



# Hands-on Introduction to Deep Learning

Graphs are everywhere

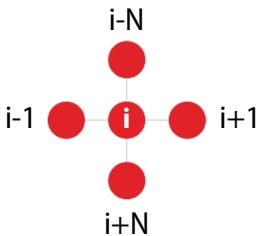
## Highly ordered data

Rebirth of Deep learning was thanks to pictures, text and speech recognition



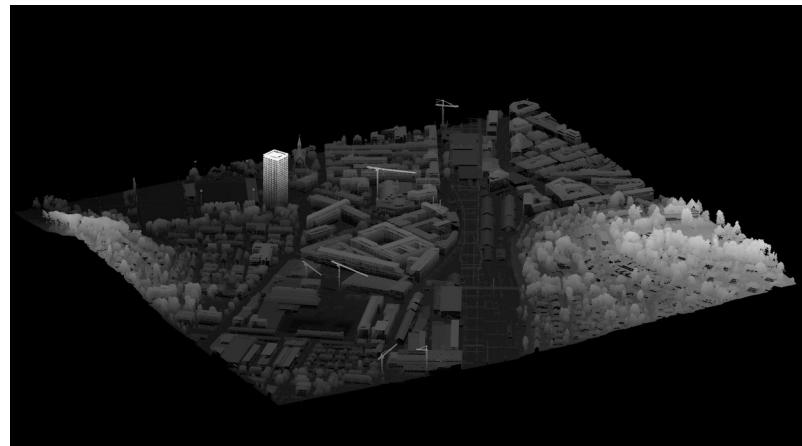
The answer to life, the universe and everything is ...

Neighborhood:

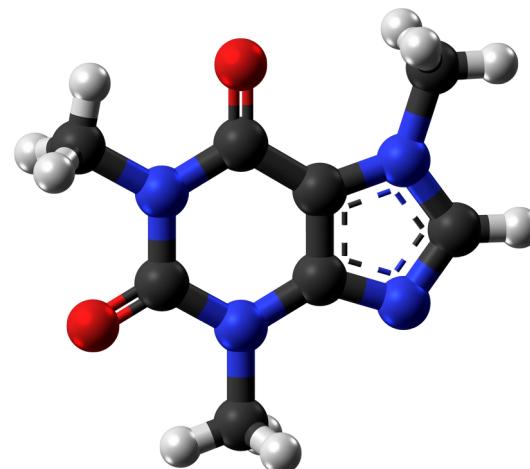


# Data structures: Data is not always euclidean

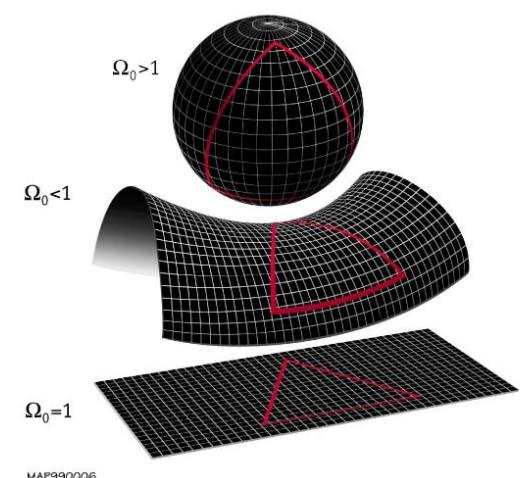
LIDAR



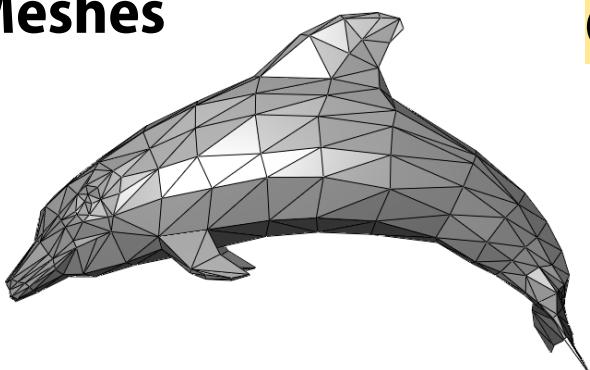
Molecules



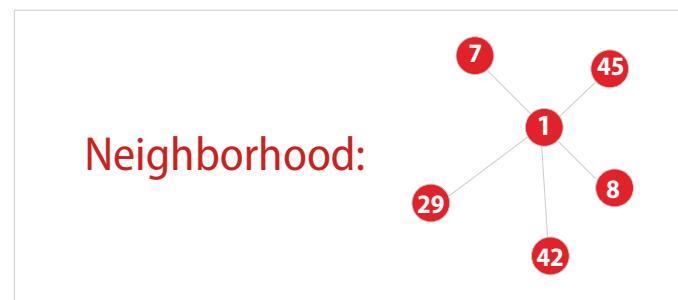
Complex geometries



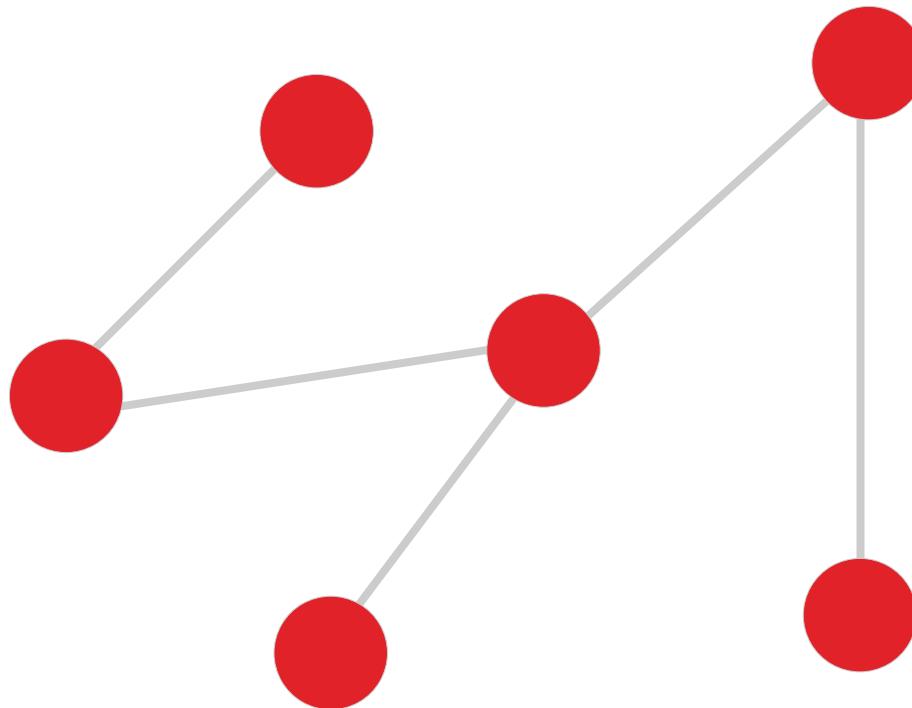
Meshes



Geometric deep learning



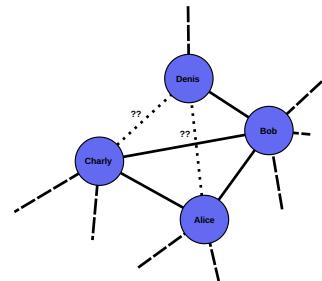
# Graphs are everywhere



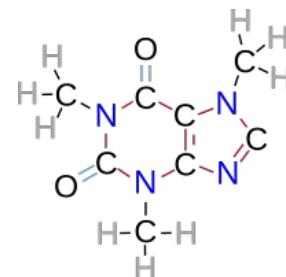
Data as a set of interconnected entities

# Graphs are everywhere

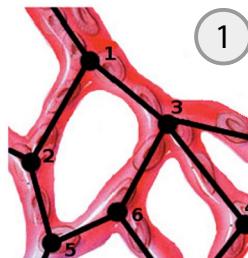
Social networks



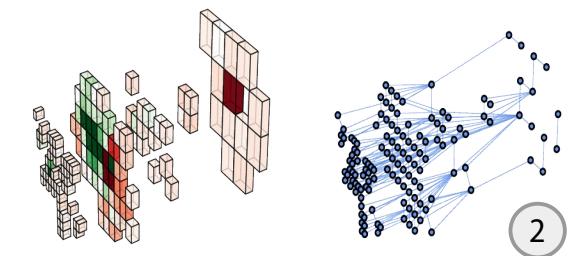
Molecules



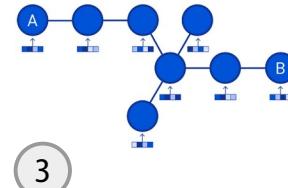
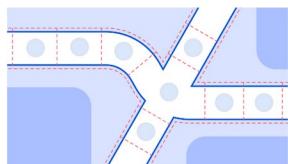
Capillary networks



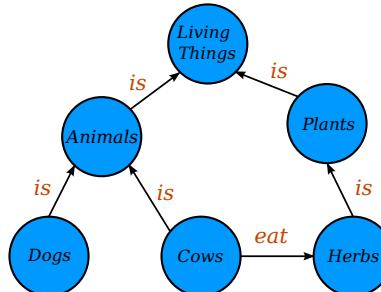
Particle physics



Directions recommendation



Knowledge graphs



Many other fields

- Biology
- Recommendation systems
- Computer vision
- Medical diagnosis
- Robotics
- ...

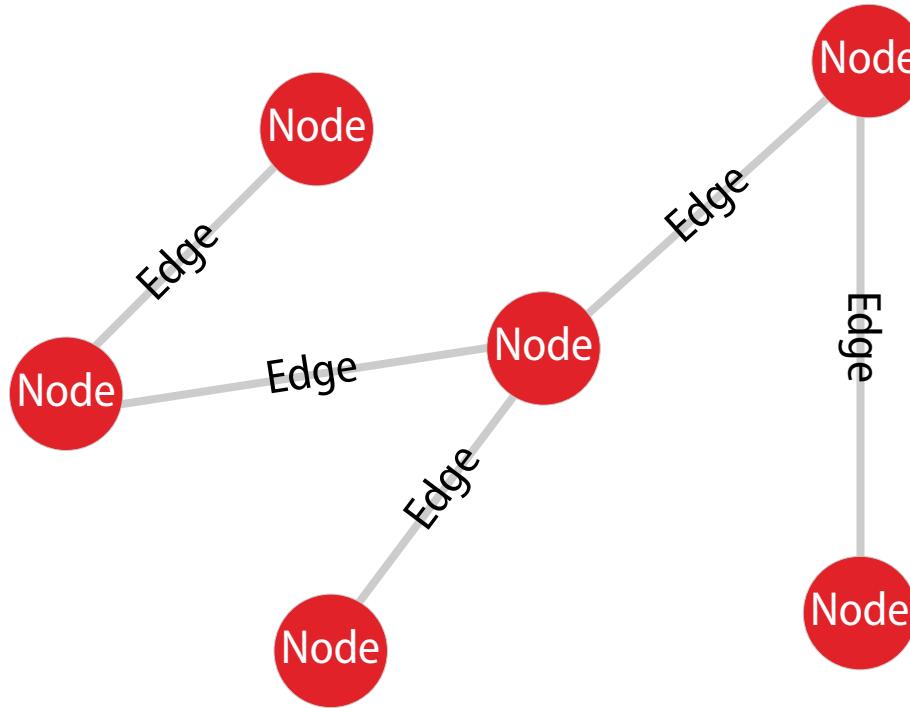
[1] Erbertseder, K., Reichold, J., Flemisch, B., Jenny, P., & Helmig, R. (2012). A coupled discrete/continuum model for describing cancer-therapeutic transport in the lung. *PLoS One*, 7(3), e31966.

[2] J. Shlomi, P. Battaglia, and J.-R. Vlimant, "Graph neural networks in particle physics," *Mach. Learn.: Sci. Technol.*, vol. 2, no. 2, p. 021001, Jan. 2021, doi: 10.1088/2632-2153/abbf9a.

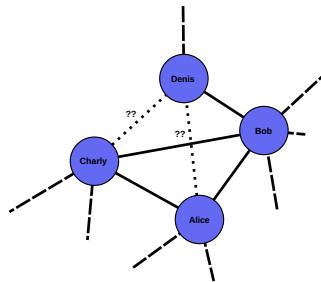
[3] A. Derrow-Pinion et al., "ETA Prediction with Graph Neural Networks in Google Maps," in *Proceedings of the 30th ACM International Conference on Information & Knowledge Management* New York, NY, USA, Oct. 2021, pp. 3767–3776. doi: 10.1145/3459637.3481916.

# Vocabulary

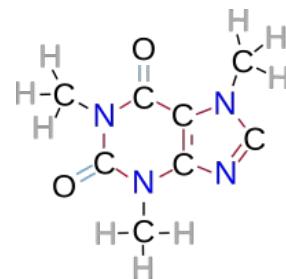
## Graph



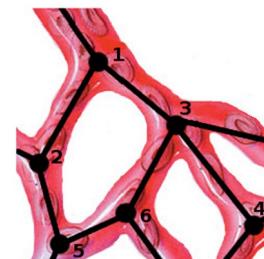
Persons



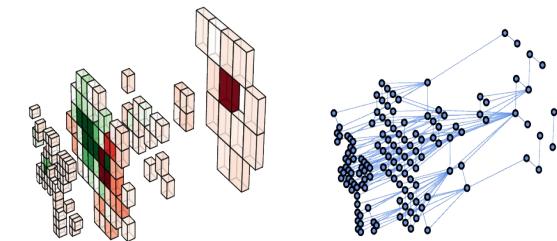
Atoms



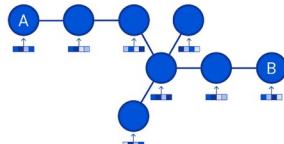
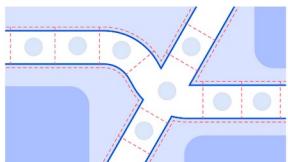
Intersections



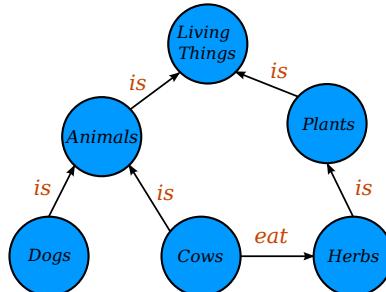
Particles



Road sections



Concepts

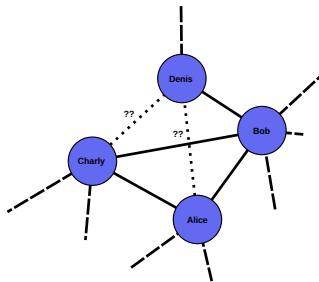


Many other fields

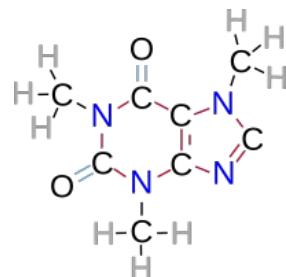
- Biology: An aminoacid in a protein
- Recommendation systems: A customer
- Computer vision: An object in a picture
- Medical diagnosis: Brain region (MRI)
- Robotics: Joints
- ...

