



Hands-on Introduction to Deep Learning

Graphs are everywhere



Highly ordered data

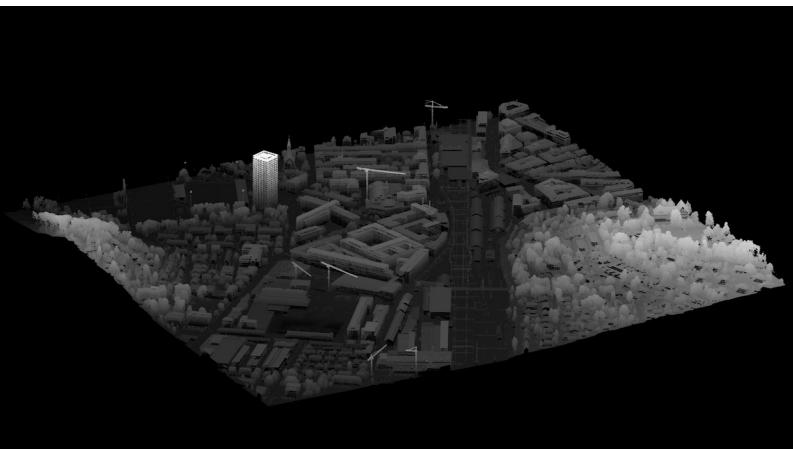


The answer to life, the universe and everything is ...

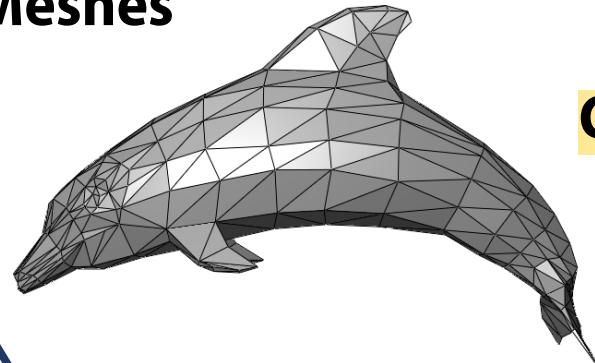
Rebirth of Deep learning was thanks to pictures, text and speech recognition

Data structures: Data is not always euclidean

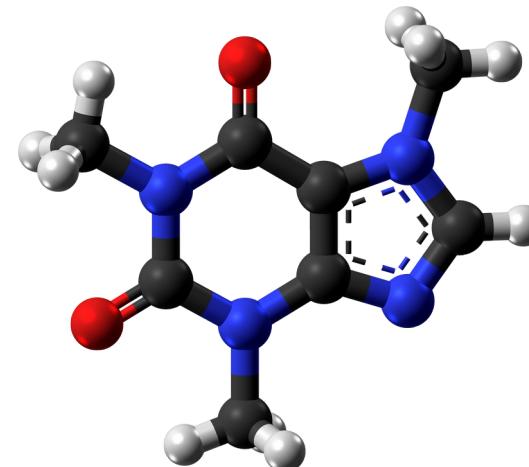
LIDAR



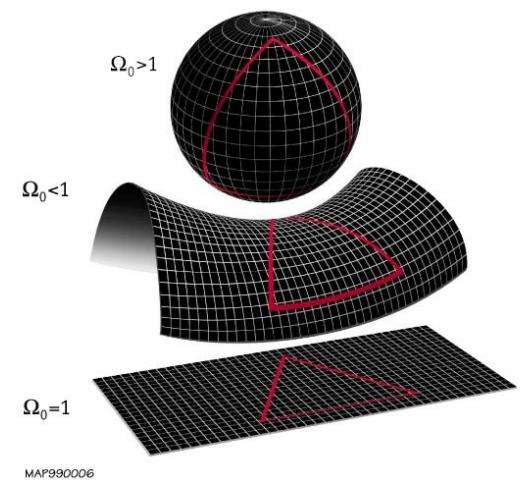
Meshes



Molecules



Complex geometries



Geometric deep learning

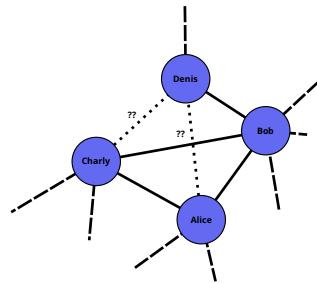
Michael M. Bronstein, et. al , Geometric Deep Learning: Grids, Groups, Graphs, Geodesics, and Gauges
<https://arxiv.org/abs/2104.13478>

Graphs are everywhere

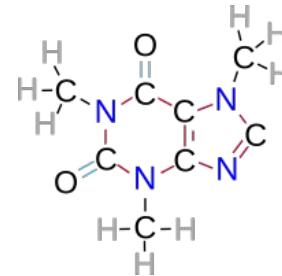
[1] A. Derrow-Pinion et al., "ETA Prediction with Graph Neural Networks in Google Maps," in Proceedings of the 30th ACM International Conference on Information & Knowledge Management New York, NY, USA, Oct. 2021, pp. 3767–3776. doi: 10.1145/3459637.3481916.

[2] J. Shlomi, P. Battaglia, and J.-R. Vlimant, "Graph neural networks in particle physics," *Mach. Learn.: Sci. Technol.*, vol. 2, no. 2, p. 021001, Jan. 2021, doi: 10.1088/2632-2153/abbf9a.

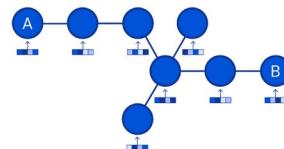
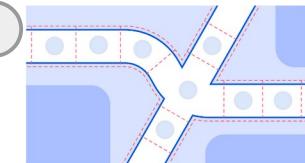
Social networks



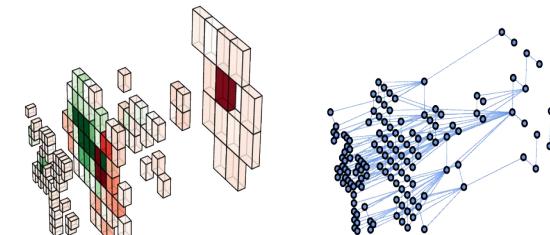
Molecules



Directions recommendation

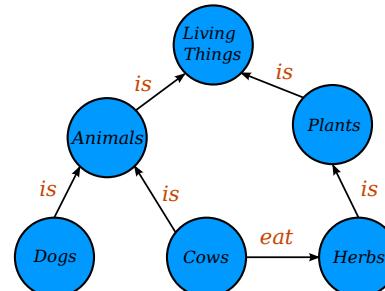


Particle physics



2

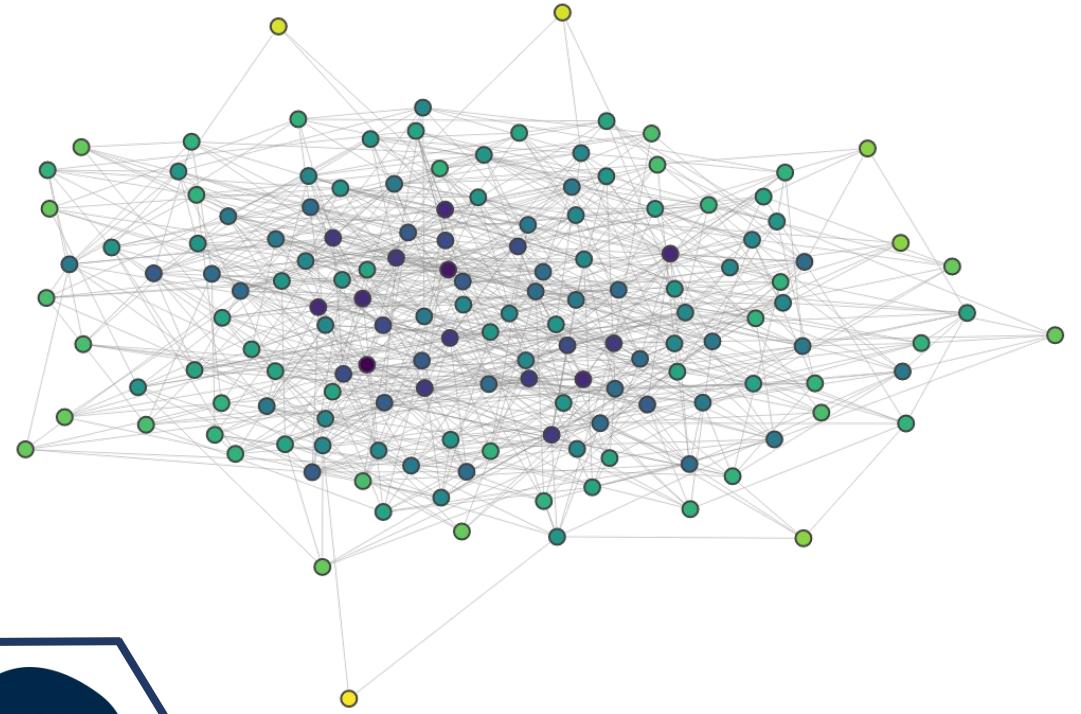
Knowledge graphs



Many other fields

- Biology
- Recommendation systems
- Computer vision
- Medical diagnosis
- Robotics
- ...

