STAT 526 HW 1

Satoshi Ido (ID: 34788706)

23 January 2023

Library import

```
library("tidyverse")
library("dplyr")
library("car")
library("ggplot2")
library("reshape2")
library("caTools")
library("Rcmdr")
library("MASS")
```

data import

```
data(Cars93, package = "MASS")
Cars93
```

##		Manufacturer	Model	Туре	Min.Price	Price	Max.Price	MPG.city
##	1	Acura	Integra	Small	12.9	15.9	18.8	25
##	2	Acura	Legend	Midsize	29.2	33.9	38.7	18
##	3	Audi	90	Compact	25.9	29.1	32.3	20
##	4	Audi	100	${\tt Midsize}$	30.8	37.7	44.6	19
##	5	BMW	535i	${\tt Midsize}$	23.7	30.0	36.2	22
##	6	Buick	Century	${\tt Midsize}$	14.2	15.7	17.3	22
##	7	Buick	LeSabre	Large	19.9	20.8	21.7	19
##	8	Buick	Roadmaster	Large	22.6	23.7	24.9	16
##	9	Buick	Riviera	${\tt Midsize}$	26.3	26.3	26.3	19
##	10	Cadillac	DeVille	Large	33.0	34.7	36.3	16
##	11	Cadillac	Seville	${\tt Midsize}$	37.5	40.1	42.7	16
##	12	Chevrolet	Cavalier	${\tt Compact}$	8.5	13.4	18.3	25
##	13	Chevrolet	Corsica	Compact	11.4	11.4	11.4	25
##	14	Chevrolet	Camaro	Sporty	13.4	15.1	16.8	19
##	15	Chevrolet	Lumina	${\tt Midsize}$	13.4	15.9	18.4	21
##	16	Chevrolet	$Lumina_APV$	Van	14.7	16.3	18.0	18
##	17	Chevrolet	Astro	Van	14.7	16.6	18.6	15
##	18	Chevrolet	Caprice	Large	18.0	18.8	19.6	17
##	19	Chevrolet	Corvette	Sporty	34.6	38.0	41.5	17
##	20	Chrylser	Concorde	Large	18.4	18.4	18.4	20
##	21	Chrysler	LeBaron	${\tt Compact}$	14.5	15.8	17.1	23
##	22	Chrysler	Imperial	Large	29.5	29.5	29.5	20
##	23	Dodge	Colt	Small	7.9	9.2	10.6	29
##	24	Dodge	Shadow	Small	8.4	11.3	14.2	23

##	25	Dodge	Spirit	Compact	11.9	13.3	14.7	22
##	26	Dodge	Caravan	Van	13.6	19.0	24.4	17
##	27	Dodge		Midsize	14.8	15.6	16.4	21
##	28	Dodge	Stealth	Sporty	18.5	25.8	33.1	18
##	29	Eagle	Summit	Small	7.9	12.2	16.5	29
##	30	Eagle	Vision	Large	17.5	19.3	21.2	20
##	31	Ford	Festiva	Small	6.9	7.4	7.9	31
##	32	Ford	Escort	Small	8.4	10.1	11.9	23
##	33	Ford			10.4	11.3	12.2	22
##	34	Ford	Mustang	Compact Sporty	10.4	15.9	21.0	22
	35	Ford	Probe		12.8	14.0	15.2	24
##	36	Ford	Aerostar	Sporty Van	14.5	19.9	25.3	15
	37	Ford		Midsize	15.6	20.2	24.8	21
##	38				20.1	20.2	24.6	18
			Crown_Victoria	Large		8.4		
##	39	Geo	Metro	Small	6.7		10.0	46
##	40	Geo	Storm	Sporty	11.5	12.5	13.5	30
##	41	Honda	Prelude	Sporty	17.0	19.8	22.7	24
##	42	Honda	Civic	Small	8.4	12.1	15.8	42
	43	Honda		Compact	13.8	17.5	21.2	24
	44	Hyundai	Excel	Small	6.8	8.0	9.2	29
	45	Hyundai	Elantra		9.0	10.0	11.0	22
##	46	Hyundai	Scoupe	Sporty	9.1	10.0	11.0	26
	47	Hyundai		Midsize	12.4	13.9	15.3	20
##	48	Infiniti	•	Midsize	45.4	47.9	50.4	17
##	49	Lexus		Midsize	27.5	28.0	28.4	18
##	50	Lexus		Midsize	34.7	35.2	35.6	18
##	51	Lincoln	Continental		33.3	34.3	35.3	17
##	52	Lincoln	Town_Car	Large	34.4	36.1	37.8	18
##	53	Mazda	323	Small	7.4	8.3	9.1	29
##	54	Mazda	Protege	Small	10.9	11.6	12.3	28
##	55	Mazda		Compact	14.3	16.5	18.7	26
##	56	Mazda	MPV	Van	16.6	19.1	21.7	18
	57	Mazda	RX-7	Sporty	32.5	32.5	32.5	17
##		Mercedes-Benz		Compact	29.0	31.9	34.9	20
##		Mercedes-Benz		Midsize	43.8	61.9	80.0	19
##	60	Mercury	Capri	Sporty	13.3	14.1	15.0	23
##		Mercury	•	Midsize	14.9	14.9	14.9	19
##		Mitsubishi	Mirage	Small	7.7	10.3	12.9	29
	63	Mitsubishi	Diamante		22.4	26.1	29.9	18
##	64	Nissan	Sentra	Small	8.7	11.8	14.9	29
##	65	Nissan	Altima	Compact	13.0	15.7	18.3	24
##	66	Nissan	Quest	Van	16.7	19.1	21.5	17
##	67	Nissan		Midsize	21.0	21.5	22.0	21
##	68	Oldsmobile		Compact	13.0	13.5	14.0	24
##	69	Oldsmobile	Cutlass_Ciera		14.2	16.3	18.4	23
##	70	Oldsmobile	Silhouette	Van	19.5	19.5	19.5	18
	71	Oldsmobile	Eighty-Eight	Large	19.5	20.7	21.9	19
	72	Plymouth	Laser	Sporty	11.4	14.4	17.4	23
	73	Pontiac	LeMans	Small	8.2	9.0	9.9	31
##	74	Pontiac		${\tt Compact}$	9.4	11.1	12.8	23
	75	Pontiac	Firebird	Sporty	14.0	17.7	21.4	19
##	76	Pontiac	${\tt Grand_Prix}$	${\tt Midsize}$	15.4	18.5	21.6	19
##	77	Pontiac	Bonneville	Large	19.4	24.4	29.4	19
##	78	Saab	900	${\tt Compact}$	20.3	28.7	37.1	20

##	79	Saturn	SL	Small	9.2	11.1	12.9	28
##		Subaru	Justy	Small	7.3	8.4	9.5	33
##		Subaru	Loyale	Small	10.5	10.9	11.3	25
##		Subaru	•	Compact	16.3	19.5	22.7	23
##		Suzuki	Swift	Small	7.3	8.6	10.0	39
##		Toyota	Tercel	Small	7.8	9.8	11.8	32
	85	Toyota	Celica	Sporty	14.2	18.4	22.6	25
	86	Toyota		Midsize	15.2	18.2	21.2	22
	87	Toyota	Previa	Van	18.9	22.7	26.6	18
	88	Volkswagen	Fox	Small	8.7	9.1	9.5	25
	89	Volkswagen	Eurovan	Van	16.6	19.7	22.7	17
##	90	Volkswagen	Passat	Compact	17.6	20.0	22.4	21
##	91	Volkswagen	Corrado	Sporty	22.9	23.3	23.7	18
##	92	Volvo	240	Compact	21.8	22.7	23.5	21
##	93	Volvo		Midsize	24.8	26.7	28.5	20
##		MPG.highway	AirBag	gs DriveTrain	Cylin	ders E	ngineSize	Horsepower
##	1	31	Nor			4	1.8	140
##	2	25 Driver	& Passenge	er Front		6	3.2	200
##	3	26	Driver onl	Ly Front		6	2.8	172
##	4	26 Driver	& Passenge	er Front		6	2.8	172
##	5	30	Driver onl	Ly Rear		4	3.5	208
##	6	31	Driver onl	Ly Front		4	2.2	110
##	7	28	Driver onl	Ly Front		6	3.8	170
##	8	25	Driver onl	Ly Rear		6	5.7	180
##	9	27	Driver onl	Ly Front		6	3.8	170
##	10	25	Driver onl	Ly Front		8	4.9	200
##	11	25 Driver	& Passenge	er Front		8	4.6	295
##	12	36	Nor	ne Front		4	2.2	110
##	13	34	Driver onl	•		4	2.2	110
	14		& Passenge	er Rear		6	3.4	160
	15	29	Nor			4	2.2	110
	16	23	Nor			6	3.8	170
	17	20	Nor			6	4.3	165
##	18	26	Driver onl	•		8	5.0	170
	19	25	Driver onl	•		8	5.7	300
	20		& Passenge			6	3.3	153
##			& Passenge			4	3.0	141
	22	26	Driver onl	-		6	3.3	147
	23	33	Nor			4	1.5	92
	24	29	Driver onl	•		4	2.2	93
	25	27	Driver onl	•		4	2.5	100
	26	21	Driver onl	•		6	3.0	142
	27	27	Driver onl	•		4	2.5	100
	28	24	Driver onl	-		6	3.0	300
	29	33	Nor			4 6	1.5	92
	30		& Passenge				3.5	214
	31 32	33 30	Nor Nor			4 4	1.3 1.8	63 127
	33	27	Nor			4	2.3	96
	34	29	Driver onl			4	2.3	105
	35	30	Driver onl	•		4	2.3	105
	36	20	Driver onl	•		6	3.0	145
	37	30	Driver onl	•		6	3.0	140
	38	26	Driver onl	•		8	4.6	190
11 TT		20	21101 0111	- <i>j</i> 16641		9	4.0	100

