STAT 524

HW 1

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Sep / 05 / 2023

Problem 1.1

$$\bar{x}_1 = \frac{1}{7}(3+4+2+6+8+2+5) = \frac{30}{7} \cong \underline{4.286}$$

$$\bar{x}_2 = \frac{1}{7}(5+5.5+4+7+10+5+7.5) = \frac{44}{7} \cong \underline{6.286}$$

$$S_{11} = \frac{1}{n-1}\sum_{i=1}^{n}(X_{ij} - \bar{X}_j)^2$$

$$= \frac{1}{6}\{(3-4.286)^2 + (4-4.286)^2 + (2-4.286)^2 + (6-4.286)^2$$
$$+ (8-4.286)^2 + (2-4.286)^2 + (5-4.286)^2\}$$

$$\cong \frac{29.4286}{6} \cong \underline{4.90}$$

$$S_{22} = \frac{1}{6}\{(5-6.286)^2 + (5.5-6.286)^2 + (4-6.286)^2$$
$$+ (7-6.286)^2 + (10-6.286)^2 + (5-6.286)^2$$
$$+ (7.5-6.286)^2\}$$

$$\cong \frac{24.9286}{6} \cong \underline{4.15}$$

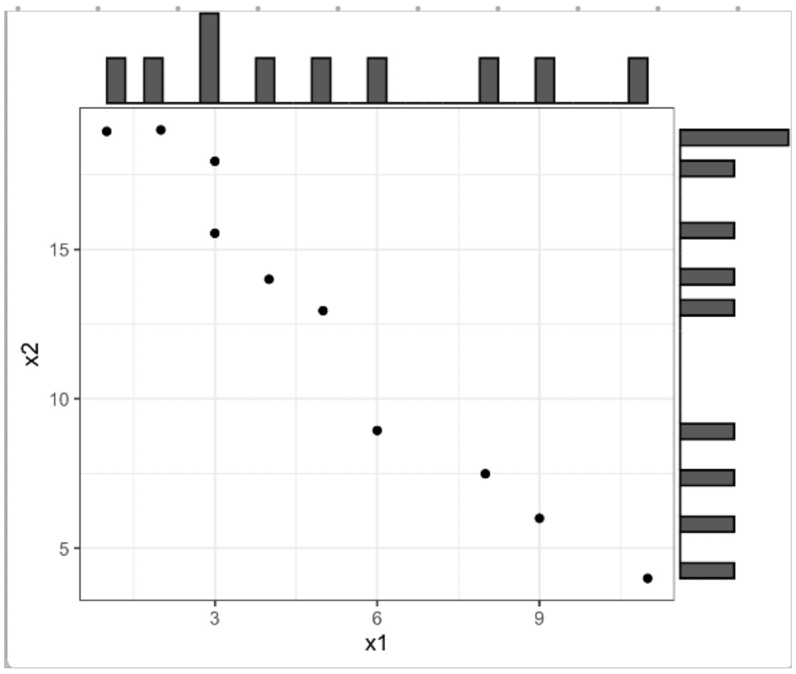
$$S_{12} = \frac{1}{7-1}\sum_{i=1}^{7}(x_{i1} - \bar{x}_{.1})(x_{i2} - \bar{x}_{.2})$$

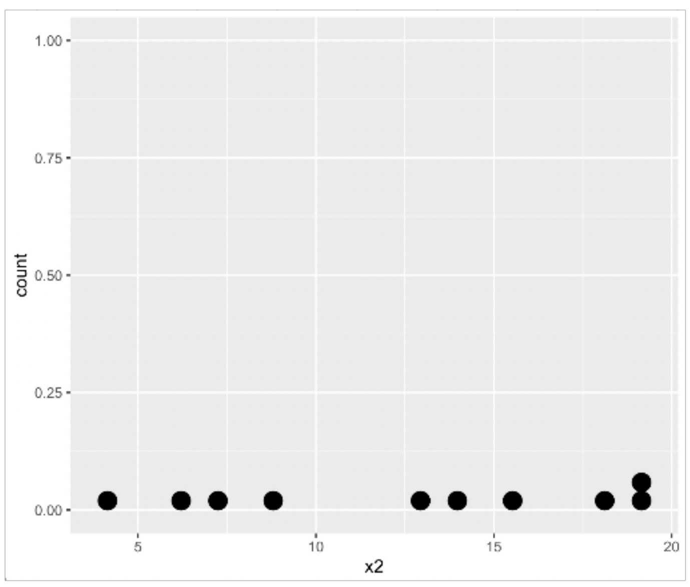
$$= \frac{1}{6}\{(3-4.286)(5-6.286) + (4-4.286)(5.5-6.286)$$
$$+ (2-4.286)(4-6.286) + (6-4.286)(7-6.286)$$
$$+ (8-4.286)(10-6.286) + (2-4.286)(5-6.286)$$
$$+ (5-4.286)(7.5-6.286)\}$$

