

Development of Topics in Electrical  
Engineering 1 -  
Reproducing “Valadarsky, Asaf, et al.  
Learning to Route 2017 “Results

**Report**

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## Introduction

The main purpose of this report is about reproducing the achieved results from [\[1\]](#).

The authors introduce a new approach of how to use machine learning techniques in order to solve one of the fundamental network control challenges, namely traffic routing.

Both deep supervised learning and deep reinforcement learning techniques are introduced in the paper for handling this challenge. Supervised learning is used for predicting the next traffic demands matrix based on the past compared to focusing on solving the routing control problem directly using reinforcement learning agent. The supervised learning model is a good starting point but did not achieve noticeable results, on the other hand, the performance of the reinforcement learning agent is better, moreover, for some traffic patterns even beats known routing schemes.

The reproduction of the results starts with generating traffic patterns as the authors described and continues with training a reinforcement learning agent.

Another motivation for this work is gaining new knowledge about routing algorithms (Optimal Routing, Oblivious Routing), new heuristics methods, useful programming packages, yet the main goal is to develop a new direction for a continuing research.

## Definitions

Network topology- following the paper, for the control routing problem the network topology is represented by a directional graph which is based on unidirectional graph with edges' capacity function (in Mb/s).

The directional graph is simply made by considering each edge as an independent bidirectional link.

Synthetic Traffic Generation – to examine the developed techniques a synthetic traffic generation is necessary.

The traffic demand is represent by a square matrix where the cell  $i, j$  is the flow demand from node  $i$  to node  $j$ .

Another property of the matrix is sparsity, which is the percentage of source, destination pairs that demand traffic, for example, a topology with 10 nodes includes 90 difference pairs of source destination, with sparsity of 30% only 27 randomly chosen pairs are included in the traffic.

**Two different types of traffic are generated for evaluation:**

- **Gravity Traffic**: traffic with correlation to the links' capacities connected to the source node and the destination node, calculated by the formula:  
$$\frac{[\text{source out links' capacity}] \times [\text{destination out links' capacity}]}{\text{total nodes out links' capacity}}$$
- **Bimodal Traffic [4]**: each flow demand of the traffic is sampled by some probability (biased coin flip) from two independent Gaussian distributions. One distribution represents mice flows and the other elephant flows. For all the evaluations the Gaussian distributions are:  $N(400, 20)$  for elephant flows and  $N(150, 20)$  for mice flows.

**The Objective**: finding a routing scheme for the traffic demands with the objective of minimization of maximum link utilization, also referred to as the minimax problem, and load balancing flows in the network.

## Baselines - Optimal Routing

To evaluate the models and techniques that were developed by the authors they define a reference baseline that is based on **load balancing**.

The optimal routing criterion is defined as the **minimization of maximum link utilization**, i.e., minimize the most congested link, for which the mathematic expression is:

$$\min_{e \in E} \left\{ \max \frac{f_e}{C_e} \right\}$$

$C_e$  – capacity of link  $e$

$f_e$  – total flow in link  $e$

This can be formulated as an optimization problem for a traffic demand matrix with flow in the  $i, j$  cell as follows:

*Objective : Minimize  $r$*

$$\sum_i \sum_{j \neq i} g_e(i, j) \leq C_e \cdot r \quad \forall e \in Edges$$

where  $g_e(i, j)$  is a fraction of the flow  $i \rightarrow j$  that link  $e$  carries

The routing scheme constartions:

For each existing demend  $i \rightarrow j$  :

$$\sum_{e \in IN(v)} g_e(i, v) - \sum_{e \in OUT(v)} g_e(i, v) = demand(i, v) \Big| v = j, \text{destination constraint}$$

$$\sum_{e \in OUT(v)} g_e(v, j) - \sum_{e \in IN(v)} g_e(v, j) = demand(v, j) \Big| v = i, \text{source constraint}$$

$$\sum_{e \in OUT(v)} g_e(i, j) - \sum_{e \in IN(v)} g_e(i, j) = 0 \Big| v \neq i, j, \text{transit constraint}$$

$$g_e(i, j) \geq 0 \quad \forall i, j$$

$$r \geq 0$$

By the definition of this optimization problem, all the constraints are linear expressions therefore, a linear programing solver (like "IBM – CPLEX" or "Gurobi") can be used to solve it, ("Gurobi" has been used, because it easy to set multiple objectives much easier so as to prevent the tool from creating unnecessary loops of flows).

The linear programming problem includes  $O(|edges| \times |nodes|^2)$  variables and

$O(|edges| + |nodes|^2)$  constrains.

## Baseline - Optimal Oblivious Routing

Another baseline that the authors used is **optimal oblivious routing**. As its name implies, oblivious routing **does not depend on traffic patterns but only on the topology**. This routing technique was presented in several papers in the early 2000s.

The oblivious performance ratio of a routing scheme  $f$  [2] is define as follows:

The congestion of routing  $D$  using  $f$  is the maximum link congestion, that is

$$CONGESTION(f, D) = \max_{e \in E} \frac{Flow(e, f, D)}{C_e}$$

The oblivious performance ratio of a routing  $f$ ;

which is the maximum performance ratio it can obtain, over all demand matrices, that is

$$OBLIV - PERF - RATIO(f) = \sup_D \frac{CONGESTION(f, D)}{OPT(D)}$$

The performance ratio of an optimal oblivious routing is denoted by

$$OBLIVE - OPT(G) = \min_f OBLIV - PERF - RATIO(f)$$

Using the result of [3], the optimal oblivious routing problem can be formulated as a single optimization problem as follows:

*Objective : Minimize  $r$*

$f$  is valid routing scheme

$\forall$  edges  $e$ :

$$\sum_{h \in E} C_h \cdot \pi_e(h) \leq r$$

$$\forall \text{ pairs } i \rightarrow j: \frac{f_e(i, j)}{C_e} \leq p_e(i, j)$$

$$\forall \text{ node } i, \forall \text{ edge } a=(j, k):$$

$$\pi_e(a) + p_e(i, j) - p_e(i, k) \geq 0$$

$$\forall \text{ edge } h \in E: \pi_e(h) \geq 0$$

$$\forall \text{ node } i: p_e(i, i) = 0$$

$$\forall \text{ node } i, j: p_e(i, j) \geq 0$$

As one can see, because all the constrains are linear, this is also a linear programming problem with  $O(|edges| \times |nodes|^2)$  variables and  $O(|edges|^2 \times |nodes|)$  constrains, where  $\pi_e(h)$  is a variable that represents an existing weight such that for every pair of edges  $e, h$ :  $\sum_{h \in E} C_h \cdot \pi_e(h) \leq r$  and  $p_e(i, j)$  represents the length of the shortest path from node  $i$  to node  $j$  according the edge weights  $\pi_e(h)$ .

## Reproducing the Baseline Results

The reproduction process starts with reproducing all the baselines, namely optimal routing based on averaged history and Oblivious Routing.

Motivated by the reinforcement learning method of observing a state of the environment then taking an action, the authors created a baseline which is based on an history of traffic matrices sequence to approximate the next traffic demand.

By calculating the average of the last  $K$  traffic matrices (component-wise), creating a routing scheme for it by solving the optimal routing optimization problem and using it for routing the next traffic demand i.e., create a routing scheme in advance, based on an already seen traffic sequence, for the next in line traffic matrix.

One of the critical challenges that needed to be considered is that a new demand by nodes  $i, j$  can be in the next traffic demand but not in the past i.e., that demand does not exists in the approximate average traffic matrix (based on the history traffic sequence), but does in the next traffic matrix (this is more common in low matrix sparsity); the solution is to use an ECMP routing policy with equal weights for these new traffic demand that result an equally divided flows among all shortest paths between the source and destination.

The result are normalized with respect to the most congested link utilization when applying the optimal routing scheme.

