A SHORT INTRODUCTION TO PYTHON

Numpy, Scipy

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Slides adapted from a presentation by

ENTHOUGHT
SCIENTIFIC COMPUTING SOLUTIONS

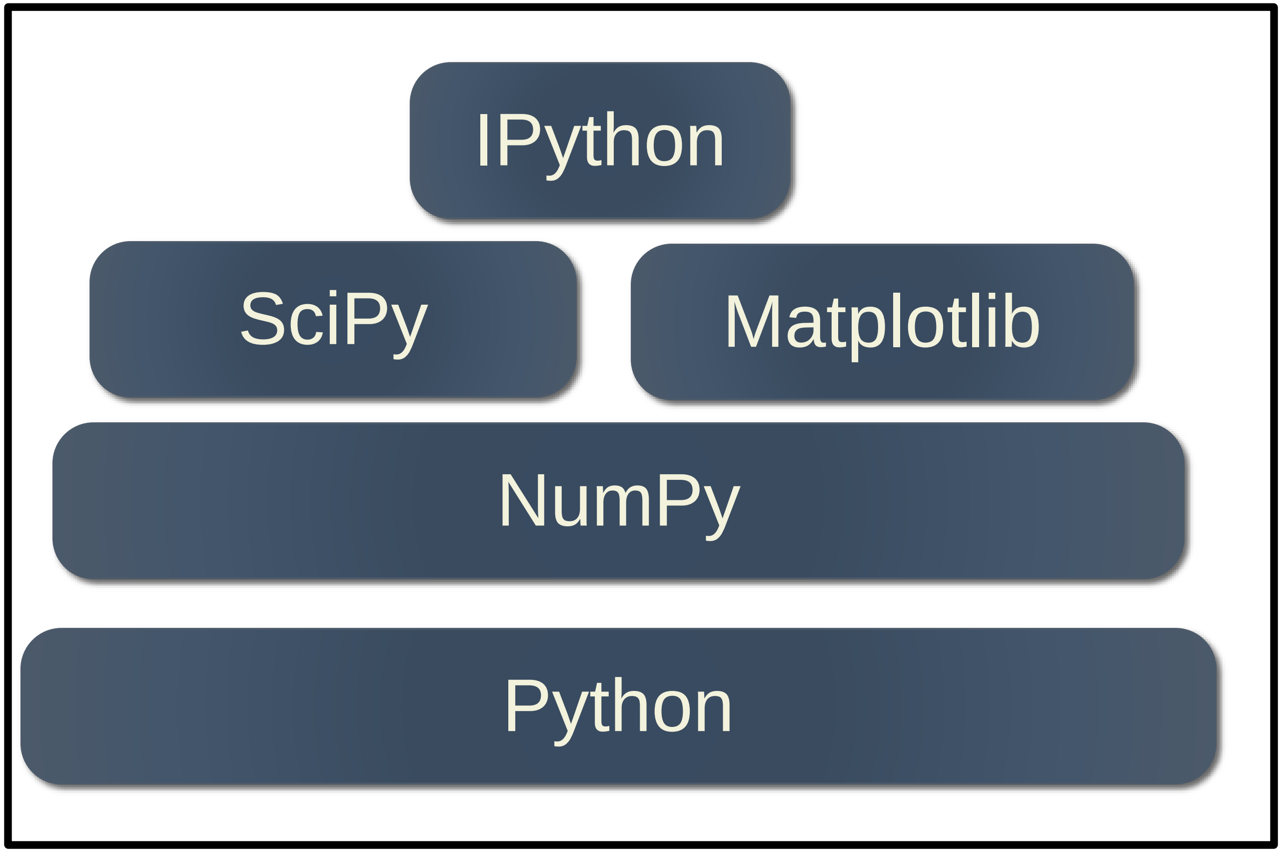
Essential Python Extensions

- The following packages extend Python with extra features
  - NumPy – Fast, multidimensional arrays
  - SciPy – Libraries of reliable, tested scientific functions

- Additional packages for Data Science (not covered today)
  - Wide range of learning algorithms (scikit-learn)
  - Tools for data manipulation (Pandas)
  - Plotting tools (Matplotlib)
  - Direct connection to R (rpy2)

PyLab

Sometimes the union of the 5 packages is called pylab



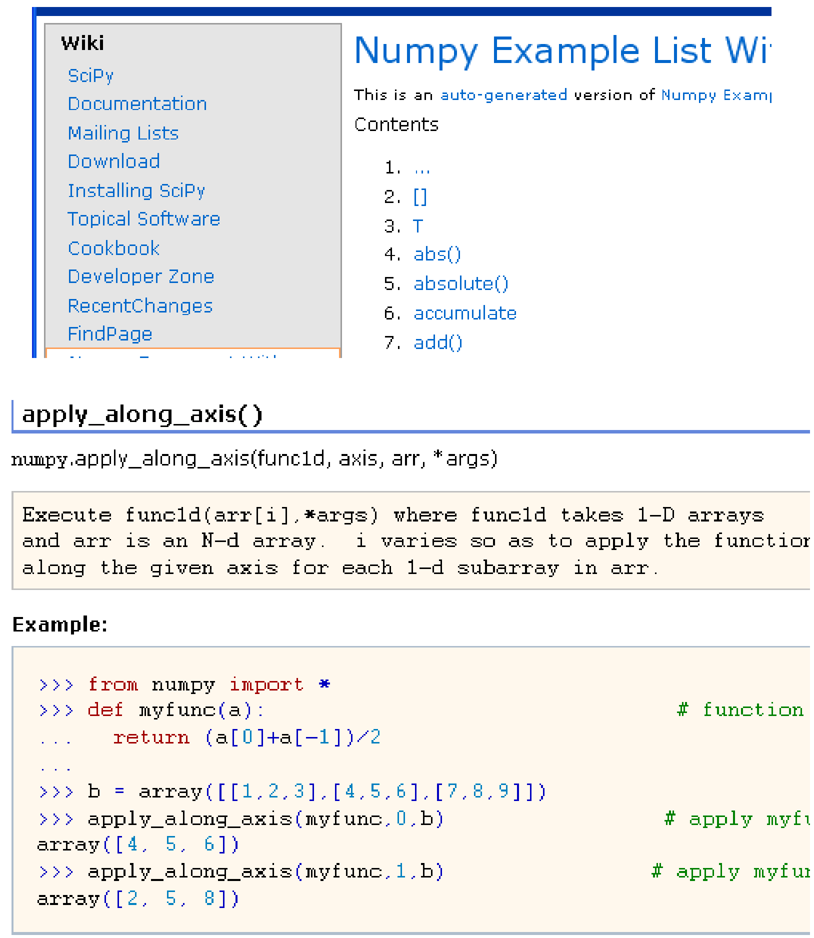
Helpful Sites

SCIPY DOCUMENTATION PAGE

http://www.scipy.org/Documentation

NUMPY EXAMPLES

http://www.scipy.org/Numpy_Example_List_With_Doc



NUMPY

Numerical Python

What is NumPy?

