvironment
11 type SIRAgentBehaviourIgnoreEnv = ReactiveBehaviourIgnoreEnv SIRAgentState SIRMMsg SIREnvironment
12 type SIRAgentIn = AgentIn SIRAgentState SIRMMsg SIREnvironment
13 type SIRAgentOut = AgentOut SIRAgentState SIRMMsg SIREnvironment
14 type SIRAgentObservable = AgentObservable SIRAgentState
15
16 type SIREventSource = EventSource SIRAgentState SIRMMsg SIREnvironment
17
18 -----
19 infectivity :: Double
20 infectivity = 0.05
21
22 contactRate :: Double
23 contactRate = 5
24
25 illnessDuration :: Double
26 illnessDuration = 15
27
28 contactSS :: Int
29 contactSS = 20
30
31 illnessTimeoutSS :: Int
32 illnessTimeoutSS = 2
33
34 -----
35 createSIRNumInfected :: Int -> Int -> IO ([SIRAgentDef], SIREnvironment)
36 createSIRNumInfected agentCount numInfected = do
37   let agentIds = [0 .. (agentCount-1)]
38   let infectedIds = take numInfected agentIds
39   let susceptibleIds = drop numInfected agentIds
40
41   adefsSusceptible <- mapM (sirAgent Susceptible) susceptibleIds
42   adefsInfected <- mapM (sirAgent Infected) infectedIds
43
44   return (adefsSusceptible ++ adefsInfected, agentIds)
45
46 sirAgent :: SIRState -> AgentId -> IO SIRAgentDef
47 sirAgent initS aid = do
48   rng <- newStdGen
49   let beh = sirAgentBehaviour rng initS
50   let adef = AgentDef {
51     adId = aid
52     , adState = initS
53     , adBeh = beh
54     , adInitMessages = NoEvent
55     , adConversation = Nothing
56     , adRng = rng
57   }
58
59   return adef
60
61 -----
62 -- UTILITIES
63 gotInfected :: SIRAgentIn -> Rand StdGen Bool
64 gotInfected ain = onMessageM gotInfectedAux ain False
65   where
66     gotInfectedAux :: Bool -> AgentMessage SIRMMsg -> Rand StdGen Bool
67     gotInfectedAux False (_, Contact Infected) = randomBoolM infectivity
68     gotInfectedAux x _ = return x
69
70
71
```



```

72 respondToContactWith :: SIRState -> SIRAgentIn -> SIRAgentOut -> SIRAgentOut
73 respondToContactWith state ain ao = onMessage respondToContactWithAux ain ao
74   where
75     respondToContactWithAux :: AgentMessage SIRMsg -> SIRAgentOut -> SIRAgentOut
76     respondToContactWithAux (senderId, Contact _) ao = sendMessage (senderId, Contact state) ao
77
78 -- SUSCEPTIBLE
79 sirAgentSuceptible :: RandomGen g => g -> SIRAgentBehaviour
80 sirAgentSuceptible g =
81   transitionOnEvent
82     sirAgentInfectedEvent
83     (readEnv $ sirAgentSusceptibleBehaviour g)
84     (sirAgentInfected g)
85
86 sirAgentInfectedEvent :: SIREventSource
87 sirAgentInfectedEvent = proc (ain, ao) -> do
88   let (isInfected, ao') = agentRandom (gotInfected ain) ao
89   infectionEvent <- edge -< isInfected
90   returnA -< (ao', infectionEvent)
91
92 sirAgentSusceptibleBehaviour :: RandomGen g => g -> SIRAgentBehaviourReadEnv
93 sirAgentSusceptibleBehaviour g = proc (ain, e) -> do
94   let ao = agentOutFromIn ain
95   ao1 <- doOnce (setDomainState Susceptible) -< ao
96   ao2 <- sendMessageOccasionallySrcSS
97     g
98     (1 / contactRate)
99     contactSS
100    (randomAgentIdMsgSource (Contact Susceptible) True) -< (ao1, e)
101   returnA -< ao2
102
103 -- INFECTED
104 sirAgentInfected :: RandomGen g => g -> SIRAgentBehaviour
105 sirAgentInfected g =
106   transitionAfterExpSS
107     g
108     illnessDuration
109     illnessTimeoutSS
110     (ignoreEnv $ sirAgentInfectedBehaviour g)
111     sirAgentRecovered
112
113 sirAgentInfectedBehaviour :: RandomGen g => g -> SIRAgentBehaviourIgnoreEnv
114 sirAgentInfectedBehaviour g = proc ain -> do
115   let ao = agentOutFromIn ain
116   ao1 <- doOnce (setDomainState Infected) -< ao
117   let ao2 = respondToContactWith Infected ain ao1
118   returnA -< ao2
119
120 -- RECOVERED
121 sirAgentRecovered :: SIRAgentBehaviour
122 sirAgentRecovered = setDomainStateR Recovered
123
124 -- INITIAL CASES
125 sirAgentBehaviour :: RandomGen g => g -> SIRState -> SIRAgentBehaviour
126 sirAgentBehaviour g Susceptible = sirAgentSuceptible g
127 sirAgentBehaviour g Infected = sirAgentInfected g
128 sirAgentBehaviour _ Recovered = sirAgentRecovered
129
130 -----
131 runSIR :: IO ()
132 runSIR = do
133   -- no collapsing/updating of environment
134   params <- initSimulation updateStrategy Nothing Nothing shuffleAgents (Just rngSeed)
135   (initAdefs, initEnv) <- createSIRNumInfected agentCount numInfected
136   let dynamics = simulateAggregateTime initAdefs initEnv params dt t aggregate
137   print dynamics
138
139 aggregate :: (Time, [SIRAgentObservable], SIREnvironment) -> (Time, Double, Double, Double)
140 aggregate (t, aobs, _) = (t, susceptibleCount, infectedCount, recoveredCount)
141   where
142     susceptibleCount = fromIntegral $ length $ filter ((Susceptible==) . snd) aobs
143     infectedCount = fromIntegral $ length $ filter ((Infected==) . snd) aobs
144     recoveredCount = fromIntegral $ length $ filter ((Recovered==) . snd) aobs

```

APPENDIX B
FUNCTIONAL-REACTIVE CODE OF THE SYSTEM DYNAMICS SIR MODEL

