BTC Equity Swaps

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Setup

There are two agents: the platform (denoted P), and the client (denoted C.) The client enters into a swap with the platform by which each day the client is paid daily returns off an ETF if the returns are positive, and the client pays daily returns if the returns are negative. Regardless of it the ETF rose or fell, the client pays a funding rate. All returns and the funding rate are applied to a constant Bitcoin notional and exchanged in Bitcoin.

Notation

Let S_t be the daily closing price of the underlying ETF. Let X_t be the daily closing exchange rate for BTC/USD (i.e. BTC is base currency.) Let N be the constant BTC notional value of the swap. And let f be the funding rate (fee.) We then say the daily percent returns on the underlying are

$$R_t = \frac{S_t - S_{t-1}}{S_{t-1}}$$

The matrix of daily cashflows in Bitcoin between P and C is

	P pays C daily	C pays P daily
if $R_t \geq 0$	NR_t	Nf
if $R_t < 0$	0	$N(f - R_t)$

Each of these can be multiplied by X_t to get the daily cashflows in USD.

Valuation and Hedging

We consider everything from the platform's point-of-view. P will be valuing and hedging the swap, aspects that we claim C is not concerned with. We also say that P uses USD as their accounting currency. The total cashflow in USD from the P's perspective is

$$V_P = \sum_{t=1}^{T} NX_t(f - R_t)$$

Note that this is when P is fully unhedged. There are two hedges that the platform makes, one for the underlying risk (spot delta), and one for the currency risk (fx delta.) For spot delta, P should hold n_t shares of the underlying upon the close of day t. We define

$$n_t = \frac{NX_t}{S_t}$$

The hedging prescription is for P to buy $n_t - n_{t-1} = N\left(\frac{X_t}{S_t} - \frac{X_{t-1}}{S_{t-1}}\right)$ shares of the underlying ETF at its closing price. The daily hedge PnL is

$$n_{t-1}(S_t - S_{t-1}) = NX_{t-1}R_t$$

Note that a day's return on the underlying is applied to the shares prescribed by yesterday's closing price. This is a mark-to-market PnL. The actual daily cashflows induced by the hedge are $(n_{t-1} - n_t)S_t$, i.e. either cash is spent by accumulating shares, or is generated by selling excess shares. We consider the mark-to-market valuation, which yields

$$\begin{aligned}
V_P &= \sum_{t=1}^{T} NX_t(f - R_t) + n_{t-1}(S_t - S_{t-1}) \\
&= N\sum_{t=1}^{T} fX_t + R_t(X_{t-1} - X_t) \\
&= N\sum_{t=1}^{T} fX_t - R_t\Delta X_t
\end{aligned}$$

By hedging against spot delta, we've changed our exposure to R_t from being proportional to the exchange rate to being proportional to the daily change in the exchange rate. This leaves us with fx delta. Note that when $R_t > f$, P is short bitcoin, and when $R_t < f$, P is long bitcoin.

We look to set f such that it sufficiently covers averse moves due to the cross risk. This can be most aptly performed by using a Monte-Carlo simulation to value the swap for a given funding rate. By revaluing for different levels of f, we can solve for the fee that produces a specified expected return. Please see the accompanying python code `swaps.py` which implements the MC valuation.