# Some example of edges

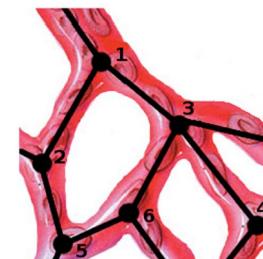
Relationship



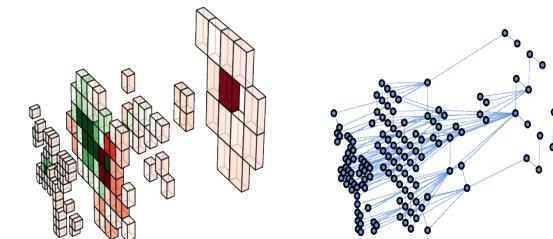
Type of bond



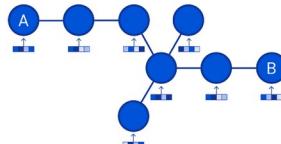
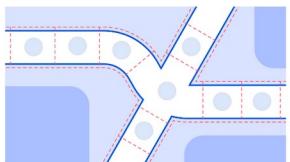
Vessel



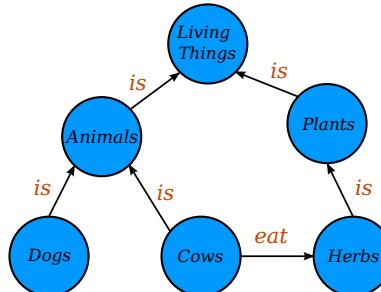
Decayed to



Time



Statement

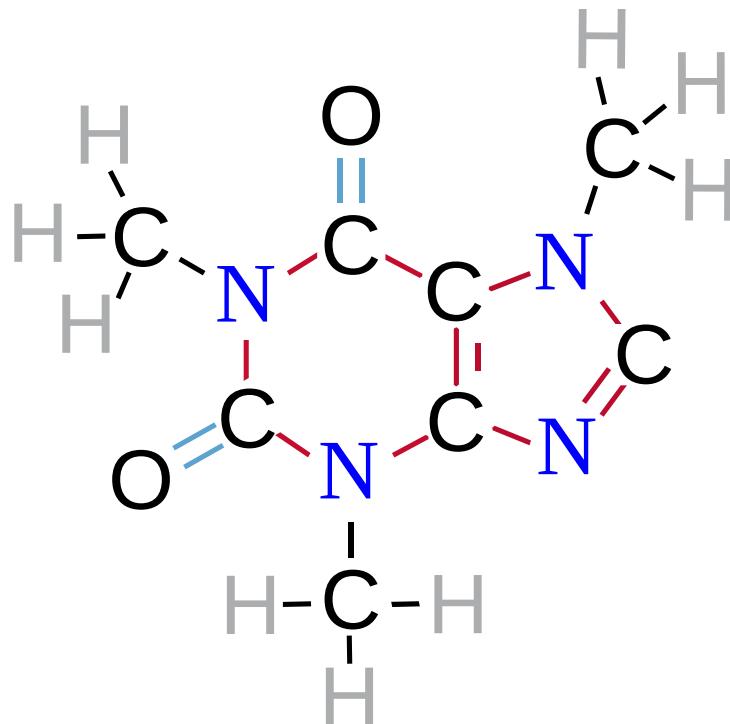


Many other fields

- Biology: Distance between residues
- Recommendation systems: Connected customers
- Computer vision: Interaction between objects
- Medical diagnosis: Interaction between brain region (MRI)
- Robotics: connection between joints
- ...

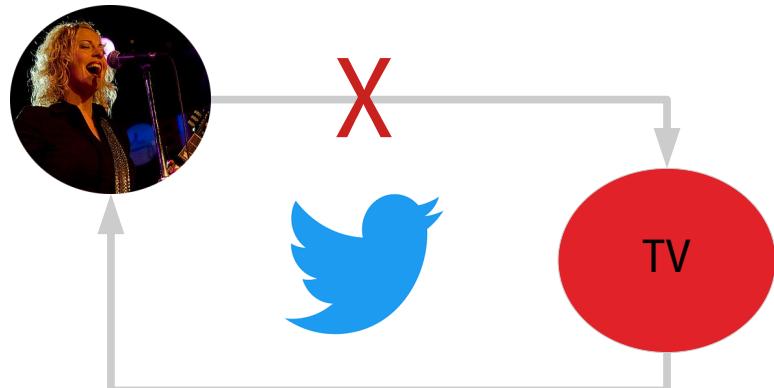
A relationship can be symmetrical or not between nodes

Undirected graphs



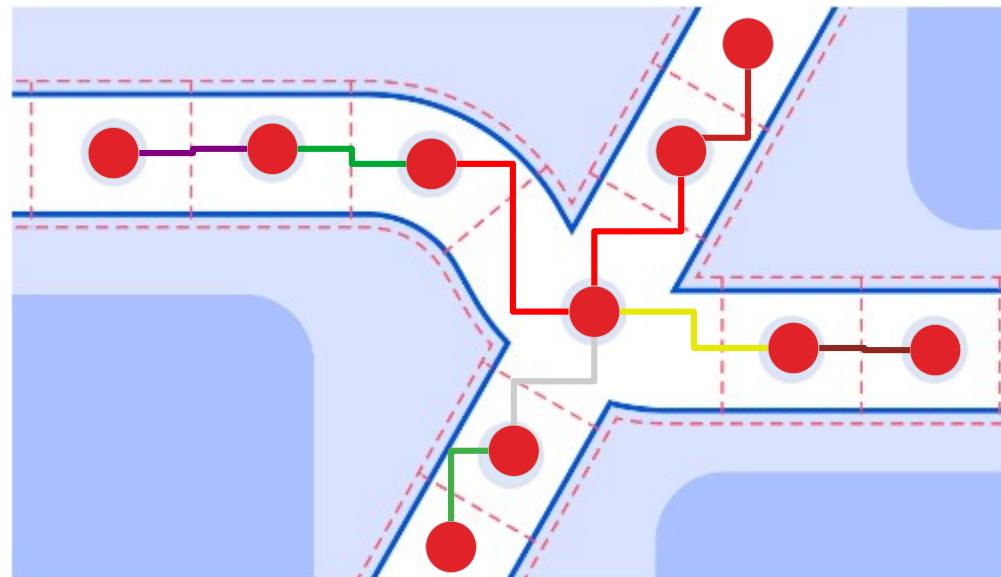
Directed graphs

Anneke van  
Giersbergen



# Edge: weight

Edges can carry information → **edge weight**



# Graphs store information: Features

Graphs can store information on **nodes**, **edges** and **globally**

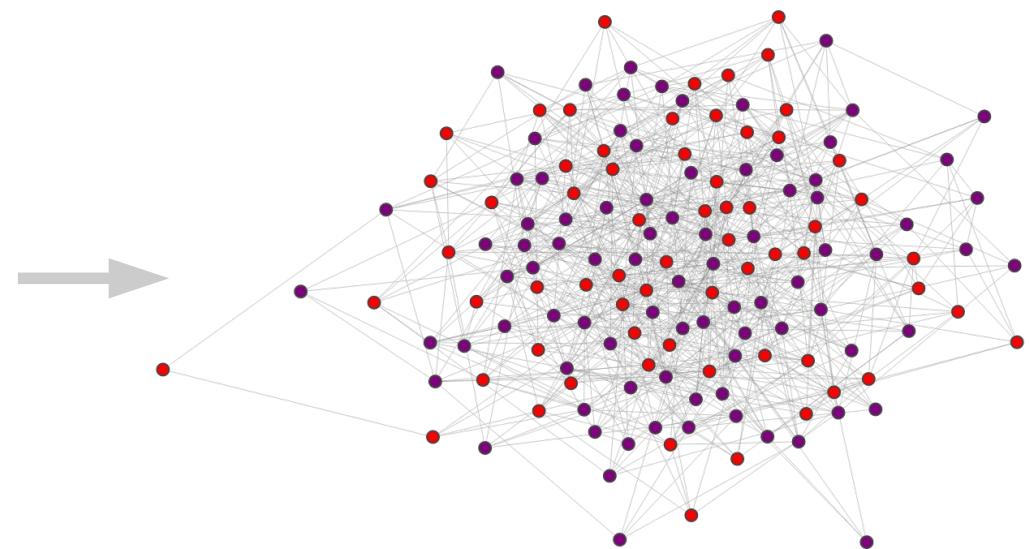
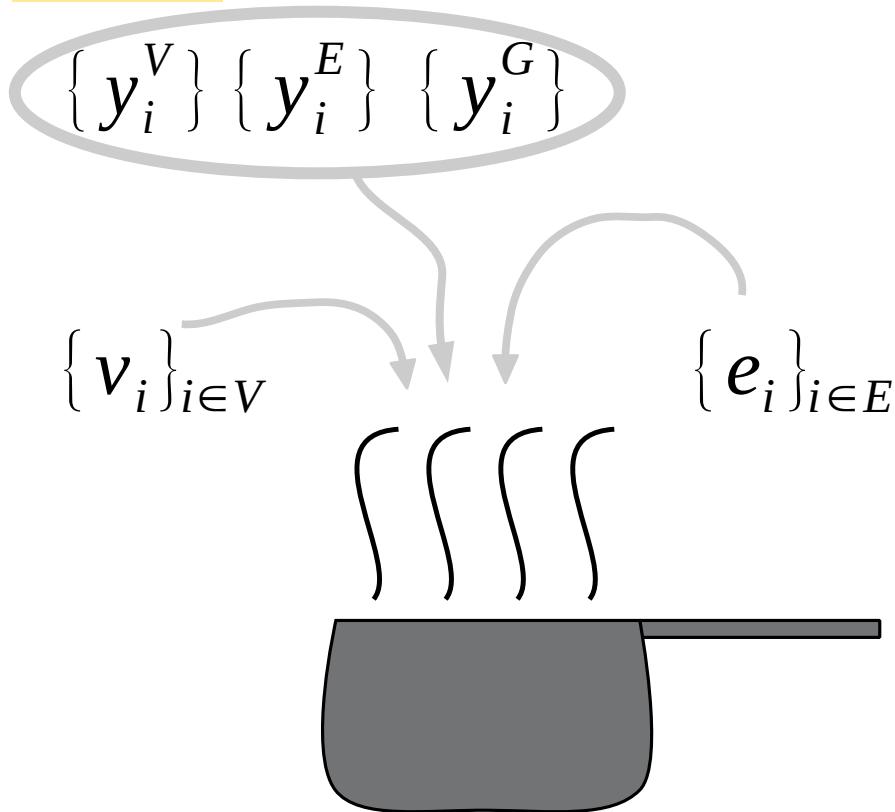
	Globally	Nodes	Edges
Social Network	Group of interest,...	Name, age, job,...	Is friend, follows, family,...
Molecule	Is a drug, energy,...	Atomic number,...	Bond order,...
Citations	Field,...	Article,...	Was cited,...
Particle physics	Experiment,...	Particle,...	Decayed to,...
Motion capture	Character,...	Joints,...	Is connected to,...
Natural language	Paragraph,...	Group of words,...	Refers to,...

It can be a number, a concept, ...