```
1  totalPopulation :: Double
2  totalPopulation = 1000
3
4  infectivity :: Double
5  infectivity = 0.05
6
7  contactRate :: Double
8  contactRate = 5
9
10 avgIllnessDuration :: Double
11 avgIllnessDuration = 15
12
13 -- Hard-coded ids for stocks & flows interaction
14 susceptibleStockId :: StockId
15 susceptibleStockId = 0
16
17 infectiousStockId :: StockId
18 infectiousStockId = 1
19
20 recoveredStockId :: StockId
21 recoveredStockId = 2
22
23 infectionRateFlowId :: FlowId
24 infectionRateFlowId = 3
25
26 recoveryRateFlowId :: FlowId
27 recoveryRateFlowId = 4
28
29 -----
30 -- STOCKS
31 susceptibleStock :: Stock
32 susceptibleStock initValue = proc ain -> do
33   let infectionRate = flowInFrom infectionRateFlowId ain
34
35   stockValue <- (initValue+) ^<< integral -< (-infectionRate)
36
37   let ao = agentOutFromIn ain
38   let ao0 = setDomainState stockValue ao
39   let ao1 = stockOutTo stockValue infectionRateFlowId ao0
40
41   returnA -< ao1
42
43 infectiousStock :: Stock
44 infectiousStock initValue = proc ain -> do
45   let infectionRate = flowInFrom infectionRateFlowId ain
46   let recoveryRate = flowInFrom recoveryRateFlowId ain
47
48   stockValue <- (initValue+) ^<< integral -< (infectionRate - recoveryRate)
49
50   let ao = agentOutFromIn ain
51   let ao0 = setDomainState stockValue ao
52   let ao1 = stockOutTo stockValue infectionRateFlowId ao0
53   let ao2 = stockOutTo stockValue recoveryRateFlowId ao1
54
55   returnA -< ao2
56
57 recoveredStock :: Stock
58 recoveredStock initValue = proc ain -> do
59   let recoveryRate = flowInFrom recoveryRateFlowId ain
60
61   stockValue <- (initValue+) ^<< integral -< recoveryRate
62
63   let ao = agentOutFromIn ain
64   let ao' = setDomainState stockValue ao
65
66   returnA -< ao'
```

```

72 -----
73 -- FLOWS
74 infectionRateFlow :: Flow
75 infectionRateFlow = proc ain -> do
76     let susceptible = stockInFrom susceptibleStockId ain
77     let infectious = stockInFrom infectiousStockId ain
78
79     let flowValue = (infectious * contactRate * susceptible * infectivity) / totalPopulation
80
81     let ao = agentOutFromIn ain
82     let ao' = flowOutTo flowValue susceptibleStockId ao
83     let ao'' = flowOutTo flowValue infectiousStockId ao'
84
85     returnA -< ao''
86
87 recoveryRateFlow :: Flow
88 recoveryRateFlow = proc ain -> do
89     let infectious = stockInFrom infectiousStockId ain
90
91     let flowValue = infectious / avgIllnessDuration
92
93     let ao = agentOutFromIn ain
94     let ao' = flowOutTo flowValue infectiousStockId ao
95     let ao'' = flowOutTo flowValue recoveredStockId ao'
96
97     returnA -< ao''
98
99 -----
100 createSysDynSIR :: [SDDef]
101 createSysDynSIR =
102     [ susStock
103     , infStock
104     , recStock
105     , infRateFlow
106     , recRateFlow
107     ]
108 where
109     initialSusceptibleStockValue = totalPopulation - 1
110     initialInfectiousStockValue = 1
111     initialRecoveredStockValue = 0
112
113     susStock = createStock susceptibleStockId initialSusceptibleStockValue susceptibleStock
114     infStock = createStock infectiousStockId initialInfectiousStockValue infectiousStock
115     recStock = createStock recoveredStockId initialRecoveredStockValue recoveredStock
116
117     infRateFlow = createFlow infectionRateFlowId infectionRateFlow
118     recRateFlow = createFlow recoveryRateFlowId recoveryRateFlow
119
120 -----
121 runSysDynSIRSteps :: IO ()
122 runSysDynSIRSteps = print dynamics
123 where
124     -- SD run completely deterministic, this is reflected also in the types of
125     -- the createSysDynSIR and runSD functions which are pure functions
126     sdDefs = createSysDynSIR
127     sdObs = runSD sdDefs dt t
128
129     dynamics = map calculateDynamics sdObs
130
131 -- NOTE: here we rely on the fact the we have exactly three stocks and sort them by their id to access them
132 --     stock id 0: Susceptible
133 --     stock id 1: Infectious
134 --     stock id 2: Recovered
135 --     the remaining items are the flows
136 calculateDynamics :: (Time, [SDObservable]) -> (Time, Double, Double, Double)
137 calculateDynamics (t, unsortedStocks) = (t, susceptibleCount, infectedCount, recoveredCount)
138 where
139     stocks = sortBy (\s1 s2 -> compare (fst s1) (fst s2)) unsortedStocks
140     (_, susceptibleCount) : (_, infectedCount) : (_, recoveredCount) : _ = stocks

```