Hall Effect in a Semiconductor

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1 Abstract

In this experiment, the properties of a doped Germanium crystal were determined (University of California, Berkeley). The sample was determined to be a p-type semiconductor and the average extrinsic region was determined to be for temperatures less than 241.3 K ± 1.42 K. Furthermore, the carrier mobility, "μ" was determined to be 1.44 * 10^3 cm^2/(V*s) ± 1.12 * 10^2 cm^2/(V*s) and the average carrier concentration, "n", was determined to be 6.34 * 10^{23} m^{-3} ± 4.87 * 10^{21} m^{-3}. The average electron concentration in the extrinsic region was determined to be 6.34 * 10^{23} m^{-3} ± 4.87 * 10^{21} m^{-3}. The average electron mobility in the extrinsic region was determined to be 4.51 * 10^3 $cm^2/(V*s)$ ± 9.56*10^2 cm^2/(V*s) and the average hole mobility in the extrinsic region was determined to be 661.6 cm^2/(V*s) ± 91.0 cm^2/(V*s). Lastly, at room temperature, the average resistance for the sample was measured to be 5.50*10^2 Ω ± 36.5 Ω and the average magnetoresistance was determined to be 4.78*10^2 Ω ± 42.5 Ω.

2 Introduction

The purpose of this experiment is to determine characteristics of semiconductors such as by the density of carriers and carrier mobility utilizing the Hall Effect (1). The Hall Effect is utilized to measure the resistivity and Hall Coefficient of the sample. Additionally, information about "dopants" in a semiconductor can be determined, such as whether a doped semiconductor is a p-type or n-type semiconductor.

3 Background Research and Theory

3.1 Conduction in Metals, Semiconductors, and Insulators

Within one atom in a ground state, electrons are found in the lowest possible energy states and can travel to higher energy levels if additional energy is supplied (1). However, the structure of energy levels differs in solids, as atoms experience forces from other atoms, creating energy bands rather than distinct energy levels. Separating these energy bands are band gap energies. At 0 K in temperature, electrons in solids are found in energy states below the Fermi Energy, E_F, which is equal to k_BT, in which "k_B" is the Boltzmann constant. At temperatures of "T" that cause k_BT to be equal to E_F, electrons can travel to energy levels above E_F.

The structure of energy bands differ depending on if a solid is a metal, semiconductor, or an insulator. Diagrams of energy bands of metals, semiconductors, and insulators are shown in Figure 1 (1). Conductors allow current to travel due to the Fermi energy being in an occupied energy band and electrons realigning to pass current in the presence of electric fields. Additionally, the valence band is not completely filled.

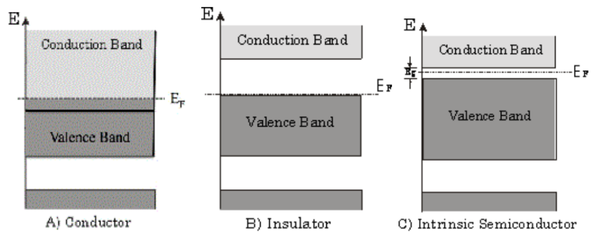

Figure 1: Figure 10 from the Laboratory Manual for this experiment (1). This figure shows the different arrangement of energy bands in conductors, insulators, and semiconductors.

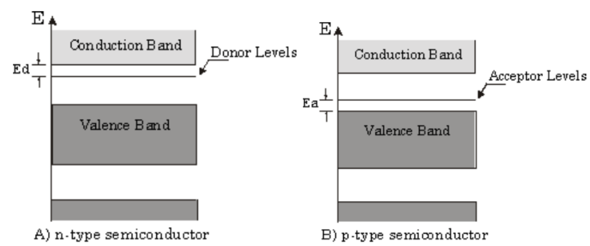

Figure 2: Figure 11 from the Laboratory Manual for this experiment (1). This figure shows the orientation of donor levels and acceptor levels in n-type and p-type semiconductors, respectively.

In insulators, a large energy gap is between the conduction band and the filled valence band (1). Thus, current does not flow because electrons can not align to allow current to flow in the presence of an electric field.

Although semiconductors are considered insulators, semiconductors have a smaller energy gap than insulators. This energy gap is proportional to k_BT (1). This small energy gap causes holes to appear in the conduction bands and electrons to appear in the conduction band depending on temperature.

A value that describes the effect of energy gaps on conductivity is resistivity, ρ, which is described in Equation 1 and has SI units of $\Omega * m$ (1). In Equation 1, "E" is the electric field and "J" is the current density. For metals, resistivity increases as temperature increases, while resistivity decreases as temperature increases for semiconductors.

$$\rho = E/J \tag{1}$$

Furthermore, semiconductors can be "doped" to decrease resistivity and are then considered to be "extrinsic". Figure 2 highlights the differences between "n-type" and "p-type" semiconductors (1). In n-type semiconductors, electrons from other elements are added and can travel to the conduction band through excitation. In p-type semiconductors, holes from other elements are added and electrons in the conduction band can become excited and fill the holes in the acceptor levels below the conduction band.

3.2 The Hall Effect

As part of this experiment in measuring carrier density and mobility, the Hall Effect is implemented, as shown in Figure 3 (1). For the Hall Effect to take place, a magnetic field in the z-direction is generated and a current through the x-direction is generated. Thus, carriers experience the Lorentz Force, which is shown in Equation 2.