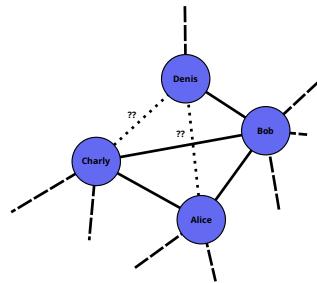
Complexity



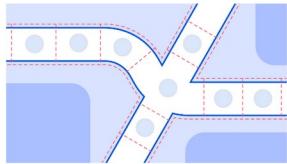
- Might have several thousand nodes/edges
- Number of edges/nodes might vary a lot
- ...

Vocabulary: Node/Vertex

Persons

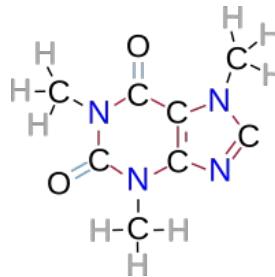


Road sections

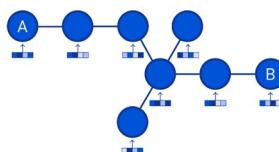


Some example of nodes

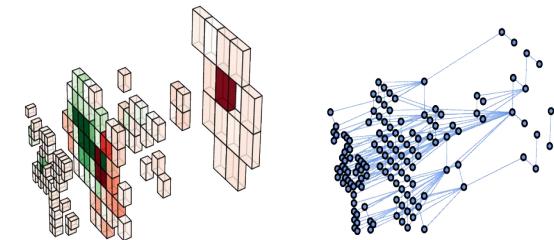
Atoms



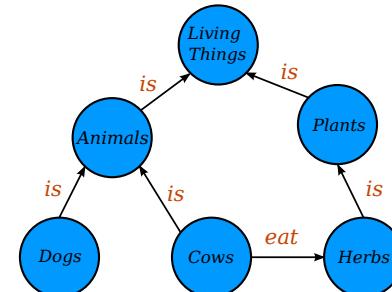
A concept



Particles



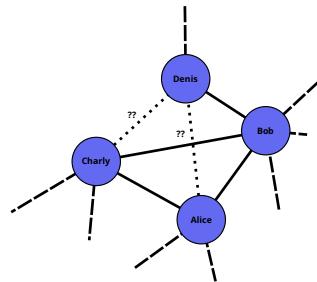
Many other fields



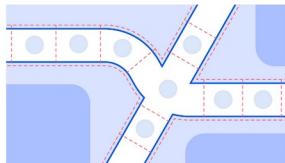
- Biology : an aminoacid in a protein
- Recommendation systems : a customer
- Computer vision : an element in a picture
- Medical diagnosis : Brain region (MRI)
- Robotics : joints
- ...

Vocabulary: Edges

Relationship

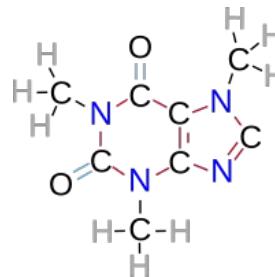


Time, connection

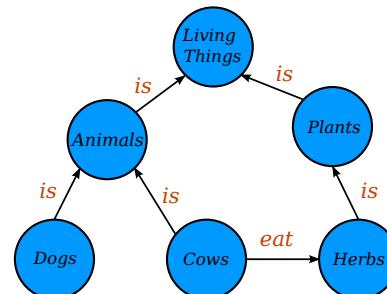


Some example of nodes

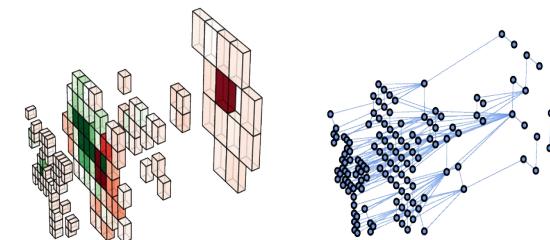
Type of bond



A statement



Decayed to

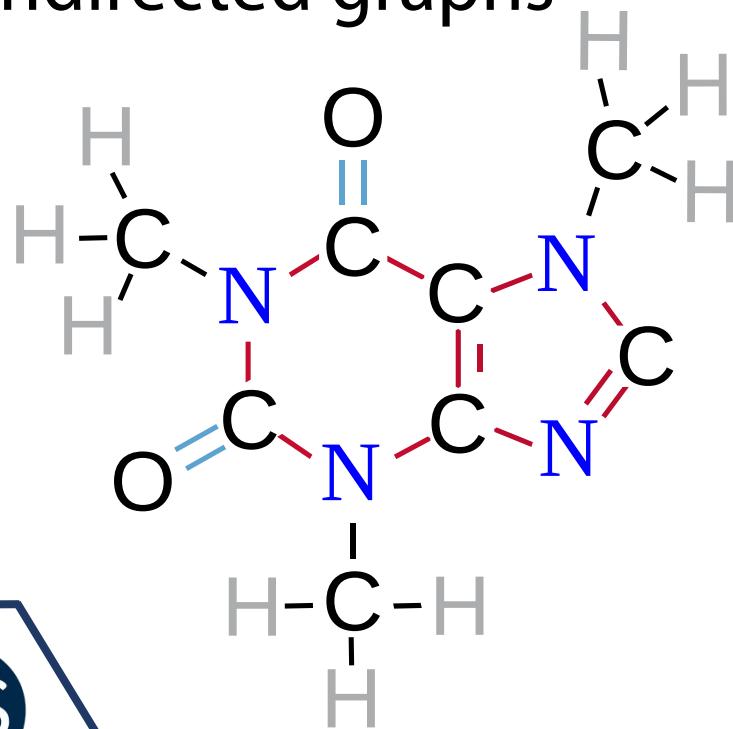


Many other fields

- Biology : distance between residues
- Recommendation systems : connected customers
- Computer vision : an interaction between elements
- Medical diagnosis : interaction between brain regions (MRI)
- Robotics : connection between joints
- ...