- NumPy is the fundamental package for scientific computing with Python
- NumPy provides a fast built-in object, ndarray, which is a multi-dimensional array of a homogeneous data-type that can be manipulated in a vectorized form
  - Numpy Offers Matlab-ish capabilities within Python
- NumPy can also be used as an efficient multi-dimensional container of generic data
  - This allows NumPy to seamlessly and speedily integrate with a wide variety of databases
- Website – http://www.numpy.org/
- Chronology
  - Initially developed by Travis Oliphant
  - NumPy 1.0 released October, 2006
  - ~20K downloads/month from Sourceforge
    - Doesn’t count distributions that that include NumPy
  - NumPy is at the core of nearly every scientific Python

Overview of NumPy

N-Dimensional ARRAY (NDARRAY)

- A NumPy array is a homogeneous collection of "items" of the same "data-type" (dtype)
  - Can be 1-dim or N-dims
- Element of the array can be C-structure or simple data-type
- Fast algorithms on machine data-types (int, float, etc.)

Universal Functions (UFUNC)

- Functions that operate element-by-element and return result
- Fast-loops registered for each fundamental data-type
  - $\sin(\boldsymbol{x}) = [\sin(x_i), i = 0 \ldots N]$
  - $\boldsymbol{x} + \boldsymbol{y} = [x_i + y_i, i = 0 \ldots N]$

Arrays in Python

- Python doesn't include a built-in multi-dimensional array
- Lists ok for storing small amounts of one-dimensional data

```
>>> a = [1,3,5,7,9]
>>> print(a[2:4])
[5, 7]
>>> b = [[1, 3, 5, 7, 9], [2, 4, 6, 8, 10]]
>>> print(b[0])
[1, 3, 5, 7, 9]
>>> print(b[1][2:4])
[6, 8]
```

```
>>> a = [1,3,5,7,9]
>>> b = [3,5,6,7,9]
>>> c = a + b
>>> print c
[1, 3, 5, 7, 9, 3, 5, 6, 7, 9]
```

- But, can't use directly with arithmetical operators (+, -, *, /, …)
- Need efficient arrays with arithmetic and better multidimensional tools

Introducing NumPy Arrays

SIMPLE ARRAY CREATION

```
>>> a = array([0,1,2,3])
>>> a
array([0, 1, 2, 3])
```

CHECKING THE TYPE

```
>>> type(a)
<type 'array'>
```

NUMERIC 'TYPE' OF ELEMENTS

```
>>> a.dtype
dtype('int32')
```

NUMBER OF DIMENSIONS

```
>>> a.ndim
1
```

ARRAY SHAPE

```
# shape returns a tuple
# listing the length of the
# array along each dimension.
>>> a.shape
(4,)
>>> shape(a)
(4,)
```

ARRAY SIZE

```
# size reports the entire
# number of elements in an
# array.
>>> a.size
4
>>> size(a)
4
```

Introducing NumPy Arrays

ARRAY COPY

```
# create a copy of the array
>>> b = a.copy()
>>> b
array([0, 1, 2, 3])
```

CONVERSION TO LIST

```
# convert a numpy array to a
# python list
>>> a.tolist()
[0, 1, 2, 3]

# For 1D arrays, list also
# works equivalently, but
# is slower
>>> list(a)
[0, 1, 2, 3]
```

Setting Array Elements

ARRAY INDEXING

```
>>> a[0]
0
>>> a[0] = 10
>>> a
[10, 1, 2, 3]
```

FILL

```
# set all values in an array
>>> a.fill(0)
>>> a
[0, 0, 0, 0]

# This also works, but may
# be slower
>>> a[:] = 1
>>> a
[1, 1, 1, 1]
```

BEWARE OF TYPE COERSION

```
>>> a.dtype
dtype('int32')

# assigning a float to
# an int32 array will
# truncate decimal part
>>> a[0] = 10.6
>>> a
[10, 1, 2, 3]

# fill has the same behavior
>>> a.fill(-4.8)
>>> a
[-4, -4, -4, -4]
```

Multi-Dimensional Arrays (ndarray)

MULTI-DIMENSIONAL ARRAYS

```
>>> a = array([[ 0, 1, 2, 3],
               [10,11,12,13]])
>>> a
array([[ 0, 1, 2, 3],
       [10,11,12,13]])
```

(ROWS,COLUMNS)

```
>>> a.shape
(2, 4)
>>> shape(a)
(2, 4)
```

ELEMENT COUNT

```
>>> a.size
8
>>> size(a)
8
```

NUMBER OF DIMENSIONS

```
>>> a.ndims
2
```

GET/SET ELEMENTS

```
>>> a[1,3]
13
```

column
row

```
>>> a[1,3] = -1
>>> a
array([[ 0, 1, 2, 3],
       [10,11,12,-1]])
```

ADDRESS FIRST ROW USING SINGLE INDEX

```
>>> a[1]
array([10, 11, 12, -1])
```

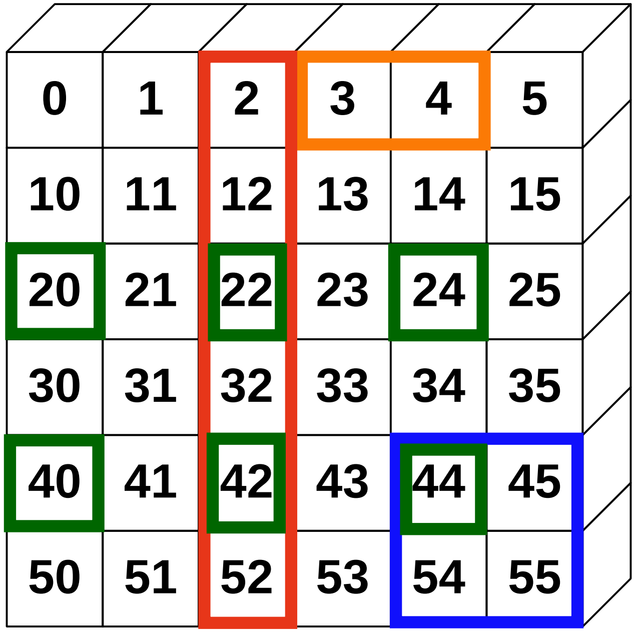
Array Slicing

SLICING WORKS MUCH LIKE STANDARD PYTHON SLICING

```
>>> a[0,3:5]
array([3, 4])

>>> a[4:,4:]
array([[44, 45],
       [54, 55]])

>>> a[:,2]
array([2,12,22,32,42,52])
```

STRIDES ARE ALSO POSSIBLE

```
>>> a[2::2,::2]
array([[20, 22, 24],
       [40, 42, 44]])
```