## Evaluation

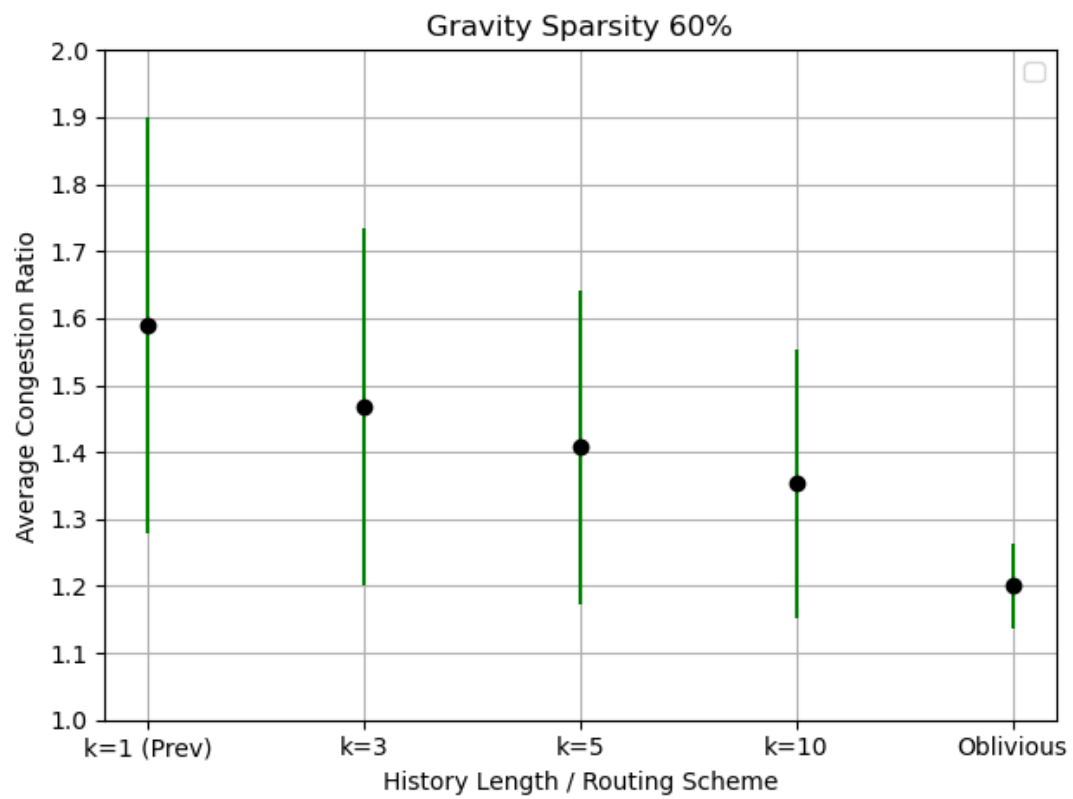
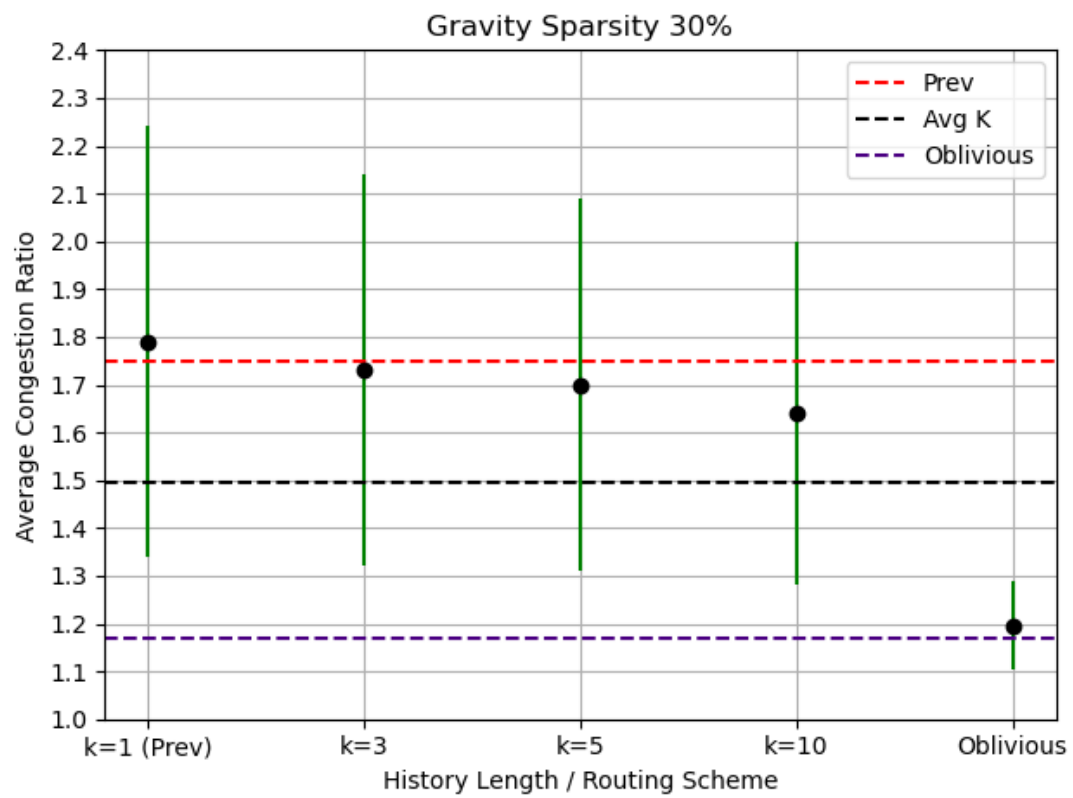
Similar to the paper, a 12-node topology with 26 edges (the original topology includes duplicate edges, this represented as double the capacity for those edges) and constant link capacity of 10,000 Mb/s.

20,000 traffic matrices dataset have been used in order to get the results.

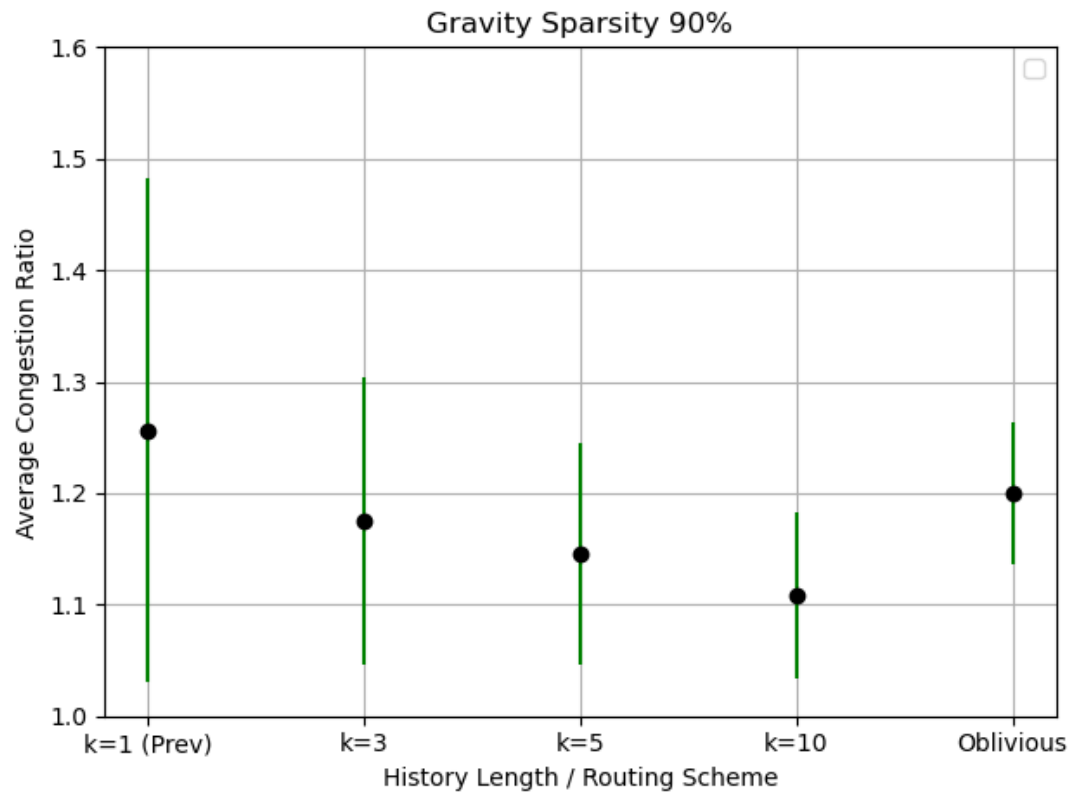
For example, for  $K=5$  matrices 1,2,3,4,5 are used for calculate an average traffic matrix then finding the optimal routing scheme for it and apply it on matrix 6.