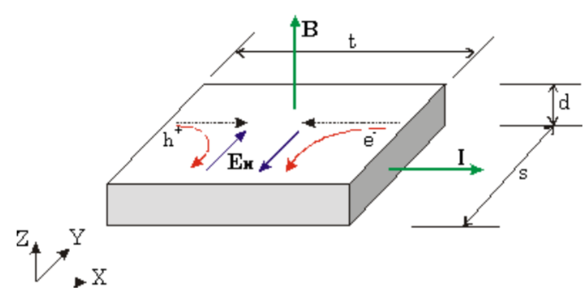

Figure 3: Figure 12 from the Laboratory Manual for this experiment (1). This figure shows the conditions for the Hall Effect to take place.

$$\vec{F} = q(\vec{v} \times \vec{B}) \tag{2}$$

In this experiment, the Lorentz Force moves carriers on the y-axis, with the charge of the carrier determining which direction on the y-axis the Lorentz force moves the carriers. As positive and negative carrier collects on opposite ends of the y-axis, an electric field is created that counteracts the Lorentz Force and facilitates current to continue moving in the same direction (1). The electric field that is created and voltage that develops from the electric field, which is called the Hall Voltage, are given in Equations 3 and 4, respectively.

$$E_H = v_x B_z \tag{3}$$

$$V_H = sE_H \tag{4}$$

Furthermore, V_H is positive for p-type semiconductors and negative for n-type semiconductors. Also, R_H, a factor of V_H that is called the Hall coefficient, is positive for p-type semiconductors and negative for n-type semiconductors (1). The equations for V_H and R_H for p-type semiconductors are given in Equations 5 and 6. The equations for V_H and R_H for n-type semiconductors are given in Equations 7 and 8.

$$V_H = -\frac{I_x B_z}{end} \tag{5}$$

$$R_H = \frac{E_H}{J_x B_z} = \frac{1}{en} \tag{6}$$

$$V_H = \frac{I_x B_z}{epd} \tag{7}$$

$$R_H = -\frac{1}{ep} \tag{8}$$

3.3 The Van Der Pauw Technique

In the Van Der Pauw Technique, current and voltage measurements of four points of a semiconductor can be used to retrieve the Hall Voltage and resistivity of a semiconductor (1). Four arrangements of current and voltage measurements are measured for one direction of current and are measured again for another direction of current, as shown in Figures 4 and 5.

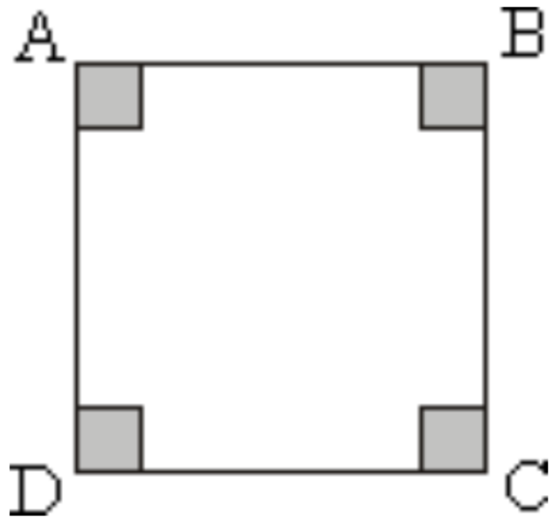

Figure 4: Figure 13 from the Laboratory Manual for this experiment (1). This figure shows the layout for the four points of the semiconductor in which current and voltage are measured.

Current	Voltage	Trans-resistance
I_{BA}	V_{DC}	$R_{BA,DC} = V_{DC}/I_{BA}$
I_{DA}	V_{BC}	$R_{DA,BC} = V_{BC}/I_{DA}$
I_{CA}	V_{DB}	$R_{CA,DB} = V_{DB}/I_{CA}$
I_{DB}	V_{AC}	$R_{DB,AC} = V_{AC}/I_{DB}$

Figure 5: A table from the Laboratory Manual for this experiment (1). This figure shows the voltage and current measurements that are measured in pairs as well as the trans-resistance value that is recorded from the current and voltage measurements.

To measure the Hall Voltage, the average trans-resistance values of $R_{CA,DB}$ and $R_{DB,AC}$ are used in Equation 9 (1). Additionally, $R_{BA,DC}$ and $R_{DA,BC}$ are used in the measurement of resistivity, which is described in Equations 10 and 11.

$$R_H = \left(\frac{R_{CA,DB} + R_{DB,AC}}{2}\right) * \left(\frac{d}{B_z}\right) \tag{9}$$

$$\rho = \frac{\pi d}{ln(2)} * \frac{R_{BA,DC} + R_{DA,BC}}{2} * f(\frac{R_{BA,DC}}{R_{DA,BC}}) \quad (10)$$

$$f(x) = \frac{1}{cosh(\frac{ln(x)}{2.403})} \quad (11)$$

4 Methods

To measure the effects of temperature on Hall Voltage and resistivity on semiconductors, a temperature range of 95 K to 350 K is used (1). Liquid nitrogen is used to decrease the temperature of the signal to 95 K and the heater heats the sample until it reaches 350 K. At various temperatures throughout the experiment, a LabVIEW program measures the voltage and current values at approximately every degree of temperature change for positive, zero, and negative magnetic field. The data that is output at every recorded temperature point is given in Figure 6.