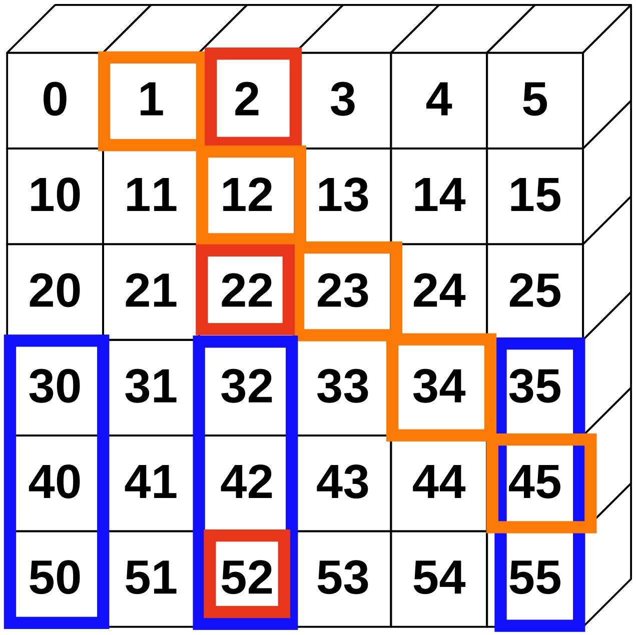
Slices Are References

Slices are references to memory in original array. Changing values in a slice also changes the original array.

```
>>> a = array((0,1,2,3,4))

# create a slice containing only the
# last element of a
>>> b = a[2:4]
>>> b[0] = 10

# changing b changed a!
>>> a
array([ 1,  2, 10, 3, 4])
```

Fancy Indexing in 2D

```
# Indexing by position
>>> a[(0,1,2,3,4),(1,2,3,4,5)]
array([ 1, 12, 23, 34, 45])

>>> a[3:,[0, 2, 5]]
array([[30, 32, 35],
       [40, 42, 45]])
       [50, 52, 55]])

# Indexing with Booleans
>>> mask = array([1,0,1,0,0,1],
                 dtype=bool)
>>> a[mask,2]
array([2,22,52])
```



Unlike slicing, fancy indexing creates copies instead of views into original arrays.

Array Calculation Methods

SUM FUNCTION

```
>>> a = array([[1,2,3],
               [4,5,6]], float)

# Sum defaults to summing all
# *all* array values.
>>> sum(a)
21.

# supply the keyword axis to
# sum along the 0th axis.
>>> sum(a, axis=0)
array([5., 7., 9.])

# supply the keyword axis to
# sum along the last axis.
>>> sum(a, axis=-1)
array([6., 15.])
```

SUM ARRAY METHOD

```
# The a.sum() defaults to
# summing *all* array values
>>> a.sum()
21.

# Supply an axis argument to
# sum along a specific axis.
>>> a.sum(axis=0)
array([5., 7., 9.])
```

PRODUCT

```
# product along columns.
>>> a.prod(axis=0)
array([ 4., 10., 18.])

# functional form.
>>> prod(a, axis=0)
array([ 4., 10., 18.])
```

Min/Max

MIN

```
>>> a = array([2.,3.,0.,1.])
>>> a.min(axis=0)
0.
# use Numpy's amin() instead
# of Python's builtin min()
# for speed operations on
# multi-dimensional arrays.
>>> amin(a, axis=0)
0.
```

ARGMIN

```
# Find index of minimum value.
>>> a.argmin(axis=0)
2
# functional form
>>> argmin(a, axis=0)
2
```

MAX

```
>>> a = array([2.,1.,0.,3.])
>>> a.max(axis=0)
3.
# functional form
>>> amax(a, axis=0)
3.
```

ARGMAX

```
# Find index of maximum value.
>>> a.argmax(axis=0)
1
# functional form
>>> argmax(a, axis=0)
1
```

Statistics Array Methods

MEAN

```
>>> a = array([[1,2,3],
               [4,5,6]], float)

# mean value of each column
>>> a.mean(axis=0)
array([ 2.5,  3.5,  4.5])
>>> mean(a, axis=0)
array([ 2.5,  3.5,  4.5])
>>> average(a, axis=0)
array([ 2.5,  3.5,  4.5])

# average can also calculate
# a weighted average
>>> average(a, weights=[1,2],
...         axis=0)
array([ 3., 4., 5.])
```

STANDARD DEV./VARIANCE

```
# Standard Deviation
>>> a.std(axis=0)
array([ 1.5,  1.5,  1.5])

# Variance
>>> a.var(axis=0)
array([2.25, 2.25, 2.25])
>>> var(a, axis=0)
array([2.25, 2.25, 2.25])
```

Other Array Methods

CLIP

```
# Limit values to a range
>>> a = array([[1,2,3],
               [4,5,6]], float)

# Set values < 3 equal to 3.
# Set values > 5 equal to 5.
>>> a.clip(3,5)
array([[ 3.,  3.,  3.],
       [ 4.,  5.,  5.]])
```

POINT TO POINT

```
# Calculate max – min for
# array along columns
>>> a.ptp(axis=0)
array([ 3.0,  3.0,  3.0])
# max – min for entire array.
>>> a.ptp(axis=None)
5.0
```

ROUND

```
# Round values in an array.
# Numpy rounds to even, so
# 1.5 and 2.5 both round to 2.
>>> a = array([1.35, 2.5, 1.5])
>>> a.round()
array([ 1.,  2.,  2.])

# Round to first decimal place.
>>> a.round(decimals=1)
array([ 1.4,  2.5,  1.5])
```

Universal Functions (ufunc)

- ufuncs are objects that rapidly evaluate a function element-by-element over an array.
- Core piece is a 1-d loop written in C that performs the operation over the largest dimension of the array
- For 1-d arrays it is equivalent to but much faster than list comprehension

```
>>> type(np.exp)
<type 'numpy.ufunc'>
>>> x = array([1,2,3,4,5])
>>> print np.exp(x)
[2.71828, 7.38905, 20.08553, 54.59815, 148.41315]
>>> print [math.exp(val) for val in x]
[2.71828, 7.38905, 20.08553, 54.59815, 148.41315]

# note: values reformatted to fit slide
```

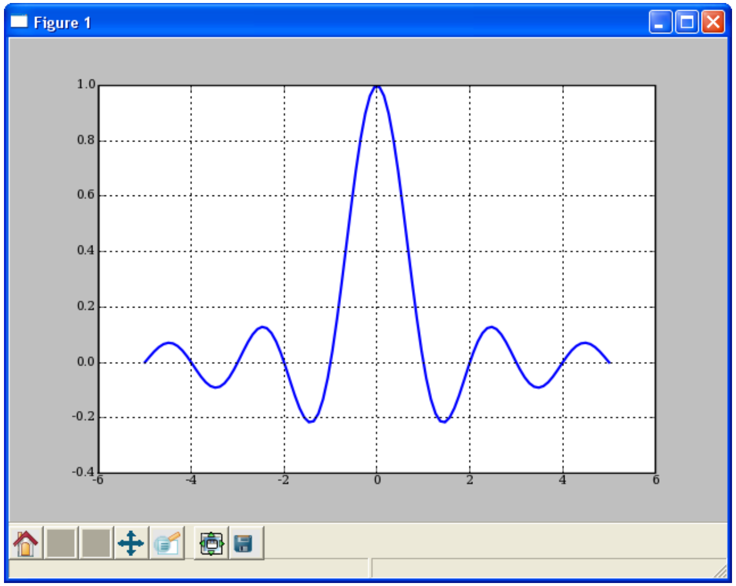
Vectorizing Functions

VECTORIZING FUNCTIONS

Example

```
# special.sinc already available
# This is just for show.
def sinc(x):
    if x == 0.0:
        return 1.0
    else:
        w = pi*x
        return sin(w) / w
```

```
# attempt
>>> sinc([1.3,1.5])
TypeError: can't multiply
sequence to non-int
>>> x = r_[-5:5:100j]
>>> y = vsinc(x)
>>> plot(x, y)
```