##	39	50	None	Front	3	1.0	55
	40	36	Driver only	Front	4	1.6	90
##			ver & Passenger	Front	4	2.3	160
	42	46	Driver only	Front	4	1.5	102
	43		ver & Passenger	Front	4	2.2	140
	44	33	None	Front	4	1.5	81
##		29	None	Front	4	1.8	124
##		34	None	Front	4	1.5	92
	47	27	None	Front	4	2.0	128
	48	22	Driver only	Rear	8	4.5	278
	49	24	Driver only	Front	6	3.0	185
	50		ver & Passenger	Rear	6	3.0	225
	51		ver & Passenger	Front	6	3.8	160
	52		ver & Passenger	Rear	8	4.6	210
##		37	None	Front	4	1.6	82
	54	36	None	Front	4	1.8	103
	55	34	Driver only	Front	4	2.5	164
	56	24	None	4WD	6	3.0	155
##		25	Driver only	Rear	rotary	1.3	255
##		29	Driver only	Rear	10tary 4	2.3	130
##			ver & Passenger	Rear	6	3.2	217
##		26 DI1	Driver only	Front	4	1.6	100
##		26	None	Rear	6	3.8	140
	62	33	None	Front	4	1.5	92
	63	24		Front	6	3.0	202
	64	33	Driver only	Front	4	1.6	110
##		30	Driver only Driver only	Front	4	2.4	150
	66	23	None		6	3.0	150
	67	26		Front	6	3.0	160
	68	31	Driver only None	Front	4	2.3	155
	69	31		Front Front	4	2.3	110
##		23	Driver only None	Front	6	3.8	170
	71	28	Driver only	Front	6	3.8	170
	72	30	None	4WD	4	1.8	92
##		41	None	Front	4	1.6	74
	74	31	None	Front	4	2.0	110
##					6	3.4	160
			ver & Passenger	Rear	_	3.4	200
## ##		27	None & Paggangar	Front	6 6	3.8	170
##		26	ver & Passenger Driver only	Front	4	2.1	140
##		38	-	Front	4	1.9	85
##		37	Driver only None	Front 4WD	3	1.9	73
##		30			4		
	82	30	None	4WD 4WD	4	1.8 2.2	90
	83	43	Driver only None		3	1.3	130 70
	84	37	Driver only	Front	4	1.5	82
			•	Front			
## ##		32	Driver only	Front	4	2.2	135
##		29	Driver only	Front	4	2.2	130
		22	Driver only	4WD	4	2.4	138
	88	33	None	Front	4	1.8	81
##		21	None	Front	5 4	2.5	109 134
##		30	None	Front	6	2.0	134
##		25	None	Front		2.8	178
##	92	28	Driver only	Rear	4	2.3	114

##	93		28 Driver	r & Passenger	Front 5	2.4	168
##	50	R.PM		•	Fuel.tank.capacity		
##	1	6300	2890	Yes	13.2	5	177
##		5500	2335	Yes	18.0	5	195
##	3	5500	2280	Yes	16.9	5	180
##	4	5500	2535	Yes	21.1	6	193
##	5	5700	2545	Yes	21.1	4	186
##	6	5200	2565	No	16.4	6	189
##	7	4800	1570	No	18.0	6	200
##	8	4000	1320	No	23.0	6	216
##	9	4800	1690	No	18.8	5	198
##		4100	1510	No	18.0	6	206
##		6000	1985	No	20.0	5	204
##		5200	2380	Yes	15.2	5	182
		5200	2665	Yes	15.6	5	184
		4600	1805	Yes	15.5	4	193
		5200	2595	No	16.5	6	198
		4800	1690	No	20.0	7	178
		4000	1790	No	27.0	8	194
		4200	1350	No	23.0	6	214
		5000	1450	Yes	20.0	2	179
		5300	1990	No No	18.0	6	203
		5000 4800	2090	No	16.0 16.0	6 6	183 203
		6000	1785 3285	No Yes	13.2	5	203 174
		4800	2595	Yes	14.0	5	174
		4800	2535	Yes	16.0	6	181
		5000	1970	No	20.0	7	175
		4800	2465	No	16.0	6	192
		6000	2120	Yes	19.8	4	180
		6000	2505	Yes	13.2	5	174
		5800	1980	No	18.0	6	202
		5000	3150	Yes	10.0	4	141
##	32	6500	2410	Yes	13.2	5	171
##	33	4200	2805	Yes	15.9	5	177
		4600	2285	Yes	15.4	4	180
##	35	5500	2340	Yes	15.5	4	179
		4800	2080	Yes	21.0	7	176
		4800	1885	No	16.0	5	192
		4200	1415	No	20.0	6	212
		5700	3755	Yes	10.6	4	151
		5400	3250	Yes	12.4	4	164
		5800	2855	Yes	15.9	4	175
		5900	2650	Yes	11.9	4	173
		5600	2610	Yes	17.0	4	185
		5500	2710	Yes	11.9	5	168
		6000	2745	Yes	13.7	5	172 166
		5550 6000	2540 2335	Yes Yes	11.9 17.2	4 5	166 184
		6000	1955	No	22.5	5	200
		5200	2325	Yes	18.5	5	188
		6000	2510	Yes	20.6	4	191
		4400	1835	No	18.4	6	205
		4600	1840	No	20.0	6	219

		5000	2370		Yes		13.2	4	164
		5500	2220		Yes		14.5	5	172
##		5600	2505		Yes		15.5	5	184
##		5000	2240		No		19.6	7	
##		6500	2325		Yes		20.0	2	
##	58	5100	2425		Yes		14.5	5	175
##	59	5500	2220		No		18.5	5	187
##	60	5750	2475		Yes		11.1	4	166
##	61	3800	1730		No		18.0	5	199
##	62	6000	2505		Yes		13.2	5	172
##	63	6000	2210		No		19.0	5	190
##	64	6000	2435		Yes		13.2	5	170
##	65	5600	2130		Yes		15.9	5	181
##	66	4800	2065		No		20.0	7	190
##	67	5200	2045		No		18.5	5	188
##	68	6000	2380		No		15.2	5	188
##	69	5200	2565		No		16.5	5	190
##	70	4800	1690		No		20.0	7	194
##	71	4800	1570		No		18.0	6	201
##	72	5000	2360		Yes		15.9	4	173
##		5600	3130		Yes		13.2	4	177
##		5200	2665		Yes		15.2	5	181
##		4600	1805		Yes		15.5	4	196
##		5000	1890		Yes		16.5	5	195
##		4800	1565		No		18.0	6	177
##		6000	2910		Yes		18.0	5	184
##		5000	2145		Yes		12.8	5	176
##		5600	2875		Yes		9.2	4	146
##		5200	3375		Yes		15.9	5	175
##		5600	2330		Yes		15.9	5	179
##		6000	3360		Yes		10.6	4	161
##		5200	3505		Yes		11.9	5	162
##		5400	2405		Yes		15.9	4	174
##		5400	2340		Yes		18.5	5	188
		5000	2515		Yes		19.8	7	
##		5500	2550		Yes		12.4	4	163
		4500	2915		Yes		21.1	7	
		5800	2685		Yes		18.5	5	180
		5800	2385		Yes		18.5	4	159
		5400	2215		Yes		15.8	5	190
		6200	2310		Yes		19.3	5	
##	00			circle F		room	Luggage.room		
##	1	102	68	37	tear . Beat.	26.5	11	_	non-USA
##		115	71	38		30.0	15		non-USA
##		102	67	37		28.0	14		non-USA
##		102	70	37		31.0	17		non-USA
##		100	69	39		27.0	13		non-USA
##		109	69	39 41		28.0	16	2880	USA
	о 7	111	69 74	41		30.5	17	3470	USA
##				42 45					
##		116 108	78 73	45 41		30.5	21	4105 3405	USA
			73 73			26.5	14	3495 3620	USA
##		114	73 74	43		35.0	18	3620	USA
##		111	74	44		31.0	14	3935	USA
##	12	101	66	38		25.0	13	2490	USA