(The dashed lines are approximations of the results from the paper, the vertical green lines are the standard deviations.)

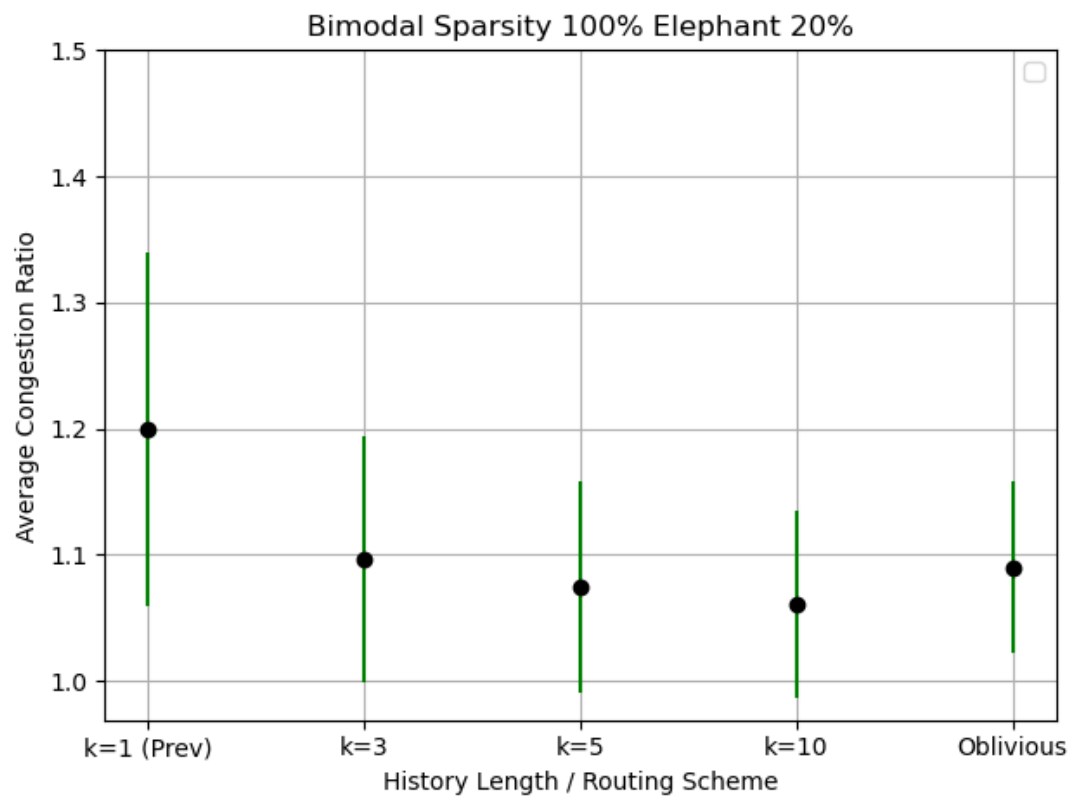
### Gravity Traffic:

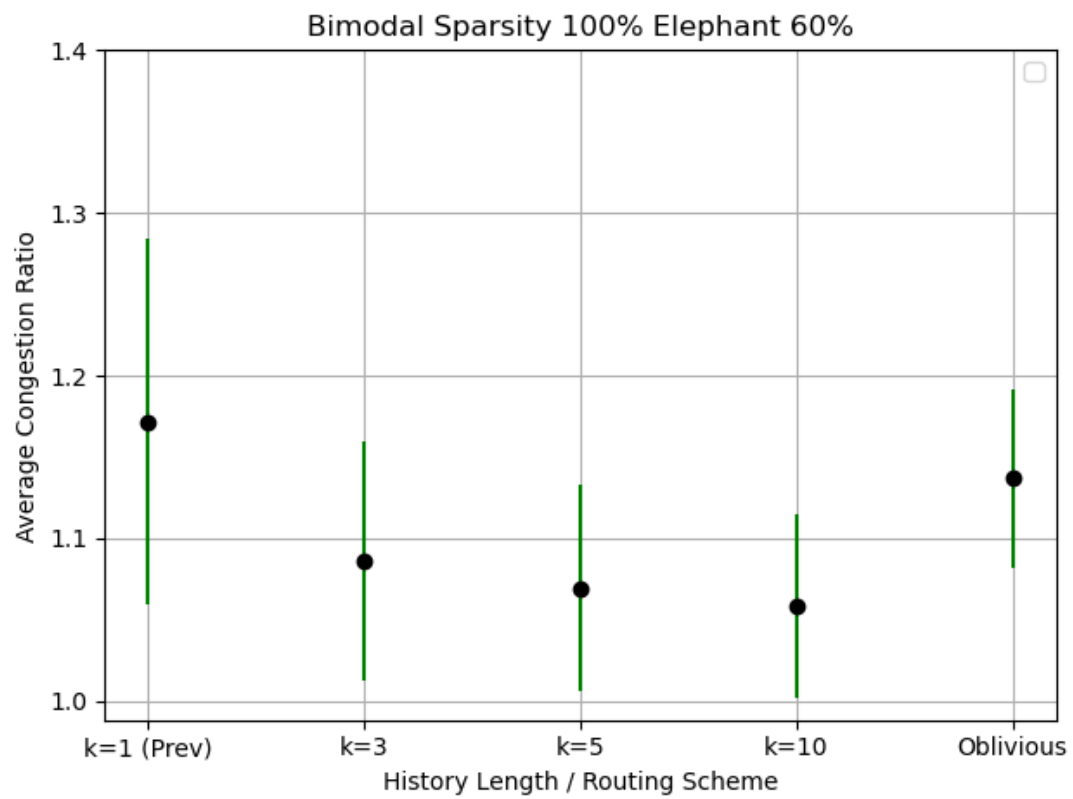
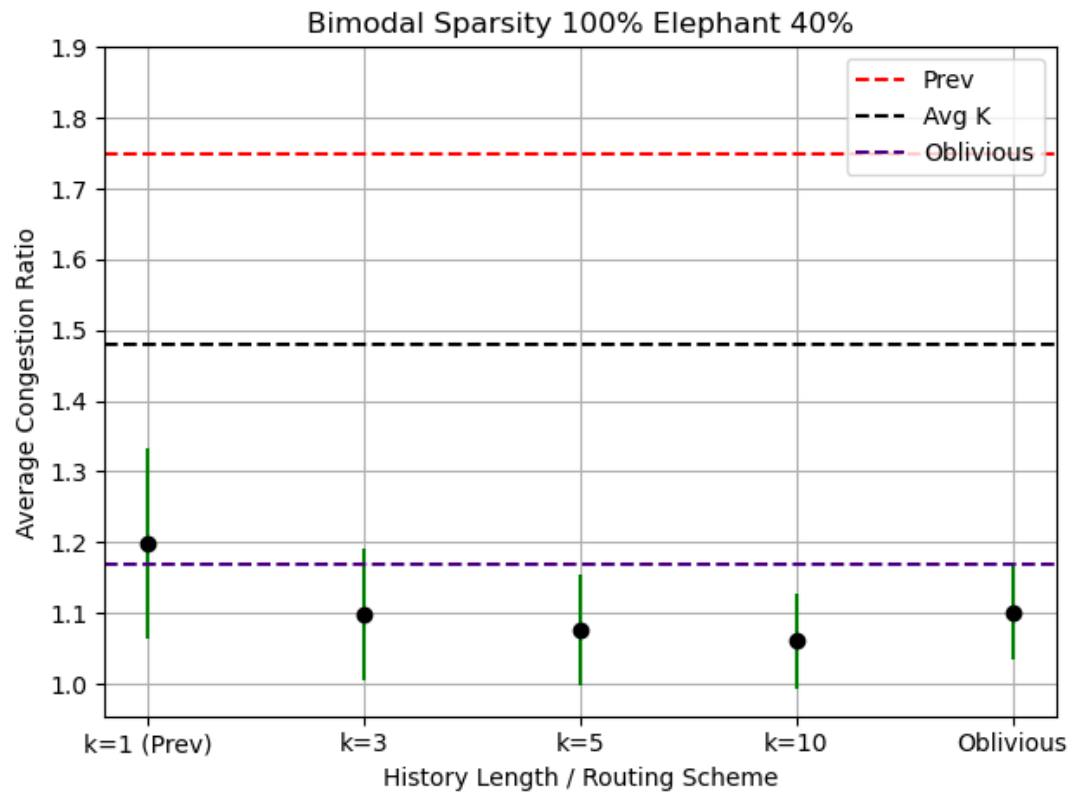