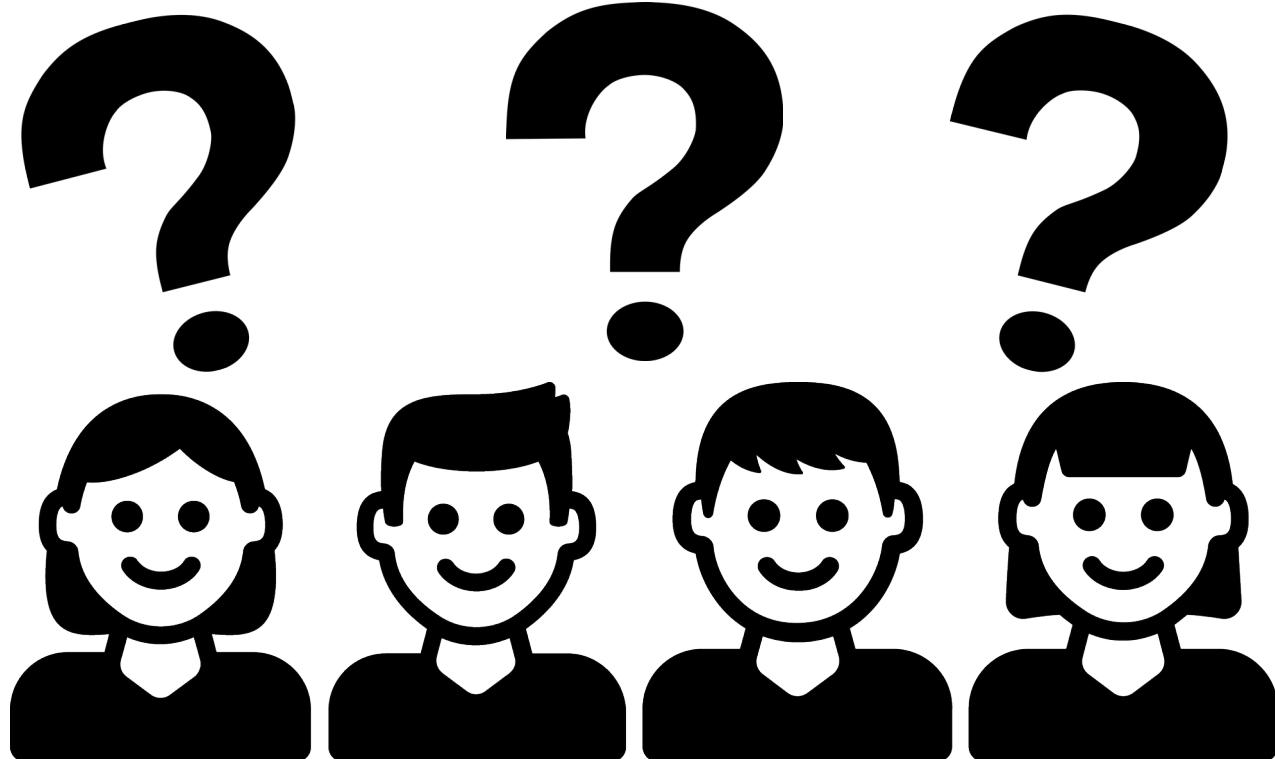
# Formal definition

**$G = (V, E)$ : a set of nodes and edges**

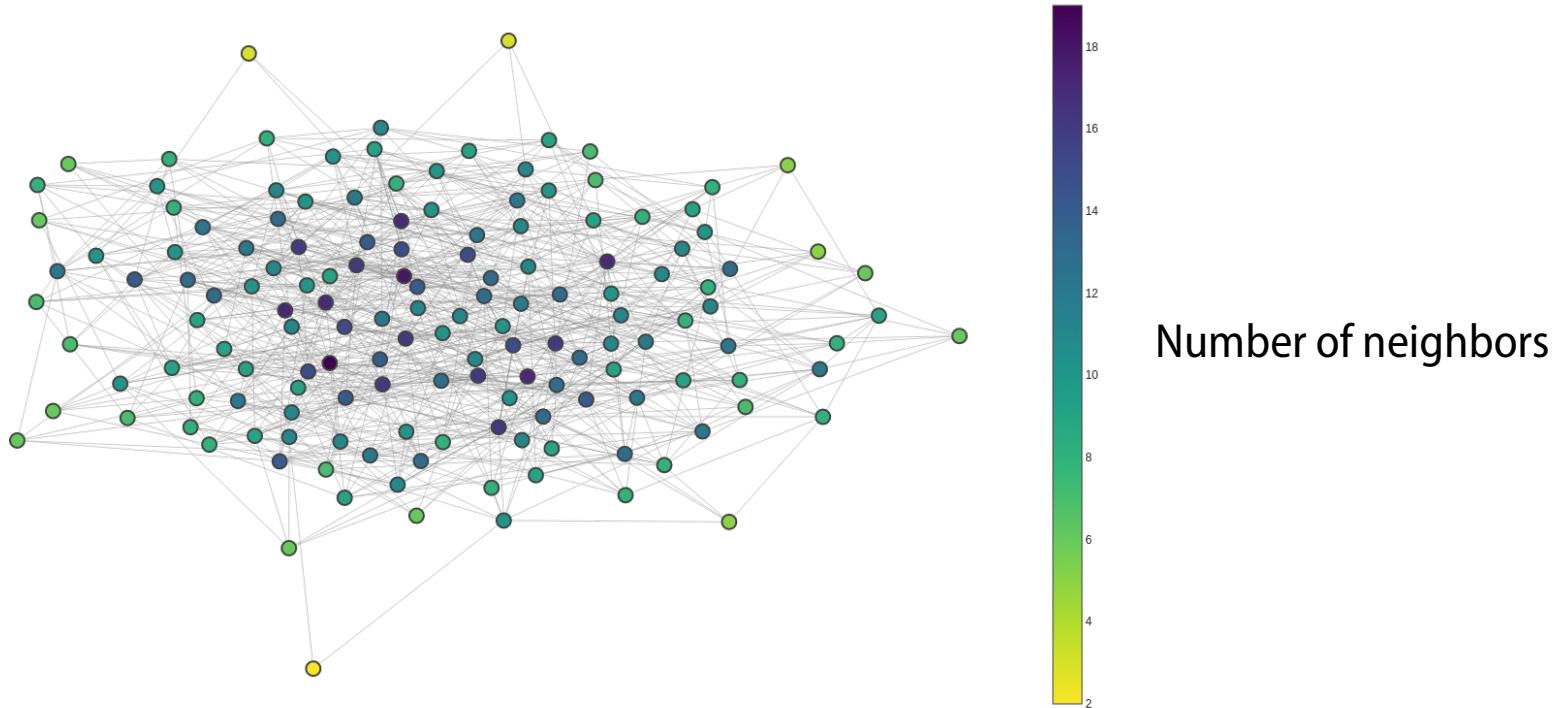
## Features



# Question break



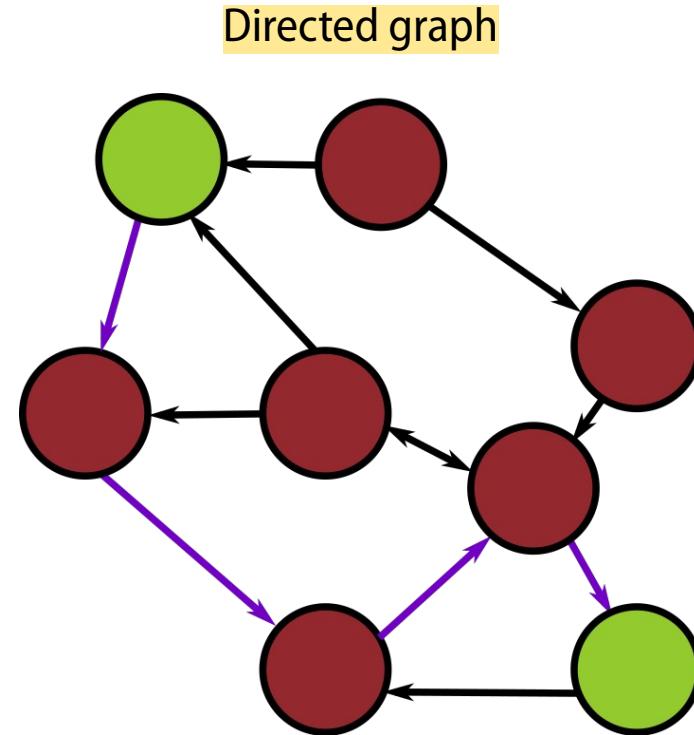
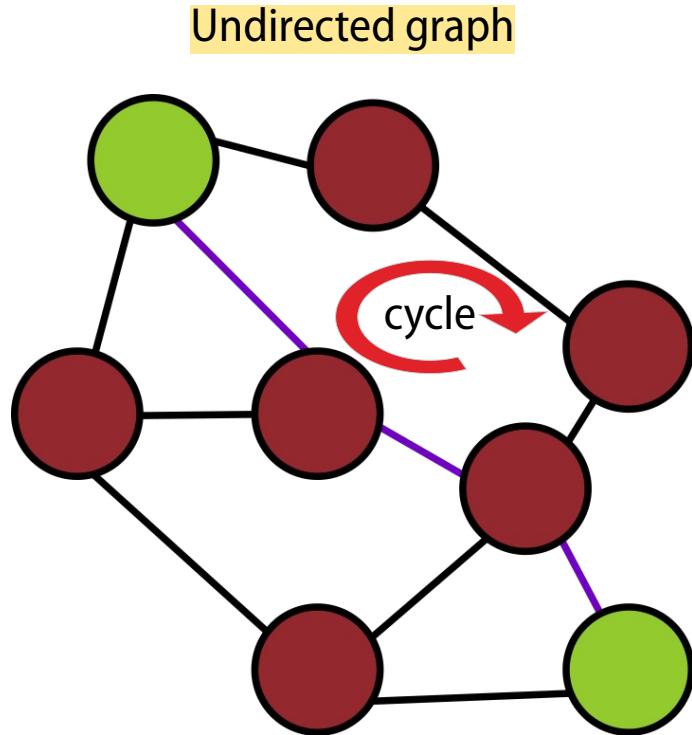
# Graph: Complexity



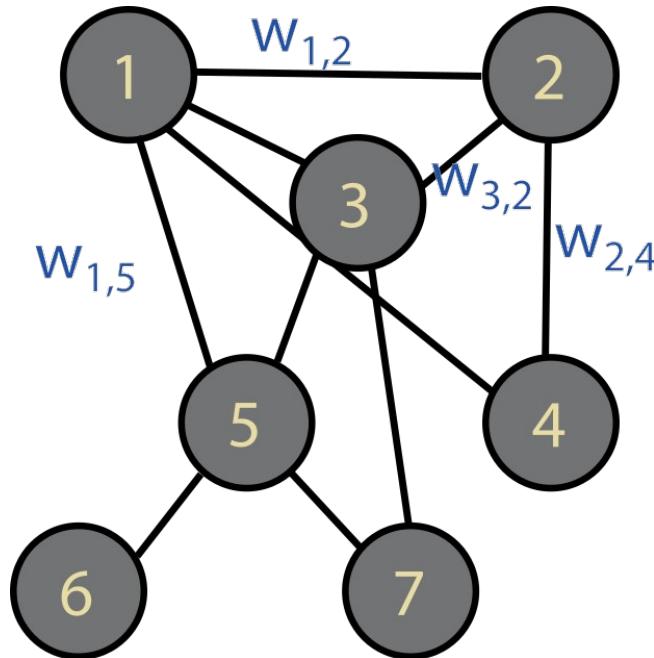
- The inner structure of a graph can vary a lot
- The number of edges/nodes might vary a lot from one graph to another
- One single graph can contain several thousand of nodes/edges
- ...