Plus Gauss Field

Sample Current Plus (+)		
Data point	I	V
1	AB	CD
2	AD	CB
3	AC	BD
4	BD	CA

Sample Current Negative (-)		
Data point	I	V
5	-AB	-CD
6	-AD	-CB
7	-AC	-BD
8	-BD	-CA

Zero (0) Gauss Field

Sample Current Plus (+)		
Data point	I	V
1	AB	CD
2	AD	CB
3	AC	BD
4	BD	CA

Sample Current Negative (-)		
Data point	I	V
5	-AB	-CD
6	-AD	-CB
7	-AC	-BD
8	-BD	-CA

Negative Gauss Field

Sample Current Plus (+)		
Data point	I	V
1	AB	CD
2	AD	CB
3	AC	BD
4	BD	CA

Sample Current Negative (-)		
Data point	I	V
5	-AB	-CD
6	-AD	-CB
7	-AC	-BD
8	-BD	-CA

Figure 6: A table from the Laboratory Manual for this experiment that provides an example of the data that is recorded at every temperature point by the LabVIEW program (1).

5 data runs were taken at the current values of 5 μA, 10 μA, 15 μA, 20 μA, and 25 μA. The "target" magnetic field used in the LabVIEW Program was 3,000 G. An example of the LabVIEW program is provided in Figure 7.

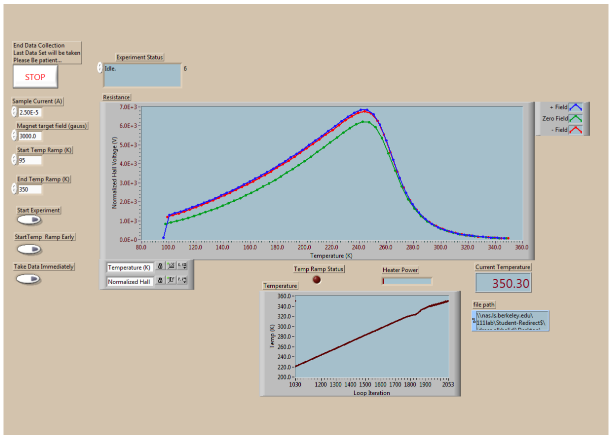

Figure 7: An example of the LabVIEW program after one data run is finished.

5 Experimental Design

In this experiment, a 10mm x 10 mm square of doped Germanium crystal that is 1.25 mm wide was placed inside of a cryostat (1). The design of the cryostat is provided in Figure 8. The semiconductor is connected to a slab of copper which holds a temperature detector on one of its sides. Four wires are connected to the temperature detector to measure current and voltage. Additionally, the slab of copper is connected to slabs of brass and copper that hold a heating coil which controls the rate of temperature change. Part of the cryostat contains a vacuum to prevent oxidation of the Germanium crystal and another part of the cryostat contains liquid nitrogen to adjust the temperature of the cryostat. Lastly, an electromagnet is used to provide the magnetic field and is cooled by water-cooling pipes.

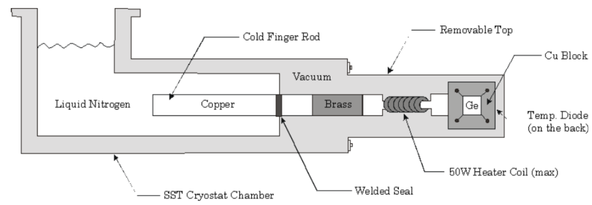

Figure 8: Figure 16 from the Laboratory Manual for this experiment which illustrates the design of the cryostat which contains the Germanium crystal (1).

An important component of this experiment is being safe when using liquid nitrogen. For example, cryogenic gloves, safety glasses, and a clear face shield are used when pouring liquid nitrogen.

6 Raw Data

The voltage readings for the data files that are created by the LabVIEW program were matched correspondingly to the voltage values that were provided in Figure 6 of this report, due to the voltage readings being labeled in a different order than given in Figure 6. The data for the five data runs are provided in Figures 9 through 13. Additionally, the voltage for each temperature for zero magnetic field was taken as the "zero" point for every voltage reading due to there being a voltage offset.

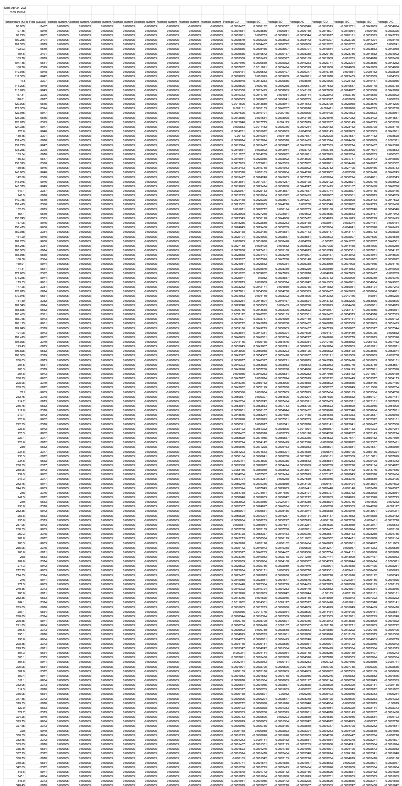

Figure 9: The data from the first data run, in which the current is 5 μA.

