

STAT 526 HW 2

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Q2(a)

$\eta = -6 + 8x$

```
# create a vector of the 51 x values which are incremented by 0.2
x <- seq(0, 1, by = 0.02)
# formula
eta <- - 6 + 8 * x

# Bernoulli trials
y <- rbinom(length(eta), 1, 1/(1+exp(-eta)))

length(y)
```

```
## [1] 51
```

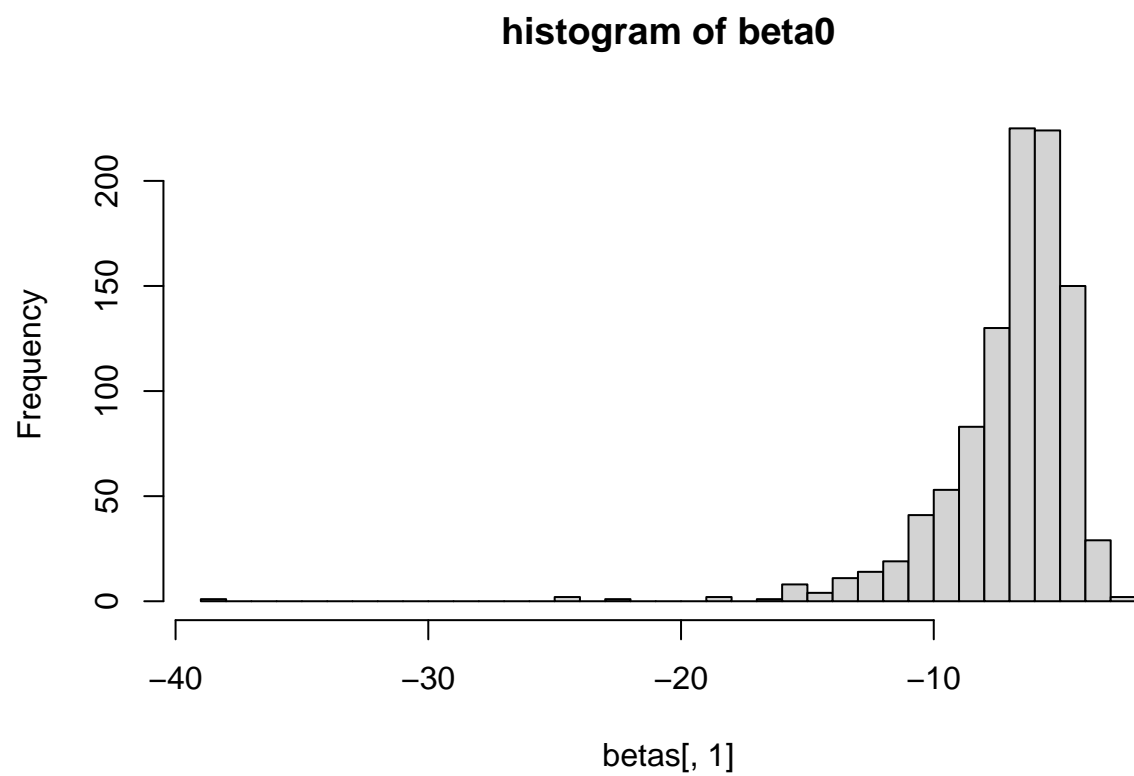
```
table(y)
```

```
## y
##  0  1
## 39 12
```

```
# check the summary to get beta0 and beta1
summary(glm(y ~ x, family = "binomial"))$coefficients[,1]
```

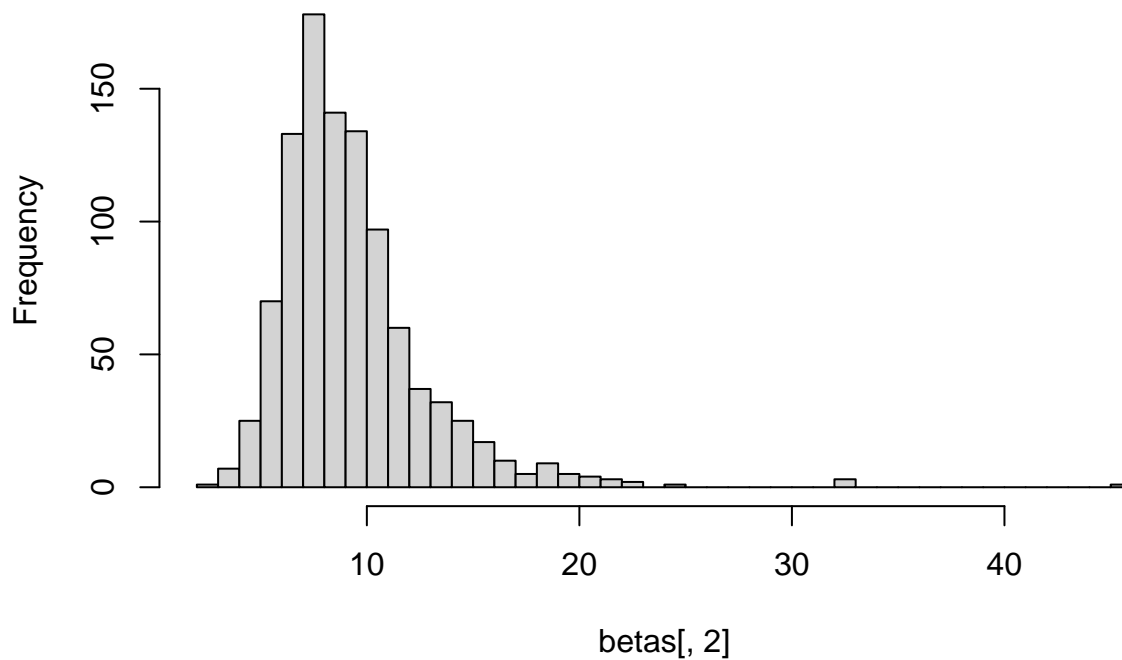
```
## (Intercept)          x
##   -5.824101    7.322365
```

```
# get histogram
betas <- matrix(NA, 1000, 2)
for (i in 1:1000) {
  y <- rbinom(length(eta), 1, 1/(1+exp(-eta)))
  betas[i, ] <- summary(glm(y ~ x, family = "binomial"))$coefficients[, 1]
}
## histogram of Intercept
hist(betas[, 1], br = 50, main = "histogram of beta0")
```



```
## histogram of x  
hist(betas[, 2], br = 50, main = "histogram of beta1")
```