# Graph: Paths

A **path** is a sequence of edges connecting 2 nodes



# Graph: Node proximity and centrality



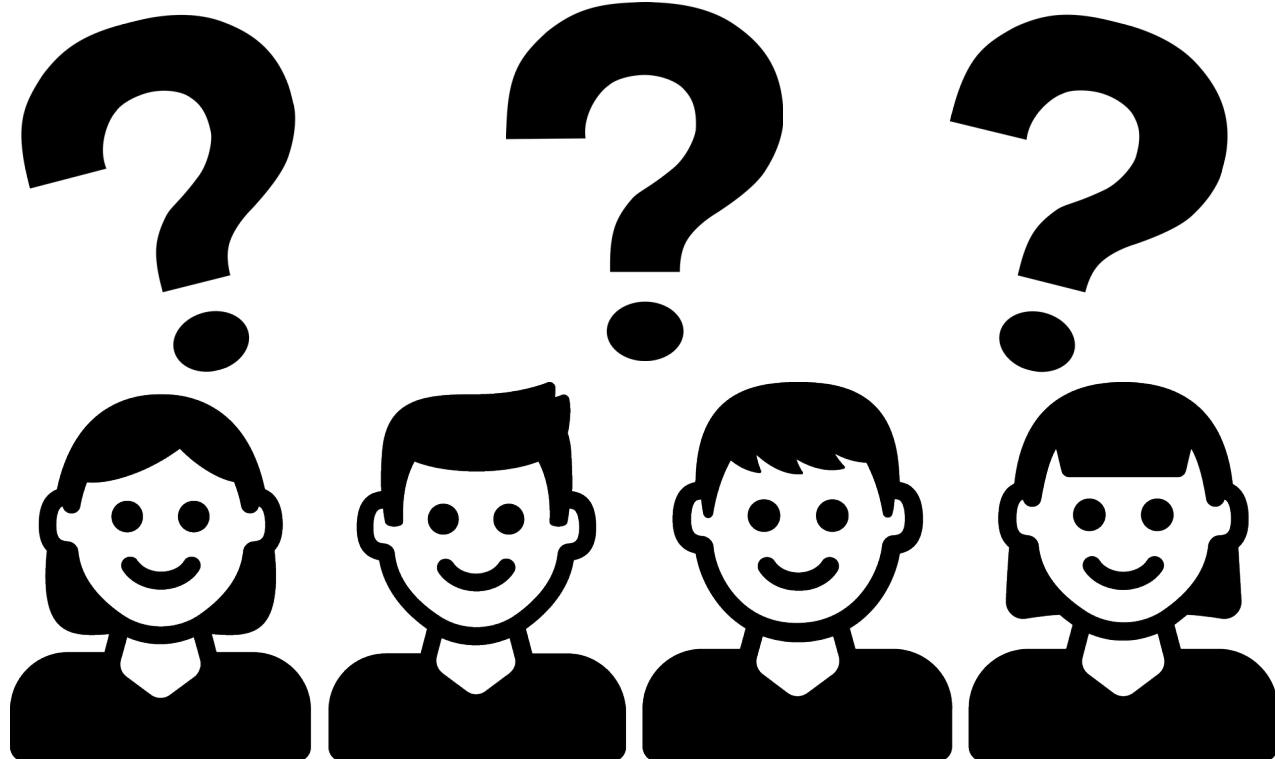
## Node centrality

Measure how many paths goes through the node

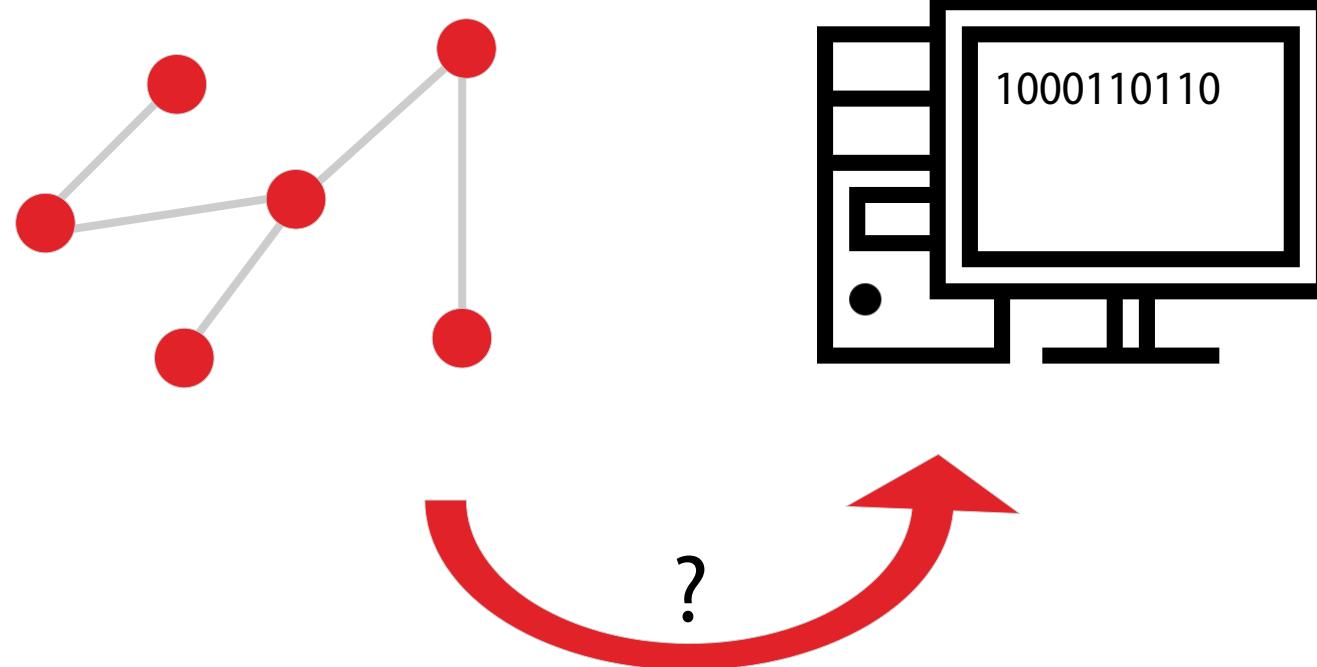
## Node proximity

- 1st order:  $w_{i,j}$  between node i and j
- 2nd order: similarity of neighborhood structure
- Higher orders possible

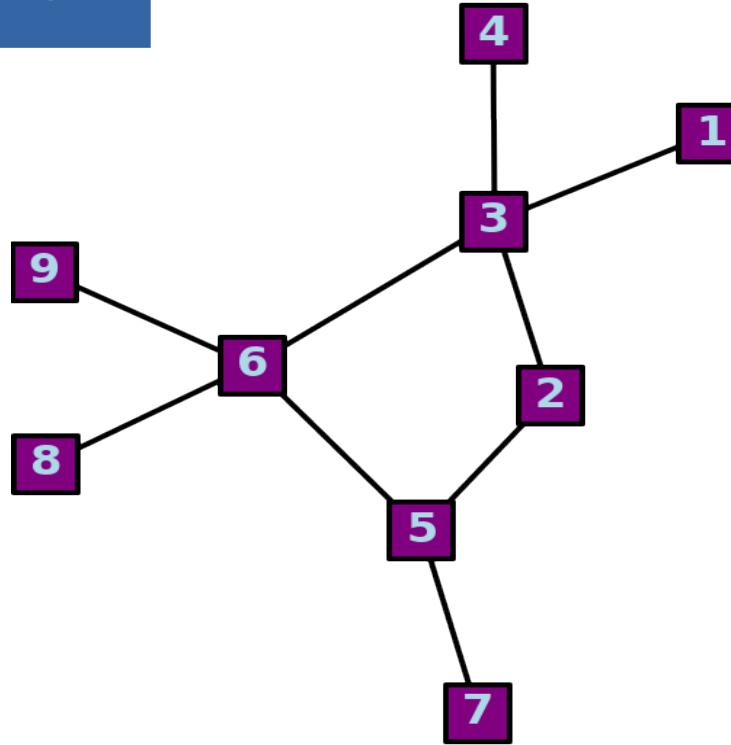
# Question break



# Graph representation



# Graph representation

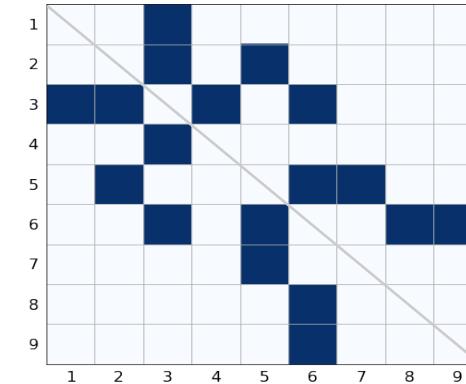
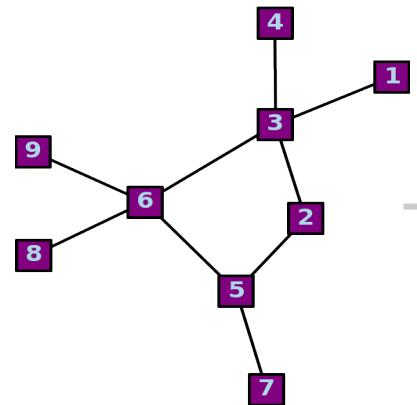


**Random numbering of nodes**

# Graph representation

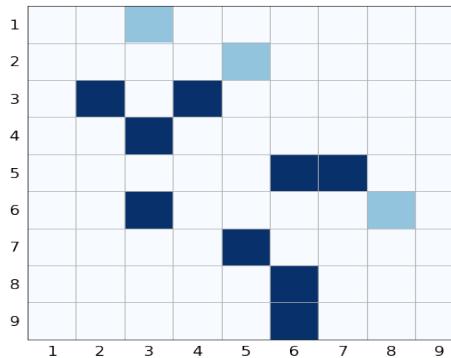
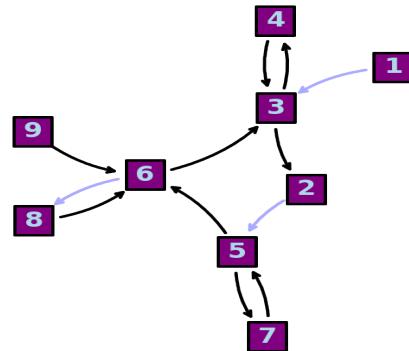
**Adjacency matrix**  $W_{(i,j)} = \begin{cases} w_{i,j} & \text{if there is an edge} \\ 0 & \text{if not} \end{cases}$

**Undirected**



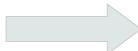
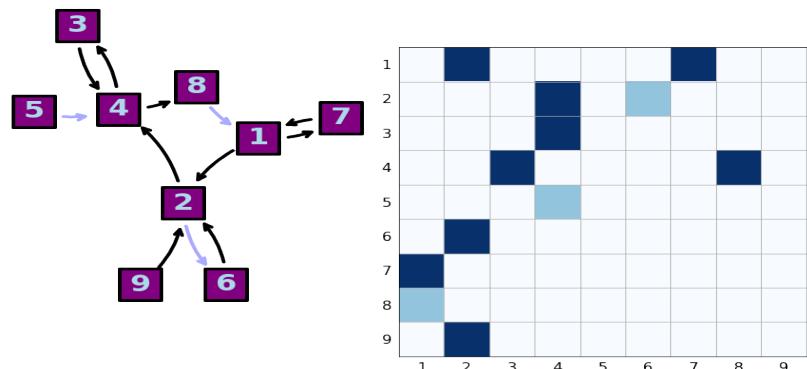
**Symmetric**

**Directed**



# Graph representation

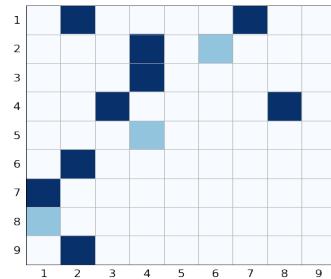
## Adjacency list



Adjacency list: [[5, 4],  
[8, 1],  
[4, 8], [4,3],  
[3, 4],  
[1, 7], [1, 2],  
[2, 4], [2, 6],  
[7,1],  
[6, 2],  
[9, 2]]

Edges: [0.4, 0.4, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 0.4, 1.0, 1.0, 1.0]

# Graph representation



Adjacency list: [[5, 4],  
[8, 1],  
[4, 8], [4,3],  
[3, 4],  
[1, 7], [1, 2],  
[2, 4], [2, 6],  
[7,1],  
[6, 2],  
[9, 2]]

Edges: [0.4, 0.4, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0,  
0.4, 1.0, 1.0, 1.0]

- Scale  $V^2 \rightarrow$  lot of space
- Might be sparse
- Easy to find an edge
- Scale  $E \rightarrow$  less space
- Might be difficult to find an edge

$V =$  number of nodes/vertices

$E =$  number of edges

# Graph representation

- Edge weights are stored either directly in the adjacency matrix, or in an independent tensor.

Adjacency matrix

1		1.0							
2				1.2					
3	3.7		8.0						
4		4.0							
5				5.0	1.0				
6		1.0					3.0		
7			4.2						
8				1.0					
9					7.0				
1	2	3	4	5	6	7	8	9	

Adjacency list: [[5, 4],  
[8, 1],  
[4, 8], [4, 3],  
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[1, 7], [1, 2],  
[2, 4], [2, 6],  
[7, 1],  
[6, 2],  
[9, 2]]

Edges: [0.4, 1.4, 2.4, 9.0, 1.0, 5.0, 1.7, 3.0,  
0.4, 1.3, 7.0, 6.2]

- Information (features) on nodes and graphs will also be stored in independent tensors.

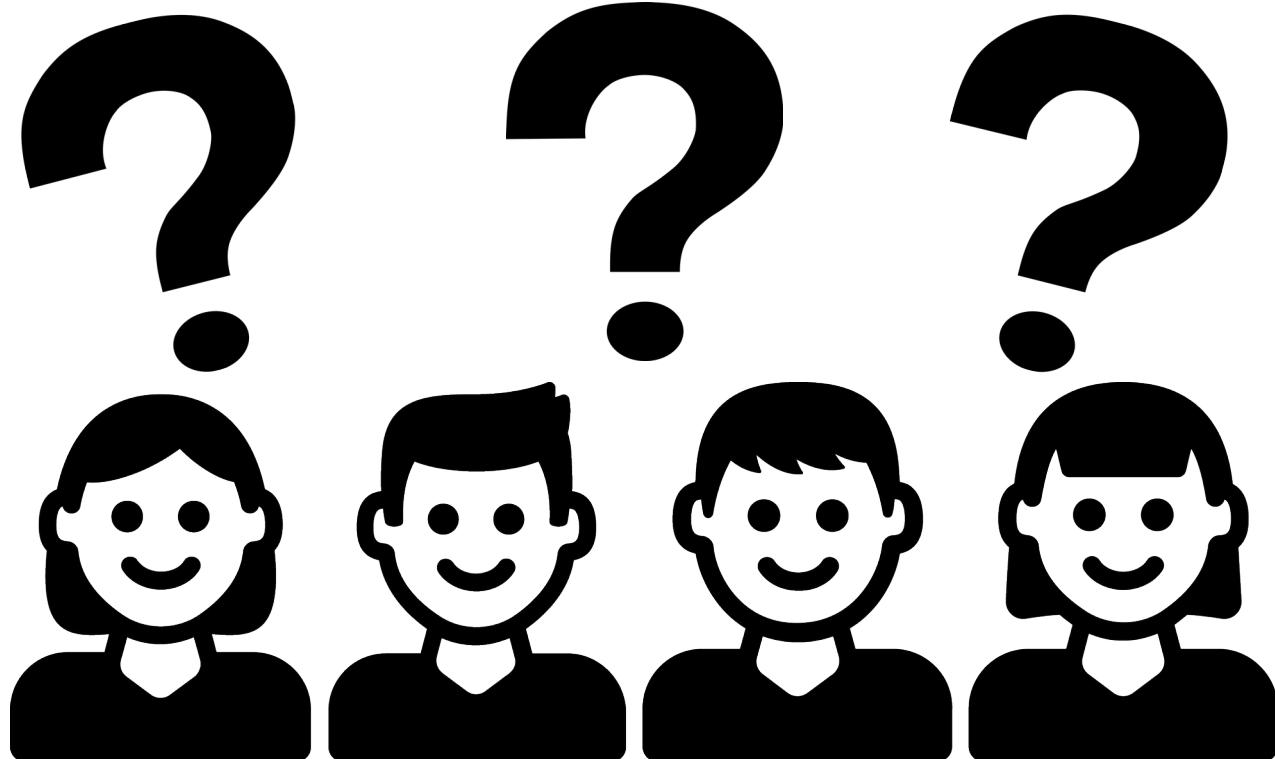
Nodes: [4.1, 4.2, 6.4, 1.0, 1.0, 5.0, 1.7,  
3.0, 5.0]

Graph: [8.0]

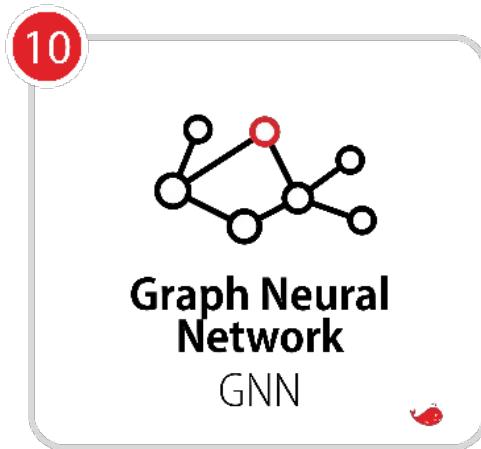
# Useful Matrices

Adjacency	$W$	Weight of edges
Degree	$D$	Diagonal matrix with number of edges for each node
Laplacian	$L$	$D - W$
Node Features	$X$	Information stored

# Question break



# Roadmap



9.1

## Graphs are everywhere

- Complex data structures
- Basics of graph theory

9.2

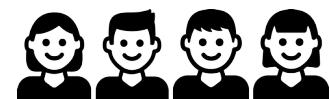
## Learning on Graphs

- Graph embedding
- Transductive and inductive learning
- Tasks on graph learning