SOLUTION

```
>>> from numpy import vectorize
>>> vsinc = vectorize(sinc)
>>> vsinc([1.3,1.5])
array([-0.1981, -0.2122])
```

Mathematic Binary Operators *element by element*

```
a + b →   add(a,b)              a * b  →  multiply(a,b)
a - b →   subtract(a,b)         a / b  →  divide(a,b)
a % b →   remainder(a,b)        a ** b →  power(a,b)
```

MULTIPLY BY A SCALAR

```
>>> a = array((1,2))
>>> a*3.
array([3., 6.])
```

ELEMENT BY ELEMENT ADDITION

```
>>> a = array([1,2])
>>> b = array([3,4])
>>> a + b
array([4, 6])
```

ADDITION USING AN OPERATOR FUNCTION

```
>>> add(a,b)
array([4, 6])
```

IN PLACE OPERATION

```
# Overwrite contents of a.
# Saves array creation
# overhead
>>> add(a,b,a) # a += b
array([4, 6])
>>> a
array([4, 6])
```

Comparison and Logical Operators

```
equal          (==)    not_equal   (!=)    greater     (>)
greater_equal  (>=)    less        (<)     less_equal  (<=)
logical_and            logical_or          logical_xor
logical_not
```

2D EXAMPLE

```
>>> a = array(((1,2,3,4),(2,3,4,5)))
>>> b = array(((1,2,5,4),(1,3,4,5)))
>>> a == b
array([[True, True, False, True],
       [False, True, True, True]])
# functional equivalent
>>> equal(a,b)
array([[True, True, False, True],
       [False, True, True, True]])
```

Bitwise Operators *work only on Integer arrays*

`bitwise_and (&)`	`invert (~)`	`right_shift(a,shifts)`
`bitwise_or (|)`	`bitwise_xor`	`left_shift (a,shifts)`

BITWISE EXAMPLES

```
>>> a = array((1,2,4,8))
>>> b = array((16,32,64,128))
>>> bitwise_or(a,b)
array([ 17,  34,  68, 136])

# bit inversion
>>> a = array((1,2,3,4), uint8)
>>> invert(a)
array([254, 253, 252, 251], dtype=uint8)

# left shift operation
>>> left_shift(a,3)
array([ 8, 16, 24, 32], dtype=uint8)
```

Matrix

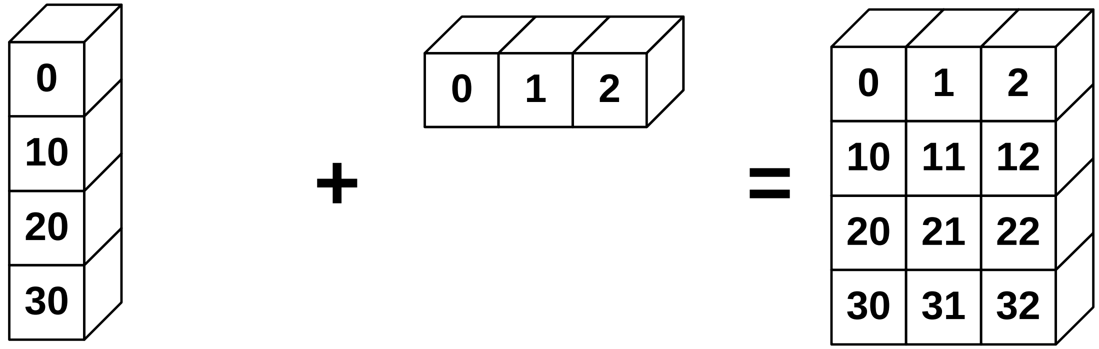
- For two dimensional arrays NumPy defined a special matrix class in module matrix
  - Objects are created either with `matrix()` or `mat()` or converted from an array with method `asmatrix()`

```
>>> import numpy
>>> m = numpy.mat([[1,2],[3,4]])
# or
>>> a = numpy.array([[1,2],[3,4]])
>>> m = numpy.mat(a)
# or
>>> a = numpy.array([[1,2],[3,4]])
>>> m = numpy.asmatrix(a)
```

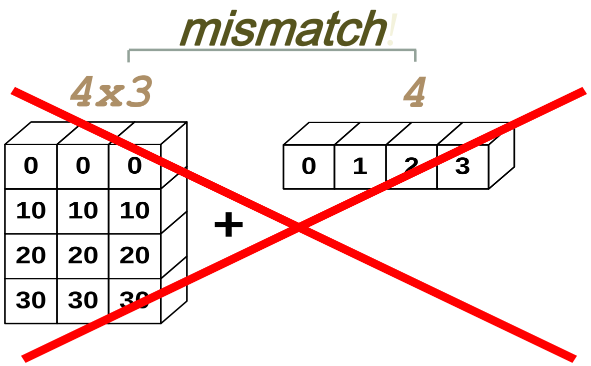
  - Note that the statement `m = mat(a)` creates a copy of array 'a‘, whereas, method `m = asmatrix(a)` returns a new reference to the same data

Broadcasting

- Multiple inputs must be "broadcasted" to the same shape
  - All arrays are promoted to the same number of dimensions
  - All dimensions of length 1 are expanded as needed



- The *trailing* axes of both arrays must either be 1 or have the same size for broadcasting to occur



Matrix Objects

STRING CONSTRUCTION

```
>>> from numpy import mat
>>> a = mat('[1,3,5;2,5,1;2,3,6]')
>>> a
matrix([[1, 3, 5],
        [2, 5, 1],
        [2, 3, 6]])
```

TRANSPOSE ATTRIBUTE

```
>>> a.T
matrix([[1, 2, 2],
        [3, 5, 3],
        [5, 1, 6]])
```

INVERTED ATTRIBUTE

```
>>> a.I
matrix([[-1.1739, 0.1304,  0.956],
        [ 0.4347, 0.1739, -0.391],
        [ 0.1739, -0.130,  0.0434]
     ])
```

DIAGONAL

```
>>> a.diagonal()
matrix([[1, 5, 6]])
>>> a.diagonal(-1)
matrix([[3, 1]])
```

SOLVE

```
>>> b = mat('10;8;3')
>>> a.I*b
matrix([[-7.82608696],
        [ 4.56521739],
        [ 0.82608696]])

>>> from scipy import linalg
>>> linalg.solve(a,b)
matrix([[-7.82608696],
        [ 4.56521739],
        [ 0.82608696]])
```

Matrix vs. Array

- Operator *, dot(), and multiply():
  - Array – '*' means element-wise multiplication; `dot()` is used for matrix mul.
  - Matrix – '*' means matrix multiplication; `multiply()` is used for element-wise mul.
- Handling of vectors (rank-1 arrays)
  - Array – the vector shapes 1xN, Nx1 are different things. Operations like A[:,1] return a rank-1 of shape N, not a rank-2 of shape Nx1. Transpose a rank-1 array does nothing
  - Matrix – rank-1 arrays are always upgraded to 1xN or Nx1 matrices (row or column vectors). A[:,1] returns a rank-2 matrix of shape Nx1
- Handling of higher-rank arrays (rank > 2)
  - Array objects can have rank > 2
  - Matrix objects always have exactly rank 2
- Convenience attributes
  - Array has a `.T` attribute, which returns the transpose of the data
  - Matrix has `.T`, `.H`, `.I`, and `.A` attributes, which return the conjugate transpose, inverse, and `asarray()` of the matrix, respectively.
- Convenience constructor
  - Array constructor takes (nested) Python sequences as initializers
  - Matrix constructor additionally takes a convenient string initializer

