Bayesian Causal Inference for Differentially Privatized Data: Application to Cash Transfer Program in Columbia.

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Overview: Rubin Causal Model

Simple example (Vaccine trial).

- Treatment $Z_i \in \{0 \text{ (Placebo)}, 1 \text{ (Vaccinated)}\}$
- Covariates X_i: Age, Gender,...
- Potential Outcomes: $Y_i(0), Y_i(1) \in \{0 \text{ (Not Infected)}, 1 \text{ (Infected)}\}$:

units	Z_i	X_i	$Y_i(0)$	$Y_i(1)$
1	0	$\mathbf{x}_1$	1	$\times$
2	1	$\mathbf{x}_2$	$\times$	0
3	0	$\mathbf{x}_3$	0	$\times$
4	1	$\mathbf{x}_4$	$\times$	0
5	1	$\mathbf{x}_5$	$\times$	1

Causal estimand τ: Causal effects of interest.

- A function of potential outcomes, covariates and treatments.
- e.g.) $\mathbb{E}[Y_i(1) - Y_i(0)]$, $\bar{Y}(1) - \bar{Y}(0)$, $\text{median}\{Y_i(1) - Y_i(0)\}$

Bayesian Causal Inference

- Bayesian methodologies for causal inference were introduced by Imbens and Rubin (1997).
- Obtain the posterior distribution $p\left(\tau \mid \mathbf{X}, \mathbf{Z}, \mathbf{Y}^{\mathrm{obs}}\right)$.
- Data augmentation approach: Impute missing potential outcomes and model parameters iteratively.

Assumption

1. *(Positivity) The probability of treatment assignment given the covariates is bounded away from zero and one:* $0 < p(Z_i = 1) < 1$.
2. *(Random Assignment) The potential outcomes are independent of treatment assignment:* $\{Y_i(0), Y_i(1)\} \perp\!\!\!\perp Z_i$.
3. *(SUTVA) There is neither interference nor hidden versions of treatment. The observed outcome is formally expressed as:* $Y_i = Z_i Y_i(1) + (1 - Z_i) Y_i(0)$.

Overview: Bayesian Causal Inference

Consider the finite-sample causal estimand: $\tau = \bar{Y}(1) - \bar{Y}(0)$.

$$
\begin{aligned}
& p\left(\tau \mid \mathbf{X}, \mathbf{Z}, \mathbf{Y}^{\mathrm{obs}}\right) \\
& \propto p\left(\mathbf{X}, \mathbf{Z}, \mathbf{Y}^{\mathrm{obs}}, \mathbf{Y}^{\mathrm{mis}}\right) \\
& = p\left(\mathbf{X}, \mathbf{Z}, \mathbf{Y}(\mathbf{0}), \mathbf{Y}(\mathbf{1})\right) \\
& = \int p(\theta) \prod_{i=1}^{N} p\left(X_i, Z_i, Y_i(0), Y_i(1) \mid \theta\right) d\theta \\
& = \int p(\theta) \prod_{i=1}^{N} \{p\left(X_i\right) p\left(Z_i \mid X_i\right) p\left(Y_i(0), Y_i(1) \mid X_i, Z_i, \theta\right)\} d\theta
\end{aligned}
$$

Three inputs are necessary.

- $p\left(Z_i \mid X_i\right)$: Treatment assignment mechanism (propensity score). Known under randomized experiments.
- $p\left(Y_i(0), Y_i(1) \mid X_i, Z_i, \theta\right)$: Outcome models.
- $p(\theta)$: Prior distribution.

Differential Privacy

DP is a mathematical framework that provides a probabilistic guarantee that protects private information about individuals when publishing statistics about a dataset. This probabilistic guarantee is often achieved by **adding random noise to the data**.

ex.) Let ϵ be a prespecified parameter of the privacy level that the privacy mechanisms guarantee.

- Laplace mechanism: Let $Y_i \in [0, 1]$. $\tilde{Y}_i = Y_i + \eta_i$ where $\eta_i \sim \mathrm{Laplace}(0, 1/\epsilon)$.
- Randomized response mechanism: Let $Z_i \in \{0, 1\}$ be a binary variable. The randomized response mechanism is defined as

$$M(Z_i) = \begin{cases} Z_i & \text{w.p. } \frac{\exp(\epsilon)}{1+\exp(\epsilon)} \\ 1 - Z_i & \text{w.p. } \frac{1}{1+\exp(\epsilon)}, \end{cases}$$

Bayesian Causal Inference for Differentially Privatized Data

- For brevity, we consider privatizing outcomes variables Y and treatment assignment variable Z.
- Challenge: We do not observe true values of Y and Z. Instead, we observe the noisy variables $\tilde{Y}$ and $\tilde{Z}$.
- Additionally, it is uncertain whether $Y(0)$ or $Y(1)$ has been privatized to yield $\tilde{Y}$.