9.3

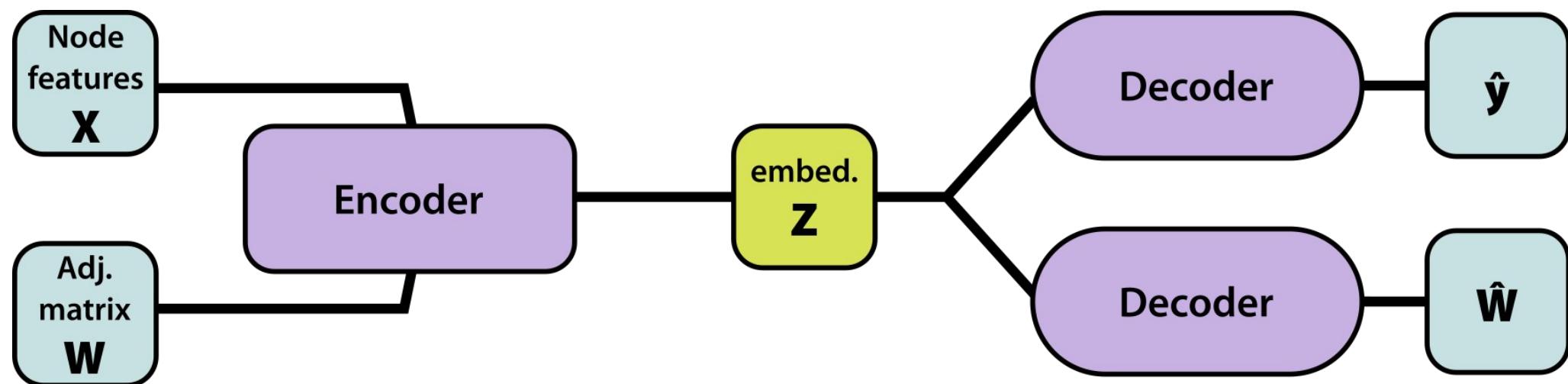
## A few examples

- Taxonomy of methods
- Graph convolution
- Message passing
- Graph Transformer

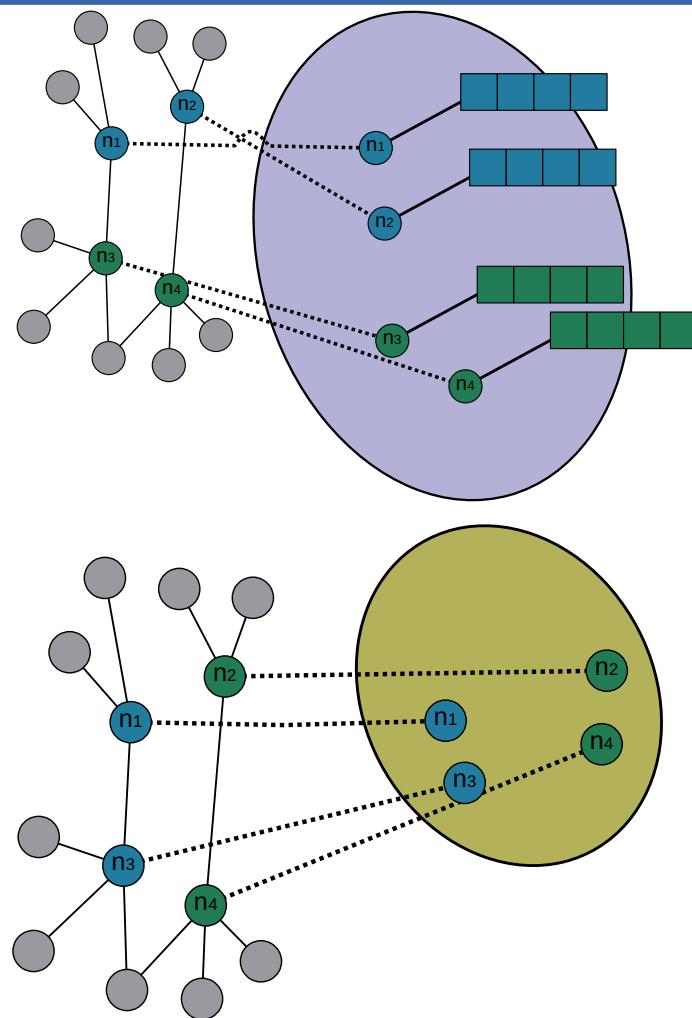


# Graph embedding

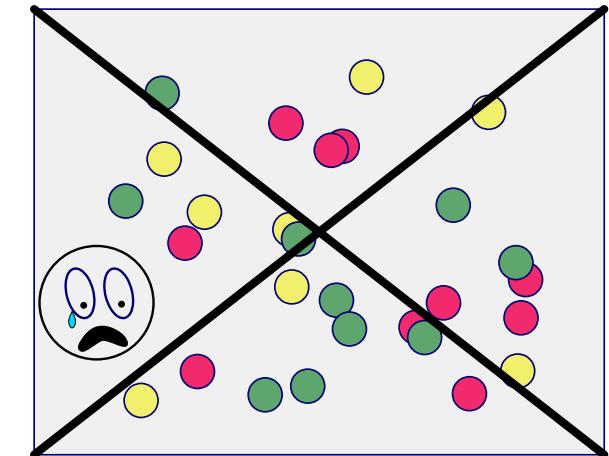
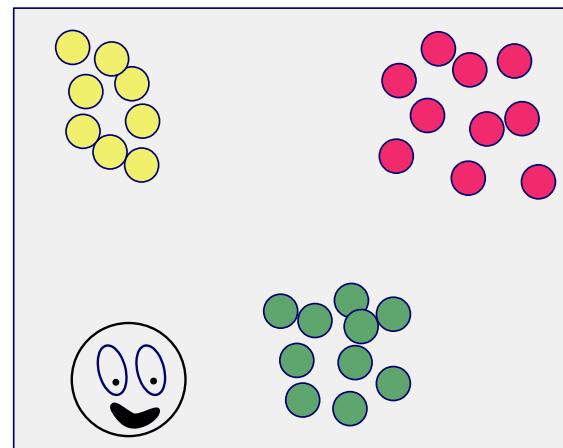
- We need to find a representation of the graph that is processable



# Graph embedding



- Features stored in nodes/edges/graphs are not easily processed.
- We transform the features into a vector in the latent space (**Dimension is a hyperparameter**).
- The embedding has to be suited for the task → **Learnable**.