**Bimodal Traffic:**





## Reproducing the Reinforcement Learning Results

The outstanding of [1] is the proof of concept that a reinforcement learning agent can be useful to produce a good routing scheme that minimizes the congestion ratio as well as other known routing techniques.

The authors of [1] set up an environment in order to simulate the network nodes and links history of traffic demand sequence and in every timestep the arrival of new traffic demand in a shape of a traffic matrix that should be routed in the network topology.

The main goal of the process of learning is to let the agent learn how to **map** out of the **history of traffic matrices appropriate weights for each network link**, so that routing is performed per these weights used for the next traffic matrix. That is, using these weights, the environment build new weighted directional graph and calculate a shortest path for each destination specified by the demand from every other node. The final step is creating a square matrix ( $|nodes| \times |nodes|$ ) of these costs of these paths and a square matrix these link weights by entering and leaving edges, summing the matrices, reshape the results to a vector and plugging it (component wise) to the Soft-Min function. The Soft-Min function can be regarded as a probability distribution therefore, it creates for each node the percentage of flow carried by each leaving link.

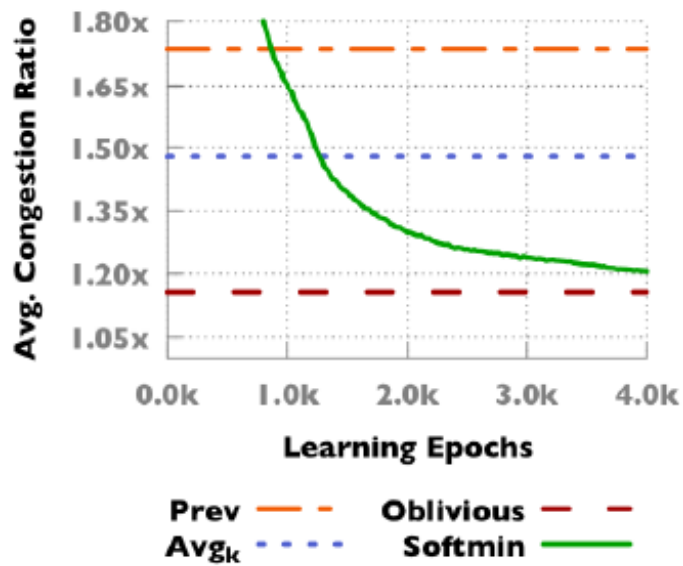
The final step is to simulate each demand of traffic around the network links until all of the flow (99.99%) reaches its destination and calculate the most congested link for the final congestion ratio of the traffic matrix.

## Evaluation

Similarly to [1], a 12-node topology with 26 edges (the original topology includes duplicate edges, this represented as double the capacity for those edges) and constant link capacity of 10,000 Mb/s.

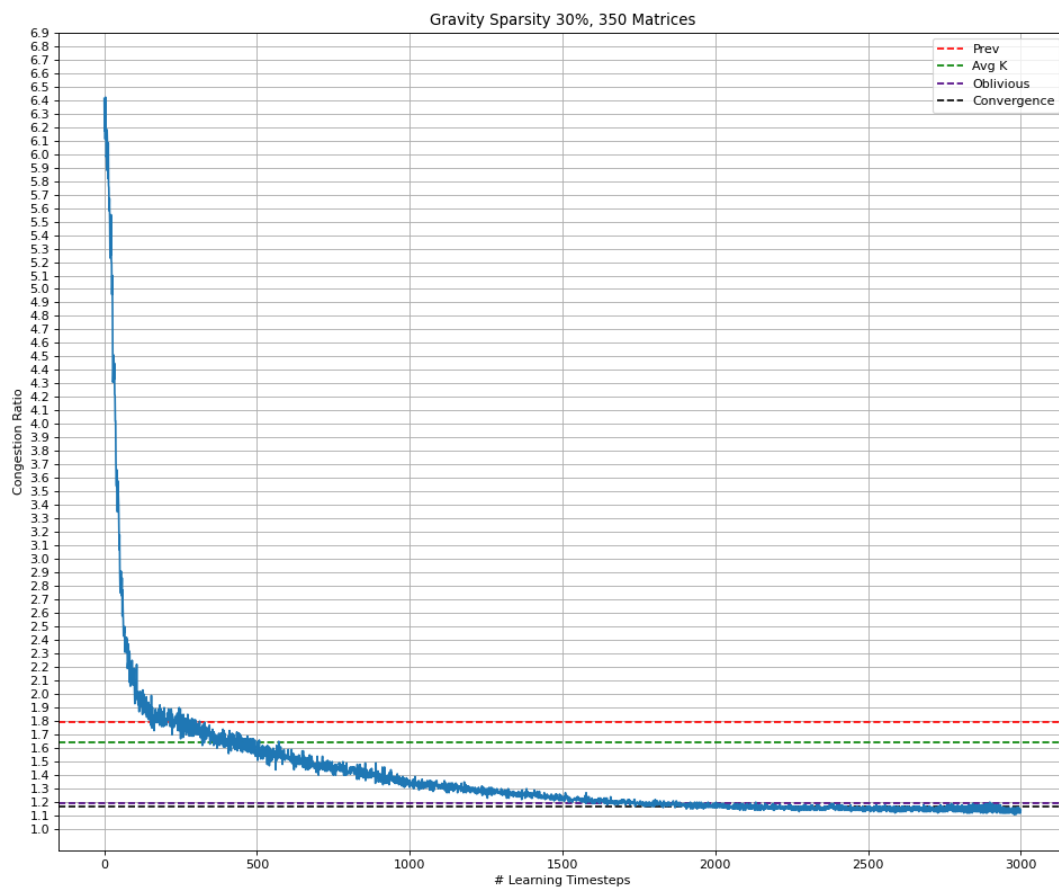
The evaluated agent parameters are: fully connected neural network with 3 layers of [128, 64, 64], the state is the history of traffic with length of 10 (like the paper), discount factor equal to 0 and episode length of 1. Because flows are not continued more than one timestep (flows can't start in time  $t$  and continue to time  $t+1$ ), therefore the assumption that a myopic approach to minimize the congestion of the current traffic matrix is a good start because agent current decisions has no effect on future ones.