Consider:

$$
\begin{aligned}
& p(\mathbf{Y}(0), \mathbf{Y}(1), \mathbf{Z}, \tilde{\mathbf{Y}}, \tilde{\mathbf{Z}}) \\
& = \int p(\boldsymbol{\theta}) \prod_i (Y_i(0), Y_i(1), W_i, \tilde{Y}_i, \tilde{Z}_i \mid \boldsymbol{\theta}) d\boldsymbol{\theta} \\
& = \int p(\boldsymbol{\theta}) \prod_i p(Z_i) p(\tilde{Z}_i \mid Z_i) p(Y_i(0), Y_i(1) \mid \boldsymbol{\theta}) p(\tilde{Y}_i \mid Y_i(0), Y_i(1), Z_i) d\boldsymbol{\theta}
\end{aligned}
$$

- Data augmentation: Impute Z, $Y(0)$, $Y(1)$ and θ iteratively.
- $p(Y_i(0), Y_i(1) \mid \boldsymbol{\theta})$ needs to be specified (Later).

Empirical Analysis: Cash Transfer Program in Columbia

- We analyzed a randomized experiment that examined the impact of a cash transfer program on students' attendance rates (Barrera-Osorio 2011), Conducted in Colombia.
- The study recruited households with one to five school children, randomly assigning children to either participate in the cash transfer program or not with probability $p = 0.628$.
- The number of recruited students is $N = 5240$.
- With a known treatment assignment, we assessed the treatment effect of the program on the attendance rate of the students.
- $Z_i = 1$ if student i attend the program. Y_i is the attendance rate of the student.
- We only observe privatized variables $\tilde{Y}_i$ and $\tilde{Z}_i$ with different privacy budgets $\epsilon = \{0.1, 0.3, 1, 3, 10\}$.

Model Specifications

- We adopt the Dirichlet Process Mixture (DPM) to model $\Pr(Y_i(0), Y_i(1) \mid W_i, \boldsymbol{\theta})$ for its flexibility.
- The DPM fits our needs that require estimate the model without assuming strong parametric forms.
- The DPM is specified as:

$$\begin{aligned} \{Y_1(0), Y_1(1)\}, \ldots, \{Y_N(0), Y_N(1)\} \mid \Phi_1, \ldots, \Phi_N &\sim p(Y_i(0), Y_i(1) | \Phi_i), \\ \Phi_1, \ldots, \Phi_N | H &\sim H, \\ H &\sim DP(\alpha, H_0). \end{aligned}$$

- In particular, the outcome model is specified by the following model.

$$\Pr(Y_i(w) | \boldsymbol{\mu}, \boldsymbol{\Sigma}) \propto \sum_{k=1}^{\infty} u_k \mathrm{TN}(\mu_w^k, \Sigma_w^k, 0, 1),$$

- For inference, we adopt an approximated blocked Gibbs sampler based on a truncation of the stick-breaking representation of the DP proposed by Ishwaran and Zarepour (2000).

- Compare our private inference with the standard non-private frequentist and Bayesian inference.
- In the "Non-private" columns, "Freq" represents the standard difference-in-means estimator, while "Bayes" represents the standard Dirichlet process mixture models for non-private data.

Results:

	Non-private						Private		
	Freq			Bayes			Bayes		
ϵ_{tot}	Mean	2.5%	97.5%	Mean	2.5%	97.5%	Mean	2.5%	97.5%
0.1	0.006	0.001	0.009	0.005	0.001	0.008	0.011	-0.178	0.145
0.3	0.006	0.001	0.009	0.005	0.001	0.008	0.049	-0.082	0.190
1.0	0.006	0.001	0.009	0.005	0.001	0.008	0.041	-0.022	0.111
3.0	0.006	0.001	0.009	0.005	0.001	0.008	0.018	-0.007	0.034
10.0	0.006	0.001	0.009	0.005	0.001	0.008	0.007	0.000	0.015

References

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