histogram of beta1



```
summary(betas[, 1])
```

```
##      Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
## -38.888  -7.844   -6.414   -6.963  -5.301   -2.482
```

```
summary(betas[, 2])
```

```
##      Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
##   2.455   7.076   8.610   9.294  10.586   45.750
```

```
eta2 = -3 + 4x
```

```
eta2 <- -3 + 4 * x
# Bernoulli trials
y2 <- rbinom(length(eta), 1, 1/(1+exp(-eta)))
```

```
length(y2)
```

```
## [1] 51
```

```
table(y2)
```

```
## y2
##  0  1
## 38 13
```

```
# check the summary to get beta0 and beta1
summary(glm(y2 ~ x, family = "binomial"))$coefficients[, 1]
```

```
## (Intercept)          x
##   -5.339185    6.882525
```

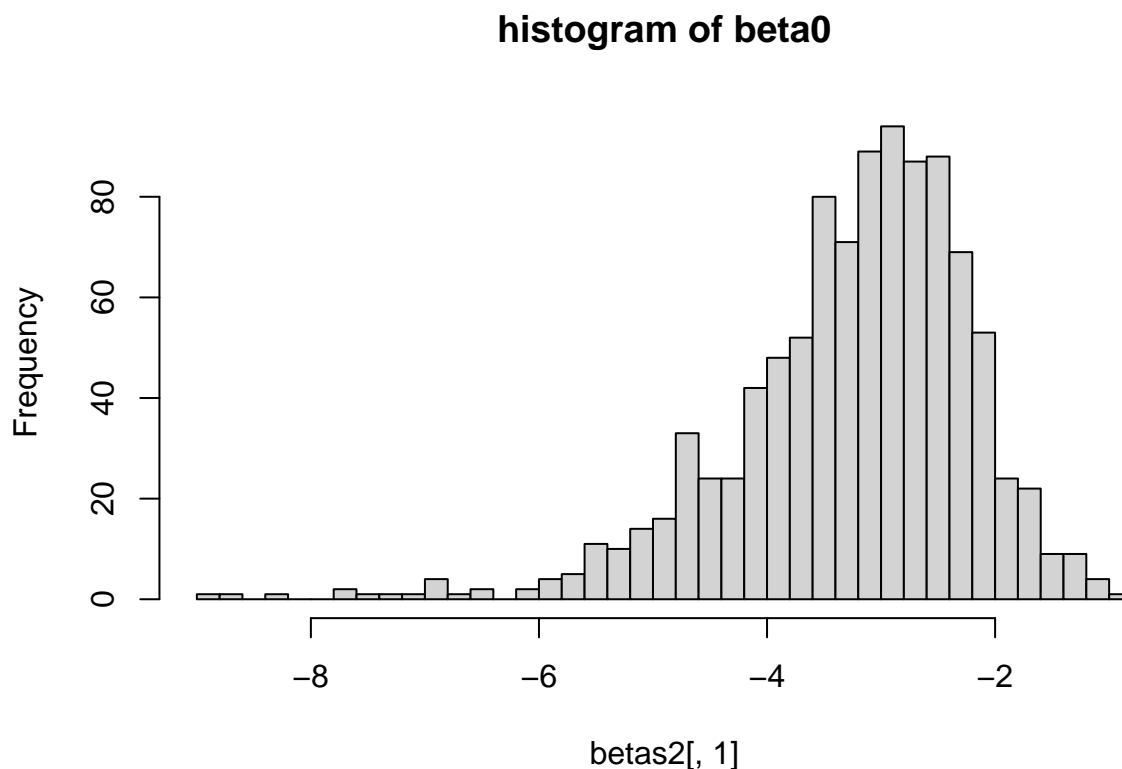
```
# get histogram
betas2 <- matrix(NA, 1000, 2)
for(i in 1:1000) {
  y2 <- rbinom(length(eta), 1, 1/(1+exp(-eta2)))
  betas2[i, ] <- summary(glm(y2 ~ x, family = "binomial"))$coefficients[, 1]
}
summary(betas2[, 1])
```

```
##      Min. 1st Qu.  Median      Mean 3rd Qu.      Max.
## -8.9939 -3.7820 -3.0866 -3.2578 -2.5376 -0.8993
```