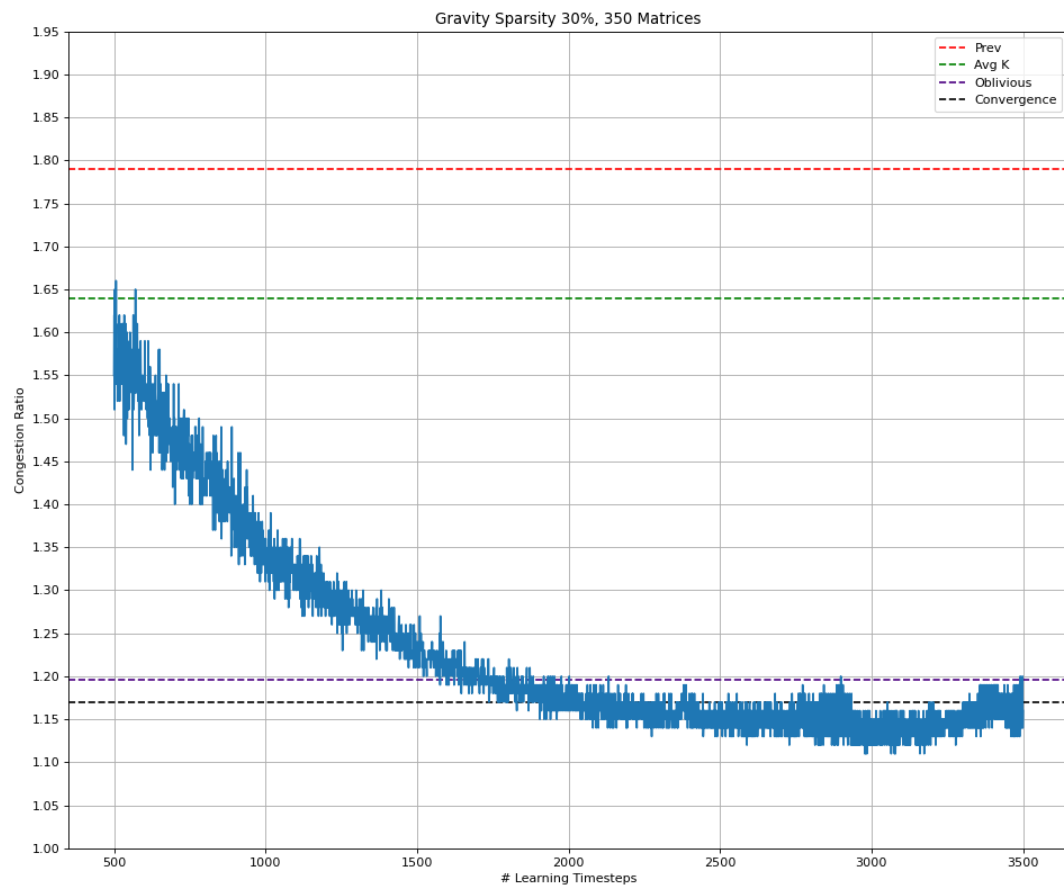
### Gravity Traffic Paper results:



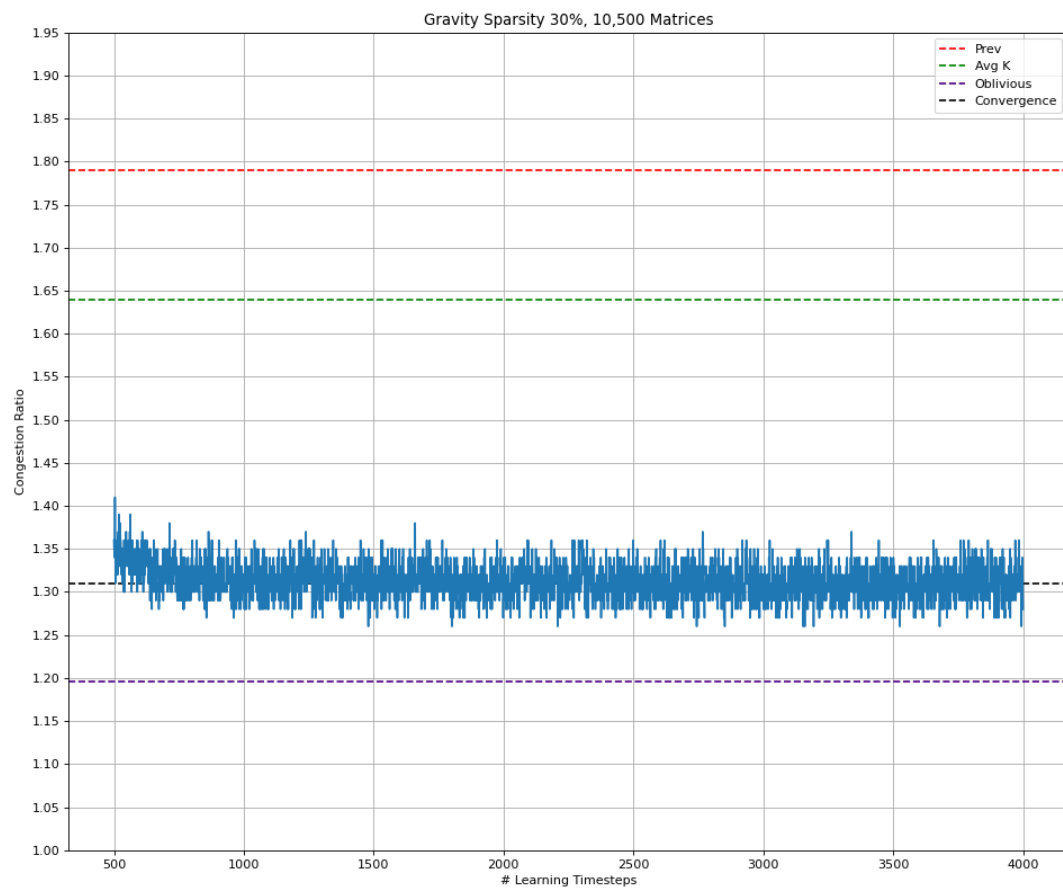
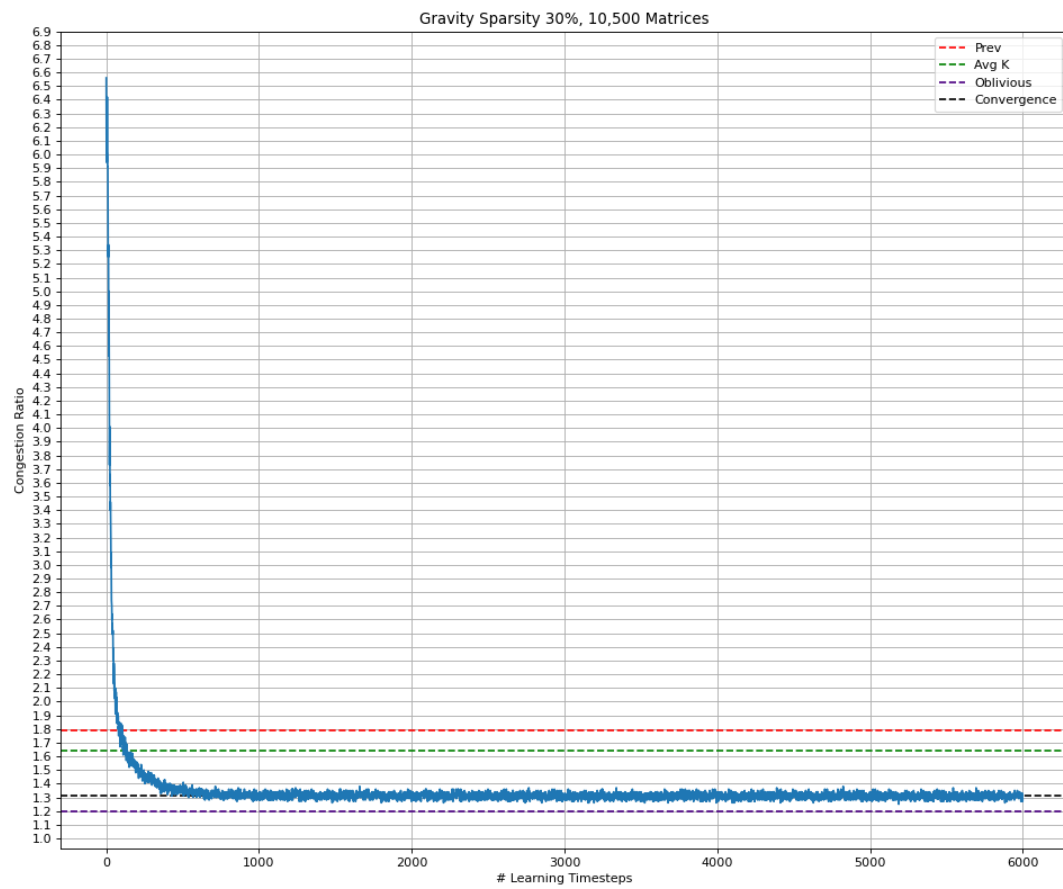
(a) Congestion ratio for sparse ( $p = 0.3$ ) gravity DM sequences