A relationship can be symmetrical or not between nodes

Undirected graphs

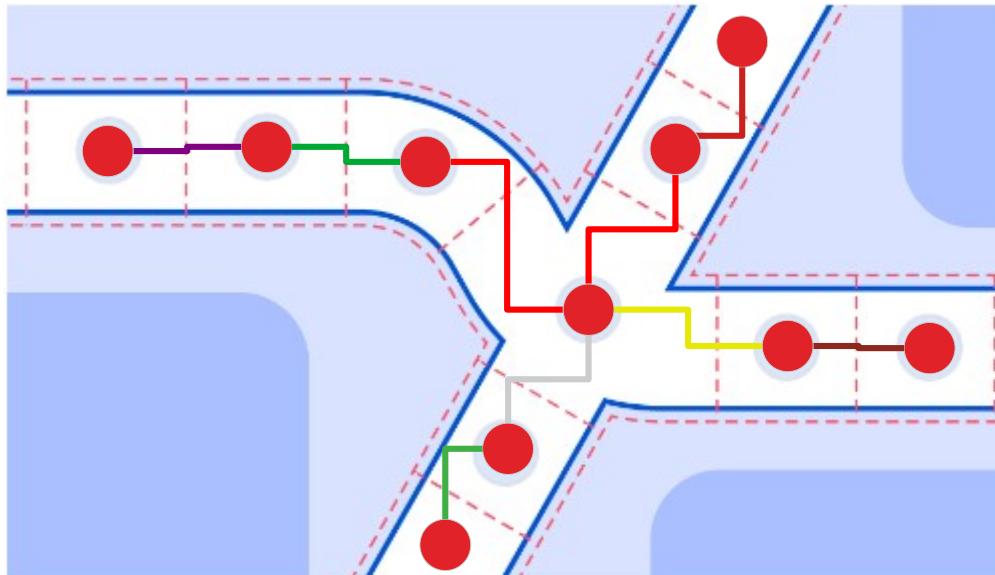


Directed graphs



Vocabulary: Edges weight

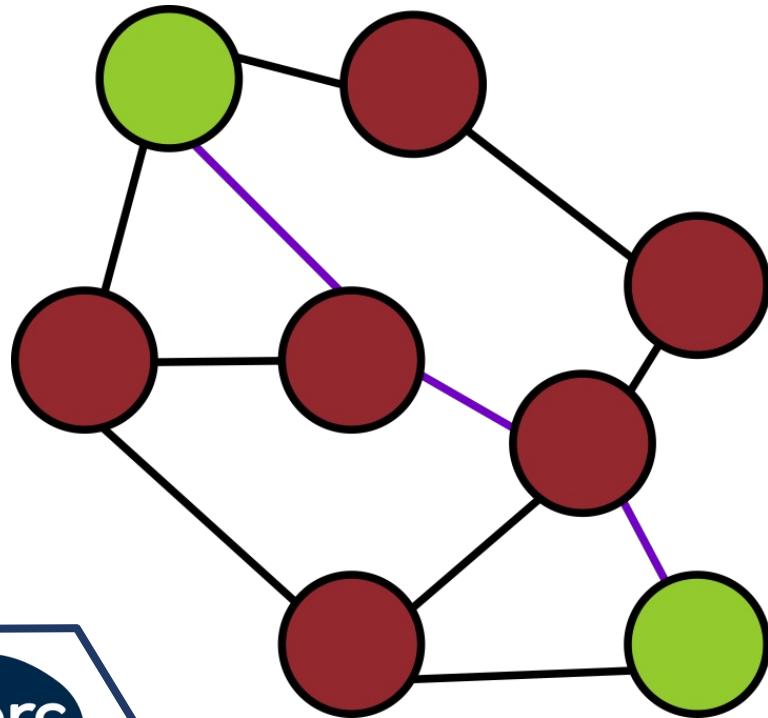
Edges can carry more information



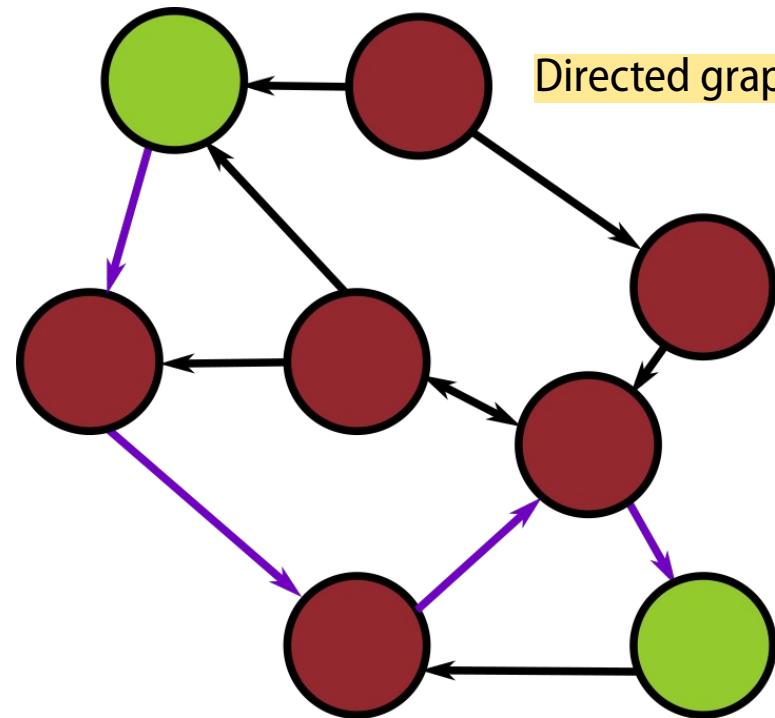
Vocabulary: Paths

A path is a sequence of edges connecting 2 nodes

Undirected graph



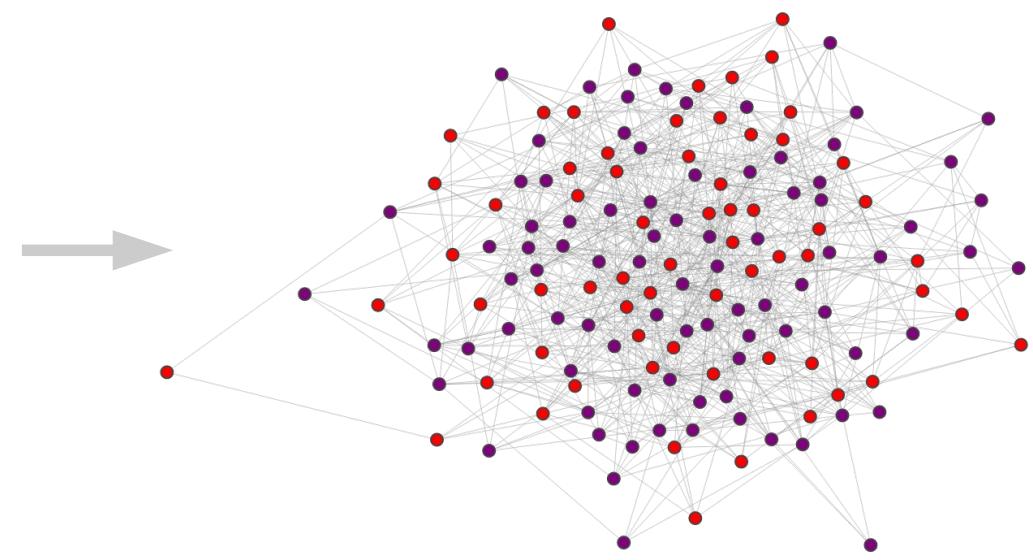
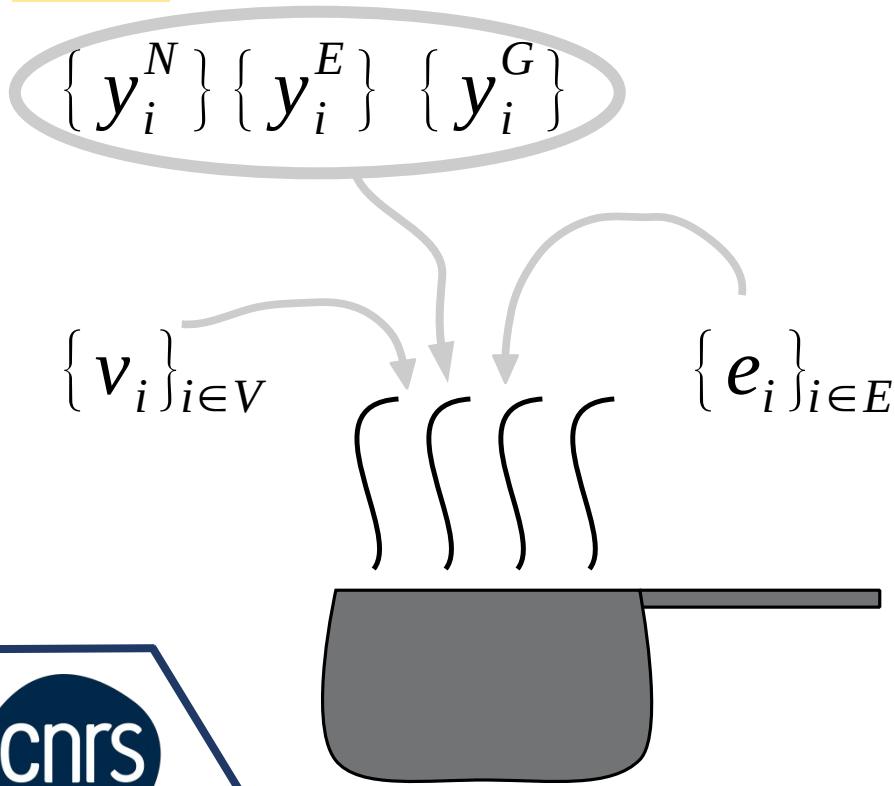
Directed graph



Formal definition

$G = (V, E)$: a set of nodes and edges

Labels



Graphs store information: Labels

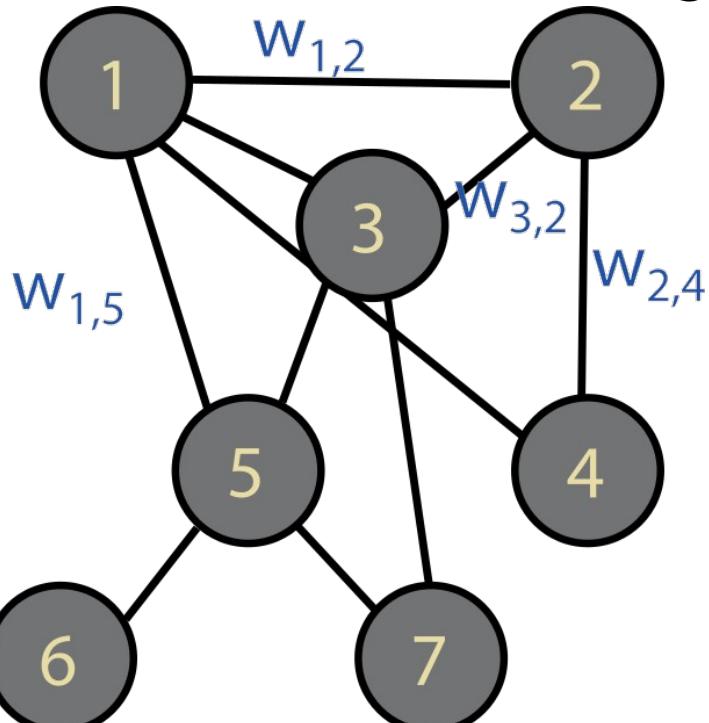
- Graphs can store information (features) on **nodes**, **edges** and **globally**

	Globally	Nodes	Edges
Social Network	Group of interest, ...	Name, Age, Job, ...	Is friend, follows, family, ...
Molecule	Is a drug, Energy, ...	Atomic number, ...	Bond order, ...
Citations	Field, ...	Article, ...	Was cited, ...
Particle physics	Experiment	Particle	Decayed to, ...
Motion capture	Character	Joints	Is connected to, ...
Natural language	Paragraph, ...	Group of words, ...	Refers to, ...