## 13	103	68	39	26.0	14	2785	USA
## 14	101	74	43	25.0	13	3240	USA
## 15	108	71	40	28.5	16	3195	USA
## 16	110	74	44	30.5	NA	3715	USA
## 17	111	78	42	33.5	NA	4025	USA
## 18	116	77	42	29.5	20	3910	USA
## 19	96	74	43	NA	NA	3380	USA
## 20	113	74	40	31.0	15	3515	USA
## 21	104	68	41	30.5	14	3085	USA
## 22	110	69	44	36.0	17	3570	USA
## 23	98	66	32	26.5	11	2270	USA
## 24	97	67	38	26.5	13	2670	USA
## 25	104	68	39	30.5	14	2970	USA
## 26	112	72	42	26.5	NA	3705	USA
## 27	105	69	42	30.5	16	3080	USA
## 28	97	72	40	20.0	11	3805	USA
## 29	98	66	36	26.5	11	2295	USA
## 30	113	74	40	30.0	15	3490	USA
## 31	90	63	33	26.0	12	1845	USA
## 32	98	67	36	28.0	12	2530	USA
## 33	100	68	39	27.5	13	2690	USA
## 34	101	68	40	24.0	12	2850	USA
## 35	103	70	38	23.0	18	2710	USA
## 36	119	72	45	30.0	NA	3735	USA
## 37	106	71	40	27.5	18	3325	USA
## 38	114	78	43	30.0	21	3950	USA
## 39	93	63	34	27.5	10		non-USA
## 40	97	67	37	24.5	11		non-USA
## 41	100	70	39	23.5	8		non-USA
## 42	103	67	36	28.0	12		non-USA
## 43	107	67	41	28.0	14		non-USA
## 44	94	63	35	26.0	11		non-USA
## 45	98	66	36	28.0	12		non-USA
## 46	94	64	34	23.5	9		non-USA
## 47	104	69	41	31.0	14		non-USA
## 48	113	72	42	29.0	15		non-USA
## 49	103	70	40	27.5	14		non-USA
## 50	106	71	39	25.0	9		non-USA
## 51	109	73	42	30.0	19	3695	USA
## 52	117	77	45	31.5	22	4055	USA
## 53	97	66	34	27.0	16		non-USA
## 54	98	66	36	26.5	13		non-USA
## 55	103	69	40	29.5	14		non-USA
## 56	110	72	39	27.5	NA		non-USA
## 57	96	69	37	NA	NA		non-USA
## 58	105	67	34	26.0	12		non-USA
## 59	110	69	37	27.0	15		non-USA
## 60	95	65	36	19.0	6	2450	USA
## 61	113	73	38	28.0	15	3610	USA
## 62	98	67	36	26.0	11		non-USA
## 63	107	70	43	27.5	14		non-USA
## 64	96	66	33	26.0	12		non-USA
## 65	103	67	40	28.5	14		non-USA
## 66	112	74	41	27.0	NA	4100	non-USA

##		104 69	41	28.5	14		non-USA
	68	103 67	39	28.0	14	2910	USA
	69	105 70	42	28.0	16	2890	USA
	70	110 74	44	30.5	NA	3715	USA
	71	111 74	42	31.5	17	3470	USA
	72	97 67	39	24.5	8	2640	USA
	73	99 66	35	25.5	17	2350	USA
	74	101 66	39	25.0	13	2575	USA
	75	101 75	43	25.0	13	3240	USA
##	76	108 72	41	28.5	16	3450	USA
##	77	111 74	43	30.5	18	3495	USA
##	78	99 67	37	26.5	14		non-USA
##	79	102 68	40	26.5	12	2495	USA
	80	90 60	32	23.5	10		non-USA
	81	97 65	35	27.5	15		non-USA
	82	102 67	37	27.0	14		non-USA
	83	93 63	34	27.5	10		non-USA
	84	94 65	36	24.0	11		non-USA
##		99 69	39	23.0	13		non-USA
	86	103 70	38	28.5	15		non-USA
##		113 71	41	35.0	NA		non-USA
	88	93 63	34	26.0	10		non-USA
##		115 72	38	34.0	NA		non-USA
##		103 67	35	31.5	14		non-USA
##		97 66	36	26.0	15		non-USA
	92	104 67	37	29.5	14		non-USA
##	93	105 69	38	30.0	15	3245	non-USA
##		Make					
	1	Acura Integra					
	2	Acura Legend					
	3	Audi 90					
	4	Audi 100					
	5	BMW 535i					
	6	Buick Century					
	7	Buick LeSabre					
	8	Buick Roadmaster					
##		Buick Riviera					
##		Cadillac DeVille					
##		Cadillac Seville					
	12	Chevrolet Cavalier					
##		Chevrolet Corsica					
	14	Chevrolet Camaro					
##		Chevrolet Lumina					
##		Chevrolet Lumina_APV					
##		Chevrolet Astro					
	18	Chevrolet Caprice					
	19	Chevrolet Corvette					
	20	Chrylser Concorde					
	21	Chrysler LeBaron					
	22	Chrysler Imperial					
##		Dodge Colt					
##		Dodge Shadow					
##		Dodge Spirit					
##	26	Dodge Caravan					

	27	Dodge Dynasty
	28	Dodge Stealth
	29	Eagle Summit
##	30	Eagle Vision
	31	Ford Festiva
##	32	Ford Escort
##	33	Ford Tempo
##	34	Ford Mustang
##	35	Ford Probe
##	36	Ford Aerostar
##	37	Ford Taurus
##	38	Ford Crown_Victoria
##	39	Geo Metro
##	40	Geo Storm
##	41	Honda Prelude
##	42	Honda Civic
##	43	Honda Accord
##	44	Hyundai Excel
##	45	Hyundai Elantra
##	46	Hyundai Scoupe
##	47	Hyundai Sonata
##	48	Infiniti Q45
	49	Lexus ES300
	50	Lexus SC300
	51	Lincoln Continental
	52	Lincoln Town_Car
	53	Mazda 323
	54	Mazda Protege
	55	Mazda 11000g0
	56	Mazda MPV
	57	Mazda RX-7
	58	Mercedes-Benz 190E
	59	Mercedes-Benz 300E
	60	Mercury Capri
	61	Mercury Cougar
	62	Mitsubishi Mirage
	63	Mitsubishi Diamante
	64	Nissan Sentra
	65	Nissan Altima
	66	Nissan Quest
	67	Nissan Quest Nissan Maxima
	68	Oldsmobile Achieva
	69	Oldsmobile Cutlass_Ciera
	70	Oldsmobile Silhouette
	70 71	
		0,0
	72 73	Plymouth Laser Pontiac LeMans
	74 75	Pontiac Sunbird
	75 76	Pontiac Firebird
	76 77	Pontiac Grand_Prix
	77	Pontiac Bonneville
	78	Saab 900
	79	Saturn SL
##	80	Subaru Justy

```
## 81
                  Subaru Lovale
## 82
                  Subaru Legacy
## 83
                   Suzuki Swift
## 84
                  Toyota Tercel
## 85
                  Toyota Celica
## 86
                   Toyota Camry
## 87
                  Toyota Previa
## 88
                 Volkswagen Fox
## 89
            Volkswagen Eurovan
## 90
             Volkswagen Passat
## 91
            Volkswagen Corrado
## 92
                      Volvo 240
## 93
                      Volvo 850
```

data display

```
View(Cars93)
# number of rows
nrow(Cars93)
```

[1] 93

```
# data type of each column
str(Cars93)
```

```
## 'data.frame':
                    93 obs. of 27 variables:
                        : Factor w/ 32 levels "Acura", "Audi", ...: 1 1 2 2 3 4 4 4 4 5 ...
##
   $ Manufacturer
##
                        : Factor w/ 93 levels "100", "190E", "240", ...: 49 56 9 1 6 24 54 74 73 35 ...
   $ Model
##
  $ Type
                        : Factor w/ 6 levels "Compact", "Large", ...: 4 3 1 3 3 3 2 2 3 2 ...
## $ Min.Price
                              12.9 29.2 25.9 30.8 23.7 14.2 19.9 22.6 26.3 33 ...
                        : num
                               15.9 33.9 29.1 37.7 30 15.7 20.8 23.7 26.3 34.7 ...
##
   $ Price
                        : num
##
   $ Max.Price
                              18.8 38.7 32.3 44.6 36.2 17.3 21.7 24.9 26.3 36.3 ...
                        : num
  $ MPG.city
                               25 18 20 19 22 22 19 16 19 16 ...
                        : int
                              31 25 26 26 30 31 28 25 27 25 ...
##
   $ MPG.highway
                        : int
##
   $ AirBags
                        : Factor w/ 3 levels "Driver & Passenger",..: 3 1 2 1 2 2 2 2 2 2 ...
## $ DriveTrain
                        : Factor w/ 3 levels "4WD", "Front", ...: 2 2 2 2 3 2 2 3 2 2 ...
                        : Factor w/ 6 levels "3", "4", "5", "6", ...: 2 4 4 4 2 2 4 4 4 5 ...
  $ Cylinders
                              1.8 3.2 2.8 2.8 3.5 2.2 3.8 5.7 3.8 4.9 ...
##
   $ EngineSize
                        : num
   $ Horsepower
##
                        : int
                               140 200 172 172 208 110 170 180 170 200 ...
## $ RPM
                        : int
                               6300 5500 5500 5500 5700 5200 4800 4000 4800 4100 ...
##
                               2890 2335 2280 2535 2545 2565 1570 1320 1690 1510 ...
   $ Rev.per.mile
                        : int
                        : Factor w/ 2 levels "No", "Yes": 2 2 2 2 2 1 1 1 1 1 ...
##
   $ Man.trans.avail
##
   $ Fuel.tank.capacity: num
                               13.2 18 16.9 21.1 21.1 16.4 18 23 18.8 18 ...
##
   $ Passengers
                        : int
                               5 5 5 6 4 6 6 6 5 6 ...
                               177 195 180 193 186 189 200 216 198 206 ...
##
   $ Length
                        : int
##
   $ Wheelbase
                               102 115 102 106 109 105 111 116 108 114 ...
                        : int
##
   $ Width
                               68 71 67 70 69 69 74 78 73 73 ...
                        : int
  $ Turn.circle
                               37 38 37 37 39 41 42 45 41 43 ...
##
                        : int
  $ Rear.seat.room
                        : num
                               26.5 30 28 31 27 28 30.5 30.5 26.5 35 ...
##
##
                               11 15 14 17 13 16 17 21 14 18 ...
   $ Luggage.room
                        : int
## $ Weight
                               2705 3560 3375 3405 3640 2880 3470 4105 3495 3620 ...
                        : int
                        : Factor w/ 2 levels "USA", "non-USA": 2 2 2 2 2 1 1 1 1 1 ...
## $ Origin
                        : Factor w/ 93 levels "Acura Integra",..: 1 2 4 3 5 6 7 9 8 10 ...
## $ Make
```

```
# if there is null values in the dataset
# is.na(Cars93)
# name of the columns in the dataset
names(Cars93)
```

```
"Model"
                                                    "Type"
##
   [1] "Manufacturer"
##
   [4] "Min.Price"
                              "Price"
                                                    "Max.Price"
  [7] "MPG.city"
                              "MPG.highway"
                                                    "AirBags"
##
## [10] "DriveTrain"
                              "Cylinders"
                                                    "EngineSize"
                              "RPM"
## [13] "Horsepower"
                                                    "Rev.per.mile"
## [16] "Man.trans.avail"
                              "Fuel.tank.capacity"
                                                   "Passengers"
## [19] "Length"
                              "Wheelbase"
                                                    "Width"
## [22] "Turn.circle"
                              "Rear.seat.room"
                                                    "Luggage.room"
## [25] "Weight"
                              "Origin"
                                                    "Make"
```