Example – Array and Matrix Calc.

```
>>> A = np.array([[n+m*10 for n in range(5)] for m in range(5)])
>>> v1 = arange(0, 5)
>>> A
array([[ 0, 1, 2, 3, 4],
[10, 11, 12, 13, 14],
[20, 21, 22, 23, 24],
[30, 31, 32, 33, 34],
[40, 41, 42, 43, 44]])
>>> v1
array([0, 1, 2, 3, 4])
>>> np.dot(A,A)
array([[ 300,  310,  320,  330,  340],
       [1300, 1360, 1420, 1480, 1540],
       [2300, 2410, 2520, 2630, 2740],
       [3300, 3460, 3620, 3780, 3940],
       [4300, 4510, 4720, 4930, 5140]])
>>> np.dot(A,v1)
array([ 30, 130, 230, 330, 430])
>>> np.dot(v1,v1)
30
```

Examples – Array and Matrix Calc.

```
# Alternatively, we can cast the array objects to the type
matrix. This # changes the behavior of the standard
arithmetic operators +, -, * to # use matrix algebra.
>>> M = np.matrix(A)
>>> v = np.matrix(v1).T
>>> v
matrix([[0],
        [1],
        [2],
        [3],
        [4]])
>>> M*v
matrix([[ 30],
        [130],
        [230],
        [330],
        [430]])
>>> v.T * v   # inner product
matrix([[30]])
```

Concluding Remarks

- Using arrays wisely
  - Array operations are implemented in C or Fortran
    - Optimized algorithms - i.e. fast!
  - Python loops (i.e. for i in a:…) are much slower
    - Prefer array operations over loops, especially when speed important
  - Also produces shorter code, often more readable

- Matrix or Array, which one to use?
  - Short answer – Use Array
    - They are the standard vector/matrix/tensor type of NumPy. Many NumPy functions return arrays, not matrices
    - There is a clear distinction between element-wise and linear algebra operations
    - You can have standard vectors or row/column vectors if you like
  - The main disadvantage of using the array type is that you will have to use `dot()` instead of '*' matrix multiplication

- NumPy for Matlab Users
  - http://wiki.scipy.org/NumPy_for_Matlab_Users

SCIPY

Scientific Python

SciPy Overview

- Available at www.scipy.org

CURRENT PACKAGES

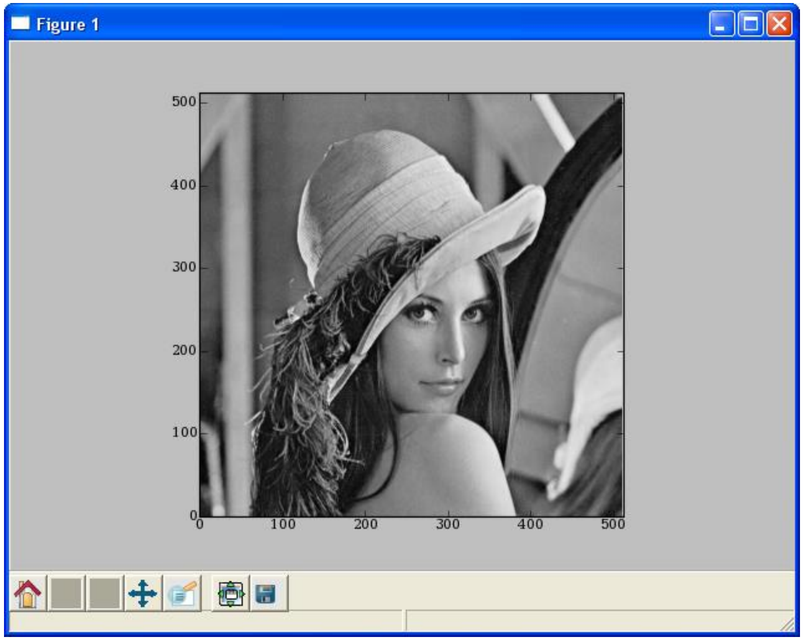
- Special Functions (scipy.special)
- Signal Processing (scipy.signal)
- Image Processing (scipy.ndimage)
- Fourier Transforms (scipy.fftpack)
- Optimization (scipy.optimize)
- Numerical Integration (scipy.integrate)
- Linear Algebra (scipy.linalg)

- Input/Output (scipy.io)
- Statistics (scipy.stats)
- Fast Execution (scipy.weave)
- Clustering Algorithms (scipy.cluster)
- Sparse Matrices (scipy.sparse)
- Interpolation (scipy.interpolate)
- More (e.g. scipy.odr, scipy.maxentropy)

Image Processing

```
# The famous lena image is packaged with scipy
>>> from scipy import lena, signal
>>> lena = lena().astype(float32)
>>> imshow(lena, cmap=cm.gray)
# Blurring using a median filter
>>> fl = signal.medfilt2d(lena, [15,15])
>>> imshow(fl, cmap=cm.gray)
```

LENA IMAGE



MEDIAN FILTERED IMAGE

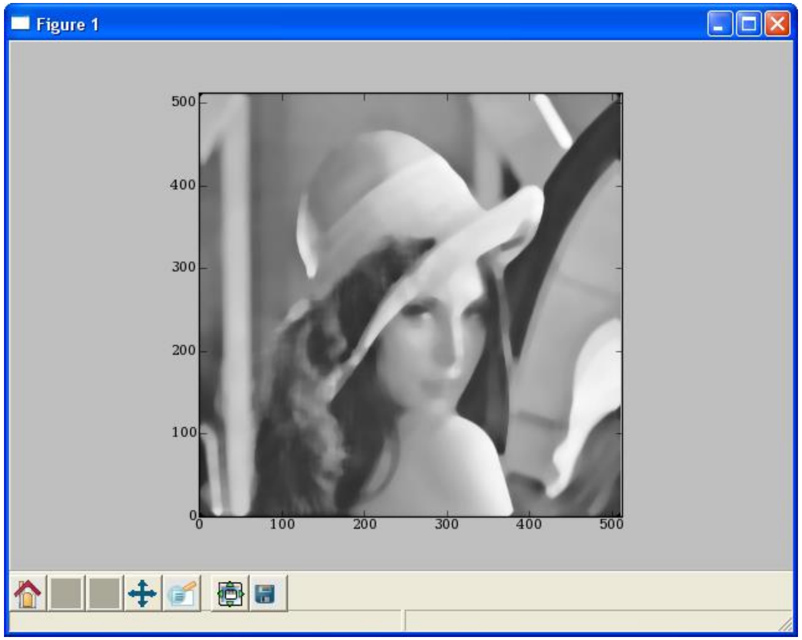
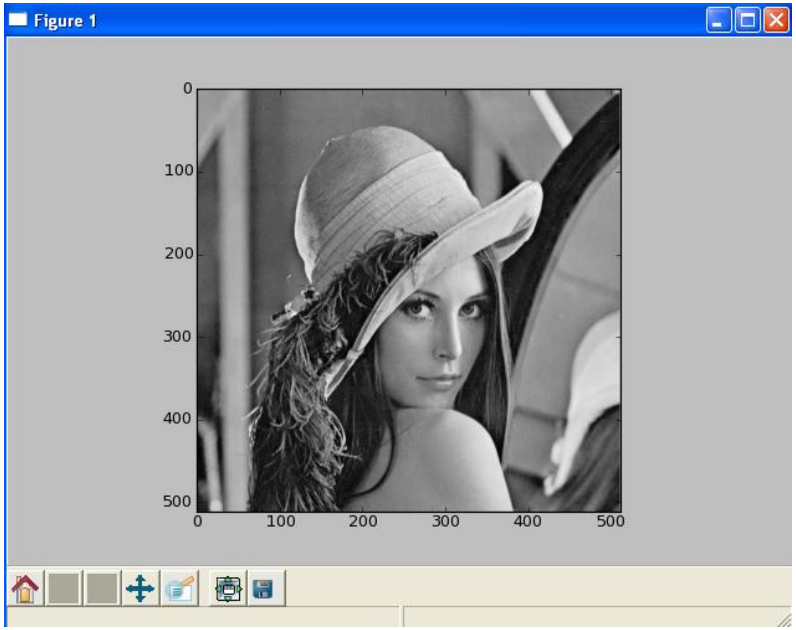