- It can be a number, a concept, ...

Node proximity and centrality

Measure of the structure of a graph



Node proximity

- 1st order: $w_{i,j}$ between node i and j
- 2nd order: similarity of neighborhood structure
- Higher orders possible

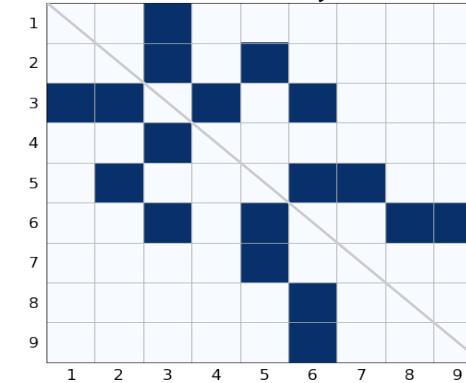
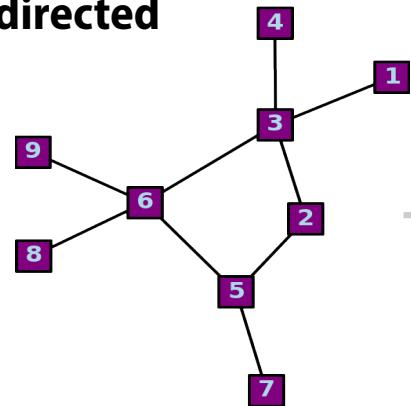
Node centrality

- Measure how many paths goes through the node

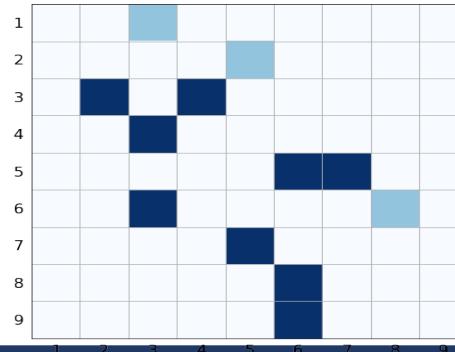
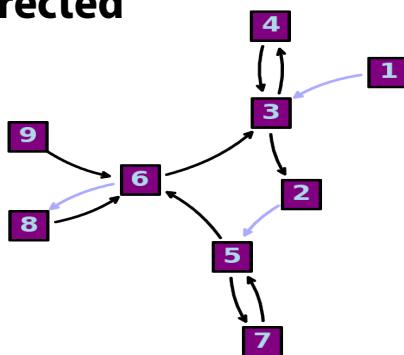
Graph representation

Adjacency matrix $W_{(i,j)} = \begin{cases} w_{i,j} & \text{if there is an edge} \\ 0 & \text{if not} \end{cases}$

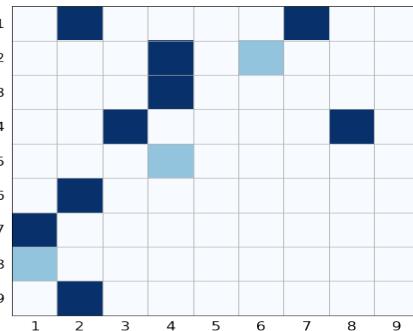
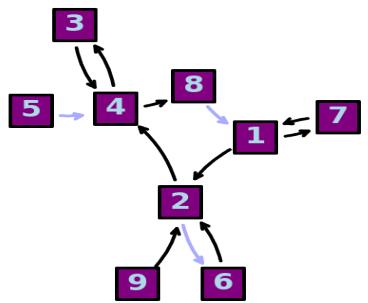
Undirected



Directed



Adjacency list



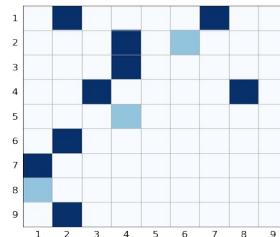
Nodes: [1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0]

Edges: [0.4, 0.4, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 0.4, 1.0, 1.0, 1.0]

Adjacency list: [[5, 4],
[8, 1],
[4, 8], [4, 3],
[3, 4],
[1, 7], [1, 2],
[2, 4], [2, 6],
[7, 1],
[6, 2],
[9, 2]]]

Global: [1.0, 1.0]

Graph representation



Adjacency list: [[5, 4],
[8, 1],
[4, 8], [4,3],
[3, 4],
[1, 7], [1, 2],
[2, 4], [2, 6],
[7,1],
[6, 2],
[9, 2]]

- Scale $V^{**}2 \rightarrow$ lot of space
- Sparse
- $N!$ permutations to represent the same graph
- Easy to find an edge
- Scale $E \rightarrow$ less space
- Might be difficult to find an edge

V = number of nodes/vertices
 E = number of edges

<https://www.geeksforgeeks.org/comparison-between-adjacency-list-and-adjacency-matrix-representation-of-graph/>

Useful Matrices

Adjacency	W	Weight of edges
Degree	D	Diagonal matrix with number of edges for each node
Laplacian	L	D - W
Node Features	X	Information stored