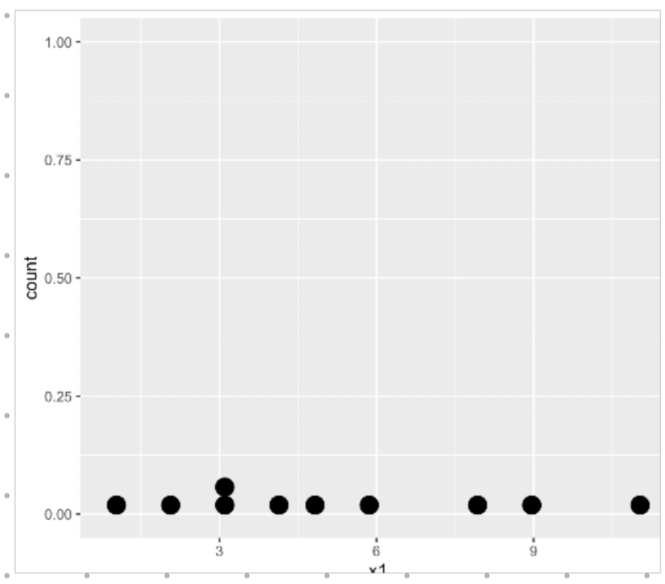
$$\cong \frac{25.929}{6} = \underline{4.321}$$

Problem 1.2

a)







(b) Since two data are negatively correlatted, sign should be negative (−).

(c) $\bar{x}_1 = \frac{1}{10}(1+2+3+3+4+5+6+8+9+11) \simeq \underline{5.2}$

$$\bar{x}_2 = \frac{1}{10}(18.95 + 19.00 + 17.95 + 15.54 + 14.00 + 12.95 + 8.94 + 7.49 + 6.00 + 3.99) \simeq \underline{12.48}$$

$$S_{11} = \frac{1}{9}\sum_{i=1}^{10}(x_{i1} - \bar{x}_1)^2$$

$$= \frac{1}{9}\{(1-5.2)^2 + (2-5.2)^2 + (3-5.2)^2 + (3-5.2)^2 + (4-5.2)^2 + (5-5.2)^2 + (6-5.2)^2 + (8-5.2)^2 + (9-5.2)^2 + (11-5.2)^2\}$$

$$= \underline{10.62}$$

$$S_{22} = \frac{1}{9}\sum_{i=1}^{10}(x_{i2} - \bar{x}_{.2})^2$$

$$= \frac{1}{9}\{(18.95 - 12.48)^2 + (19 - 12.48)^2 + (17.95 - 12.48)^2 + (15.54 - 12.48)^2$$

$$+ (14 - 12.48)^2 + (12.95 - 12.48)^2 + (8.94 - 12.48)^2$$

$$+ (7.49 - 12.48) + (6 - 12.48)^2 + (3.99 - 12.48)^2\}$$

$$= \frac{1}{9} \times 277.7 \quad = \underline{30.85}$$

$$S_{12} = \frac{1}{9}\sum_{i=1}^{10}(x_{i1} - \bar{x}_{.1})(x_{i2} - \bar{x}_{.2})$$

$$= \frac{1}{9}\{(-4.2)(6.47) + (-3.2)(6.5) + (-2.2)(5.47)$$

$$+ (-2.2)(3.06) + (-1.2)(1.5) + (-0.2)(0.47)$$

$$+ (0.8)(-3.54) + (2.8)(-4.99) + (3.8)(-6.48)$$

$$+ (5.8)(-8.49)\}$$

$$= \frac{1}{9}(-159.4)$$

$$= \underline{-17.67}$$

$$r_{12} = \frac{-17.67}{\sqrt{10.62}\sqrt{30.85}} \quad = \underline{-0.976}$$

Since $S_{11} < S_{22}$, the x_2 data is scattered more.

r_{12} is negative, so x_1 and x_2 data correlate negatively.

r_{12} is close to -1, hence x_1 and x_2 data correlate strongly.