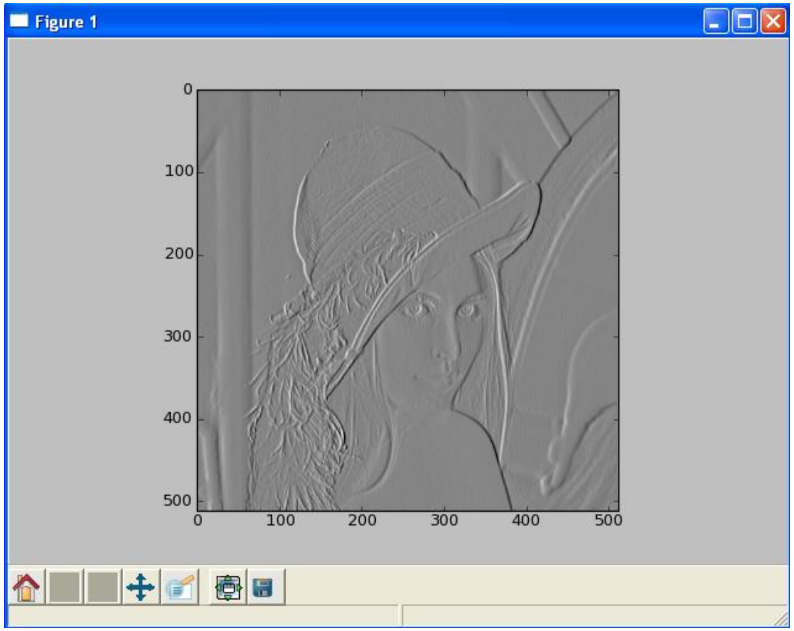
Image Processing

```
# Edge detection using Sobel filter
>>> from scipy.ndimage.filters import sobel
>>> imshow(lena)
>>> edges = sobel(lena)
>>> imshow(edges)
```

NOISY IMAGE



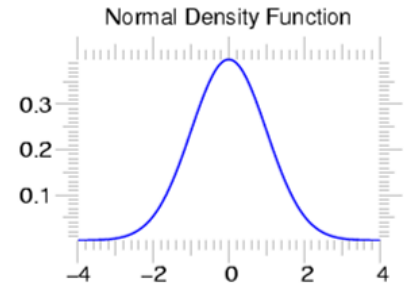
FILTERED IMAGE

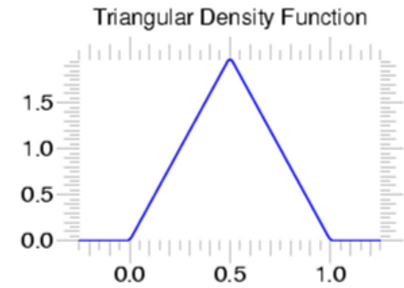


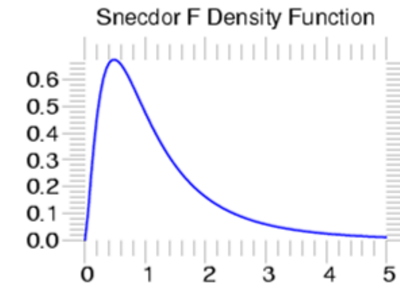
Statistics

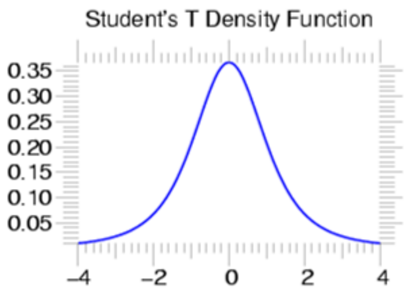
scipy.stats --- CONTINUOUS DISTRIBUTIONS

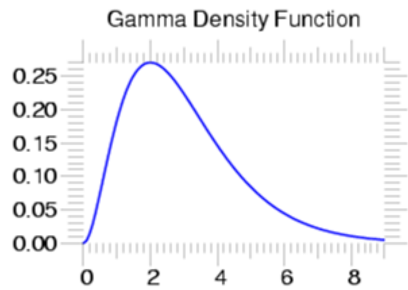
over 80 continuous distributions!