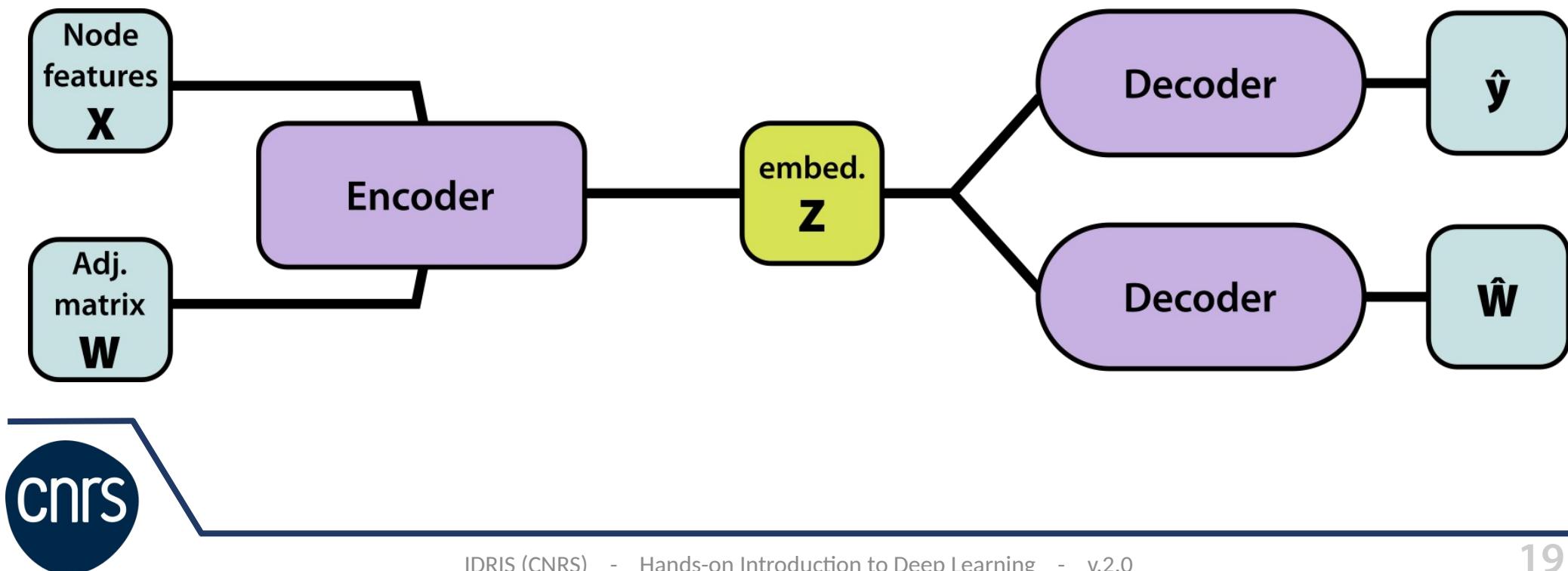


Learning on Graphs

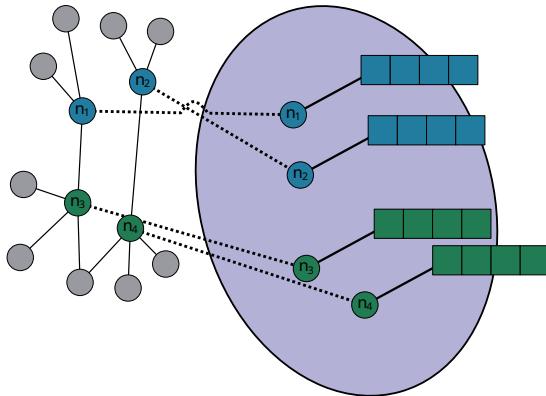


Graph embedding

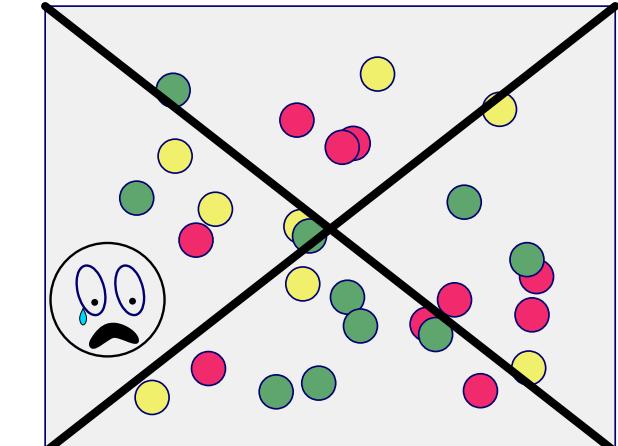
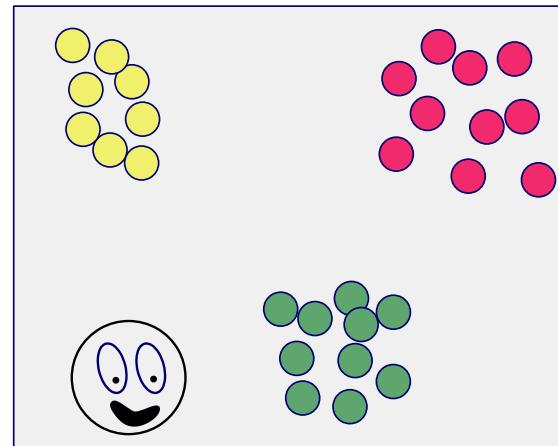
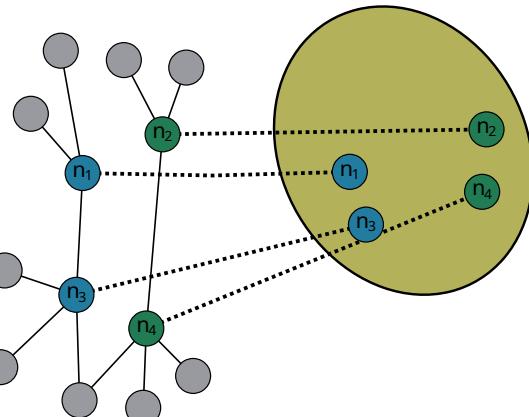
- We need to find a representation of the graph that is processable



Graph embedding



- Features stored in nodes/edges/graphs are not easily processed.
- We transform the features into a vector in the latent space (**Dimension is a hyperparameter**).
- The embedding has to be suited for the task → **Learnable**.

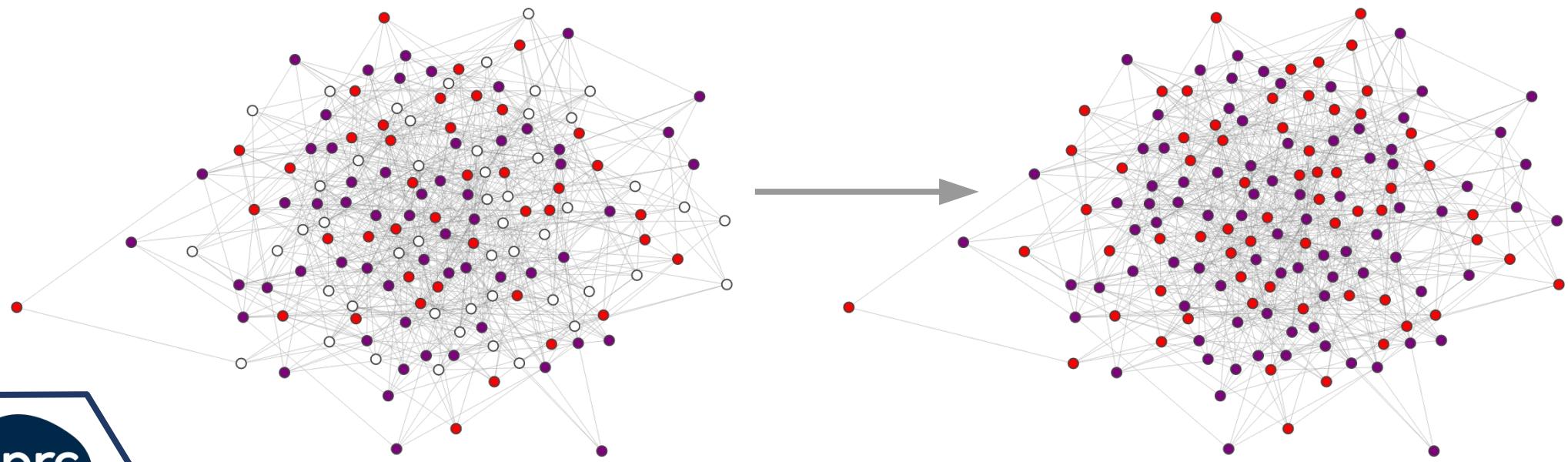


Transductive learning

The model has access to the complete graph

It is not possible to add new nodes

Node labeling

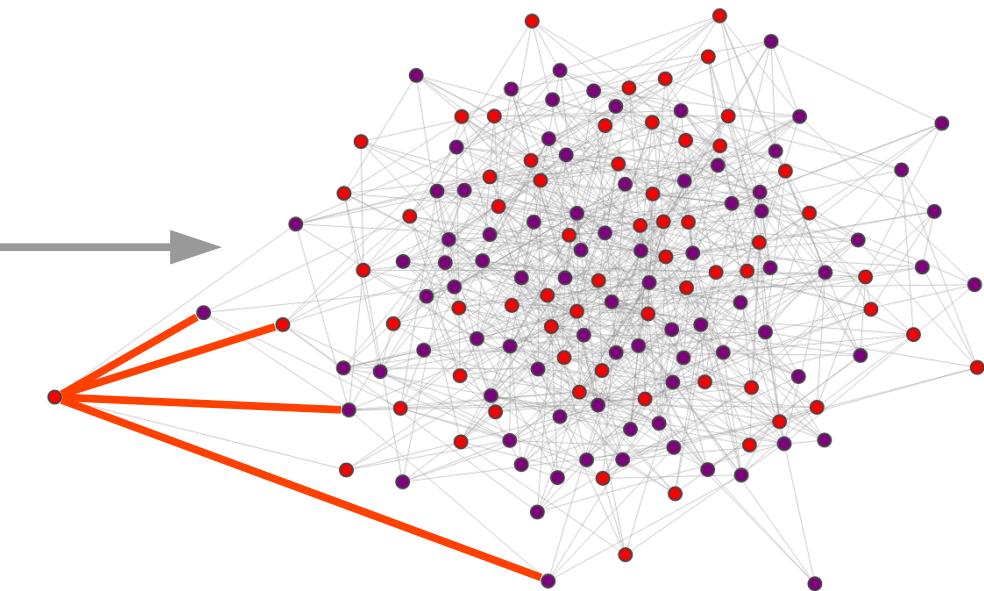
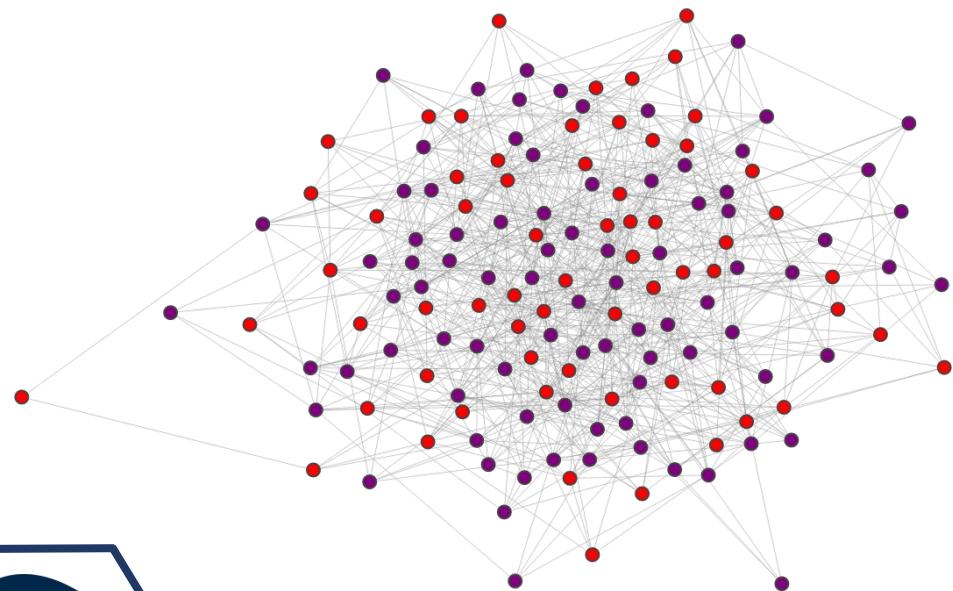