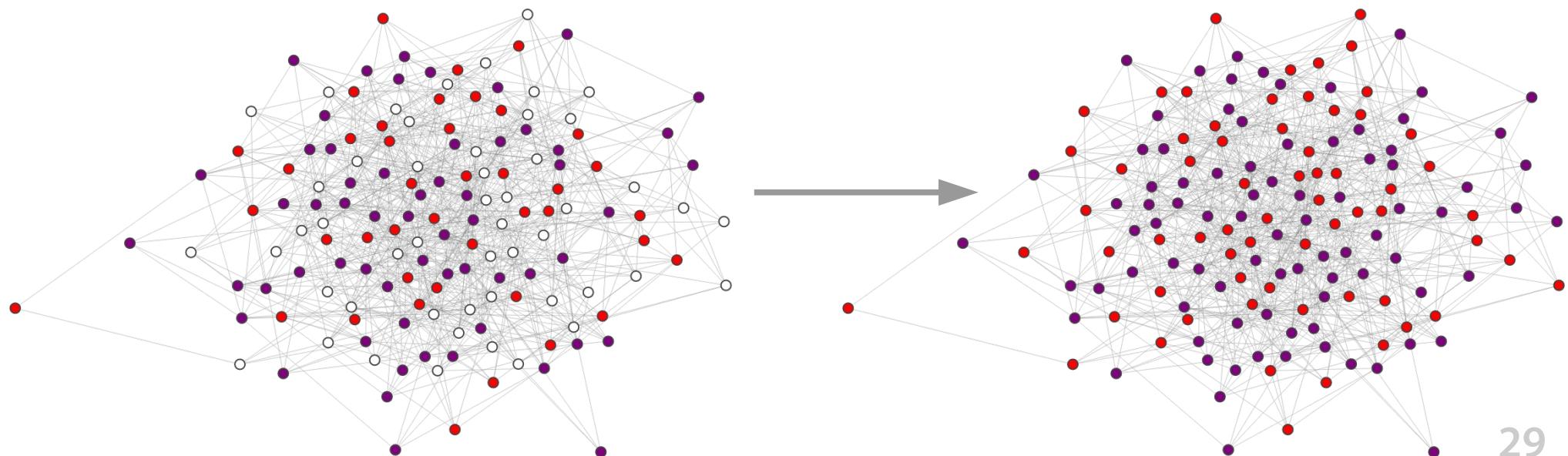


# Transductive learning

The model has access to the complete graph

It is not possible to add new nodes

Node labeling

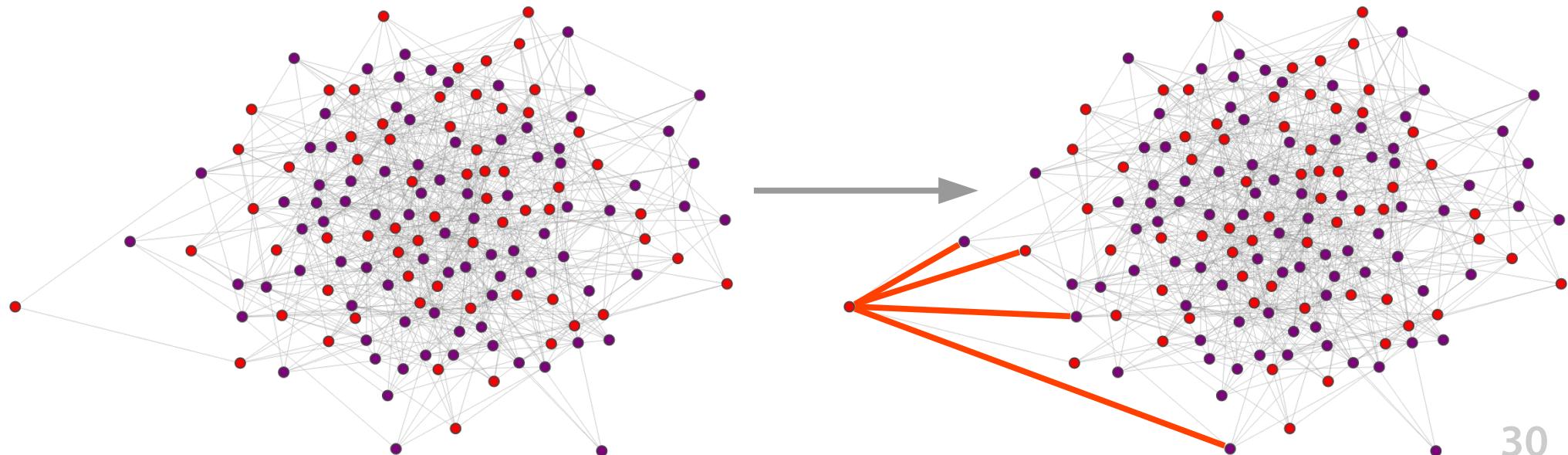


# Transductive learning

The model has access to the complete graph

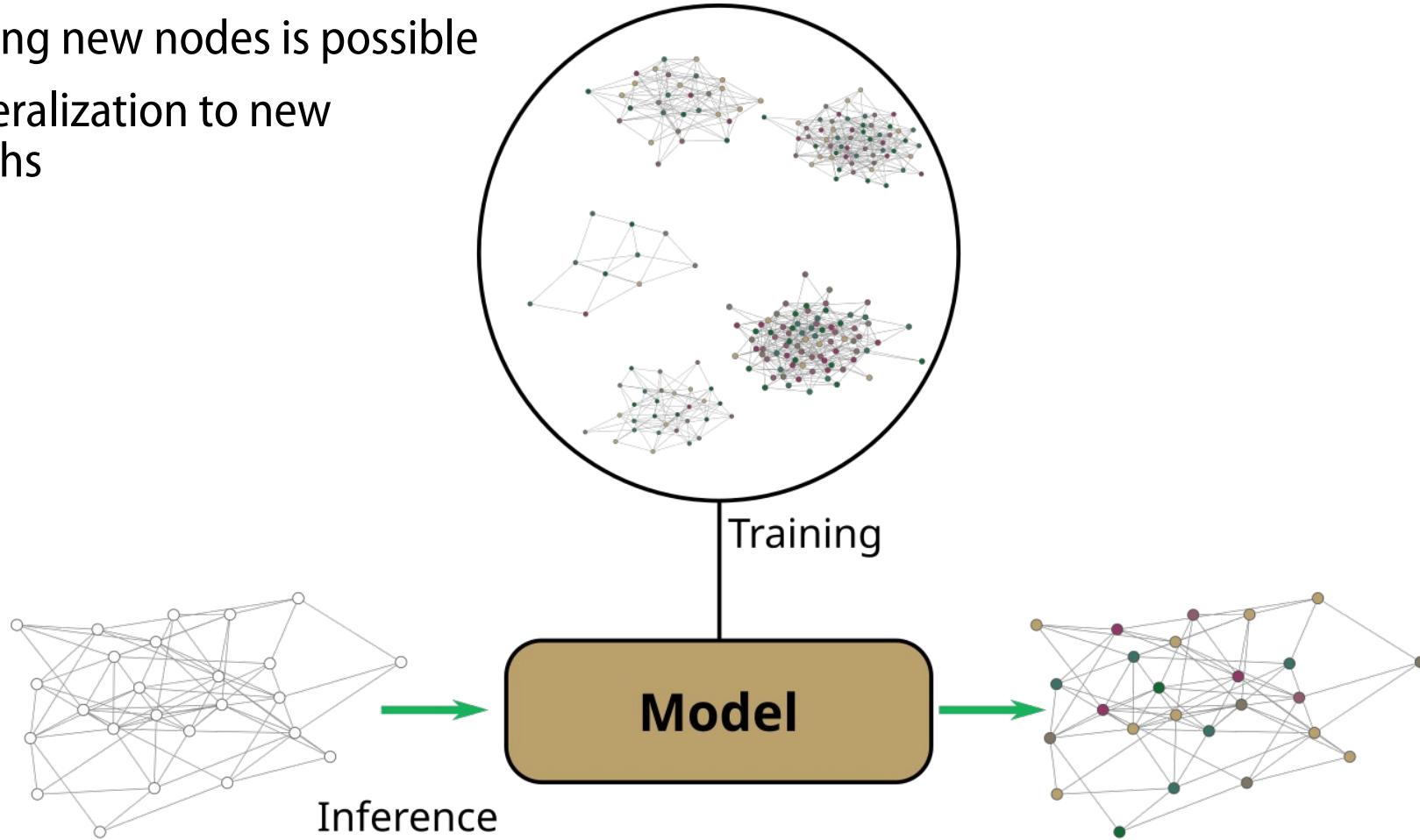
It is not possible to add new nodes

Find new edges



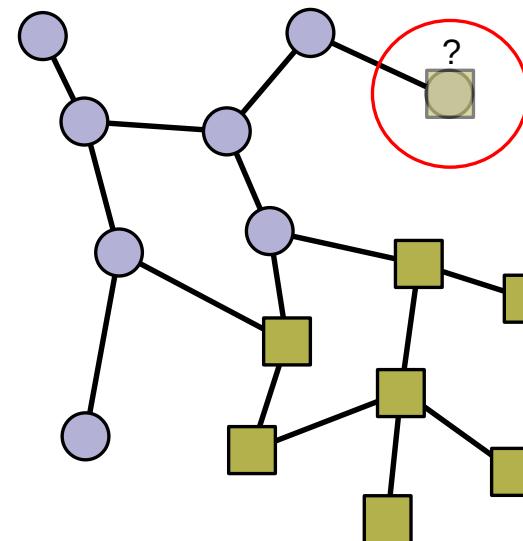
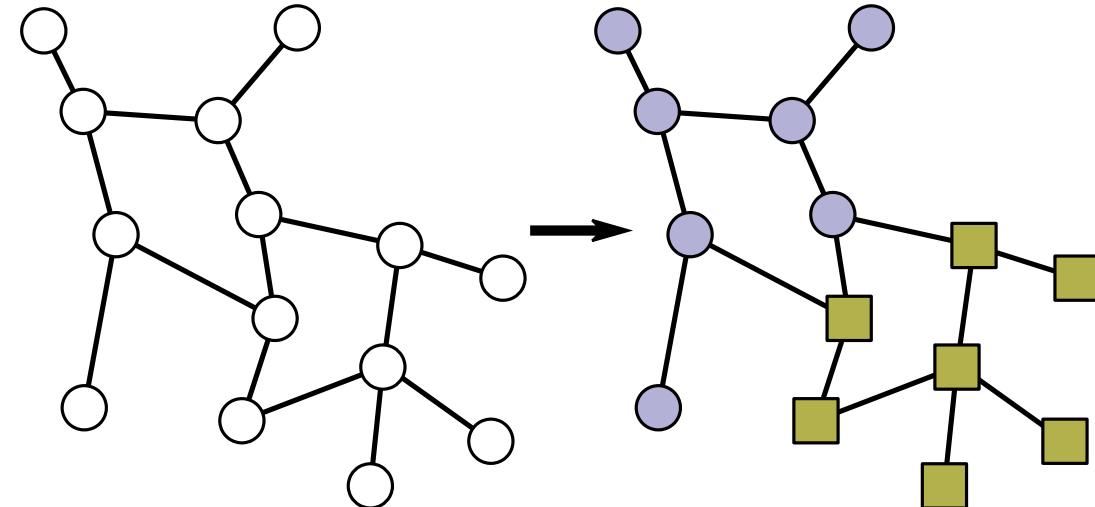
# Inductive learning

- The model has access only to a part of the graph (train set)
- Adding new nodes is possible
- Generalization to new graphs



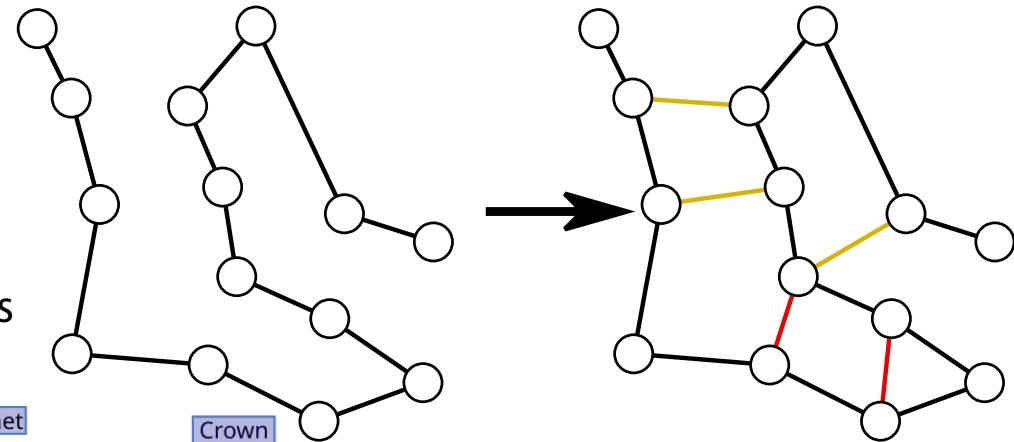
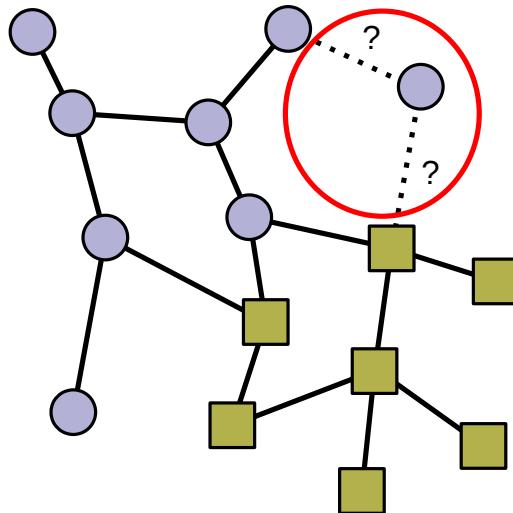
# Tasks on nodes

- Labeling nodes in a graph  
**(clustering)**
  - Find topic of a research paper (CORA, etc)
  - Find bots in a social network
  - ...
- Labeling new nodes
- Perform regression



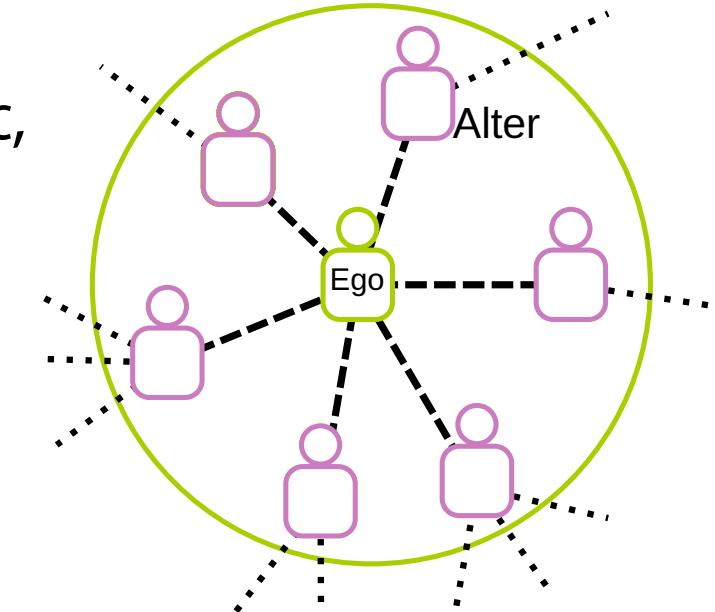
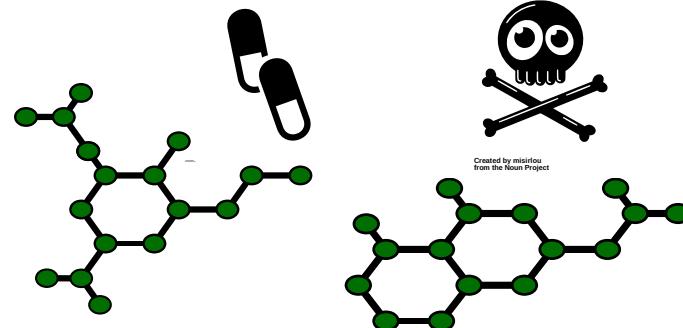
# Tasks on edges

- Find relationships
  - Contact map of aminoacids (AlphaFold)
  - Contact suggestion (social network)
  - ETA for directions (regression)
  - Relationships between segments in pictures
  - ...

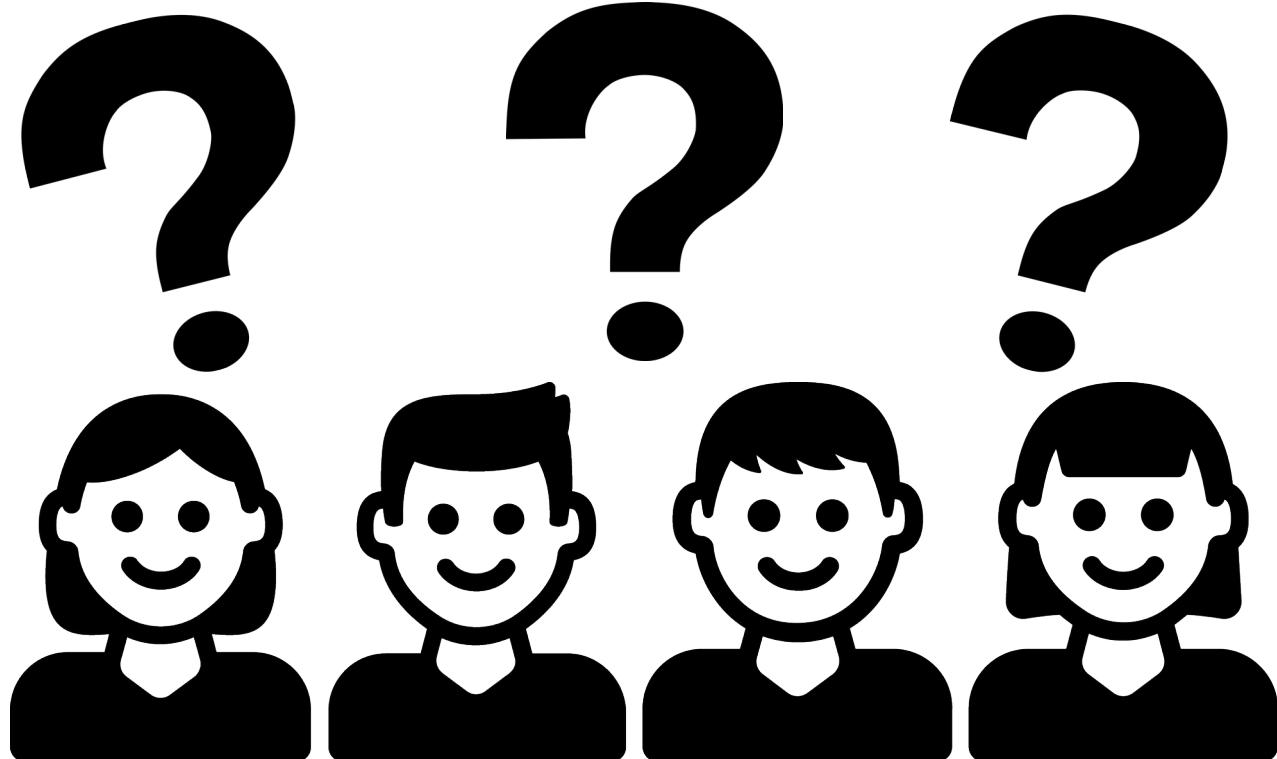


# Tasks on graphs

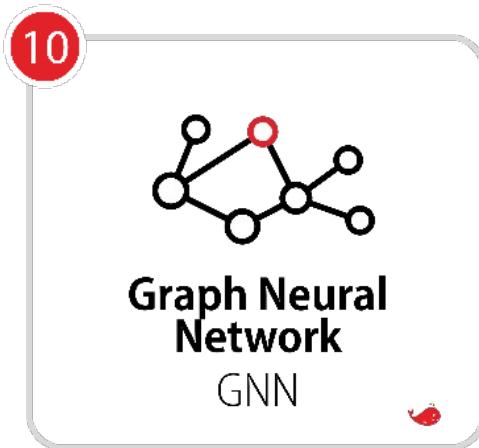
- Predict properties of graphs
  - Chemical properties (solubility, carcinogenic, possible drug)
  - Classification of the research field in an ego network
  - ...