### Gravity Traffic, Sparsity of 100% and 350 different TMs:

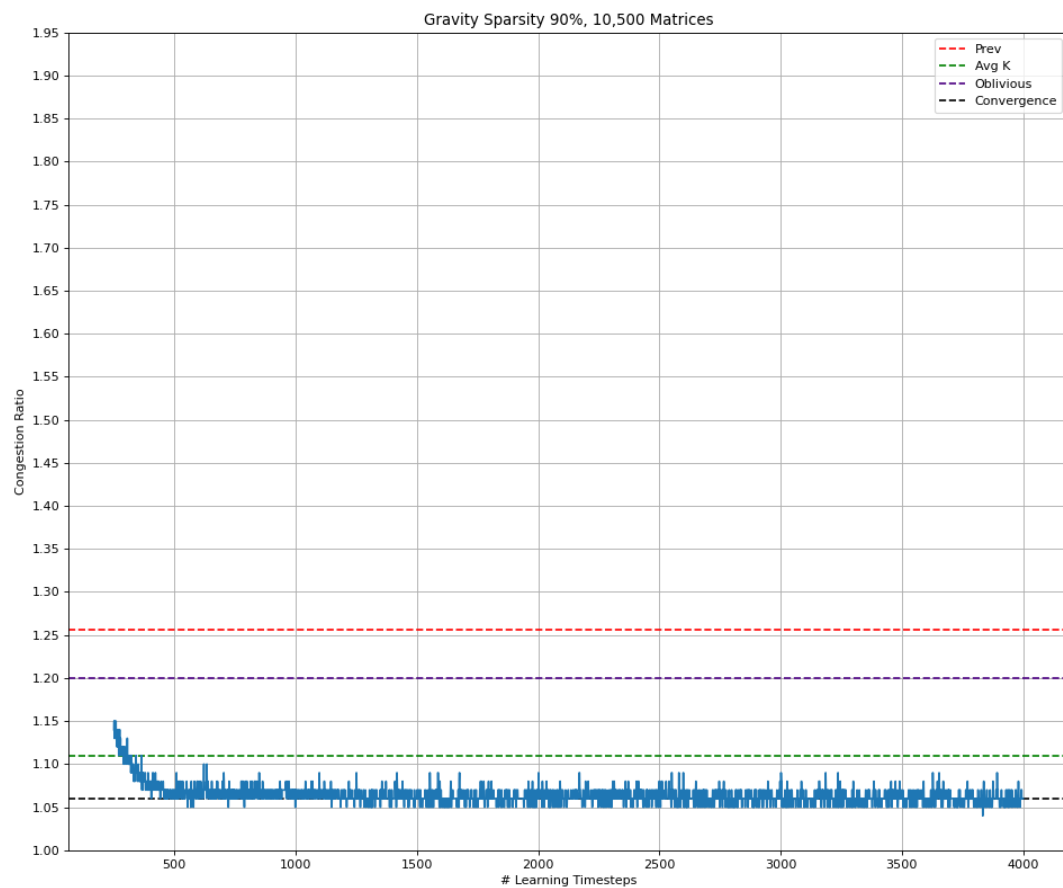
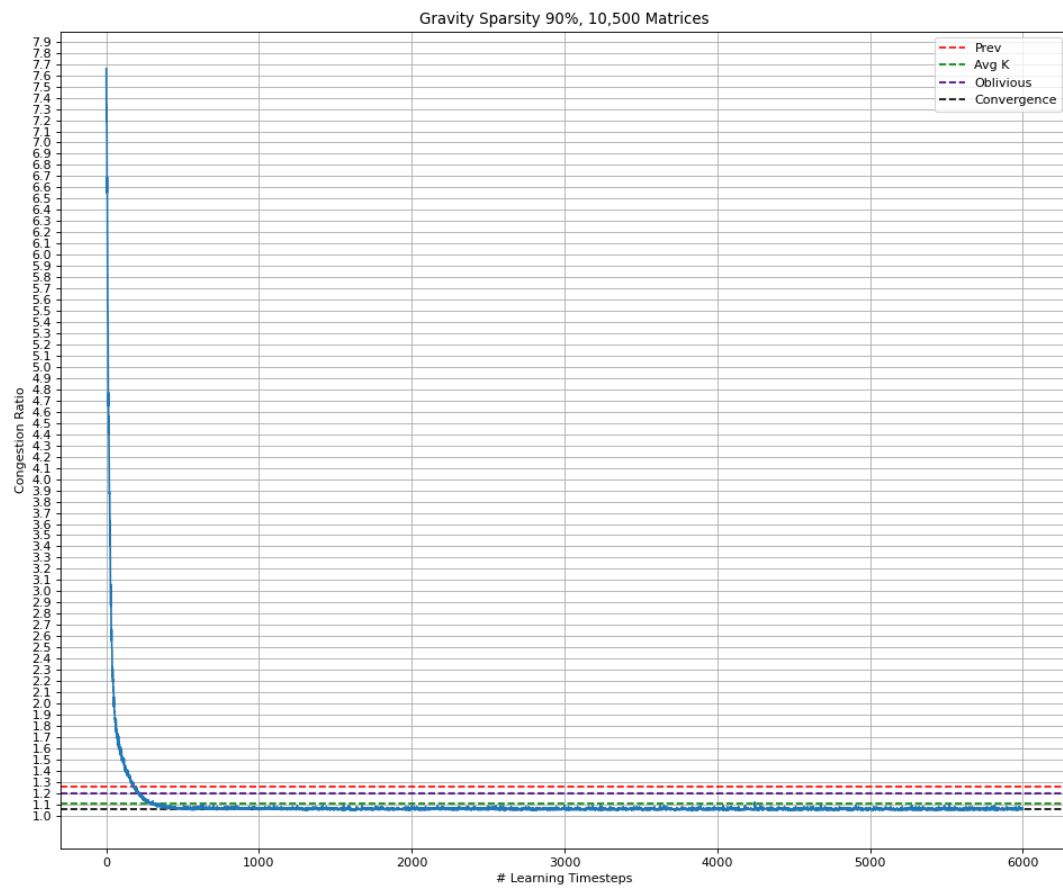




