STAT 522

HW2

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Complete Exercises 1 from Chapter 2 in the textbook. For all questions, show your work and answer all of the questions.

Helpful Hints:

For part a), use equation 2.8 from chapter 2.

For part b), first find and write the sampling distribution for the mean for each of the sampling plans.

Next for part b), calculate i to iv for each of the sampling plans using equations 2.3, 2.4, 2.5, and 2.6 from chapter 2.

1. Let $N = 6$ and $n = 3$. For purposes of studying sampling distributions, assume that all population values are known.

$$y_1 = 98 \quad y_2 = 102 \quad y_3 = 154$$
$$y_4 = 133 \quad y_5 = 190 \quad y_6 = 175$$

We are interested in $\bar{y}_U$, the population mean. Two sampling plans are proposed.

- Plan 1. Eight possible samples may be chosen.

Sample Number	Sample, $\mathcal{S}$	$P(\mathcal{S})$
1	{1,3,5}	1/8
2	{1,3,6}	1/8
3	{1,4,5}	1/8
4	{1,4,6}	1/8
5	{2,3,5}	1/8
6	{2,3,6}	1/8
7	{2,4,5}	1/8
8	{2,4,6}	1/8

- Plan 2. Three possible samples may be chosen.

Sample Number	Sample, $\mathcal{S}$	$P(\mathcal{S})$
1	{1,4,6}	1/4
2	{2,3,6}	1/2
3	{1,3,5}	1/4

(a) What is the value of $\bar{y}_U$?

(b) Let $\bar{y}$ be the mean of the sample values. For each sampling plan, find

(i) $E[\bar{y}]$; (ii) $V[\bar{y}]$; (iii) Bias($\bar{y}$); (iv) MSE($\bar{y}$).

(c) Which sampling plan do you think is better? Why?

(a) $\bar{y}_U = \frac{1}{6}(98 + 102 + 154 + 133 + 190 + 175) = 142$

(b) Plan 1

$$E[\bar{y}] = \sum_{s} \bar{y}\, P(s)$$
$$\approx \frac{1}{8}(147.33 + 142.33 + 140.33 + 135.33 + 148.66 + 143.66 + 141.66 + 136.66)$$
$$\approx 141.99$$

$$S^2 = E\left[(\bar{y} - E(\bar{y}))^2\right] = \frac{1}{6-1} \cdot 151.553 = 21.650$$

$$V[\bar{y}] = \left(1 - \frac{n}{N}\right) \cdot \frac{S^2}{n} = \left(1 - \frac{3}{6}\right) \cdot \frac{21.65}{3} = 3.608$$

$$\text{Bias}[\bar{y}] = E[\bar{y}] - \bar{y}_u = 141.9999 - 142 \cong 0$$

$$MSE[\bar{y}] = E\left[(\bar{y} - \bar{y}_u)^2\right]$$

$$= V[\bar{y}] + [\text{Bias}(\bar{y})]^2$$

$$= 3.608 + 0 = 3.608$$

Plan 2

$$E[\bar{y}] = \frac{1}{4} \cdot 135.33 + \frac{1}{2} \cdot 143.666 + \frac{1}{4} \cdot 147.333$$

$$\cong 142.499$$

$$S^2 = \frac{1}{3-1} \cdot 75.705 = 37.852$$

$$V[\bar{y}] = \left(1 - \frac{3}{6}\right) \cdot \frac{37.852}{3} = 6.3087$$

$$\text{Bias}[\bar{y}] = 142.499 - 142 \cong 0.5$$

$$MSE[\bar{y}] = 6.3087 + 0.5 = 6.8087$$

(c) Plan 1 sampling is better since Bias is 0 and MSE of sample mean is smaller. //