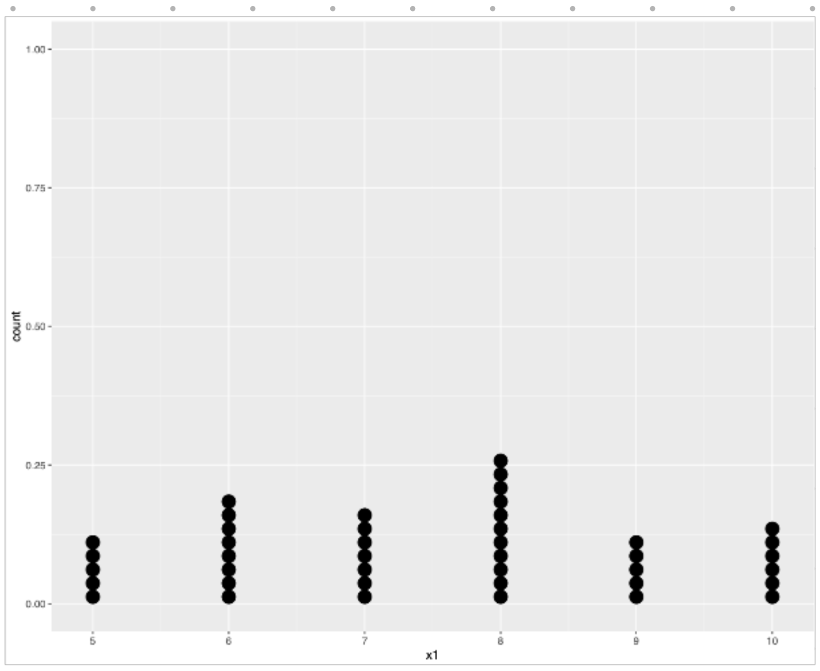
(d)

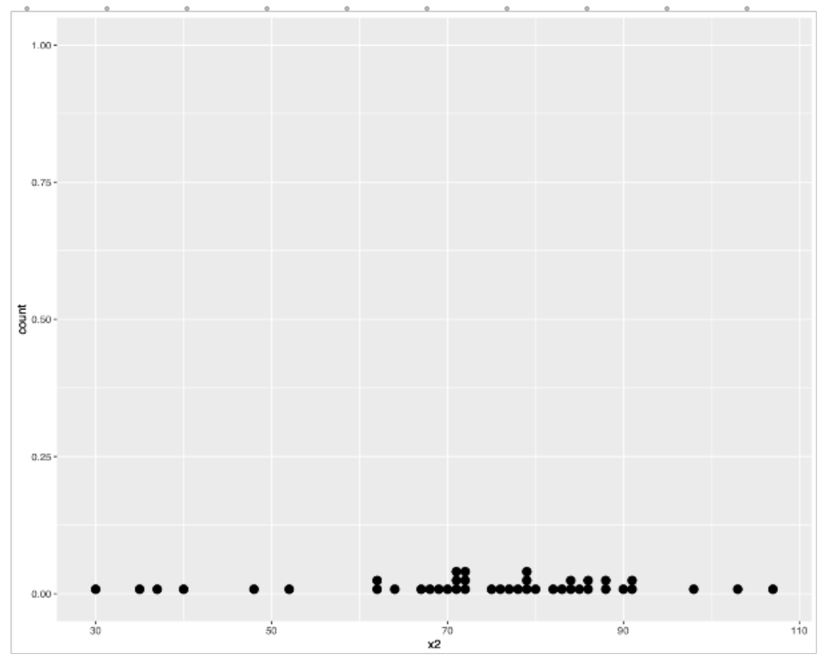
$$\bar{x} = \begin{bmatrix} \bar{x}_1 \\ \bar{x}_2 \end{bmatrix} = \begin{bmatrix} 5.2 \\ 12.48 \end{bmatrix}$$