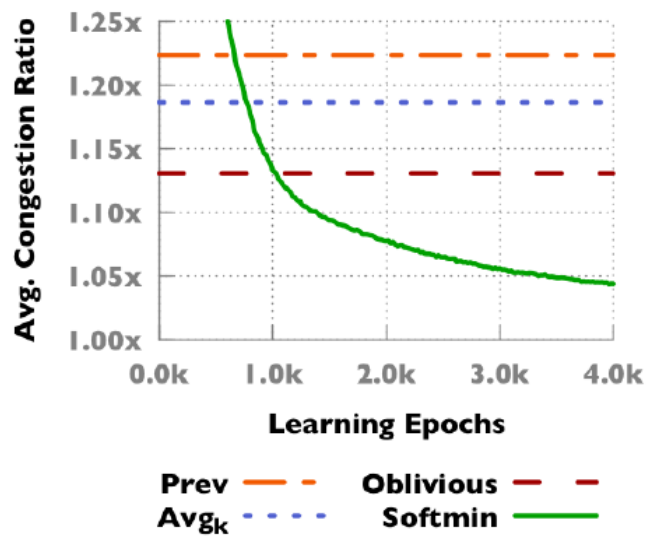
## Gravity Traffic, Sparsity of 30% and 10,500 different TMs:



## Gravity Traffic, Sparsity of 90% and 10,500 different TMs:

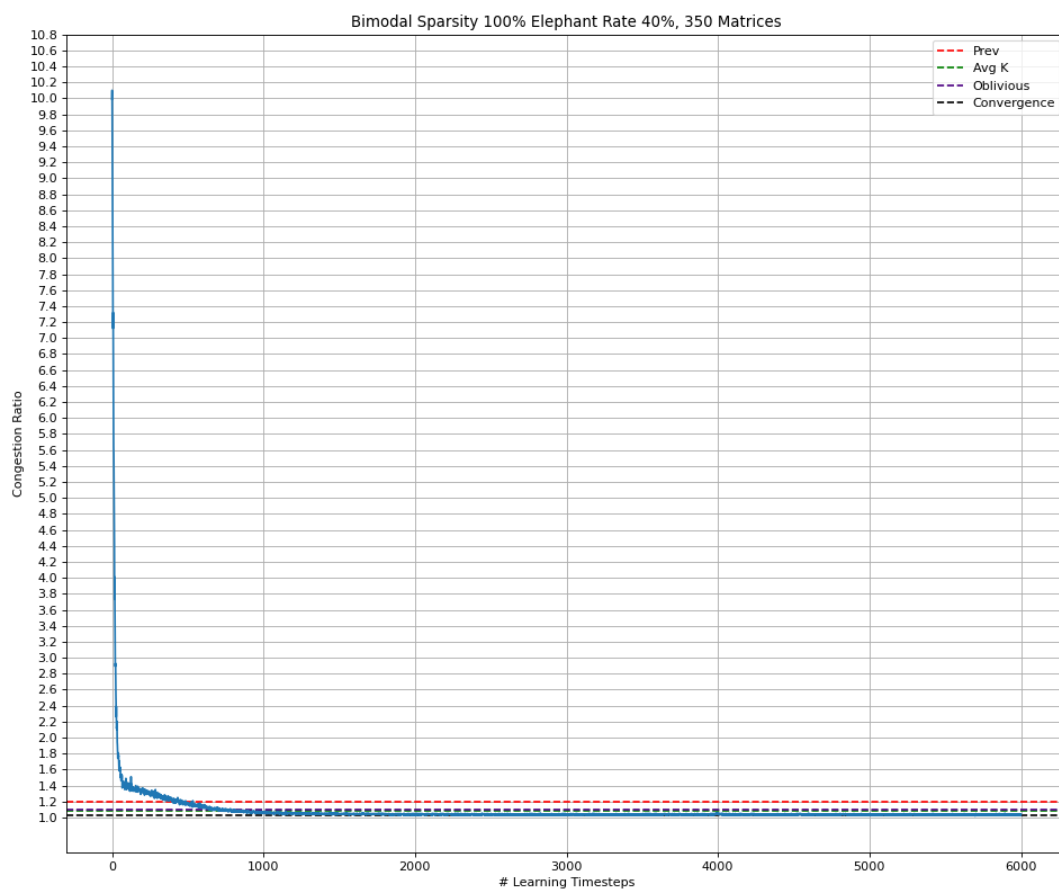


### Bimodal Traffic Paper results:

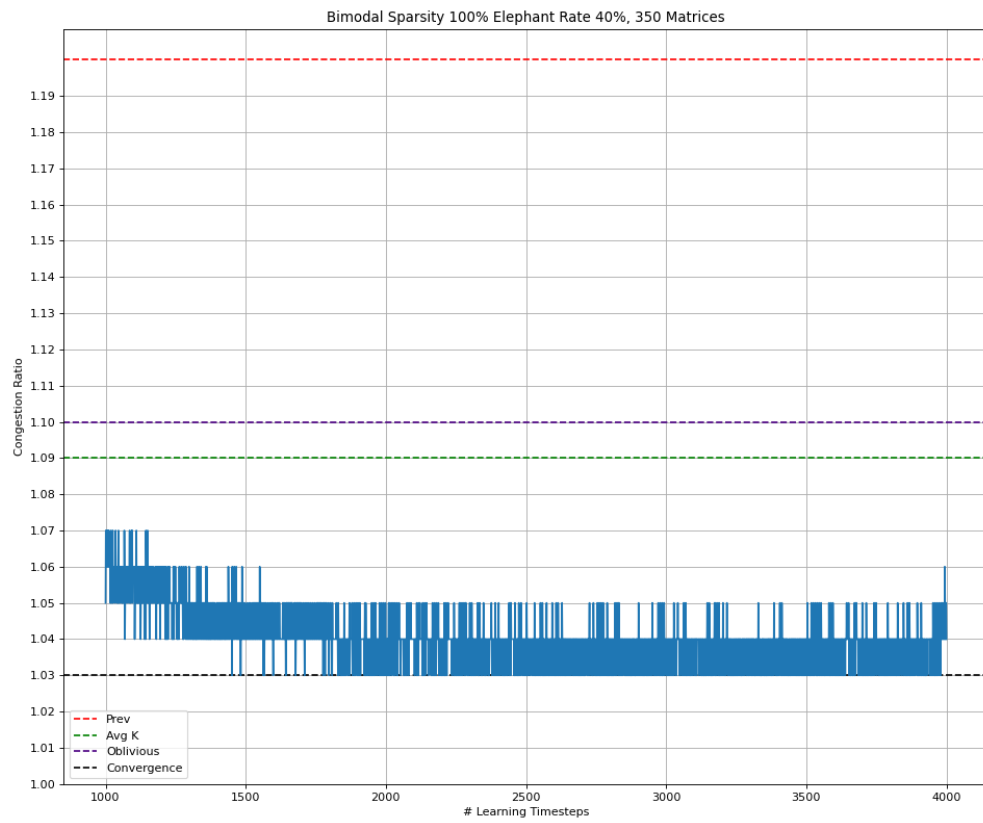


(b) Congestion ratio for non-sparse ( $p = 1.0$ ) bimodal DM sequences with 40% elephant flows