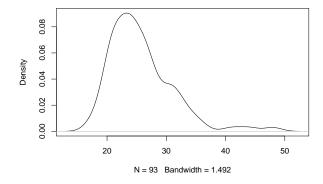
create the response variable "MPG.avg" by averaging "MPG.city" and "MPG.highway"

```
df <- Cars93 %>%
    mutate(MPG.avg = (Cars93$MPG.highway + Cars93$MPG.city) / 2)
```

check the distribution of response (histogram, qq-plot) \rightarrow box-cox

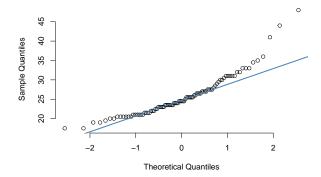
```
plot(density(df$MPG.avg))
```

density.default(x = df\$MPG.avg)



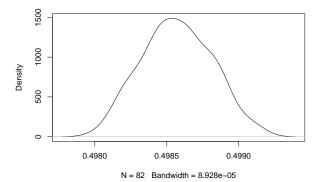
```
qqnorm(df$MPG.avg, pch = 1, frame = FALSE)
qqline(df$MPG.avg, col = "steelblue", lwd = 2)
```

Normal Q-Q Plot

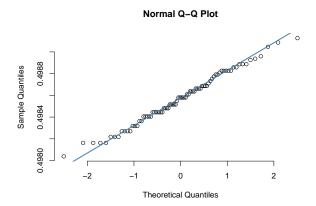


box-cox

density.default(x = df2\$MPG.avg)

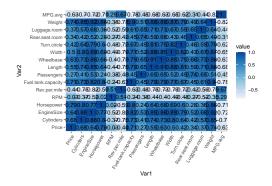


```
qqnorm(df2$MPG.avg, pch = 1, frame = FALSE)
qqline(df2$MPG.avg, col = "steelblue", lwd = 2)
```



check the Pairwise Pearson Correlations From the Person Correlation matrix, there appears to be a significant amount of correlated relations between the predictor variables. It will thus be necessary to ensure that multicollinearity can be a concern later in my model.

```
df %>%
   na.omit() %>%
   mutate(Cylinders = as.numeric(.$Cylinders)) %>%
   subset(select = -c(MPG.city, MPG.highway, Min.Price, Max.Price, Manufacturer, Model, Make)) %>%
   # remove the non-numerical variables
# https://bit.ly/3ZXRAMH
   .[, colnames(.)[!grepl("factor|logical|character", sapply(., class))]] %>%
   cor(., ) %>%
   round(., 2) %>%
   melt() %>%
   ggplot(., aes(x = Var1, y = Var2, fill = value)) +
        geom_tile() +
        scale_fill_distiller(direction = +1) +
        geom_text(aes(Var2, Var1, label = value), color = "black", size = 4) +
        theme(axis.text.x = element_text(angle = 60, hjust = 1))
```