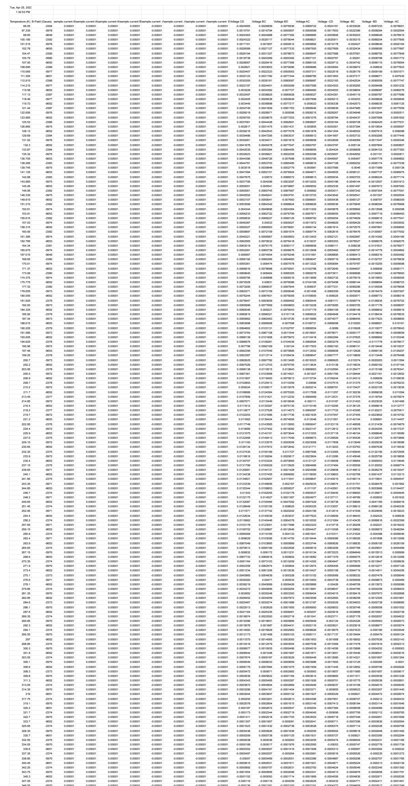

Figure 10: The data from the second data run, in which the current is 10 μA.

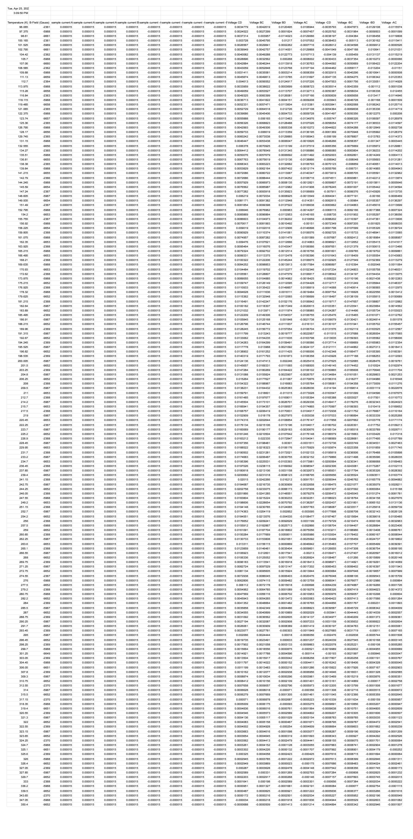

Figure 11: The data from the third data run, in which the current is 15 μA.

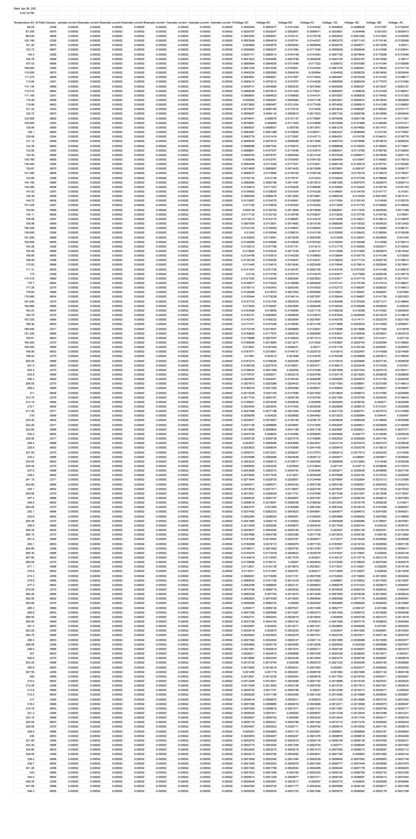

Figure 12: The data from the fourth data run, in which the current is 20 μA.

Figure 13: The data from the fifth data run, in which the current is 25 μA.

7 Data Analysis

Using Equations 10 and 11, the resistivity of the sample in each data run at room temperature was calculated. The value of resistivity calculated using each data run are shown in Tables 1 and 2.

Current	ρ (Ω*m)
5 μA	0.526
10 μA	0.647
15 μA	0.734
20 μA	0.763
25 μA	0.766

Table 1:

$<\rho>$ (Ω*m)	σ_ρ (Ω*m)
0.687	0.0457

Table 2:

Furthermore, the Hall Coefficient of the sample in each run at room temperature was measured and is shown in Tables 3 and 4.

Current	R_H (Ω*m/T)
5 μA	-0.0994
10 μA	-0.0957
15 μA	-0.0990
20 μA	-0.0999
25 μA	-0.0992

Table 3:

R_H (Ω*m/T)	σ_{R_H} (Ω*m/T)
-0.0986	0.000750

Table 4:

Additionally, the magnetic field magnitude that causes the electric field from the Hall Effect, given by Equation 12, to be the same magnitude as the electric field from resistance, given by Equation 13, can be found by setting Equations 12 and 13 equal to each other and solving for B_z, resulting in Equation 14 (1).

$$E_H = \frac{V_H}{s} = \frac{R_H I_x B_z}{sd} \tag{12}$$

$$E_R = \frac{V_R}{d} = I(\frac{\rho l}{A}) * \frac{1}{d} \tag{13}$$

$$B_z = \frac{\rho s}{dR_H} \tag{14}$$

The values of B_z found from the data at room temperature are expressed in Tables 5 and 6.

