STAT 524

Preliminary Quiz

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# Preliminary Quiz of STAT 52400

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In this quiz, we want to test whether you have enough preparation for STAT 52400. If you do not know anything about the below part, you may need to take calculus, linear algebra, and some probability courses before you take STAT 52400. You can talk to the instructor and get some advice from him.

1. Write the McLaurin sequence of $e^x$.
2. Find the differentiation of the below functions.
$$\frac{d}{dx}(\sin x) = ? \quad \frac{d}{dx}(\cos x) = ?$$
3. Find the Integration of the below functions.
$$\int \cos(x) = ?, \quad \int_1^2 x = ?$$
4. Find the results:
$$\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} + \begin{pmatrix} 5 & 6 \\ 7 & 8 \end{pmatrix} = ?$$
$$\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \times \begin{pmatrix} 5 & 6 \\ 7 & 8 \end{pmatrix} = ?,$$
$$\det \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \left( \text{or} \left| \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \right| \right) = ?$$
$$\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}^{-1} = ?$$
5. If $X \sim N(0,1)$, please write down the density function of $X$.
6. If $Y = \beta_0 + \beta_1 X + \varepsilon$ is a regression model. With $n$ observations $\{(x_i, y_i)\}$, please write down the least square estimate of $\beta_0$ and $\beta_1$ in an analytic form.
7. Do you know any approach to "check" whether a variable is normal distributed?

1. $f(x) = f(0) + f'(0)\cdot x + \frac{f''(0)}{2!}x^2 + \frac{f^3(0)}{3!}x^3 + \cdots + \frac{f^k(0)}{k!}$

$f(x) = e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots + \frac{x^k}{k!}$

2. $f(x) = \sin x = \cos\left(\frac{\pi}{2} - x\right)$

$\frac{d}{dx} f(x) = -\sin\left(\frac{\pi}{2} - x\right) \cdot \frac{d}{dx}\left(\frac{\pi}{2} - x\right)$

$= -\sin\left(\frac{\pi}{2} - x\right) \cdot (-1)$

$= \sin\left(\frac{\pi}{2} - x\right)$

$= \cos x$ //

$g(x) = \cos x = \sin\left(\frac{\pi}{2} - x\right)$

$\frac{d}{dx} g(x) = \cos\left(\frac{\pi}{2} - x\right) \cdot \frac{d}{dx}\left(\frac{\pi}{2} - x\right)$

$= \cos\left(\frac{\pi}{2} - x\right) \cdot (-1)$

$= -\sin x$ //

3. $\int \cos x \, dx$

$y = \cos x \iff \frac{dy}{dx} = -\sin x \iff dy = -\sin x \, dx$

$\sin x = \sqrt{1 - \cos^2 x}$ by trigonometric identities.

$dy = -\sqrt{1 - \cos^2 x} \, dx$

$$= -\sqrt{1-y^2}\,dx$$

$$\frac{dy}{-\sqrt{1-y^2}} = dx$$

$$\frac{\cos x\,dy}{-\sqrt{1-y^2}} = \cos x\,dx$$

$y = \cos x$

$$\frac{-y\,dy}{\sqrt{1-y^2}} = \cos x\,dx$$

$$\int \frac{-y\,dy}{\sqrt{1-y^2}} = \int \cos x\,dx$$

$$\int \frac{\frac{1}{2}du}{\sqrt{u}} = \int \cos x\,dx$$

Let $1-y^2 = u \iff -2y\,dy = du$

$$\frac{1}{2}\int u^{-\frac{1}{2}}du = \int \cos x\,dx$$

Since $\int x^n dx = \frac{x^{n+1}}{n+1} + C$

$$\frac{1}{2}\cdot\frac{u^{1/2}}{1/2} + C = \int \cos x\,dx$$

$$u^{1/2} + C = \int \cos x\,dx$$

$$(1-y^2)^{1/2} + C = \int \cos x\,dx$$

$$(1-\cos^2 x)^{1/2} + C = \int \cos x\,dx$$

$$(\sin^2 x)^{1/2} + C = \int \cos x\,dx$$

$\int \cos x\,dx = \underline{\sin x + C}$ // where $C$ is constant

$$\int_1^2 x = \frac{1}{2}x^2 \Big|_1^2$$

$$= \frac{1}{2}(4-1)$$

$$= \frac{3}{2}$$

4.

$$\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} + \begin{pmatrix} 5 & 6 \\ 7 & 8 \end{pmatrix} = \begin{pmatrix} 6 & 8 \\ 10 & 12 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \times \begin{pmatrix} 5 & 6 \\ 7 & 8 \end{pmatrix} = \begin{pmatrix} 19 & 22 \\ 43 & 50 \end{pmatrix}$$

$$\det \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} = 4 - 6 = -2$$

$$\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}^{-1} = \frac{-1}{2}\begin{pmatrix} 4 & -2 \\ -3 & 1 \end{pmatrix} = \begin{pmatrix} -2 & 1 \\ 3/2 & -1/2 \end{pmatrix}$$

5. $X \sim N(\mu, \sigma^2)$

pdf : $f(x) = \dfrac{1}{\sqrt{2\sigma^2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$

$X \sim N(0, 1)$

$$f(x) = \frac{1}{\sqrt{2\pi}} e^{\frac{-x^2}{2}}$$

6. To derive the least square estimates,

$$\sum_{i=1}^{n} (y_i - (b_0 + b_1 x))^2$$

$$b_1 = \frac{SS_{xy}}{SS_x} = \frac{\sum_{i=1}^{n} x_i y_i - \frac{1}{n}\sum_{i=1}^{n} x_i \sum_{i=1}^{n} y_i}{\sum_{i=1}^{n} x_i^2 - \frac{1}{n}\left(\sum_{i=1}^{n} x_i\right)^2}$$

$$b_0 = \bar{y} - b_1 \bar{x}.$$

7. Use Q-Q plot or residual plot and see if there is any skewness and trend to figure out whether the variable follows the normal distribution.