Linear Models to grasp the trend

```
# simple linear model # I use my intuition for now. Simply, I suppose the size of a car and engine and fuel tank are correlat
```

```
simple <- lm(MPG.avg ~ Weight + Width + Length +</pre>
                   Fuel.tank.capacity + Horsepower,
                   data = df2
                   )
# Only weight is significant
summary(simple)
##
## Call:
## lm(formula = MPG.avg ~ Weight + Width + Length + Fuel.tank.capacity +
      Horsepower, data = df2)
##
##
## Residuals:
##
                                                     Max
                     1Q
                            Median
                                            3Q
## -2.840e-04 -7.125e-05 5.640e-06 6.970e-05 3.739e-04
##
## Coefficients:
                       Estimate Std. Error t value Pr(>|t|)
##
                       4.991e-01 3.568e-04 1398.637 < 2e-16 ***
## (Intercept)
                     -3.337e-07 8.790e-08 -3.797 0.000293 ***
## Weight
## Width
                      7.138e-06 7.763e-06 0.920 0.360736
                      2.018e-06 2.150e-06 0.938 0.351016
## Length
## Fuel.tank.capacity -1.808e-05 9.370e-06 -1.930 0.057356 .
## Horsepower
                     -5.212e-07 5.308e-07 -0.982 0.329205
## Signif. codes: 0 '*** 0.001 '** 0.01 '* 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.0001096 on 76 degrees of freedom
## Multiple R-squared: 0.8035, Adjusted R-squared: 0.7906
## F-statistic: 62.17 on 5 and 76 DF, p-value: < 2.2e-16
# Regression Diagnostics
simple.infl <- influence.measures(simple)</pre>
# print(simple.infl)
# check the vif. In fact, Weight has the highest vif (16.69)
vif(simple)
##
              Weight
                                  Width
                                                     Length Fuel.tank.capacity
##
           16.692146
                               5.546023
                                                  7.275453
                                                                      5.366428
##
          Horsepower
            4.953250
##
# get rid of the weight parameter
simple_v2 <- lm(MPG.avg ~ Width + Length +
                   Fuel.tank.capacity + Horsepower,
                   data = df2
                   )
# Indeed, Fuel.tank.capacity and Horsepower became significant. Yet, this model is not preferrable in t
summary(simple_v2)
```

```
## Call:
## lm(formula = MPG.avg ~ Width + Length + Fuel.tank.capacity +
      Horsepower, data = df2)
##
## Residuals:
                            Median
                                            3Q
##
         Min
                     1Q
                                                      Max
## -3.329e-04 -8.941e-05 1.314e-05 7.026e-05 3.344e-04
##
## Coefficients:
##
                       Estimate Std. Error t value Pr(>|t|)
## (Intercept)
                      4.998e-01 3.323e-04 1503.792 < 2e-16 ***
                     -3.313e-07 8.137e-06
                                             -0.041 0.967632
## Width
                                             -1.009 0.316263
## Length
                     -2.040e-06 2.022e-06
## Fuel.tank.capacity -3.361e-05 9.135e-06
                                             -3.680 0.000431 ***
                     -1.820e-06 4.398e-07
                                             -4.137 8.89e-05 ***
## Horsepower
## ---
## Signif. codes: 0 '*** 0.001 '** 0.01 '* 0.05 '.' 0.1 ' 1
## Residual standard error: 0.0001187 on 77 degrees of freedom
## Multiple R-squared: 0.7663, Adjusted R-squared: 0.7541
## F-statistic: 63.11 on 4 and 77 DF, p-value: < 2.2e-16
# check the vif. There are none high vif variables
vif(simple_v2)
##
                Width
                                 Length Fuel.tank.capacity
                                                                   Horsepower
##
            5.189839
                               5.478482
                                                   4.343808
                                                                      2.896575
stepwise with AIC and BIC
# use stepwise methods based on AIC and BIC
m1 <- lm(data = df2, MPG.avg ~ .)
M1_BIC <- stepwise(m1, direction = "forward/backward", criterion = "BIC")
## Direction: forward/backward
## Criterion: BIC
## Start: AIC=-1363.87
## MPG.avg ~ 1
##
##
                       Df Sum of Sq
                                            RSS
                                                     AIC
## + Weight
                        1 3.5969e-06 1.0487e-06 -1481.5
## + Fuel.tank.capacity 1 3.2658e-06 1.3799e-06 -1459.0
## + Cylinders
                        1 3.1450e-06 1.5006e-06 -1452.1
## + EngineSize
                        1 3.1017e-06 1.5439e-06 -1449.8
                        1 3.0577e-06 1.5879e-06 -1447.5
## + Horsepower
## + Price
                        1 2.6052e-06 2.0404e-06 -1426.9
## + Type
                        4 2.7563e-06 1.8893e-06 -1420.0
## + Length
                        1 2.4066e-06 2.2390e-06 -1419.3
## + Width
                        1 2.3643e-06 2.2814e-06 -1417.8
## + Wheelbase
                        1 2.2472e-06 2.3985e-06 -1413.7
## + Rev.per.mile
                        1 2.1306e-06 2.5150e-06 -1409.8
```

```
## + Turn.circle
                         1 1.9762e-06 2.6695e-06 -1404.9
## + Man.trans.avail
                         1 1.1600e-06 3.4856e-06 -1383.0
## + Luggage.room
                         1 1.0447e-06 3.6010e-06 -1380.3
## + AirBags
                         2 1.0519e-06 3.5937e-06 -1376.1
## + Passengers
                         1 8.2490e-07 3.8207e-06 -1375.5
## + Rear.seat.room
                         1 6.5460e-07 3.9910e-06 -1371.9
## + DriveTrain
                         2 7.4690e-07 3.8987e-06 -1369.4
## + R.PM
                         1 3.6850e-07 4.2771e-06 -1366.2
## <none>
                                      4.6456e-06 -1363.9
## + Origin
                         1 1.7970e-07 4.4659e-06 -1362.7
## Step: AIC=-1481.51
## MPG.avg ~ Weight
##
##
                        Df Sum of Sq
                                             RSS
                                                     AIC
## + Wheelbase
                         1 1.2320e-07 9.2550e-07 -1487.3
## + Price
                         1 1.0170e-07 9.4700e-07 -1485.5
## + Cylinders
                         1 1.0160e-07 9.4720e-07 -1485.5
                         1 6.0600e-08 9.8820e-07 -1482.0
## + Length
## + Luggage.room
                         1 5.8500e-08 9.9020e-07 -1481.8
## <none>
                                      1.0487e-06 -1481.5
## + Horsepower
                         1 5.3700e-08 9.9500e-07 -1481.4
## + Fuel.tank.capacity 1 5.2700e-08 9.9600e-07 -1481.3
## + Width
                         1 4.9900e-08 9.9890e-07 -1481.1
## + Man.trans.avail
                         1 2.2700e-08 1.0261e-06 -1478.9
## + Turn.circle
                         1 1.9700e-08 1.0291e-06 -1478.7
## + Rear.seat.room
                         1 1.6600e-08 1.0321e-06 -1478.4
## + Passengers
                         1 4.7000e-09 1.0440e-06 -1477.5
## + RPM
                         1 3.3000e-09 1.0455e-06 -1477.4
## + EngineSize
                         1 3.2000e-09 1.0456e-06 -1477.3
## + Rev.per.mile
                         1 1.9000e-09 1.0468e-06 -1477.2
## + Origin
                         1 1.4000e-09 1.0473e-06 -1477.2
## + AirBags
                         2 3.7300e-08 1.0114e-06 -1475.7
                         2 1.8000e-09 1.0470e-06 -1472.8
## + DriveTrain
## + Type
                         4 7.6100e-08 9.7260e-07 -1470.1
                         1 3.5969e-06 4.6456e-06 -1363.9
## - Weight
##
## Step: AIC=-1487.35
## MPG.avg ~ Weight + Wheelbase
##
##
                        Df Sum of Sq
                                             RSS
## + Cylinders
                         1 9.2680e-08 8.3282e-07 -1491.6
## + Price
                         1 8.8800e-08 8.3670e-07 -1491.2
## + Fuel.tank.capacity 1 5.4600e-08 8.7090e-07 -1487.9
## <none>
                                      9.2550e-07 -1487.3
## + EngineSize
                         1 2.6110e-08 8.9939e-07 -1485.3
## + Passengers
                         1 1.4050e-08 9.1145e-07 -1484.2
## + Horsepower
                         1 1.1400e-08 9.1409e-07 -1484.0
## + Width
                         1 1.0420e-08 9.1508e-07 -1483.9
## + Luggage.room
                         1 7.3800e-09 9.1811e-07 -1483.6
## + Rear.seat.room
                         1 6.5800e-09 9.1892e-07 -1483.5
## + Turn.circle
                         1 5.8900e-09 9.1961e-07 -1483.5
## + Length
                         1 1.6900e-09 9.2381e-07 -1483.1
## + Origin
                         1 1.4700e-09 9.2402e-07 -1483.1
```

```
## + RPM
                         1 4.2000e-10 9.2508e-07 -1483.0
## + Rev.per.mile
                         1 4.0000e-11 9.2546e-07 -1483.0
## + Man.trans.avail
                         1 2.0000e-11 9.2548e-07 -1483.0
## + AirBags
                         2 4.2190e-08 8.8331e-07 -1482.4
## - Wheelbase
                         1 1.2325e-07 1.0487e-06 -1481.5
## + DriveTrain
                         2 1.5190e-08 9.1031e-07 -1479.9
## + Type
                         4 4.6120e-08 8.7938e-07 -1473.9
                         1 1.4730e-06 2.3985e-06 -1413.7
## - Weight
##
## Step: AIC=-1491.6
## MPG.avg ~ Weight + Wheelbase + Cylinders
##
                        Df Sum of Sq
##
                                             RSS
                                                     AIC
## + Fuel.tank.capacity 1 7.4610e-08 7.5821e-07 -1494.9
## + Price
                         1 6.4180e-08 7.6864e-07 -1493.8
## <none>
                                      8.3282e-07 -1491.6
## + Width
                         1 4.0150e-08 7.9267e-07 -1491.2
## + Passengers
                         1 1.5610e-08 8.1721e-07 -1488.7
                         1 1.5240e-08 8.1758e-07 -1488.7
## + Luggage.room
## + Rev.per.mile
                         1 6.5300e-09 8.2629e-07 -1487.8
## + Rear.seat.room
                         1 4.4400e-09 8.2838e-07 -1487.6
## + Man.trans.avail
                        1 4.2100e-09 8.2861e-07 -1487.6
## + Turn.circle
                         1 3.5500e-09 8.2926e-07 -1487.5
## - Cvlinders
                         1 9.2680e-08 9.2550e-07 -1487.3
## + Length
                         1 1.4500e-09 8.3137e-07 -1487.3
## + RPM
                         1 8.6000e-10 8.3195e-07 -1487.3
## + Horsepower
                         1 8.4000e-10 8.3198e-07 -1487.3
                         1 2.0000e-11 8.3280e-07 -1487.2
## + EngineSize
## + Origin
                         1 0.0000e+00 8.3282e-07 -1487.2
## - Wheelbase
                        1 1.1436e-07 9.4718e-07 -1485.5
## + AirBags
                         2 2.6390e-08 8.0643e-07 -1485.4
## + Type
                         4 9.2140e-08 7.4067e-07 -1483.6
## + DriveTrain
                         2 6.6300e-09 8.2619e-07 -1483.4
## - Weight
                         1 5.8118e-07 1.4140e-06 -1452.6
## Step: AIC=-1494.89
## MPG.avg ~ Weight + Wheelbase + Cylinders + Fuel.tank.capacity
##
##
                        Df Sum of Sq
                                             RSS
                                                     AIC
## + Price
                         1 4.3942e-08 7.1427e-07 -1495.4
## <none>
                                      7.5821e-07 -1494.9
## + Width
                         1 3.7818e-08 7.2039e-07 -1494.7
## + Luggage.room
                         1 2.7203e-08 7.3101e-07 -1493.5
## + Passengers
                         1 1.8140e-08 7.4007e-07 -1492.5
## - Fuel.tank.capacity 1 7.4605e-08 8.3282e-07 -1491.6
## + Origin
                         1 4.9060e-09 7.5330e-07 -1491.0
## + Rear.seat.room
                         1 3.6240e-09 7.5459e-07 -1490.9
## + Length
                         1 1.5500e-09 7.5666e-07 -1490.7
## + Turn.circle
                         1 4.7100e-10 7.5774e-07 -1490.5
## + RPM
                         1 2.0900e-10 7.5800e-07 -1490.5
## + Horsepower
                         1 1.4100e-10 7.5807e-07 -1490.5
## + Man.trans.avail
                         1 1.3500e-10 7.5808e-07 -1490.5
## + Rev.per.mile
                         1 1.2200e-10 7.5809e-07 -1490.5
## + EngineSize
                         1 6.3000e-11 7.5815e-07 -1490.5
```

```
## - Cylinders
                         1 1.1269e-07 8.7090e-07 -1487.9
                         2 1.5281e-08 7.4293e-07 -1487.7
## + AirBags
                         1 1.1557e-07 8.7378e-07 -1487.7
## - Wheelbase
## + DriveTrain
                         2 6.2620e-09 7.5195e-07 -1486.8
## + Type
                         4 6.0867e-08 6.9734e-07 -1484.1
                         1 1.6390e-07 9.2211e-07 -1483.2
## - Weight
## Step: AIC=-1495.38
## MPG.avg ~ Weight + Wheelbase + Cylinders + Fuel.tank.capacity +
##
       Price
##
##
                                             RSS
                        Df Sum of Sq
                                                     AIC
## <none>
                                      7.1427e-07 -1495.4
## - Price
                         1 4.3942e-08 7.5821e-07 -1494.9
## + Origin
                         1 3.2587e-08 6.8168e-07 -1494.8
## + Passengers
                         1 2.9341e-08 6.8493e-07 -1494.4
## - Fuel.tank.capacity 1 5.4371e-08 7.6864e-07 -1493.8
## + Luggage.room
                         1 1.3624e-08 7.0065e-07 -1492.5
                         1 1.2159e-08 7.0211e-07 -1492.4
## + Horsepower
## + Width
                         1 9.8340e-09 7.0444e-07 -1492.1
## + RPM
                         1 9.5100e-09 7.0476e-07 -1492.1
## + Rev.per.mile
                         1 5.0770e-09 7.0919e-07 -1491.5
## + Rear.seat.room
                         1 3.7200e-09 7.1055e-07 -1491.4
## + Turn.circle
                         1 3.0280e-09 7.1124e-07 -1491.3
## + EngineSize
                         1 1.9850e-09 7.1228e-07 -1491.2
## + Length
                         1 1.1750e-09 7.1309e-07 -1491.1
## + Man.trans.avail
                         1 5.2000e-11 7.1422e-07 -1491.0
## + AirBags
                         2 3.5308e-08 6.7896e-07 -1490.7
## - Cylinders
                         1 8.5761e-08 8.0003e-07 -1490.5
## - Wheelbase
                         1 1.0719e-07 8.2146e-07 -1488.3
## + DriveTrain
                         2 4.1200e-09 7.1015e-07 -1487.0
## - Weight
                         1 1.3345e-07 8.4772e-07 -1485.7
## + Type
                         4 4.2409e-08 6.7186e-07 -1482.8
M1_AIC <- stepwise(m1, direction = "forward/backward", criterion = "AIC")
##
## Direction: forward/backward
## Criterion: AIC
## Start: AIC=-1366.28
## MPG.avg ~ 1
##
##
                                             RSS
                                                      AIC
                        Df Sum of Sq
## + Weight
                         1 3.5969e-06 1.0487e-06 -1486.3
## + Fuel.tank.capacity 1 3.2658e-06 1.3799e-06 -1463.8
## + Cylinders
                         1 3.1450e-06 1.5006e-06 -1456.9
## + EngineSize
                         1 3.1017e-06 1.5439e-06 -1454.6
## + Horsepower
                         1 3.0577e-06 1.5879e-06 -1452.3
## + Type
                         4 2.7563e-06 1.8893e-06 -1432.1
## + Price
                         1 2.6052e-06 2.0404e-06 -1431.7
## + Length
                        1 2.4066e-06 2.2390e-06 -1424.1
```