Current	B_z
5 μA	-42.4 T
10 μA	-54.1 T
15 μA	-59.3 T
20 μA	-61.1 T
25 μA	-61.8 T

Table 5:

B_z	σ_{B_z}
-55.7 T	3.60 T

Table 6:

Furthermore, a plot of resistivity versus inverse temperature is shown below for the data runs in Figures 14 through 16.

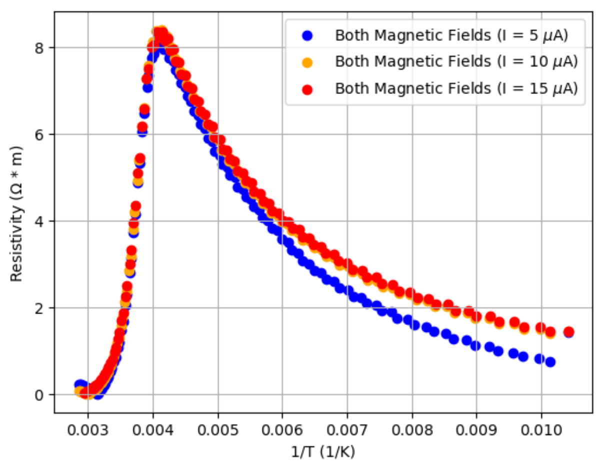

Figure 14: A graph of ρ versus 1/T for 5 μA, 10 μA, and 15 μA.

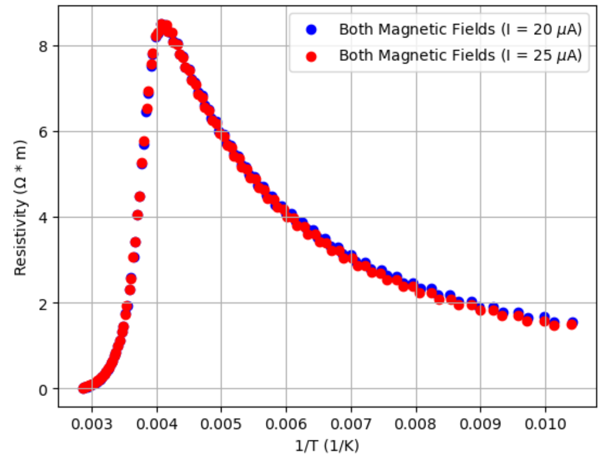

Figure 15: A graph of ρ versus 1/T for 20 μA and 25 μA.

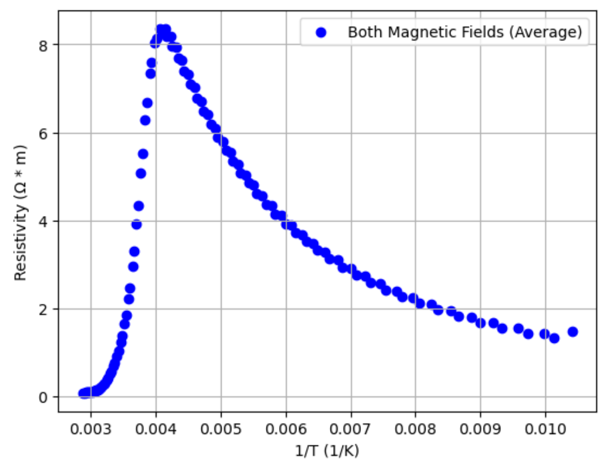

Figure 16: A graph of the average value of ρ versus 1/T for the 5 data runs.

Furthermore, the extrinsic region for each graph can be found from the region of the graphs in Figures 14 through 16 which have negative slope (Melissinos). The extrinsic regions for the data runs are given in Tables 7 and 8.

Current	Extrinsic Region
5 μA	T $<$ 241. 3 K
10 μA	T $<$ 241.85 K
15 μA	T $<$ 241.15 K
20 μA	T $<$ 245.85 K
25 μA	T $<$ 245.7 K

Table 7:

$< ExtrinsicRegion >$	$\sigma_{ExtrinsicRegion}$
T $<$ 241.3 K	1.42 K

Table 8:

Additionally, plots for the conductivity σ, which is $1/\rho$, of the data runs are given in Figures 17 through 19.

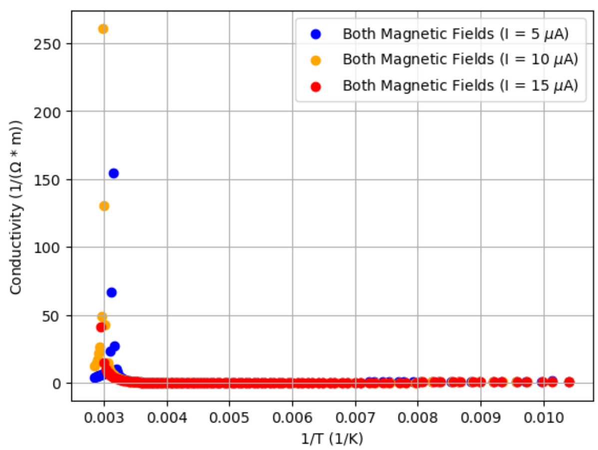

Figure 17: A graph of of σ versus 1/T for 5 μA, 10 μA, and 15 μA.