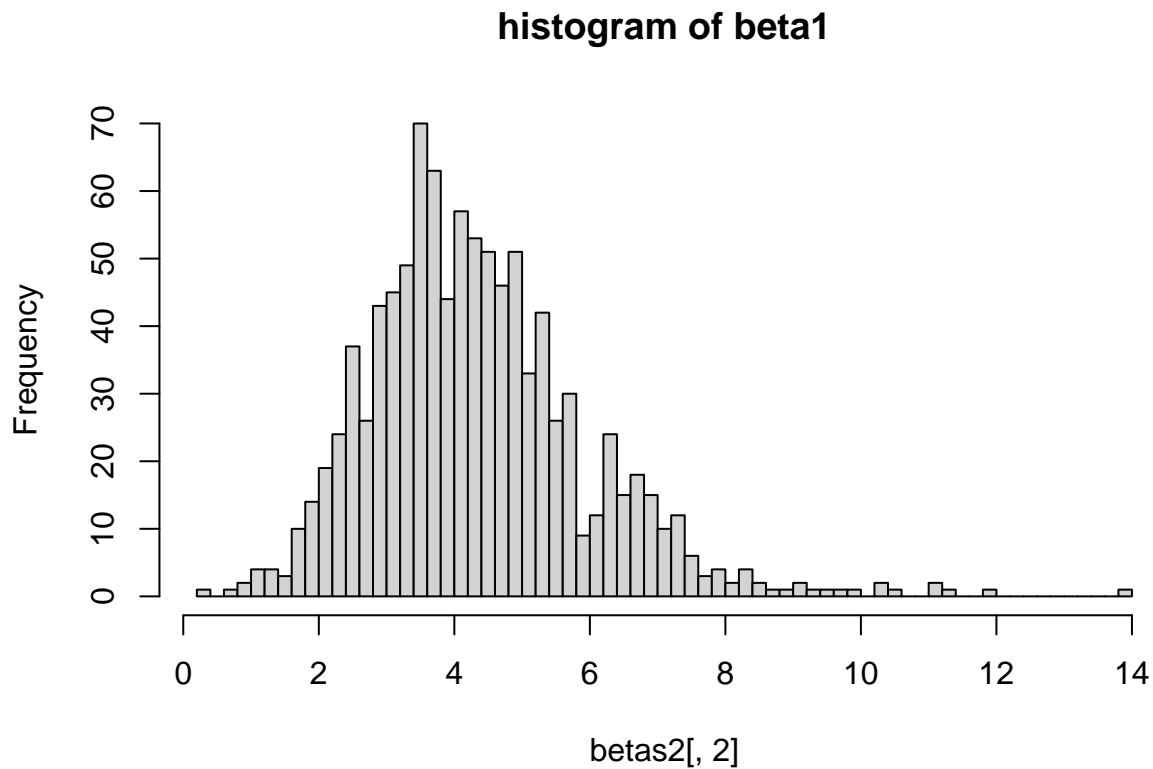
```
summary(betas2[, 2])
```

```
##      Min. 1st Qu.  Median      Mean 3rd Qu.      Max.
##  0.2314  3.2759  4.1451  4.3438  5.1992 13.8263
```

```
## histogram of Intercept
hist(betas2[, 1], br = 50, , main = "histogram of beta0")
```



```
## histogram of x
hist(betas2[, 2], br = 50, , main = "histogram of beta1")
```



Q2(b)

$\eta = -6 + 8x$

```
x <- seq(0, 1, by = 0.02)
# calculate the probability of cdf
p <- plogis(- 6 + 8 * x)
# calculate the variance
v <- p * (1 - p)

# calculate Hessian
h <- rbind(1, x) %*%
  diag(v) %*%
  cbind(1, x)
solve(h)
```

```
##           x
## 2.938188 -3.922113
## x -3.922113 5.578081
```

$\eta = -3 + 4x$

```
x <- seq(0, 1, by = 0.02)
# calculate the probability of cdf
p <- plogis(- 3 + 4 * x)
# calculate the variance
v <- p * (1 - p)

# calculate Hessian
h <- rbind(1, x) %*%
      diag(v) %*%
      cbind(1, x)
solve(h)
```

```
##                               x
##    0.7890173 -1.104672
## x -1.1046716  1.811565
```