Transductive learning

The model has access to the complete graph

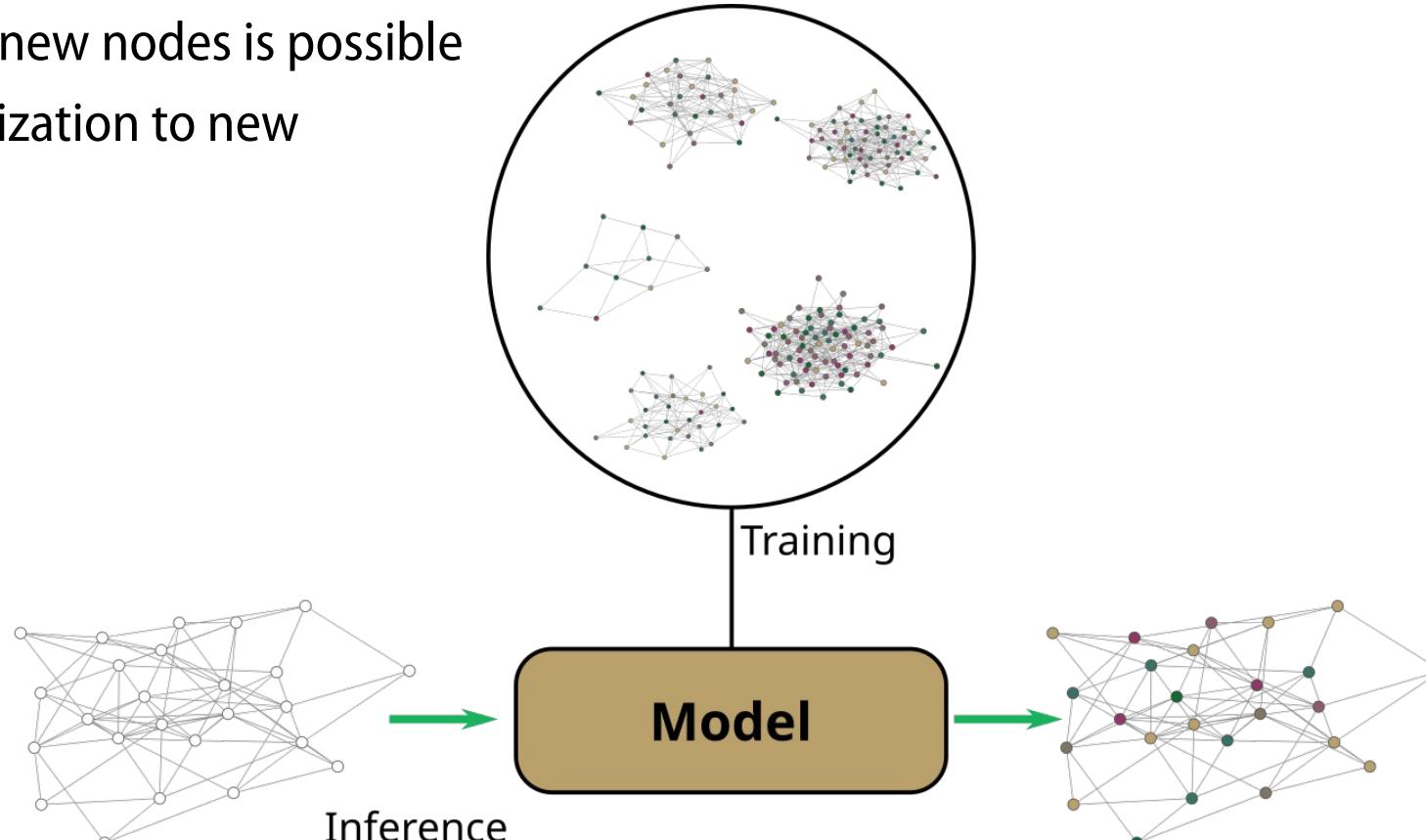
It is not possible to add new nodes

Find new edges



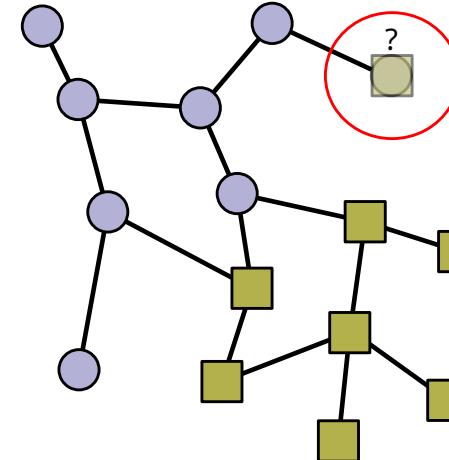
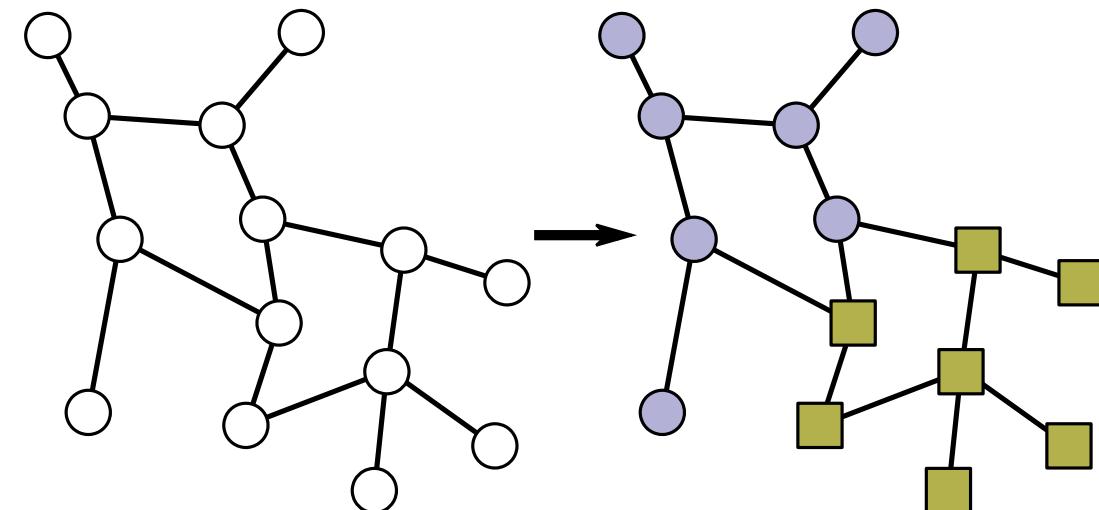
Inductive learning

- The model has access only to a part of the graph (train set)
- Adding new nodes is possible
- Generalization to new graphs



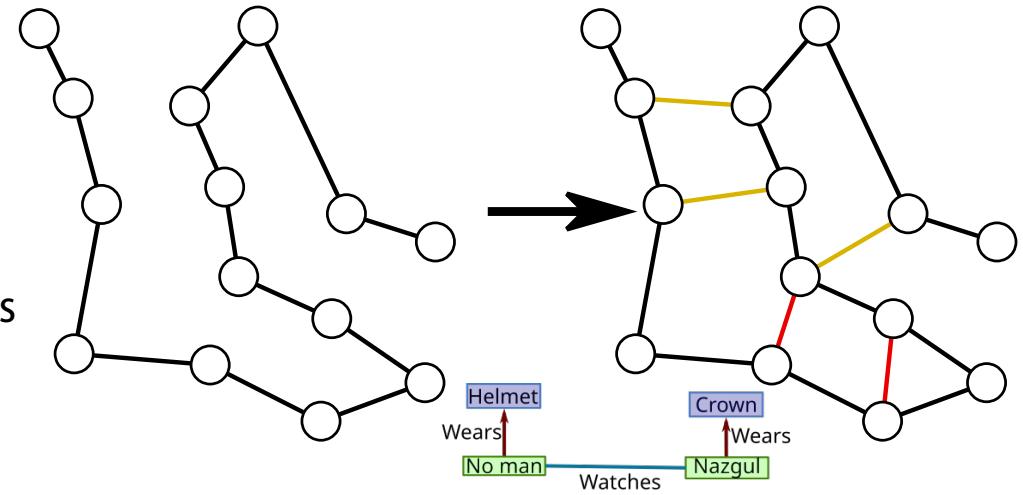
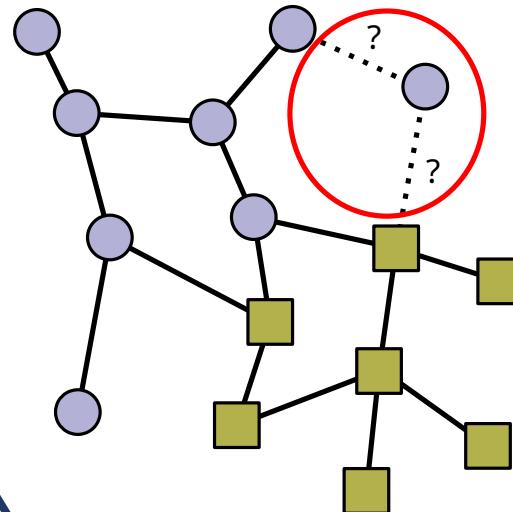
Tasks on nodes