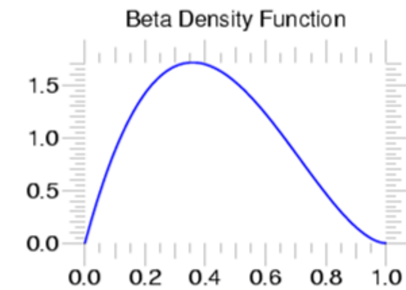


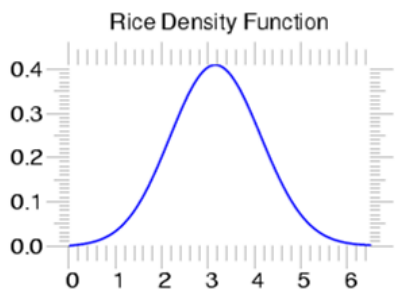


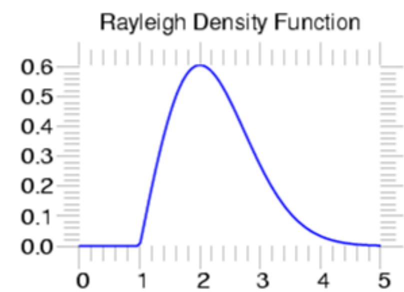


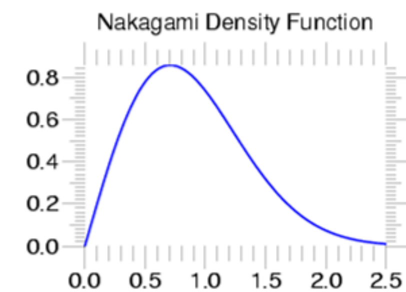












METHODS

`pdf`

`cdf`

`rvs`

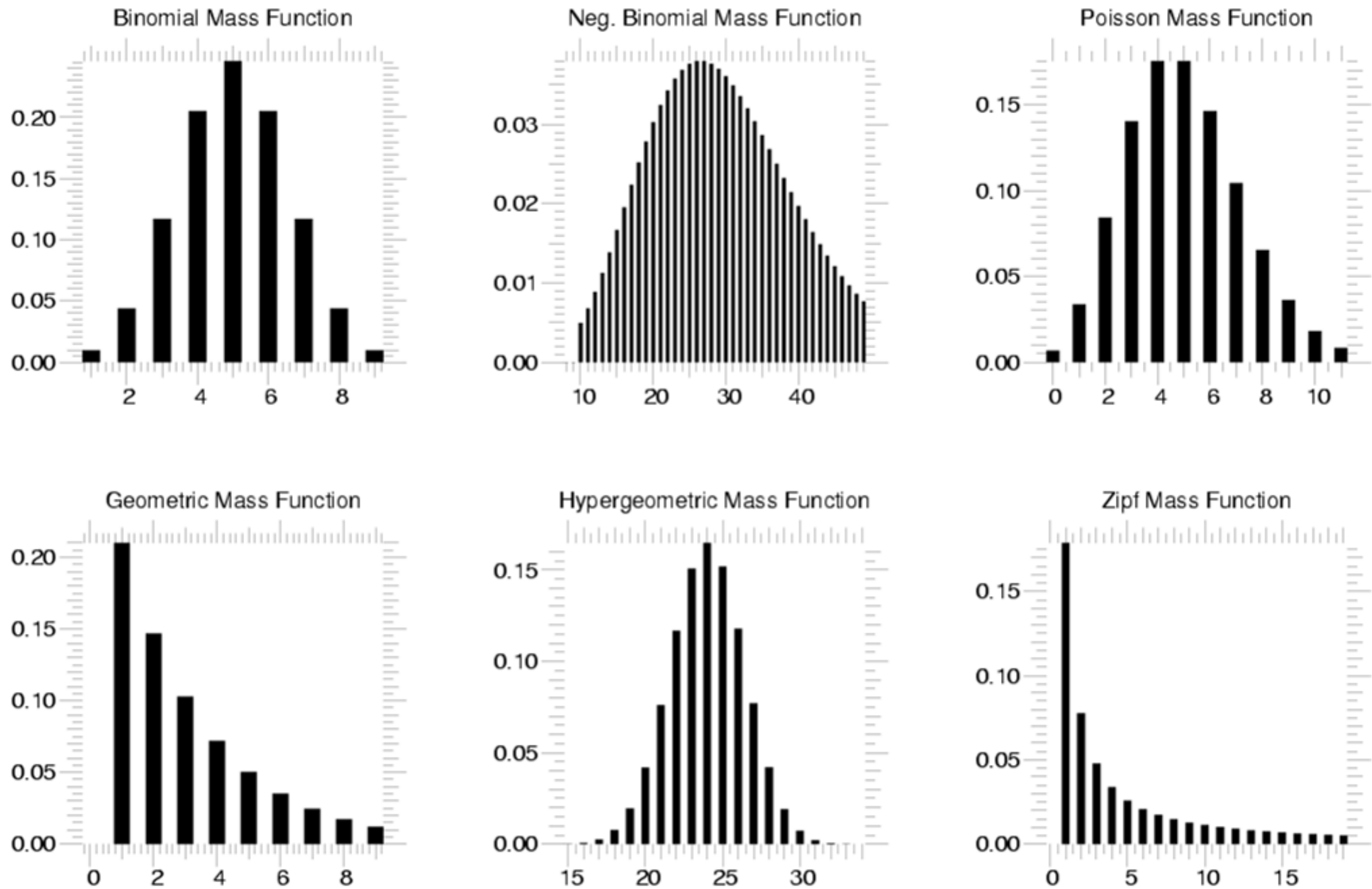
`ppf`

`stats`

Statistics

scipy.stats --- Discrete Distributions

10 standard discrete distributions (plus any arbitrary finite RV)



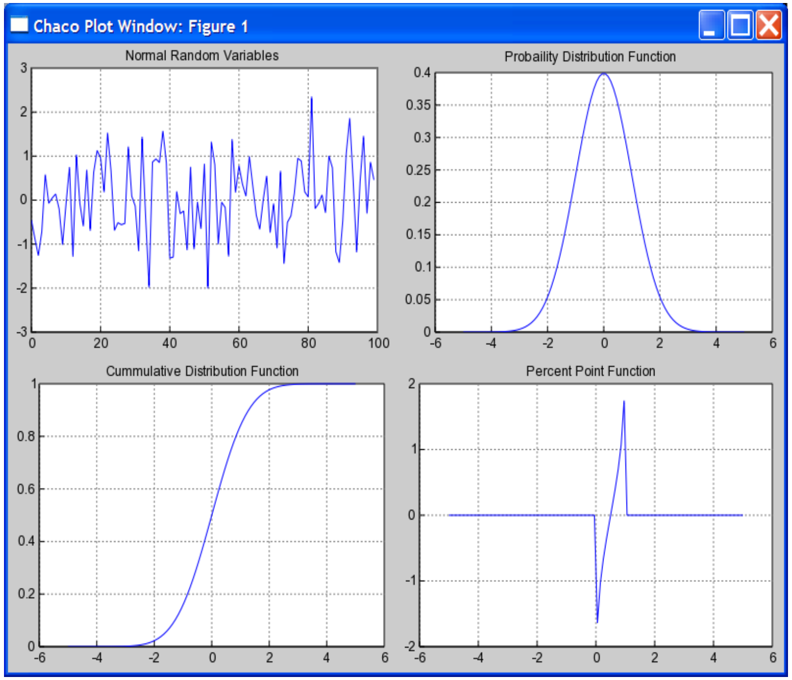
METHODS

```
pdf
cdf
rvs
ppf
stats
```

Using Stats Objects

DISTRIBUTIONS

```
# Sample normal dist. 100 times.
>>> samp = stats.norm.rvs(size=100)

>>> x = r_[-5:5:100j]
# Calculate probability dist.
>>> pdf = stats.norm.pdf(x)
# Calculate cummulative Dist.
>>> cdf = stats.norm.cdf(x)
# Calculate Percent Point Function
>>> ppf = stats.norm.ppf(x)
```



Statistics

scipy.stats --- Basic Statistical Calculations on Data

- `numpy.mean, numpy.std, numpy.var, numpy.cov`
- `stats.skew, stats.kurtosis, stats.moment`