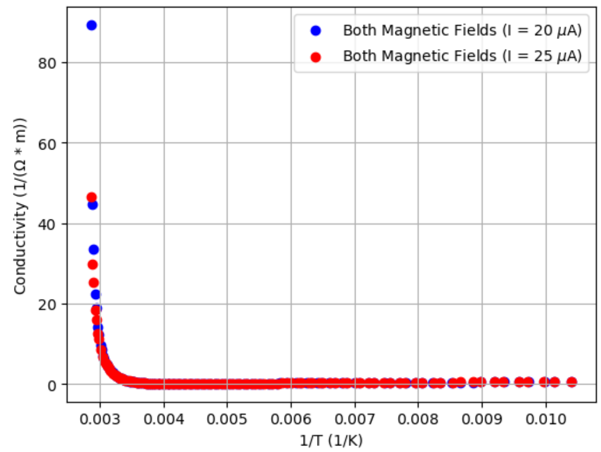

Figure 18: A graph of of σ versus 1/T for 20 μA and 25 μA.

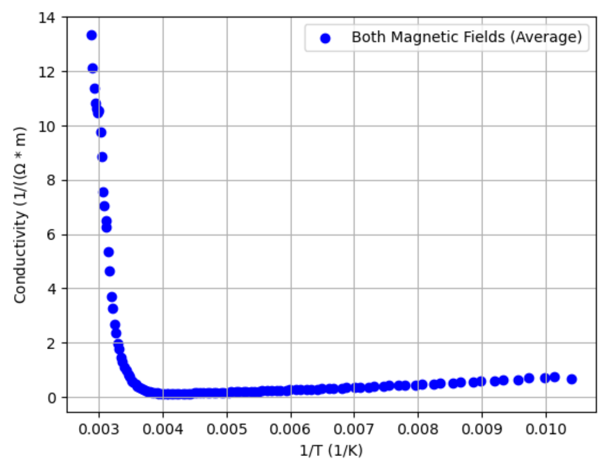

Figure 19: A graph of of the average value of σ versus 1/T.

Also, plots for R_H are given in Figures 20 through 22.

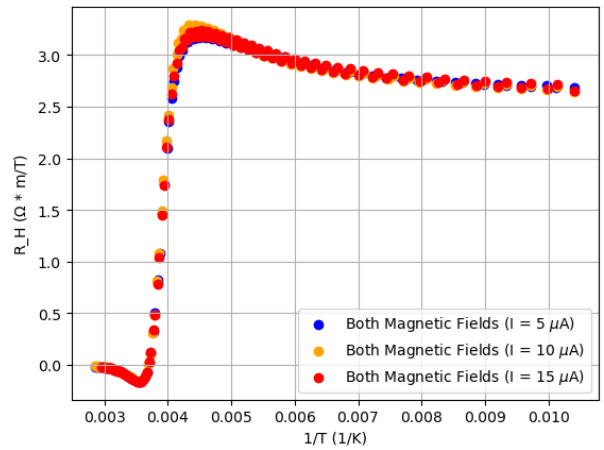

Figure 20: A graph of of R_H versus 1/T for 5 μA, 10 μA, and 15 μA.

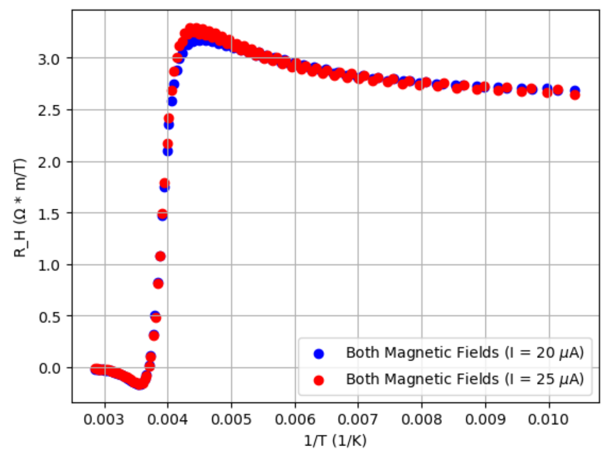

Figure 21: A graph of of R_H versus 1/T for 20 μA and 25 μA.

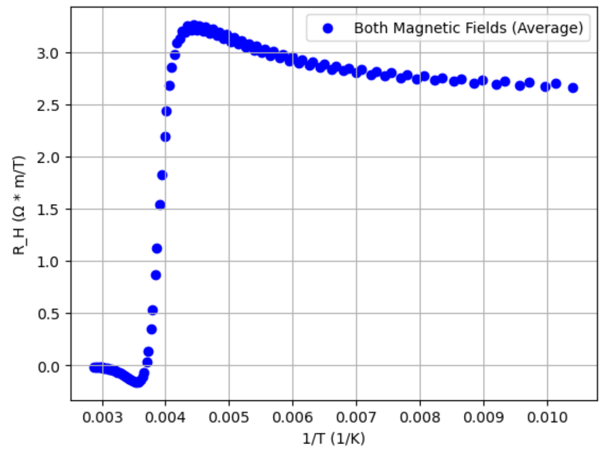

Figure 22: A graph of of the average value of R_H versus 1/T.