1 2.3643e-06 2.2814e-06 -1422.6

1 2.2472e-06 2.3985e-06 -1418.5

+ Width

+ Wheelbase

```
## + Rev.per.mile
                         1 2.1306e-06 2.5150e-06 -1414.6
## + Turn.circle
                         1 1.9762e-06 2.6695e-06 -1409.7
## + Man.trans.avail
                         1 1.1600e-06 3.4856e-06 -1387.8
## + Luggage.room
                         1 1.0447e-06 3.6010e-06 -1385.2
## + AirBags
                         2 1.0519e-06 3.5937e-06 -1383.3
## + Passengers
                         1 8.2490e-07 3.8207e-06 -1380.3
## + Rear.seat.room
                         1 6.5460e-07 3.9910e-06 -1376.7
## + DriveTrain
                         2 7.4690e-07 3.8987e-06 -1376.7
## + RPM
                         1 3.6850e-07 4.2771e-06 -1371.0
## + Origin
                         1 1.7970e-07 4.4659e-06 -1367.5
## <none>
                                      4.6456e-06 -1366.3
##
## Step: AIC=-1486.32
## MPG.avg ~ Weight
##
##
                        Df Sum of Sq
                                             RSS
                                                      AIC
## + Wheelbase
                         1 1.2320e-07 9.2550e-07 -1494.6
## + Price
                         1 1.0170e-07 9.4700e-07 -1492.7
## + Cylinders
                         1 1.0160e-07 9.4720e-07 -1492.7
## + Length
                         1 6.0600e-08 9.8820e-07 -1489.2
## + Luggage.room
                         1 5.8500e-08 9.9020e-07 -1489.0
## + Horsepower
                         1 5.3700e-08 9.9500e-07 -1488.6
## + Fuel.tank.capacity 1 5.2700e-08 9.9600e-07 -1488.5
## + Width
                         1 4.9900e-08 9.9890e-07 -1488.3
## <none>
                                      1.0487e-06 -1486.3
## + Man.trans.avail
                         1 2.2700e-08 1.0261e-06 -1486.1
## + Turn.circle
                         1 1.9700e-08 1.0291e-06 -1485.9
## + Rear.seat.room
                         1 1.6600e-08 1.0321e-06 -1485.6
## + AirBags
                         2 3.7300e-08 1.0114e-06 -1485.3
## + Passengers
                         1 4.7000e-09 1.0440e-06 -1484.7
## + RPM
                         1 3.3000e-09 1.0455e-06 -1484.6
## + EngineSize
                         1 3.2000e-09 1.0456e-06 -1484.6
## + Type
                         4 7.6100e-08 9.7260e-07 -1484.5
## + Rev.per.mile
                         1 1.9000e-09 1.0468e-06 -1484.5
## + Origin
                         1 1.4000e-09 1.0473e-06 -1484.4
## + DriveTrain
                         2 1.8000e-09 1.0470e-06 -1482.5
## - Weight
                         1 3.5969e-06 4.6456e-06 -1366.3
##
## Step: AIC=-1494.57
## MPG.avg ~ Weight + Wheelbase
##
##
                        Df Sum of Sq
                                             RSS
                                                     ATC
## + Cylinders
                         1 9.2680e-08 8.3282e-07 -1501.2
                         1 8.8800e-08 8.3670e-07 -1500.8
## + Price
## + Fuel.tank.capacity 1 5.4600e-08 8.7090e-07 -1497.6
## + EngineSize
                         1 2.6110e-08 8.9939e-07 -1494.9
## <none>
                                      9.2550e-07 -1494.6
## + AirBags
                         2 4.2190e-08 8.8331e-07 -1494.4
## + Passengers
                         1 1.4050e-08 9.1145e-07 -1493.8
## + Horsepower
                         1 1.1400e-08 9.1409e-07 -1493.6
                         1 1.0420e-08 9.1508e-07 -1493.5
## + Width
## + Luggage.room
                         1 7.3800e-09 9.1811e-07 -1493.2
## + Rear.seat.room
                         1 6.5800e-09 9.1892e-07 -1493.2
## + Turn.circle
                         1 5.8900e-09 9.1961e-07 -1493.1
```

```
## + Length
                        1 1.6900e-09 9.2381e-07 -1492.7
## + Origin
                        1 1.4700e-09 9.2402e-07 -1492.7
## + RPM
                        1 4.2000e-10 9.2508e-07 -1492.6
## + Rev.per.mile
                         1 4.0000e-11 9.2546e-07 -1492.6
## + Man.trans.avail
                        1 2.0000e-11 9.2548e-07 -1492.6
## + DriveTrain
                         2 1.5190e-08 9.1031e-07 -1491.9
## + Type
                         4 4.6120e-08 8.7938e-07 -1490.8
## - Wheelbase
                         1 1.2325e-07 1.0487e-06 -1486.3
## - Weight
                         1 1.4730e-06 2.3985e-06 -1418.5
##
## Step: AIC=-1501.22
## MPG.avg ~ Weight + Wheelbase + Cylinders
##
                        Df Sum of Sq
                                             RSS
                                                     AIC
## + Fuel.tank.capacity 1 7.4610e-08 7.5821e-07 -1506.9
## + Price
                         1 6.4180e-08 7.6864e-07 -1505.8
## + Width
                         1 4.0150e-08 7.9267e-07 -1503.3
## + Type
                         4 9.2140e-08 7.4067e-07 -1502.8
                                      8.3282e-07 -1501.2
## <none>
## + Passengers
                        1 1.5610e-08 8.1721e-07 -1500.8
## + Luggage.room
                         1 1.5240e-08 8.1758e-07 -1500.7
## + Rev.per.mile
                        1 6.5300e-09 8.2629e-07 -1499.9
## + AirBags
                         2 2.6390e-08 8.0643e-07 -1499.9
## + Rear.seat.room
                        1 4.4400e-09 8.2838e-07 -1499.7
## + Man.trans.avail
                        1 4.2100e-09 8.2861e-07 -1499.6
## + Turn.circle
                         1 3.5500e-09 8.2926e-07 -1499.6
## + Length
                         1 1.4500e-09 8.3137e-07 -1499.4
                         1 8.6000e-10 8.3195e-07 -1499.3
## + RPM
## + Horsepower
                        1 8.4000e-10 8.3198e-07 -1499.3
## + EngineSize
                        1 2.0000e-11 8.3280e-07 -1499.2
## + Origin
                         1 0.0000e+00 8.3282e-07 -1499.2
## + DriveTrain
                         2 6.6300e-09 8.2619e-07 -1497.9
## - Cylinders
                        1 9.2680e-08 9.2550e-07 -1494.6
                         1 1.1436e-07 9.4718e-07 -1492.7
## - Wheelbase
## - Weight
                         1 5.8118e-07 1.4140e-06 -1459.8
##
## Step: AIC=-1506.92
## MPG.avg ~ Weight + Wheelbase + Cylinders + Fuel.tank.capacity
##
##
                        Df Sum of Sq
                                             RSS
                                                     ATC
## + Price
                        1 4.3942e-08 7.1427e-07 -1509.8
                         1 3.7818e-08 7.2039e-07 -1509.1
## + Width
## + Luggage.room
                         1 2.7203e-08 7.3101e-07 -1507.9
## <none>
                                      7.5821e-07 -1506.9
## + Passengers
                        1 1.8140e-08 7.4007e-07 -1506.9
## + Type
                         4 6.0867e-08 6.9734e-07 -1505.8
## + Origin
                         1 4.9060e-09 7.5330e-07 -1505.5
## + Rear.seat.room
                        1 3.6240e-09 7.5459e-07 -1505.3
## + Length
                         1 1.5500e-09 7.5666e-07 -1505.1
## + Turn.circle
                         1 4.7100e-10 7.5774e-07 -1505.0
## + RPM
                         1 2.0900e-10 7.5800e-07 -1504.9
## + Horsepower
                        1 1.4100e-10 7.5807e-07 -1504.9
## + Man.trans.avail
                        1 1.3500e-10 7.5808e-07 -1504.9
## + Rev.per.mile
                         1 1.2200e-10 7.5809e-07 -1504.9
```

```
## + EngineSize
                        1 6.3000e-11 7.5815e-07 -1504.9
## + AirBags
                         2 1.5281e-08 7.4293e-07 -1504.6
## + DriveTrain
                         2 6.2620e-09 7.5195e-07 -1503.6
## - Fuel.tank.capacity 1 7.4605e-08 8.3282e-07 -1501.2
## - Cylinders
                         1 1.1269e-07 8.7090e-07 -1497.6
## - Wheelbase
                         1 1.1557e-07 8.7378e-07 -1497.3
## - Weight
                         1 1.6390e-07 9.2211e-07 -1492.9
##
## Step: AIC=-1509.82
## MPG.avg ~ Weight + Wheelbase + Cylinders + Fuel.tank.capacity +
       Price
##
##
                        Df Sum of Sq
                                             RSS
                                                     AIC
## + Origin
                         1 3.2587e-08 6.8168e-07 -1511.6
## + Passengers
                         1 2.9341e-08 6.8493e-07 -1511.3
## + AirBags
                         2 3.5308e-08 6.7896e-07 -1510.0
## <none>
                                      7.1427e-07 -1509.8
## + Luggage.room
                        1 1.3624e-08 7.0065e-07 -1509.4
                         1 1.2159e-08 7.0211e-07 -1509.2