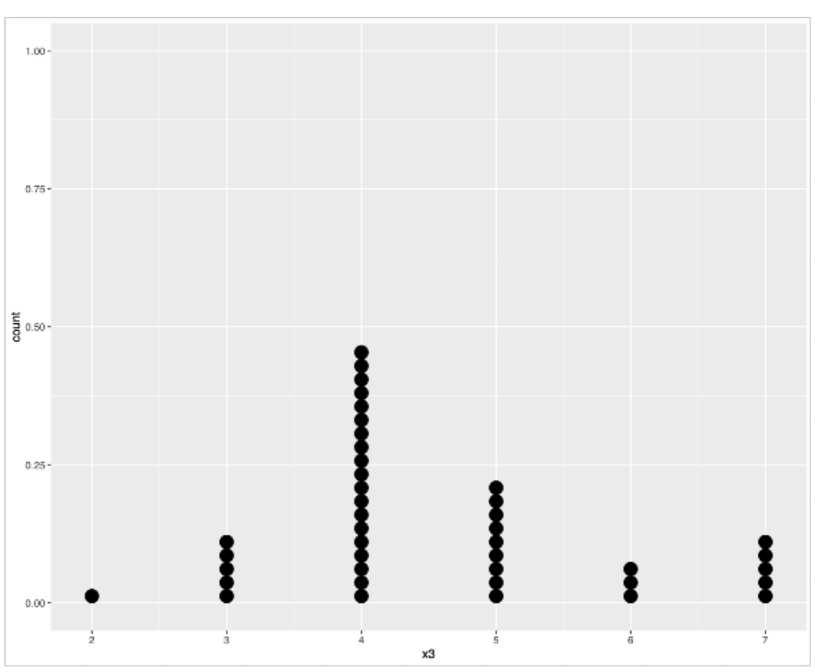
$$S_n = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} = \begin{bmatrix} 10.62 & -17.67 \\ -17.67 & 30.85 \end{bmatrix}$$

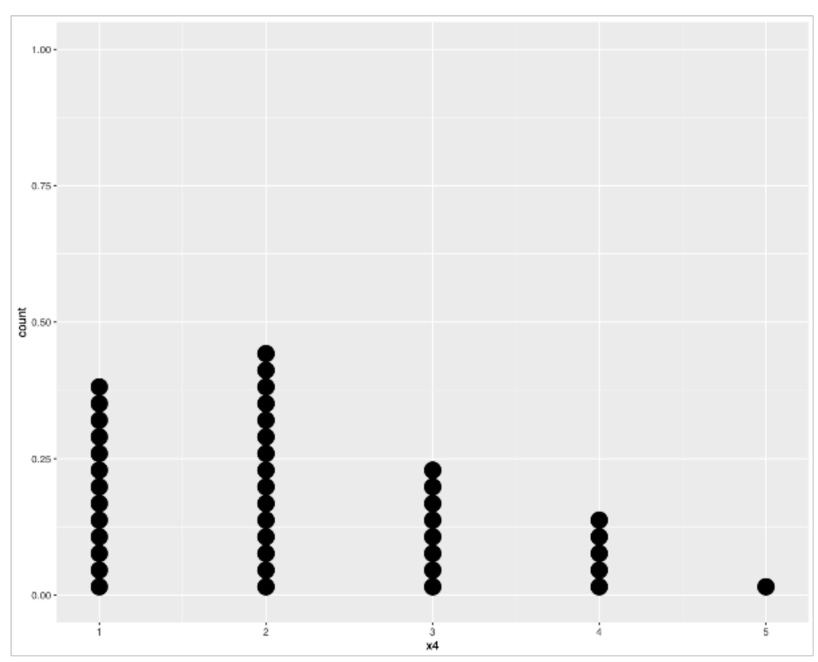
$$R = \begin{bmatrix} 1 & r_{12} \\ r_{21} & 1 \end{bmatrix} = \begin{bmatrix} 1 & -0.976 \\ -0.976 & 1 \end{bmatrix}$$