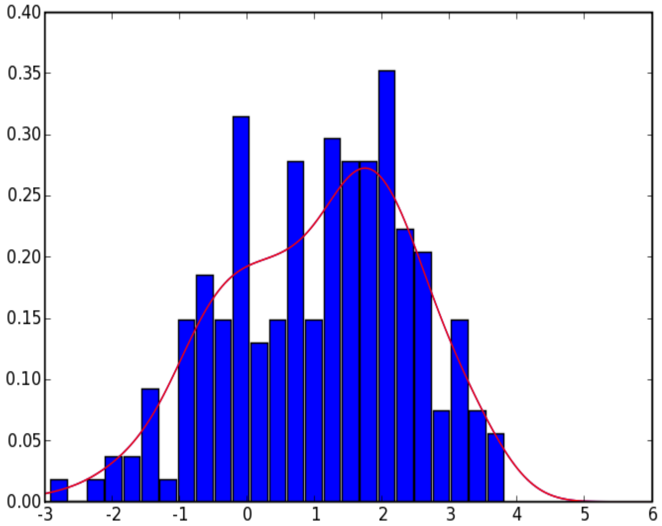
scipy.stats.bayes_mvs --- Bayesian mean, variance, and std.

```
# Create "frozen"  Gamma distribution with a=2.5
>>> grv = stats.gamma(2.5)
>>> grv.stats()       # Theoretical mean and variance
(array(2.5), array(2.5))
# Estimate mean, variance, and std with 95% confidence
>>> vals = grv.rvs(size=100)
>>> stats.bayes_mvs(vals, alpha=0.95)
((2.52887906081, (2.19560839724, 2.86214972438)),
 (2.87924964268, (2.17476164549, 3.8070215789)),
 (1.69246760584, (1.47470730841, 1.95115903475)))
#  (expected value and confidence interval for each of
# mean, variance, and standard-deviation)
```

Statistics

Continuous PDF Estimation using Gaussian Kernel Density Estimation

```
# Sample normal dist. 100 times
>>> rv1 = stats.norm()
>>> rv2 = stats.norm(2.0,0.8)
>>> samp = r_[rv1.rvs(size=100),
              rv2.rvs(size=100)]

# Kernel estimate (smoothed histogram)
>>> apdf = stats.kde.gaussian_kde(samp)
>>> x = linspace(-3,6,200)
>>> plot(x, apdf(x),'r')

# Histogram
>>> hist(x, bins=25, normed=True)
```



Linear Algebra

scipy.linalg --- FAST LINEAR ALGEBRA

- Uses ATLAS if available --- very fast
- Low-level access to BLAS and LAPACK routines in modules `linalg.fblas`, and `linalg.flapack` (FORTRAN order)
- High level matrix routines
  - Linear Algebra Basics: `inv`, `solve`, `det`, `norm`, `lstsq`, `pinv`
  - Decompositions: `eig`, `lu`, `svd`, `orth`, `cholesky`, `qr`, `schur`
  - Matrix Functions: `expm`, `logm`, `sqrtm`, `cosm`, `coshm`, `funm` (general matrix functions)

Linear Algebra

LU FACTORIZATION

```
>>> from scipy import linalg
>>> a = array([[1,3,5],
...            [2,5,1],
...            [2,3,6]])
# time consuming factorization
>>> lu, piv = linalg.lu_factor(a)

# fast solve for 1 or more
# right hand sides.
>>> b = array([10,8,3])
>>> linalg.lu_solve((lu, piv), b)
array([-7.82608696,  4.56521739,
        0.82608696])
```

EIGEN VALUES AND VECTORS

```
>>> from scipy import linalg
>>> a = array([[1,3,5],
...            [2,5,1],
...            [2,3,6]])
# compute eigen values/vectors
>>> vals, vecs = linalg.eig(a)
# print eigen values
>>> vals
array([ 9.39895873+0.j,
       -0.73379338+0.j,
        3.33483465+0.j])
# eigen vectors are in columns
# print first eigen vector
>>> vecs[:,0]
array([-0.57028326,
       -0.41979215,
       -0.70608183])
# norm of vector should be 1.0
>>> linalg.norm(vecs[:,0])
1.0
```

Optimization

scipy.optimize --- unconstrained minimization and root finding

- **Unconstrained Optimization**

  `fmin` (Nelder-Mead simplex), **`fmin_powell`** (Powell's method), **`fmin_bfgs`** (BFGS quasi-Newton method), **`fmin_ncg`** (Newton conjugate gradient), **`leastsq`** (Levenberg-Marquardt), **`anneal`** (simulated annealing global minimizer), **`brute`** (brute force global minimizer), **brent** (excellent 1-D minimizer), **golden**, **bracket**

- **Constrained Optimization**

  `fmin_l_bfgs_b`, **`fmin_tnc`** (truncated newton code), **`fmin_cobyla`** (constrained optimization by linear approximation), **`fminbound`** (interval constrained 1-d minimizer)

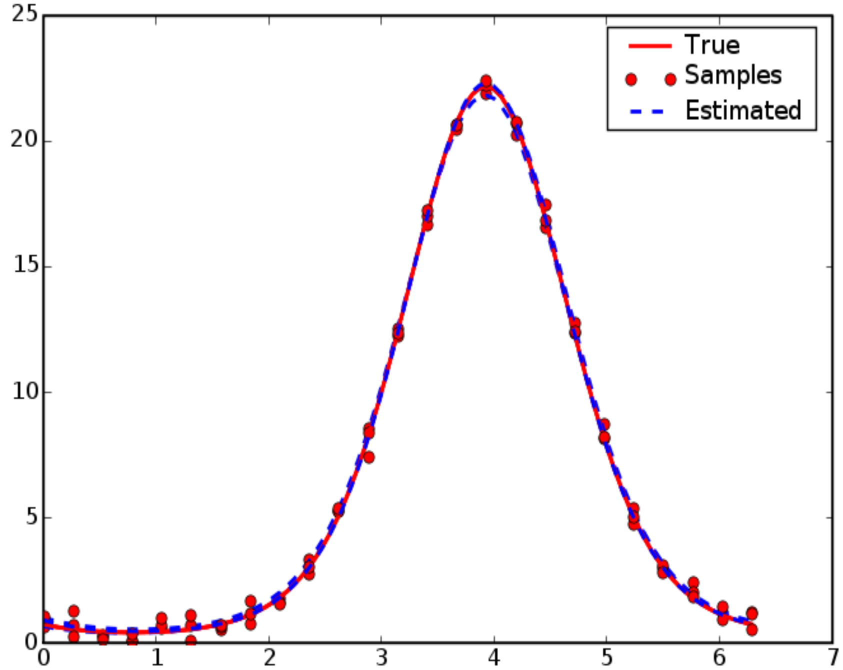
- **Root finding**

  `fsolve` (using MINPACK), **`brentq`**, **`brenth`**, **`ridder`**, **`newton`**, **`bisect`**, **`fixed_point`** (fixed point equation solver)

Optimization

EXAMPLE: Non-linear least-squares data fitting

```
# fit data-points to a curve
# demo/data_fitting/datafit.py
>>> from numpy.random import randn
>>> from numpy import exp, sin, pi
>>> from numpy import linspace
>>> from scipy.optimize import leastsq
>>> def func(x,A,a,f,phi):
        return A*exp(-a*sin(f*x+pi/4))
>>> def errfunc(params, x, data):
        return func(x, *params) - data
>>> ptrue = [3,2,1,pi/4]
>>> x = linspace(0,2*pi,25)
>>> true = func(x, *ptrue)
>>> noisy = true + 0.3*randn(len(x))
>>> p0 = [1,1,1,1]
>>> pmin, ier = leastsq(errfunc, p0, args=(x, noisy))
>>> pmin
array([3.1705, 1.9501, 1.0206, 0.7034])
```



THANK YOU