## + Horsepower
## + Width
                         1 9.8340e-09 7.0444e-07 -1509.0
## + RPM
                         1 9.5100e-09 7.0476e-07 -1508.9
## + Rev.per.mile
                        1 5.0770e-09 7.0919e-07 -1508.4
## + Rear.seat.room
                        1 3.7200e-09 7.1055e-07 -1508.2
## + Turn.circle
                         1 3.0280e-09 7.1124e-07 -1508.2
## + EngineSize
                        1 1.9850e-09 7.1228e-07 -1508.0
## + Length
                         1 1.1750e-09 7.1309e-07 -1508.0
## + Man.trans.avail
                         1 5.2000e-11 7.1422e-07 -1507.8
## - Price
                         1 4.3942e-08 7.5821e-07 -1506.9
## + Type
                         4 4.2409e-08 6.7186e-07 -1506.8
## + DriveTrain
                         2 4.1200e-09 7.1015e-07 -1506.3
## - Fuel.tank.capacity 1 5.4371e-08 7.6864e-07 -1505.8
## - Cylinders
                         1 8.5761e-08 8.0003e-07 -1502.5
## - Wheelbase
                         1 1.0719e-07 8.2146e-07 -1500.3
                         1 1.3345e-07 8.4772e-07 -1497.8
## - Weight
## Step: AIC=-1511.64
## MPG.avg ~ Weight + Wheelbase + Cylinders + Fuel.tank.capacity +
##
      Price + Origin
##
##
                                             RSS
                        Df Sum of Sq
                                                     ATC
## + Width
                        1 2.6144e-08 6.5554e-07 -1512.8
## + Passengers
                         1 2.0480e-08 6.6120e-07 -1512.2
## + AirBags
                         2 3.4964e-08 6.4672e-07 -1512.0
## + Luggage.room
                         1 1.8816e-08 6.6287e-07 -1511.9
## <none>
                                      6.8168e-07 -1511.6
## + Rear.seat.room
                        1 8.3430e-09 6.7334e-07 -1510.7
## + Horsepower
                         1 8.1790e-09 6.7350e-07 -1510.6
## + Man.trans.avail
                         1 2.3010e-09 6.7938e-07 -1509.9
## + Rev.per.mile
                         1 1.9250e-09 6.7976e-07 -1509.9
## + RPM
                         1 1.6490e-09 6.8003e-07 -1509.8
## - Origin
                        1 3.2587e-08 7.1427e-07 -1509.8
## + Length
                        1 2.2100e-10 6.8146e-07 -1509.7
## + Turn.circle
                        1 5.7000e-11 6.8163e-07 -1509.7
## + EngineSize
                         1 0.0000e+00 6.8168e-07 -1509.6
```

```
## + DriveTrain
                      2 5.6270e-09 6.7606e-07 -1508.3
                        4 3.4725e-08 6.4696e-07 -1507.9
## + Type
## - Cylinders
                       1 6.3835e-08 7.4552e-07 -1506.3
## - Fuel.tank.capacity 1 6.9588e-08 7.5127e-07 -1505.7
## - Price
                        1 7.1622e-08 7.5330e-07 -1505.5
## - Weight
                        1 1.1255e-07 7.9423e-07 -1501.1
## - Wheelbase
                        1 1.2947e-07 8.1115e-07 -1499.4
##
## Step: AIC=-1512.85
## MPG.avg ~ Weight + Wheelbase + Cylinders + Fuel.tank.capacity +
       Price + Origin + Width
##
##
                       Df Sum of Sq
                                            RSS
                                                     AIC
                        1 1.9303e-08 6.3623e-07 -1513.3
## + Luggage.room
## <none>
                                      6.5554e-07 -1512.8
## + Passengers
                        1 1.3850e-08 6.4169e-07 -1512.6
                        1 6.3430e-09 6.4919e-07 -1511.7
## + Horsepower
## - Width
                        1 2.6144e-08 6.8168e-07 -1511.6
## + Man.trans.avail
                       1 4.8680e-09 6.5067e-07 -1511.5
## + AirBags
                        2 1.8781e-08 6.3676e-07 -1511.2
## + RPM
                        1 2.8370e-09 6.5270e-07 -1511.2
## + Rev.per.mile
                        1 2.7150e-09 6.5282e-07 -1511.2
## + Rear.seat.room
                       1 2.1540e-09 6.5338e-07 -1511.1
## + Turn.circle
                        1 1.3180e-09 6.5422e-07 -1511.0
## + EngineSize
                        1 7.3100e-10 6.5481e-07 -1510.9
## - Price
                        1 3.2221e-08 6.8776e-07 -1510.9
## + Length
                        1 4.9900e-10 6.5504e-07 -1510.9
## + DriveTrain
                        2 1.1626e-08 6.4391e-07 -1510.3
## - Origin
                        1 4.8897e-08 7.0444e-07 -1509.0
## + Type
                        4 2.6404e-08 6.2913e-07 -1508.2
## - Wheelbase
                        1 7.5177e-08 7.3072e-07 -1506.0
## - Fuel.tank.capacity 1 8.0802e-08 7.3634e-07 -1505.3
## - Cylinders
                        1 8.7154e-08 7.4269e-07 -1504.6
                        1 1.3596e-07 7.9150e-07 -1499.4
## - Weight
##
## Step: AIC=-1513.3
## MPG.avg ~ Weight + Wheelbase + Cylinders + Fuel.tank.capacity +
##
      Price + Origin + Width + Luggage.room
##
##
                       Df Sum of Sq
                                                     AIC
                                            RSS
## + Passengers
                       1 2.9462e-08 6.0677e-07 -1515.2
## + Horsepower
                        1 2.0658e-08 6.1558e-07 -1514.0
## <none>
                                     6.3623e-07 -1513.3
                        1 1.9303e-08 6.5554e-07 -1512.8
## - Luggage.room
## + Rear.seat.room
                        1 1.1420e-08 6.2481e-07 -1512.8
## + RPM
                        1 1.0403e-08 6.2583e-07 -1512.7
## - Price
                        1 2.3548e-08 6.5978e-07 -1512.3
## + AirBags
                        2 2.1390e-08 6.1484e-07 -1512.1
## - Width
                        1 2.6632e-08 6.6287e-07 -1511.9
## + Rev.per.mile
                        1 3.7590e-09 6.3248e-07 -1511.8
                        1 2.1180e-09 6.3412e-07 -1511.6
## + Man.trans.avail
## + EngineSize
                        1 1.7030e-09 6.3453e-07 -1511.5
## + Turn.circle
                        1 1.5090e-09 6.3473e-07 -1511.5
## + Length
                        1 1.2390e-09 6.3500e-07 -1511.5
```

```
## - Wheelbase
                        1 3.4682e-08 6.7092e-07 -1511.0
## + DriveTrain
                        2 1.0575e-08 6.2566e-07 -1510.7
## - Origin
                        1 5.5092e-08 6.9133e-07 -1508.5
## + Type
                        4 2.1003e-08 6.1523e-07 -1508.0
## - Fuel.tank.capacity 1 9.4151e-08 7.3039e-07 -1504.0
## - Cylinders
                        1 9.8016e-08 7.3425e-07 -1503.5
## - Weight
                        1 1.2035e-07 7.5658e-07 -1501.1
##
## Step: AIC=-1515.19
## MPG.avg ~ Weight + Wheelbase + Cylinders + Fuel.tank.capacity +
       Price + Origin + Width + Luggage.room + Passengers
##
##
                       Df Sum of Sq
                                            RSS
                                                    AIC
                                     6.0677e-07 -1515.2
## <none>
## + Horsepower
                        1 1.3481e-08 5.9329e-07 -1515.0
## - Width
                        1 1.7164e-08 6.2394e-07 -1514.9
## + RPM
                        1 1.1195e-08 5.9558e-07 -1514.7
## + Man.trans.avail
                       1 1.0231e-08 5.9654e-07 -1514.6
## + AirBags
                        2 2.3228e-08 5.8354e-07 -1514.4
## + Rev.per.mile
                        1 3.5450e-09 6.0323e-07 -1513.7
## + DriveTrain
                        2 1.6993e-08 5.8978e-07 -1513.5
## + EngineSize
                       1 2.0910e-09 6.0468e-07 -1513.5
## - Price
                        1 2.8187e-08 6.3496e-07 -1513.5
## + Turn.circle
                        1 1.3750e-09 6.0540e-07 -1513.4
## - Passengers
                        1 2.9462e-08 6.3623e-07 -1513.3
## + Rear.seat.room
                       1 5.6900e-10 6.0620e-07 -1513.3
## + Length
                        1 1.2000e-11 6.0676e-07 -1513.2
## + Type
                        4 4.1545e-08 5.6523e-07 -1513.0
## - Luggage.room
                       1 3.4915e-08 6.4169e-07 -1512.6
## - Origin
                        1 3.9418e-08 6.4619e-07 -1512.0
## - Wheelbase
                        1 5.4895e-08 6.6167e-07 -1510.1
## - Fuel.tank.capacity 1 9.6051e-08 7.0282e-07 -1505.1
## - Cylinders
                        1 9.9320e-08 7.0609e-07 -1504.8
                        1 1.1637e-07 7.2314e-07 -1502.8
## - Weight
# check the summary. M1_AIC has insignificant variable; hence, do not use M1_AIC this time. Interpret M
summary(M1 BIC)
##
## Call:
## lm(formula = MPG.avg ~ Weight + Wheelbase + Cylinders + Fuel.tank.capacity +
##
      Price, data = df2)
##
## Residuals:
                     1Q
                            Median
                                           3Q
```