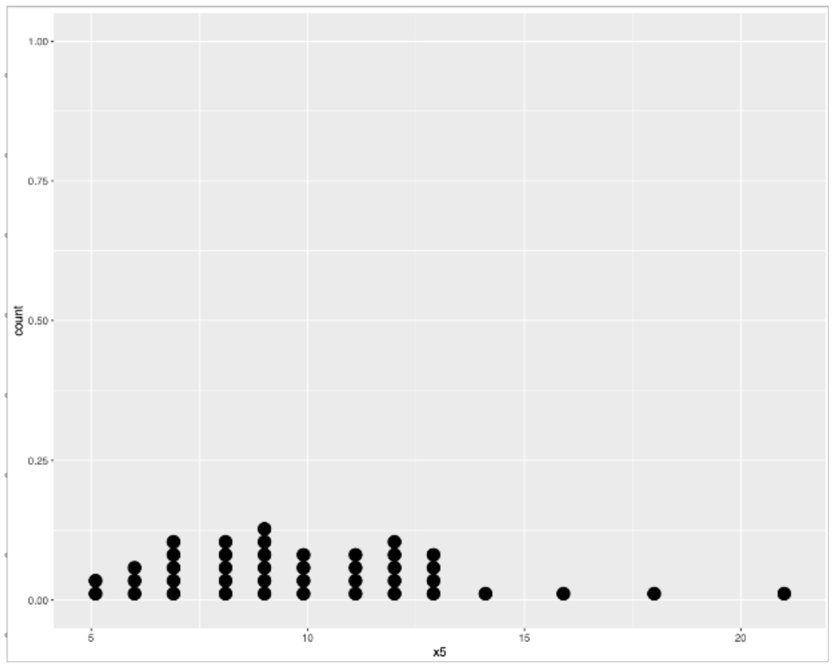
1.6 (a) Marginal dot diagrams are below:

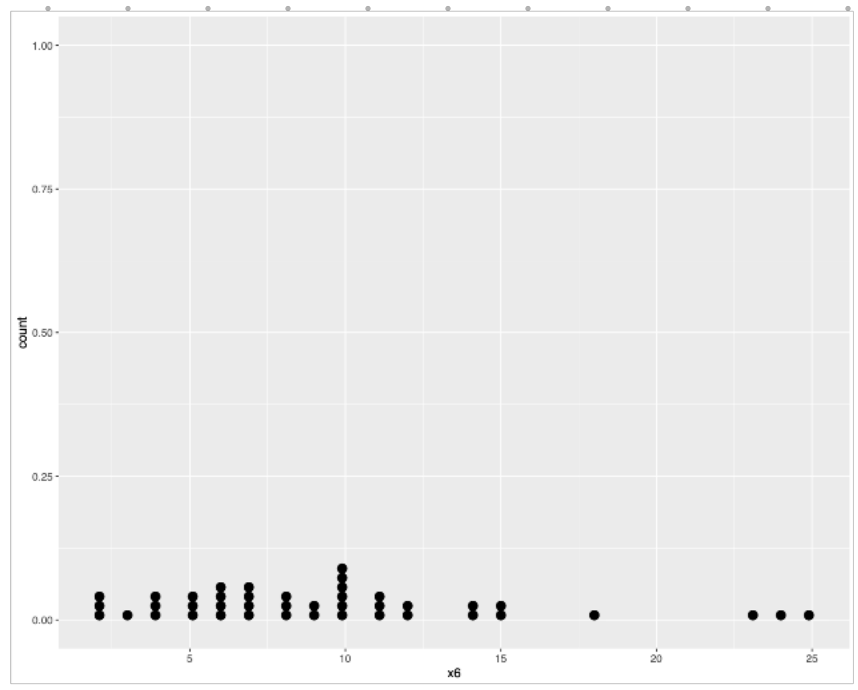


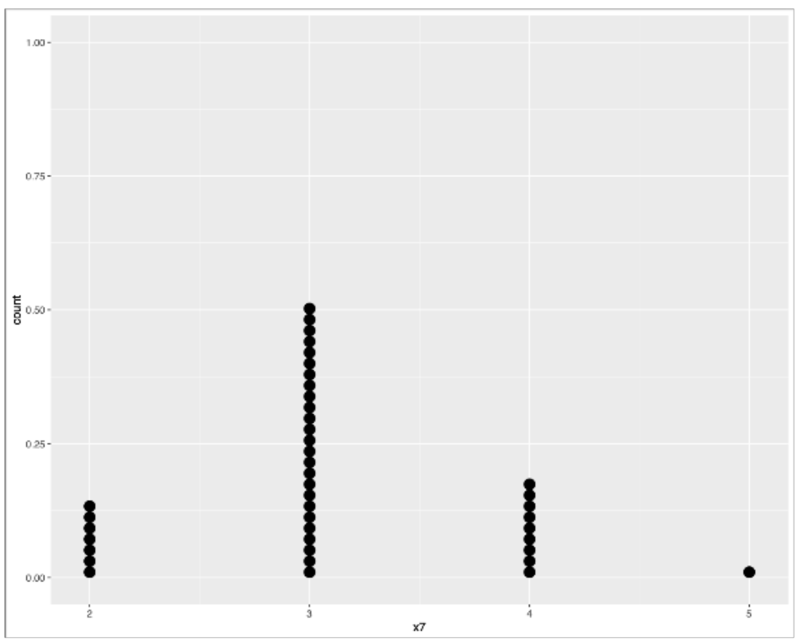












(d)

$$\bar{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \end{bmatrix} = \begin{bmatrix} 7.50 \\ 73.86 \\ 4.55 \\ 2.14 \\ 10.04 \\ 9.40 \\ 3.09 \end{bmatrix}$$

$$S_n = \begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} & S_{15} & S_{16} & S_{17} \\ S_{21} & S_{22} & S_{23} & S_{24} & S_{25} & S_{26} & S_{27} \\ S_{31} & S_{32} & S_{33} & S_{34} & S_{35} & S_{36} & S_{37} \\ S_{41} & S_{42} & S_{43} & S_{44} & S_{45} & S_{46} & S_{47} \\ S_{51} & S_{52} & S_{53} & S_{54} & S_{55} & S_{56} & S_{57} \\ S_{61} & S_{62} & S_{63} & S_{64} & S_{65} & S_{66} & S_{67} \\ S_{71} & S_{72} & S_{73} & S_{74} & S_{75} & S_{76} & S_{77} \end{bmatrix}$$

$$= \begin{bmatrix} 2.5 \\ -2.78 & 300.5 \\ -0.378 & 3.9 & 1.52 \\ -0.46 & -1.39 & 0.67 & 1.18 \\ -0.585 & 6.76 & 2.31 & 1.09 & 11.36 \\ -2.23 & 30.79 & 2.82 & -0.81 & 3.13 & 30.98 \\ 0.17 & 0.62 & 0.14 & 0.17 & 1.04 & 0.59 & 0.48 \end{bmatrix}$$

Since this is a covariance matrix, it is symmetric, meaning the data in upper right hand side is the same as that of bottom left hand side. Hence no need to write down the upper right hand side.

$$R = \begin{bmatrix} 1 & r_{12} & r_{13} & r_{14} & r_{15} & r_{16} & r_{17} \\ r_{21} & 1 & r_{23} & r_{24} & r_{25} & r_{26} & r_{27} \\ r_{31} & r_{32} & 1 & r_{34} & r_{35} & r_{36} & r_{37} \\ r_{41} & r_{42} & r_{43} & 1 & r_{45} & r_{46} & r_{47} \\ r_{51} & r_{52} & r_{53} & r_{54} & 1 & r_{56} & r_{57} \\ r_{61} & r_{62} & r_{63} & r_{64} & r_{65} & 1 & r_{67} \\ r_{71} & r_{72} & r_{73} & r_{74} & r_{75} & r_{76} & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ -0.10 & 1 \\ -0.19 & 0.183 & 1 \\ -0.26 & -0.07 & 0.50 & 1 \\ -0.11 & 0.12 & 0.56 & 0.297 & 1 \\ -0.25 & 0.32 & 0.41 & -0.13 & 0.17 & 1 \\ 0.16 & 0.05 & 0.17 & 0.23 & 0.45 & 0.15 & 1 \end{bmatrix}$$

Since this is a correlation matrix, it is symmetric, meaning the data in upper right hand side is the same as that of bottom left hand side. Hence no need to write down the upper right hand side.

It is hard to compare cross-columns covariance since the units and average values vary column by column.
However, as we take a look at the correlation matrix, we can see variable "Wind (x_1)" has mostly negative correlation with other variables, yet, most correlation values are relatively low, meaning the linear relationships are mostly weak.