# Question break



# Roadmap



9.1

## Graphs are everywhere

- Complex data structures
- Basics of graph theory

9.2

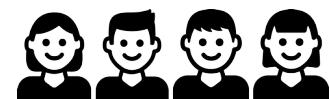
## Learning on Graphs

- Graph embedding
- Transductive and inductive learning
- Tasks on graph learning

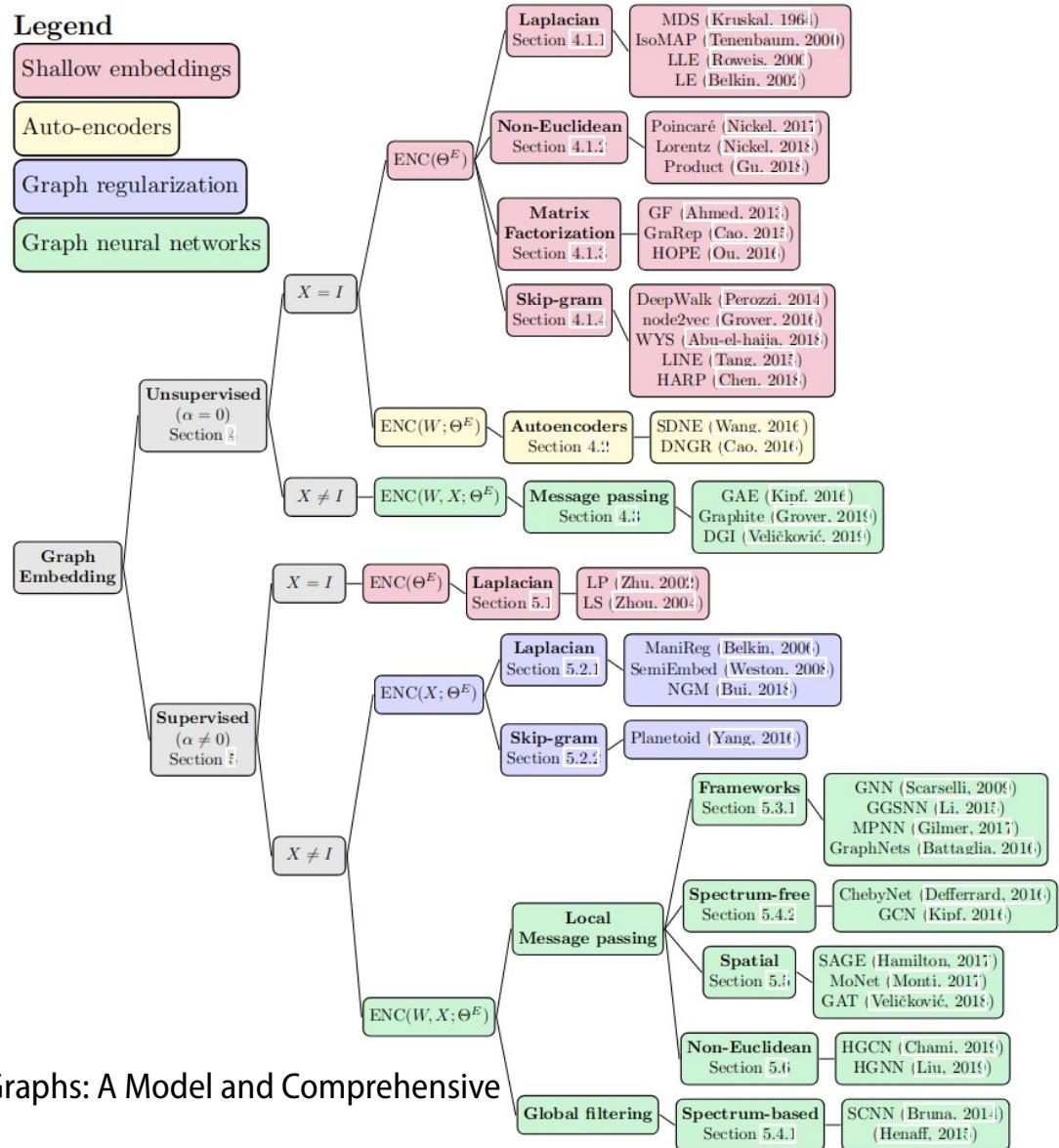
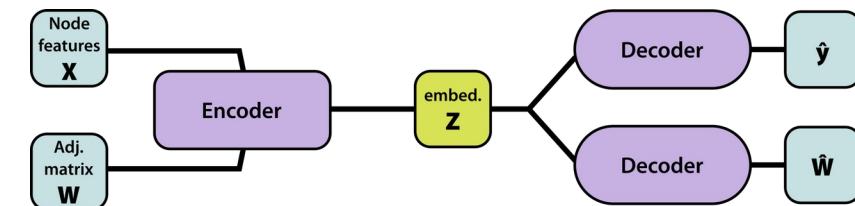
9.3

## A few examples

- Taxonomy of methods
- Graph convolution
- Message passing
- Graph Transformer

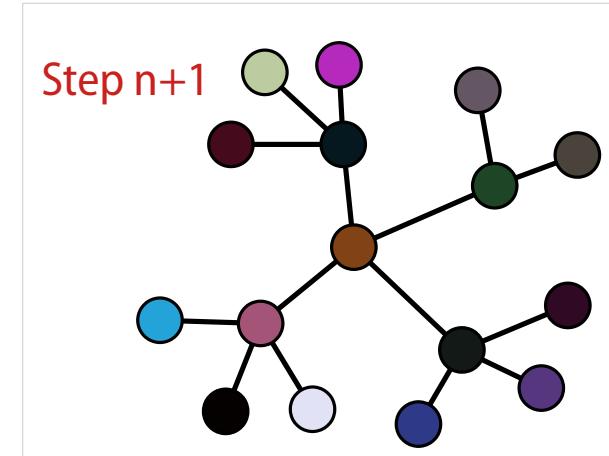
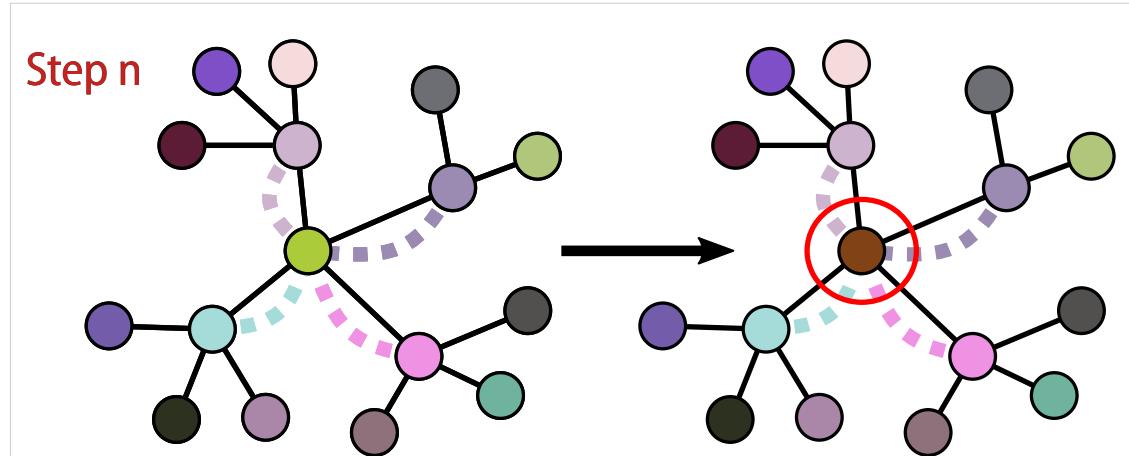


# Taxonomy of methods



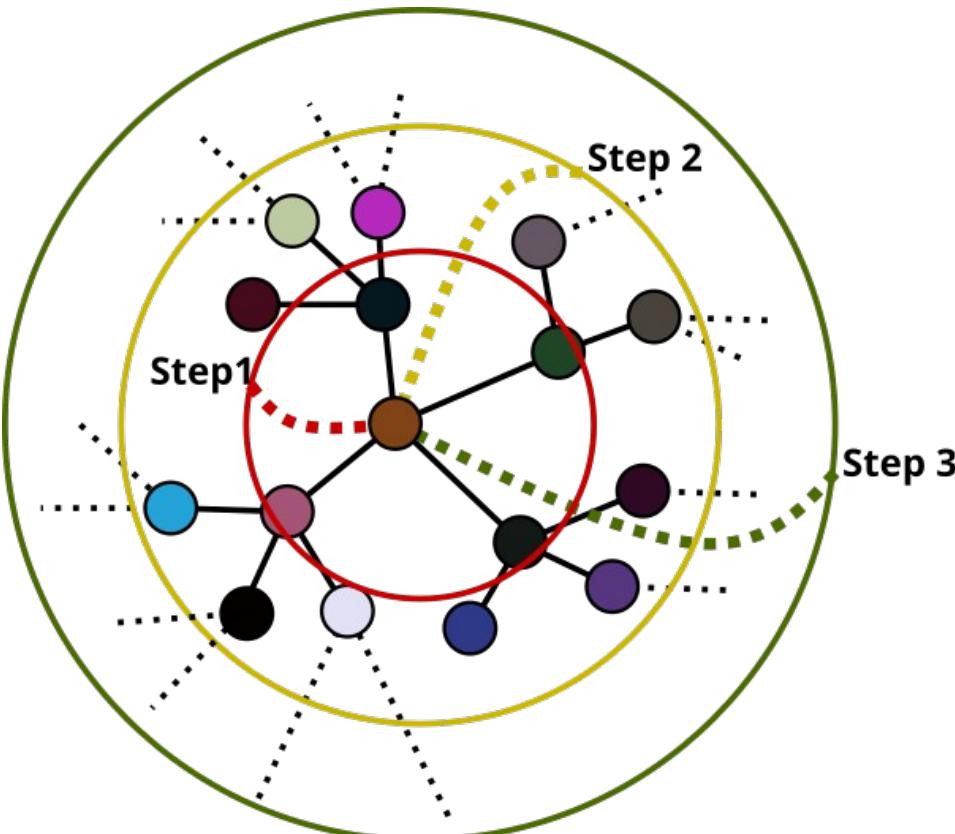
# Graph convolution

- Just like for images we can learn from neighborhood with a convolution.



- A bit more complex since the number of neighbors is unlikely to be constant.
- We want the operator to be **permutation invariant**.

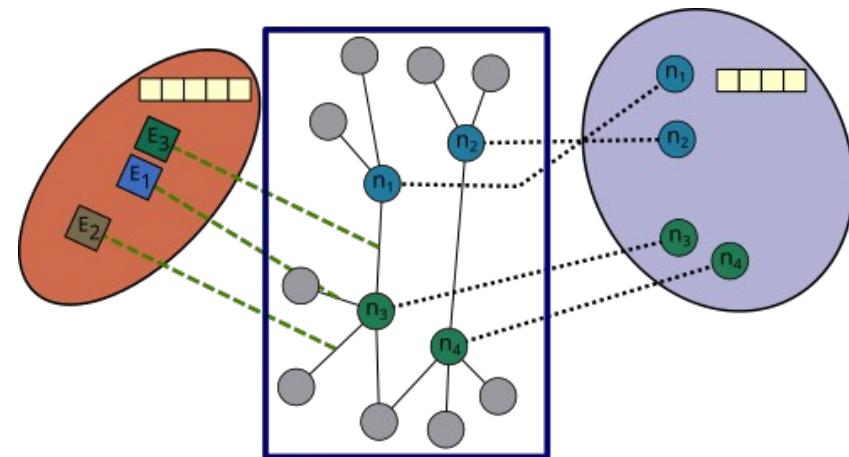
# Graph convolution



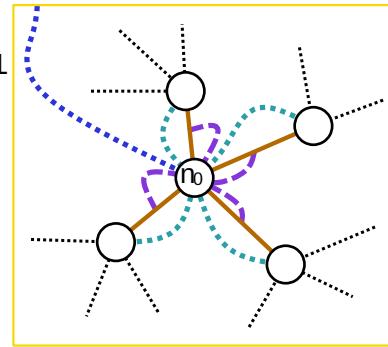
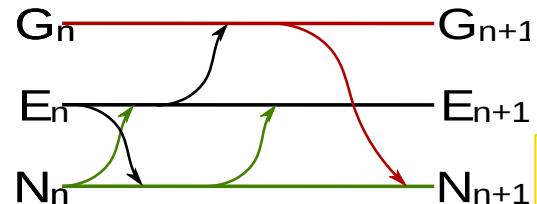
- Several steps are needed to retrieve information for distant nodes.
- For large graphs → a **cutoff**
- It is possible to use a **virtual node** connected to all other nodes. But in practice this becomes quickly intractable.

# Message passing

- We have embeddings for each part of the graph (possibly different vector sizes).
- Each part can learn from the others via a transformation.



$$\begin{array}{c} \text{Edge embedding} \\ \boxed{\text{yellow yellow yellow}} \end{array} \times \begin{array}{c} \text{Learnable transformation} \\ \boxed{\text{purple purple purple}} \end{array} = \begin{array}{c} \text{Node embedding} \\ \boxed{\text{yellow yellow yellow}} \end{array}$$

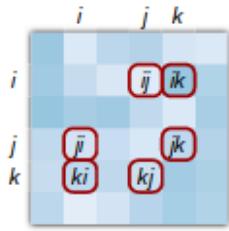


$$n_0 = (f_{NN}, f_{EN}, f_{GN})$$

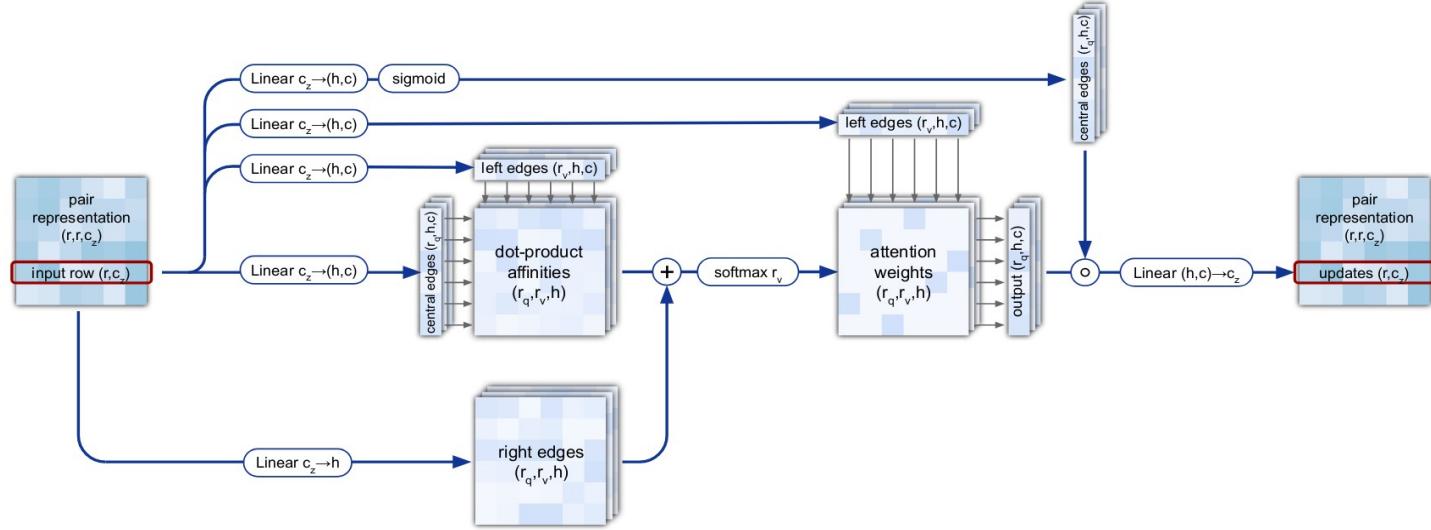
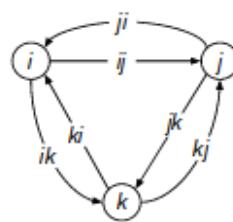
- Information is aggregated to form a message that the node/edge will send to others.