- Labeling nodes in a graph
(clustering)
 - Find topic of a research paper
(CORA, etc)
 - Find bots in a social network
 - ...
- Labeling new nodes
- Perform regression



Tasks on edges

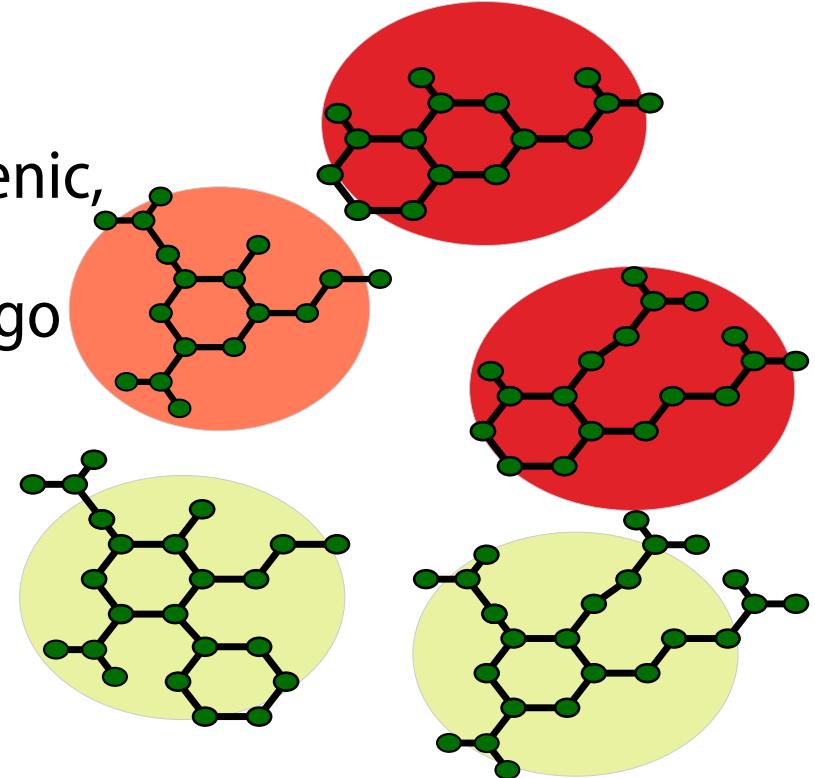
- Find relationships
 - Contact map of aminoacids (AlphaFold)
 - Contact suggestion (social network)
 - ETA for directions (regression)
 - Relationships between segments in pictures
 - ...



Scene Graph Generation
<https://cs.stanford.edu/~danfei/scene-graph/>

Tasks on graphs

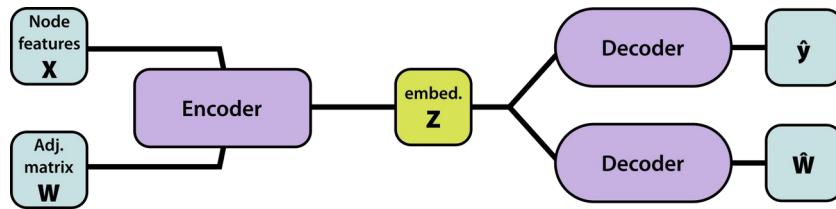
- Predict properties of graphs
 - Chemical properties (solubility, carcinogenic, possible drug)
 - Classification of the research field in an ego network
 - ...