Estimate Std. Error t value Pr(>|t|) 4.987e-01 2.641e-04 1888.156 < 2e-16 ***

-2.547e-07 6.760e-08 -3.768 0.000323 *** 1.182e-05 3.499e-06 3.377 0.001157 **

-5.507e-05 1.823e-05 -3.021 0.003433 **

-2.579e-04 -5.886e-05 1.269e-05 6.431e-05 2.312e-04

Fuel.tank.capacity -2.021e-05 8.402e-06 -2.405 0.018595 *

Coefficients:

(Intercept)
Weight

Wheelbase

Cylinders

```
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
## Residual standard error: 9.694e-05 on 76 degrees of freedom
## Multiple R-squared: 0.8462, Adjusted R-squared: 0.8361
## F-statistic: 83.66 on 5 and 76 DF, p-value: < 2.2e-16
summary(M1_AIC)
## Call:
## lm(formula = MPG.avg ~ Weight + Wheelbase + Cylinders + Fuel.tank.capacity +
      Price + Origin + Width + Luggage.room + Passengers, data = df2)
##
## Residuals:
         Min
                     1Q
                            Median
                                           3Q
                                                     Max
## -2.312e-04 -5.271e-05 2.142e-05 6.387e-05 2.091e-04
## Coefficients:
                       Estimate Std. Error t value Pr(>|t|)
## (Intercept)
                      4.981e-01 4.225e-04 1178.954 < 2e-16 ***
## Weight
                     -2.547e-07 6.854e-08 -3.716 0.000397 ***
## Wheelbase
                      1.108e-05 4.341e-06
                                             2.552 0.012826 *
## Cylinders
                     -6.612e-05 1.926e-05 -3.433 0.000993 ***
## Fuel.tank.capacity -2.828e-05 8.378e-06 -3.376 0.001188 **
## Price
                     -3.540e-06 1.935e-06 -1.829 0.071563 .
                                            2.163 0.033885 *
## Originnon-USA
                      5.764e-05 2.665e-05
## Width
                     1.121e-05 7.858e-06 1.427 0.157861
## Luggage.room
                     1.139e-05 5.597e-06 2.035 0.045487 *
                     -3.956e-05 2.116e-05 -1.870 0.065583 .
## Passengers
## ---
## Signif. codes: 0 '*** 0.001 '** 0.01 '* 0.05 '.' 0.1 ' ' 1
## Residual standard error: 9.18e-05 on 72 degrees of freedom
## Multiple R-squared: 0.8694, Adjusted R-squared: 0.8531
## F-statistic: 53.25 on 9 and 72 DF, p-value: < 2.2e-16
# vif
vif(M1_BIC)
##
              Weight
                              Wheelbase
                                                 Cylinders Fuel.tank.capacity
##
           12.614474
                               4.412349
                                                  3.680002
                                                                    5.513076
##
               Price
            2.324593
\# since Weight's vif is higher than 10 so I will try to remove some of variables while maintaing R-squa
m2 <- lm(data = df2, MPG.avg ~ Price + Cylinders + Wheelbase + Weight)
M2_BIC <- stepwise(m2, direction = "forward/backward", criterion = "BIC")
##
## Direction: forward/backward
```

-3.566e-06 1.649e-06 -2.162 0.033741 *

Price

```
## Criterion: BIC
##
## Start: AIC=-1363.87
## MPG.avg ~ 1
##
              Df Sum of Sq
                                   RSS
                                           AIC
              1 3.5969e-06 1.0487e-06 -1481.5
## + Weight
## + Cylinders 1 3.1450e-06 1.5006e-06 -1452.1
## + Price
               1 2.6052e-06 2.0404e-06 -1426.9
## + Wheelbase 1 2.2472e-06 2.3985e-06 -1413.7
## <none>
                            4.6456e-06 -1363.9
##
## Step: AIC=-1481.51
## MPG.avg ~ Weight
##
##
              Df Sum of Sq
                                   RSS
## + Wheelbase 1 1.2320e-07 9.2550e-07 -1487.3
## + Price
            1 1.0170e-07 9.4700e-07 -1485.5
## + Cylinders 1 1.0160e-07 9.4720e-07 -1485.5
## <none>
                            1.0487e-06 -1481.5
              1 3.5969e-06 4.6456e-06 -1363.9
## - Weight
## Step: AIC=-1487.35
## MPG.avg ~ Weight + Wheelbase
##
              Df Sum of Sq
                                   RSS
## + Cylinders 1 9.2680e-08 8.3282e-07 -1491.6
## + Price
               1 8.8800e-08 8.3670e-07 -1491.2
## <none>
                            9.2550e-07 -1487.3
## - Wheelbase 1 1.2325e-07 1.0487e-06 -1481.5
## - Weight
               1 1.4730e-06 2.3985e-06 -1413.7
##
## Step: AIC=-1491.6
## MPG.avg ~ Weight + Wheelbase + Cylinders
##
              Df Sum of Sq
##
                                   RSS
## + Price
              1 6.4180e-08 7.6864e-07 -1493.8
## <none>
                            8.3282e-07 -1491.6
## - Cylinders 1 9.2680e-08 9.2550e-07 -1487.3
## - Wheelbase 1 1.1436e-07 9.4718e-07 -1485.5
## - Weight
               1 5.8118e-07 1.4140e-06 -1452.6
##
## Step: AIC=-1493.77
## MPG.avg ~ Weight + Wheelbase + Cylinders + Price
##
              Df Sum of Sq
                                   RSS
                                           AIC
                            7.6864e-07 -1493.8
## <none>
## - Price
               1 6.4180e-08 8.3282e-07 -1491.6
## - Cylinders 1 6.8060e-08 8.3670e-07 -1491.2
## - Wheelbase 1 1.0474e-07 8.7338e-07 -1487.7
              1 3.9963e-07 1.1683e-06 -1463.8
## - Weight
# vif of Weight become under 10.
vif(M2_BIC)
```

```
## Weight Wheelbase Cylinders Price
## 8.062370 4.411181 3.597503 2.256325
```

```
# check the summary
summary(M2_BIC)
##
## Call:
## lm(formula = MPG.avg ~ Weight + Wheelbase + Cylinders + Price,
##
      data = df2)
## Residuals:
                     1Q
                           Median
                                          3Q
## -2.330e-04 -6.264e-05 5.455e-06 6.208e-05 2.238e-04
## Coefficients:
               Estimate Std. Error t value Pr(>|t|)
##
## (Intercept) 4.986e-01 2.720e-04 1833.399 < 2e-16 ***
             -3.524e-07 5.570e-08 -6.327 1.51e-08 ***
## Weight
             1.168e-05 3.605e-06
## Wheelbase
                                     3.239 0.00177 **
## Cylinders -4.851e-05 1.858e-05 -2.611 0.01084 *
## Price
              -4.245e-06 1.674e-06 -2.536 0.01325 *
## ---
## Signif. codes: 0 '*** 0.001 '** 0.01 '* 0.05 '.' 0.1 ' 1
##
```

```
# check the normality by residual plot
plot(df2$Weight, resid(M2_BIC), ylab = "Residuals", xlab = "Weight", main = "Car Efficiency")
abline(0, 0)
```

Residual standard error: 9.991e-05 on 77 degrees of freedom
Multiple R-squared: 0.8345, Adjusted R-squared: 0.8259
F-statistic: 97.1 on 4 and 77 DF, p-value: < 2.2e-16</pre>

Car Efficiency