# AlphaFold transformer

**b** Pair representation ( $r, r, c$ )

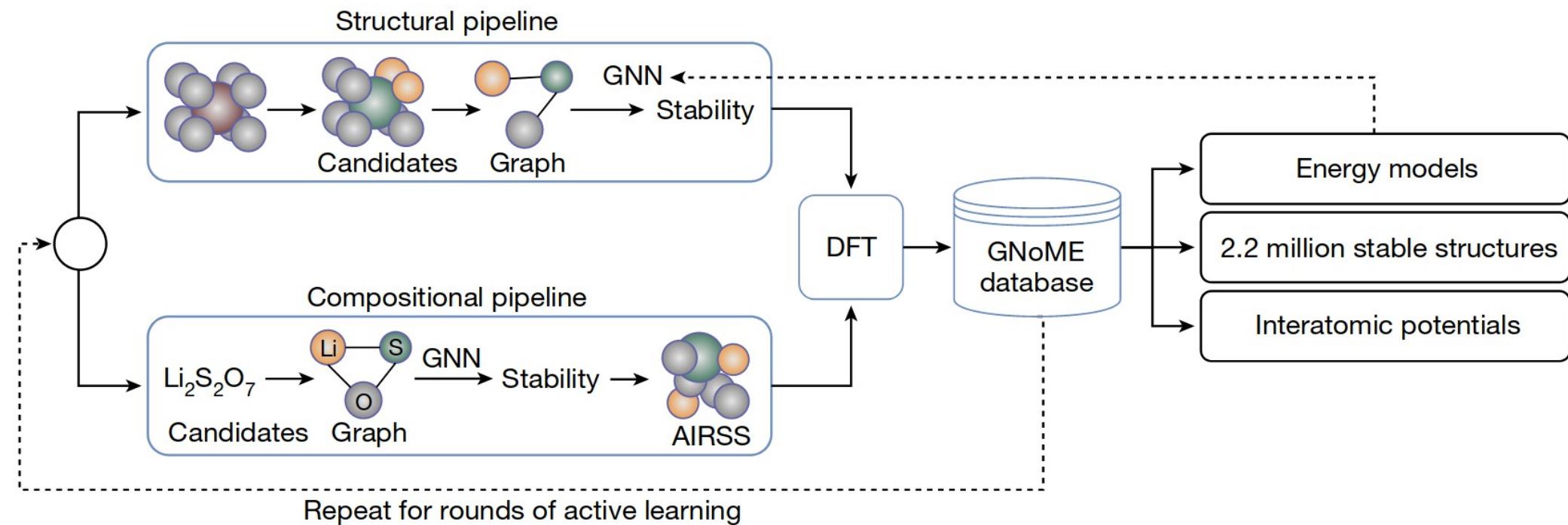


Corresponding edges in a graph



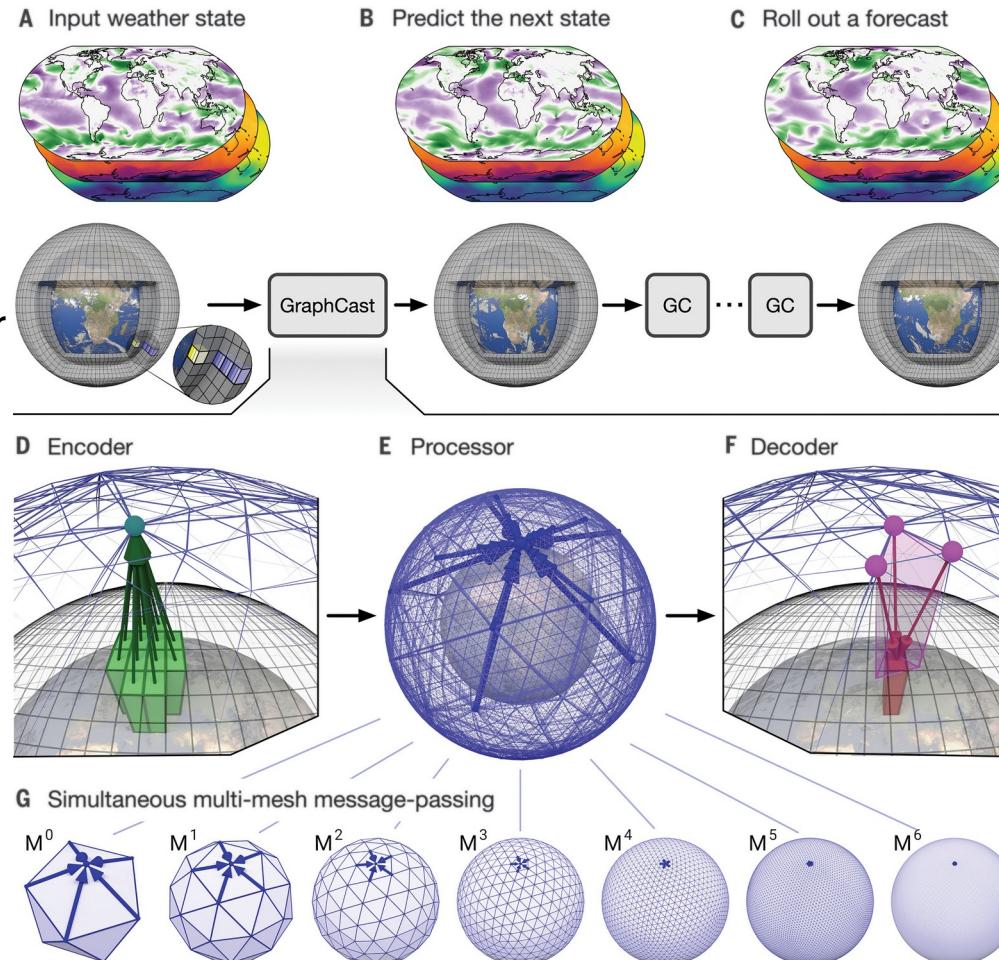
**Supplementary Figure 7 |** Triangular self-attention around starting node. Dimensions:  $r$ : residues,  $c$ : channels,  $h$ : heads

## Generation of novel crystal structures

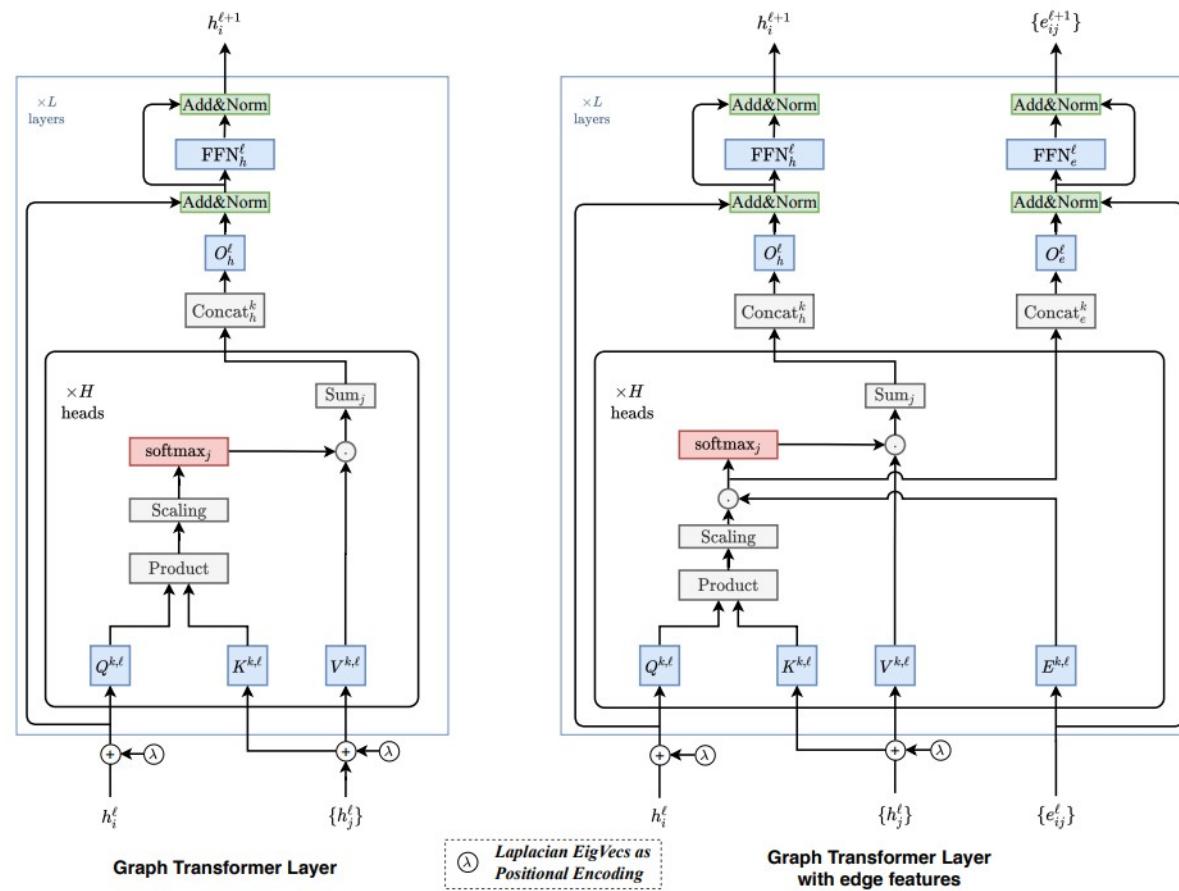


# GraphCast

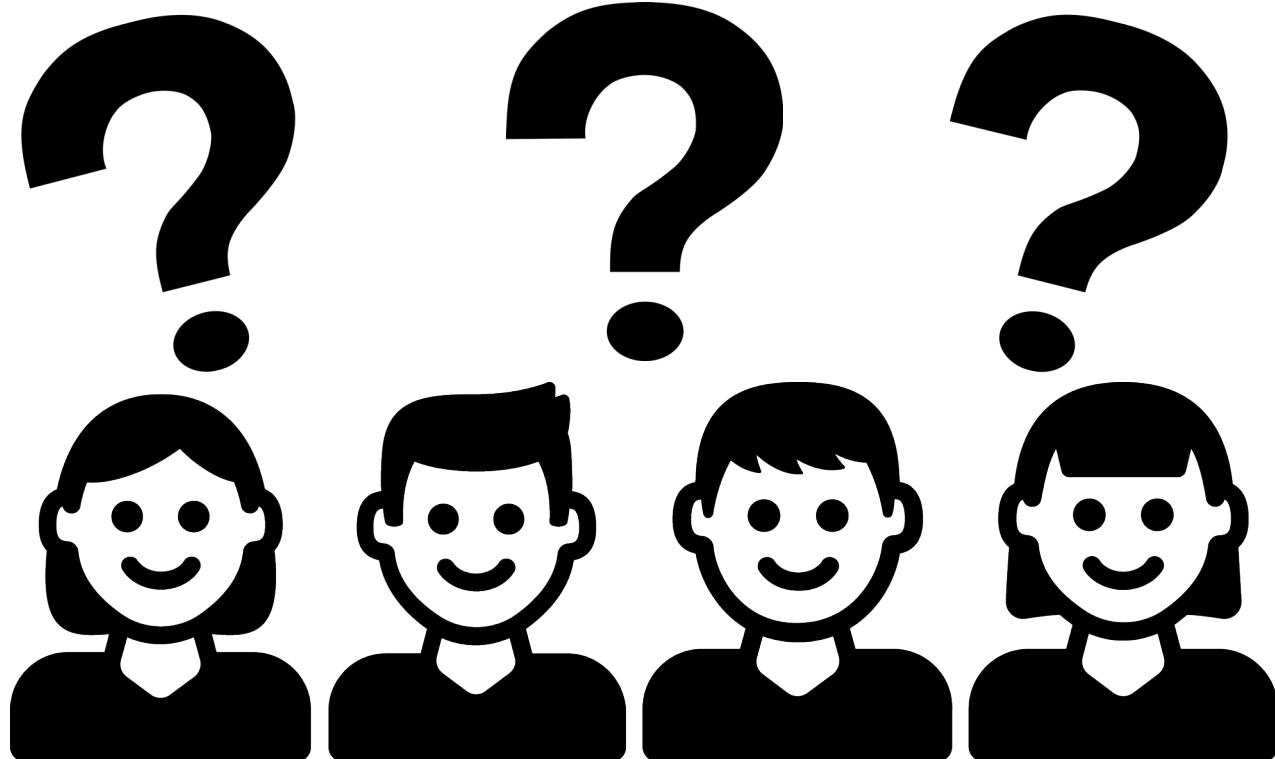
Prediction of the weather  
with temporal graphs



# Graph Transformer Network



# Question break



## Libraries

- Pytorch Geometric
- Deep Graph Library
- Graph Nets
- Spektral
- ...

- <https://logconference.org/>
- <https://ogb.stanford.edu/>

## Tutorials

- [https://antoniolonga.github.io/Pytorch\\_geometric\\_tutorials/](https://antoniolonga.github.io/Pytorch_geometric_tutorials/)
- <https://docs.dgl.ai/tutorials/blitz>

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