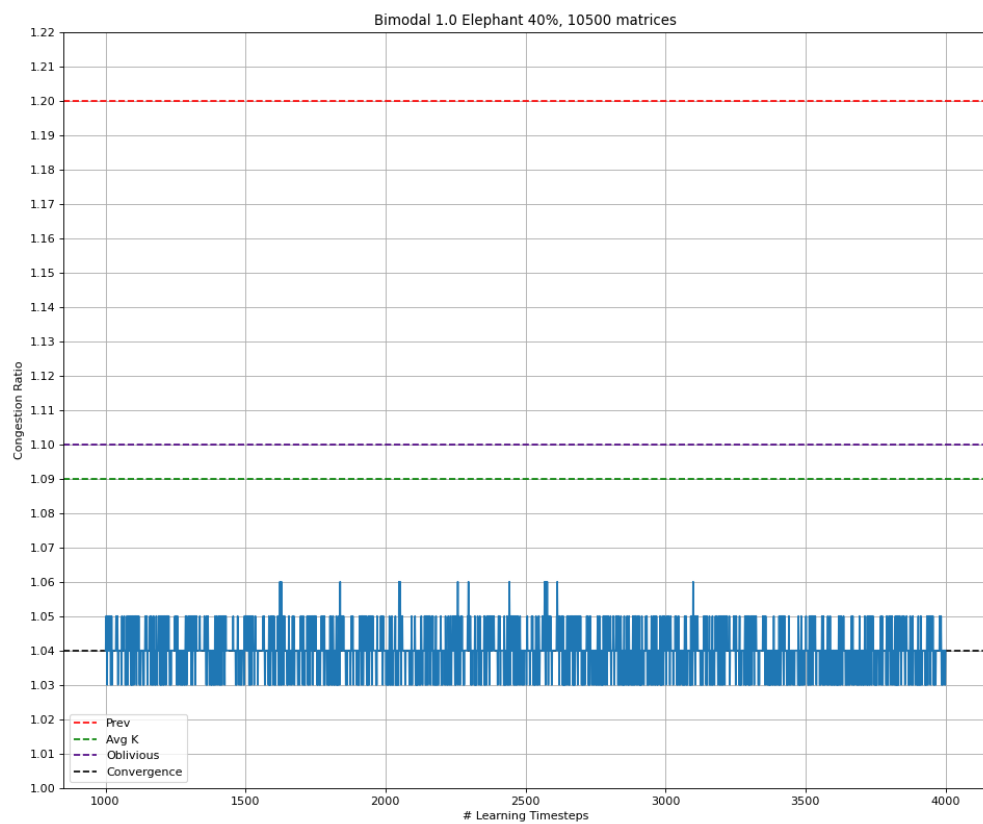
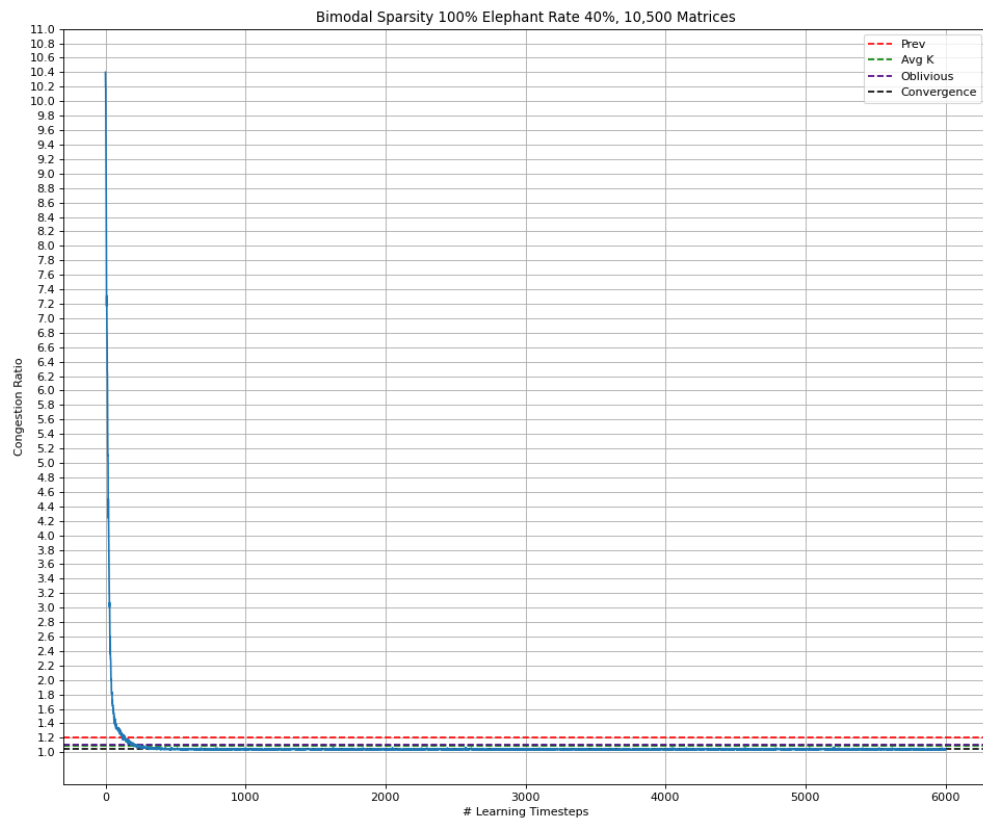
### Bimodal Traffic, Sparsity of 100% and 350 different TMs:







## Bimodal Traffic, Sparsity of 100% and 10,500 different TMs:



## Conclusions and Summary

First, I must say the results reproduction process helps me, as a young researcher, to understand much better the algorithms and methods of evaluation used in the paper that probably I am about to use in my own research.

Moreover, the practical experience with some of the Python packages for this task help me improve my machine learning programming skills, especially for reinforcement learning, for example, how to write a RL environment and how to configure an agent such as choosing the hyperparameters.

This reproduction process and its outputs help me to notice some conclusions that may be helpful for next research directions.

As seen in the baselines results, the oblivious routing scheme is becoming less effective when the sparsity of the matrix is higher. I think the reason for that is because with higher sparsity the average matrix includes more relevant flows that appears in the next routed traffic matrix, so this makes the average one a better approximation of the next one.

The first seen flows (as described earlier) have a big impact on the performance because they are routed by static ECMP with equal link weight, therefore, a more sophisticated way can be useful for allocating the weights, for example making each link weight be  $\frac{1}{Capacity}$ . The motivation behind it is to prefer paths with higher link

capacities Another approach is to route these flows by another routing algorithm such as oblivious routing or even consider an altogether different routing algorithm such as one that considers the bottleneck value i.e., the path with the minimum value of the max weight along it.

By inspection of the Bimodal traffic matrices evaluations specifically, the effectiveness of the percentage of elephant flows on the performance is low (the reason may be because the means of the Gaussians distributions for mice and elephant flows should be of different scales, like 100).

The reinforcement learning agent achieves results similar to those presented in [XXX] and it seems the convergence of the congestion ratio is faster; the reason is probably because the change in the learning algorithm from TRPO to PPO which is good as much as TRPO.

The same trends the paper shows can be seen also in the reproduced results, as follows:

For gravity traffic with low sparsity (30%) the oblivious scheme is still better than the agent-based scheme, but for the bimodal traffic with sparsity of 100% and for gravity traffic with sparsity of 90% the agent-based scheme beats all the baselines.

The reasons for better performance, namely lower congestion ratio, in those cases are for the bimodal is the Gaussian distributions' low standards deviation (the values of demands do not change too much) and for the gravity model is that for source destination pair of nodes traffic demand are constants (calculating using topology

static capacities). These two factors combining the high sparsity (most of the nodes demand traffic) make the history traffic matrices sequence better approximation of the next traffic matrix. Another evidence for that statement is that there is no change in the performance when number of different traffic matrices to be learned from is increasing.

To summarize, I enjoyed examining and reproducing the results from the paper, that helped me to start gain some knowledge and experience about the topics and challenges my research is about to handle with.

Also, I would like to thank Asaf Valadarsky (one of the paper authors) for helping me and sharing with me parts of his code.

## References

- [1] Valadarsky, A., Schapira, M., Shahaf, D., & Tamar, A. (2017, November). Learning to route. In Proceedings of the 16th ACM workshop on hot topics in networks (pp. 185-191).
- [2] Azar, Y., Cohen, E., Fiat, A., Kaplan, H., & Räcke, H. (2004). Optimal oblivious routing in polynomial time. *Journal of Computer and System Sciences*, 69(3), 383-394.
- [3] Applegate, D., & Cohen, E. (2006). Making routing robust to changing traffic demands: algorithms and evaluation. *IEEE/ACM Transactions on Networking*, 14(6), 1193-1206.
- [4] Roughan, Matthew, et al. "Experience in measuring internet backbone traffic variability: Models metrics, measurements and meaning." *Teletraffic Science and Engineering*. Vol. 5. Elsevier, 2003. 379-388.