Also, at room temperature, the carrier mobility "μ" and the carrier concentration, "n", can be found through Equations 15 and 16, respectively (2). The values calculated for the carrier mobility and concentration are given in Tables 9 through 12.

$$\mu = |R_H|\sigma = \frac{R_H}{\rho} \tag{15}$$

$$n = \frac{1}{R_H e} \tag{16}$$

Current	μ (cm^2/(V*s))
5 μA	1.89 * 10^3
10 μA	1.48 * 10^3
15 μA	1.35 * 10^3
20 μA	1.31 * 10^3
25 μA	1.30 * 10^3

Table 9:

μ (cm^2/(V*s))	σ_μ (cm^2/(V*s))
1.44 * 10^3	1.12 * 10^2

Table 10:

Current	n (m^{-3}))
5 μA	6.29 * 10^{23}
10 μA	6.53 * 10^{23}
15 μA	6.31 * 10^{23}
20 μA	6.26 * 10^{23}
25 μA	6.30 * 10^{23}

Table 11:

$< n >$ (m^{-3})	σ_n (m^{-3})
6.34 * 10^{23}	4.87 * 10^{21}

Table 12:

Additionally, the Hall mobility, which is provided by Equation 17, is graphed for the data runs in Figures 23 through 25. Additionally, the temperature where the Hall Coefficient becomes zero, the inversion temperature T_0, is given in Tables 13 and 14 (1).

$$\mu_H = R_H \sigma \tag{17}$$

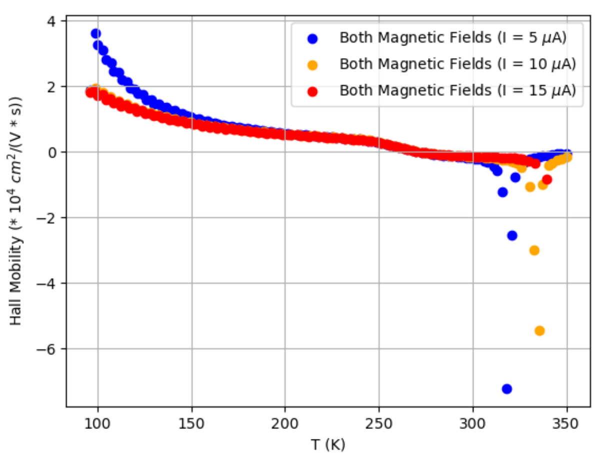

Figure 23: A graph of of μ_H versus T for 5 μA, 10 μA, and 15 μA.

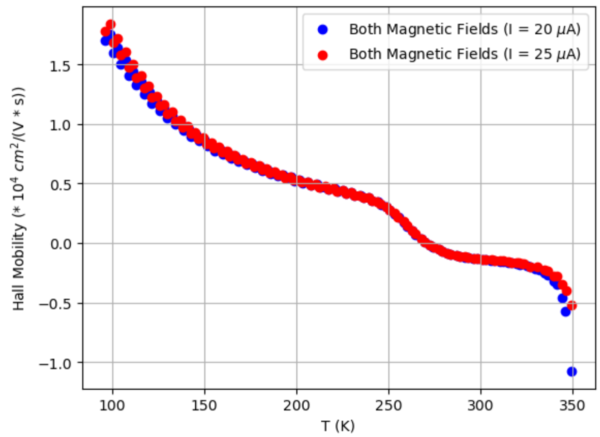

Figure 24: A graph of of μ_H versus T for 20 μA and 25 μA.

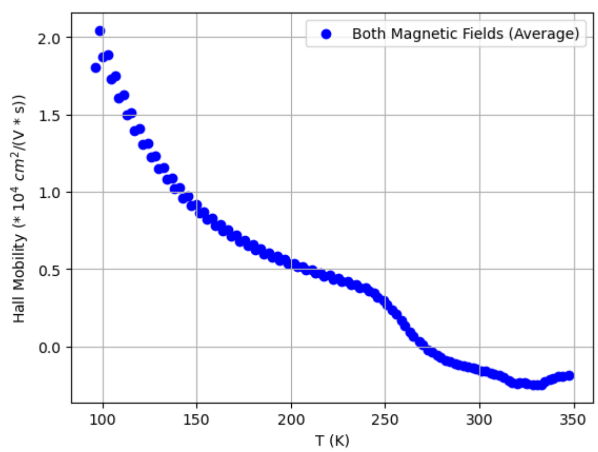

Figure 25: A graph of of the average value of μ_H versus T.

Current	T_0
5 μA	269.8 K
10 μA	270.35 K
15 μA	269.75 K
20 μA	269.55 K
25 μA	269.55 K

Table 13:

$< T_0 >$	σ_{T_0}
269.8 K	0.147 K

Table 14:

Because the Hall Coefficient changes signs, the Germanium sample used in this experiment is concluded to be a p-type semiconductor, as Equation 18 highlights that because μ_e is often greater than μ_h, R_H is only negative if the density of holes, "p", is greater than the density of electrons, "e" (2)

$$R_H = \frac{\mu_h^2 p - \mu_e^2 n}{e(\mu_h p + \mu_e n)^2} \tag{18}$$

Furthermore, the electron and hole concentrations as functions of temperature are given by Equations 19 and 20 (2).

$$n = (\frac{2\pi m_e k_B T}{h^2})^{3/2} e^{\frac{E_F}{k_B T}} \tag{19}$$

$$p = (\frac{2\pi m_h k_B T}{h^2})^{3/2} e^{\frac{-(E_g+E_F)}{k_B T}} \tag{20}$$

Because μ_H is represented by Equation 21, the value of C can be found by fitting the data for this experiment and is shown in Tables 15 and 16 (2). The units of "C" are $Tesla^-1 * Kelvin^{3/2}$.

$$\mu_H = CT^{-3/2} \tag{21}$$

Current	C ($T^{-1}K^{3/2}$)
5 μA	466.1
10 μA	334.6
15 μA	280.7
20 μA	270.6
25 μA	274.9

Table 15:

$< C >$ ($T^{-1}K^{3/2}$)	σ_C ($T^{-1}K^{3/2}$)
239.2	56.8

Table 16:

Also, because $\mu_H = R_H\sigma$ and μ_H = C$T^{-3/2}$, R_H can be represented by Equation 22.

$$R_H = \frac{CT^{-3/2}}{\sigma} \tag{22}$$

Because there is one type of carrier in the extrinsic region, the order of magnitude for the density of impurity carriers can be calculated by using Equation 6. For example, using the average value of R_H from the first temperature point recorded, the density is equal to 2.35 * $10^{22} m^{-3}$ (2).

Furthermore, by substituting the inversion temperature, T_0, into Equation 21, the mobility of holes in the extrinsic region, μ_H(h) can be calculated and is shown given in Tables 17 and 18.

Current	μ_h (V*s/cm^2)
5 μA	1.052 * 10^3
10 μA	752.7
15 μA	633.6
20 μA	611.5
25 μA	621.8

Table 17:

$< \mu_h >$ (V*s/cm^2)	σ_{μ_h} (V*s/cm^2)
661.6	91.0

Table 18:

Furthermore, the ratio between mobilities is given in Equation 23 (2). R_e(T = T_0) is the resistance of the sample at the inversion point using a linear, fitted function of the resistivity function in the extrinsic region and R_0 is the resistance of the sample at T_0. The fitted function of resistivity is given in Table 19. The values of the ratio between mobilities is given in Tables 20 and 21 and the value of μ_e that is calculated from Equation 23 is given in Tables 22 and 23.

$$b = \frac{\mu_e}{\mu_h} = \frac{R_e(T = T_0}{R_e(T = T_0) - R_0} \tag{23}$$

Current	Fitted Function of ρ
5 μA	ρ = (-350.0 Ω*m*K)T^{-1} + (5.52 Ω*m)
10 μA	ρ = (-341.8 Ω*m*K)T^{-1} + (5.91 Ω*m)
15 μA	ρ = (-339.3 Ω*m*K)T^{-1} + (5.94 Ω*m)
20 μA	ρ = (-339.1 Ω*m*K)T^{-1} + (6.01 Ω*m)
25 μA	ρ = (-340.2 Ω*m*K)T^{-1} + (5.96 Ω*m)
Average	ρ = (-342.1 Ω*m*K)T^{-1} + (5.87 Ω*m)

Table 19:

Current	b
5 μA	8.67
10 μA	5.39
15 μA	6.45
20 μA	6.79
25 μA	6.90

Table 20:

$< b >$	σ_b
6.81	0.53

Table 21:

Current	μ_e (cm^2/(V*s))
5 μA	8.72 * 10^3
10 μA	4.06 * 10^3
15 μA	4.09 * 10^3
20 μA	4.15 * 10^3
25 μA	4.29 * 10^3

Table 22:

$< \mu_e >$ (cm^2/(V*s))	σ_{μ_e} (cm^2/(V*s))
4.51 * 10^3	9.56 * 10^2

Table 23:

Also, at room temperature, the resistance for the sample at zero magnetic field and the magnetoresistance, which is the resistance measured in the presence of a magnetic field, are given in Tables 24 through 27 (1)

Current	$R(B \neq 0)(\Omega)$
5 μA	4.21 * 10^2
10 μA	5.18 * 10^2
15 μA	5.87 * 10^2
20 μA	6.10 * 10^2
25 μA	6.13 * 10^2

Table 24:

$< R(B \neq 0) > (\Omega)$	$\sigma_R(B \neq 0)$ (Ω)
5.50 * 10^2	36.5

Table 25:

Current	R(B=0)(Ω)
5 μA	3.24 * 10^2
10 μA	4.21 * 10^2
15 μA	5.16 * 10^2
20 μA	5.39 * 10^2
25 μA	5.41 * 10^2

Table 26:

$< R(B=0) >$ (Ω)	$\sigma_{R(B=0)}$ (Ω)
$4.78 * 10^2$	42.5

Table 27:

8 Bibliography

(1) Berkeley, University of California. "SHE - Hall Effect in a Semiconductor". http://experimentationlab.berkeley.edu/sites/default/ /writeups/SHE.pdf.

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