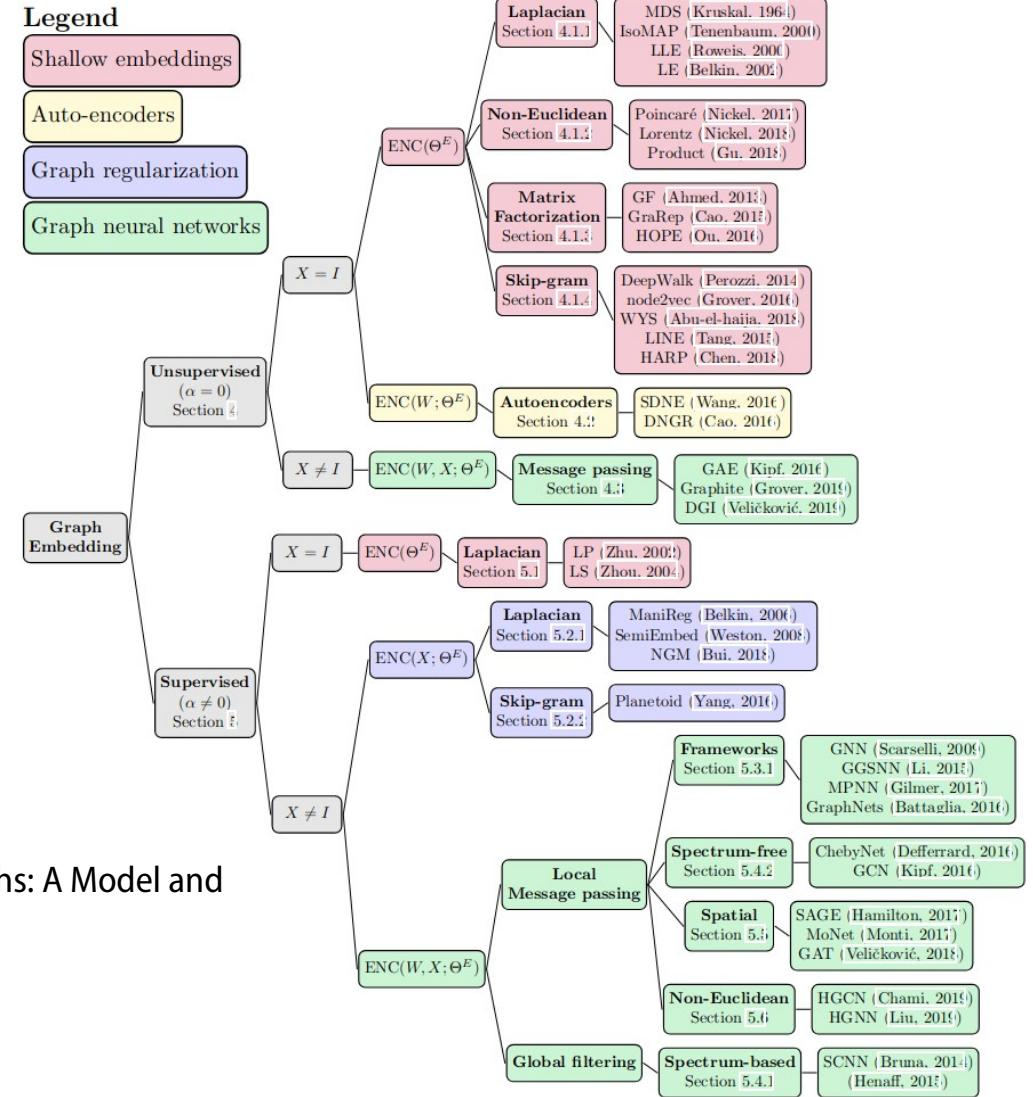
A few examples



Taxonomy of methods

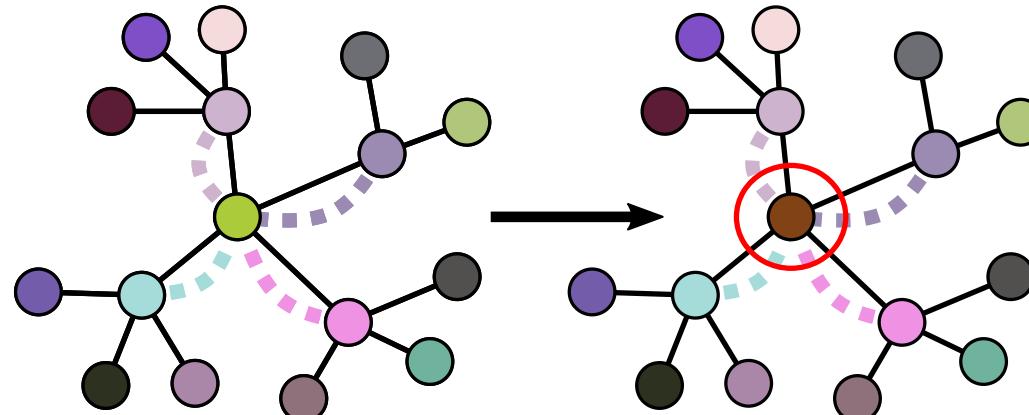


I. Chami, S. Abu-El-Haija, and B. Perozzi, "Machine Learning on Graphs: A Model and Comprehensive Taxonomy".



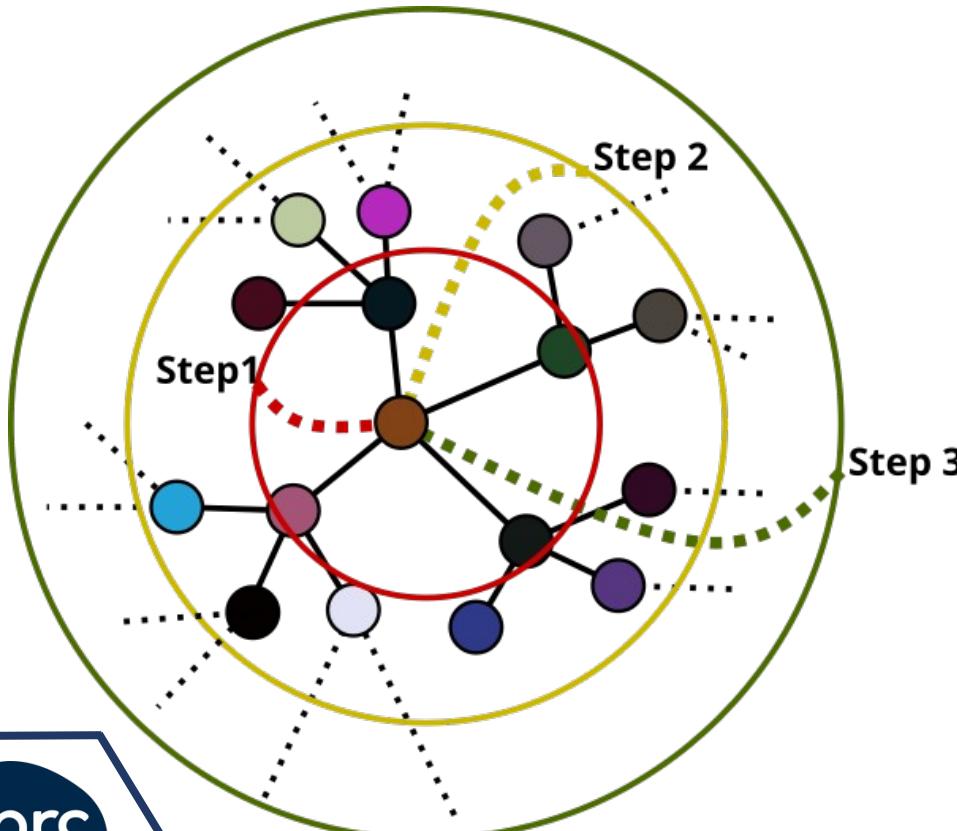
Graph convolution

- Just like for images we can learn from neighborhood with a convolution operator.



- A bit more complex since the number of neighbors is unlikely to be constant.
- We want the operator to be permutation invariant.

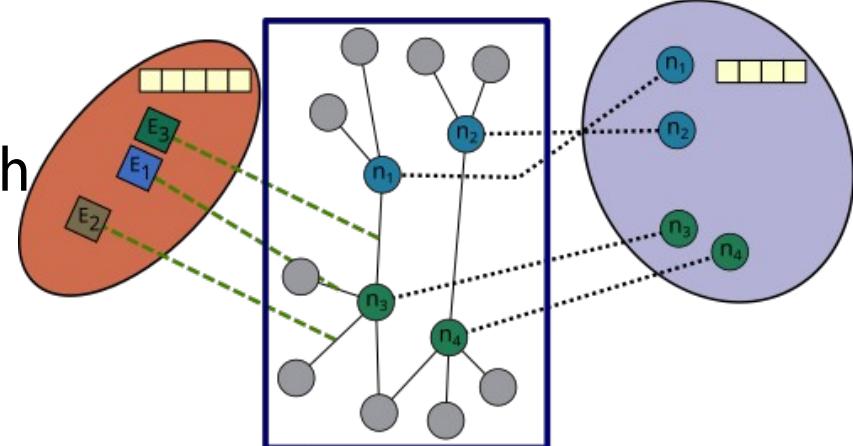
Graph convolution



- Several steps are needed to retrieve information for distant nodes.
- For large graphs → a **cutoff**
- It is possible to use a **virtual node** connected to all other nodes. But in practice this becomes quickly intractable.

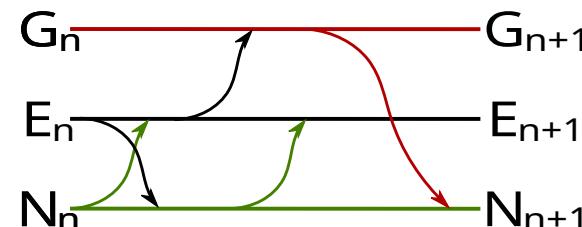
Message passing

- We have embeddings for each part of the graph (possibly different vector sizes).
- Each part can learn from the others via a transformation.

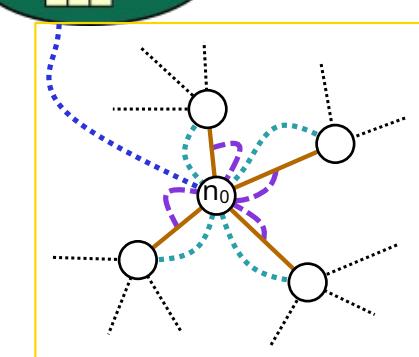


$$\begin{array}{c} \text{Edge embedding} \\ \text{Learnable transformation} \\ \times \end{array} = \begin{array}{c} \text{Node embedding} \end{array}$$

A diagram illustrating the learnable transformation of edge embeddings into node embeddings. An edge embedding (a row of yellow squares) is multiplied by a learnable transformation matrix (a purple grid) to produce a node embedding (another row of yellow squares).



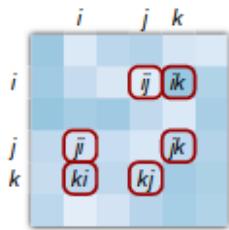
- Information is aggregated to form a message that the node/edge will send to others.



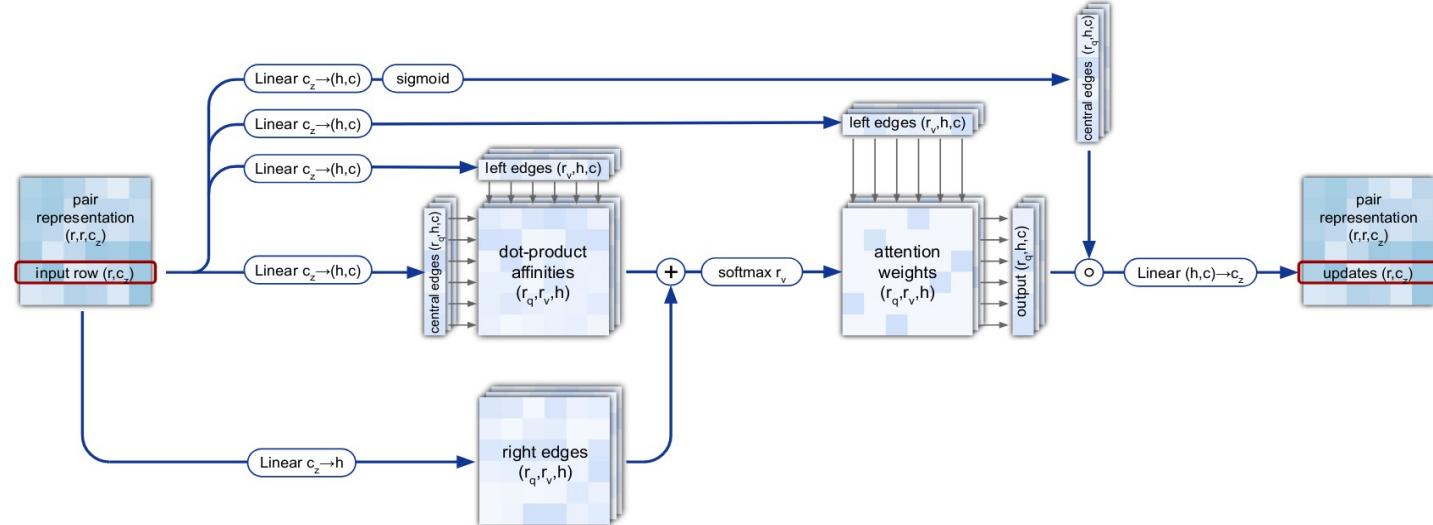
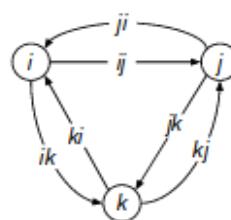
$$n_0 = (f_{NN}, f_{EN}, f_{GN})$$

AlphaFold transformer

b Pair representation (r, r, c)

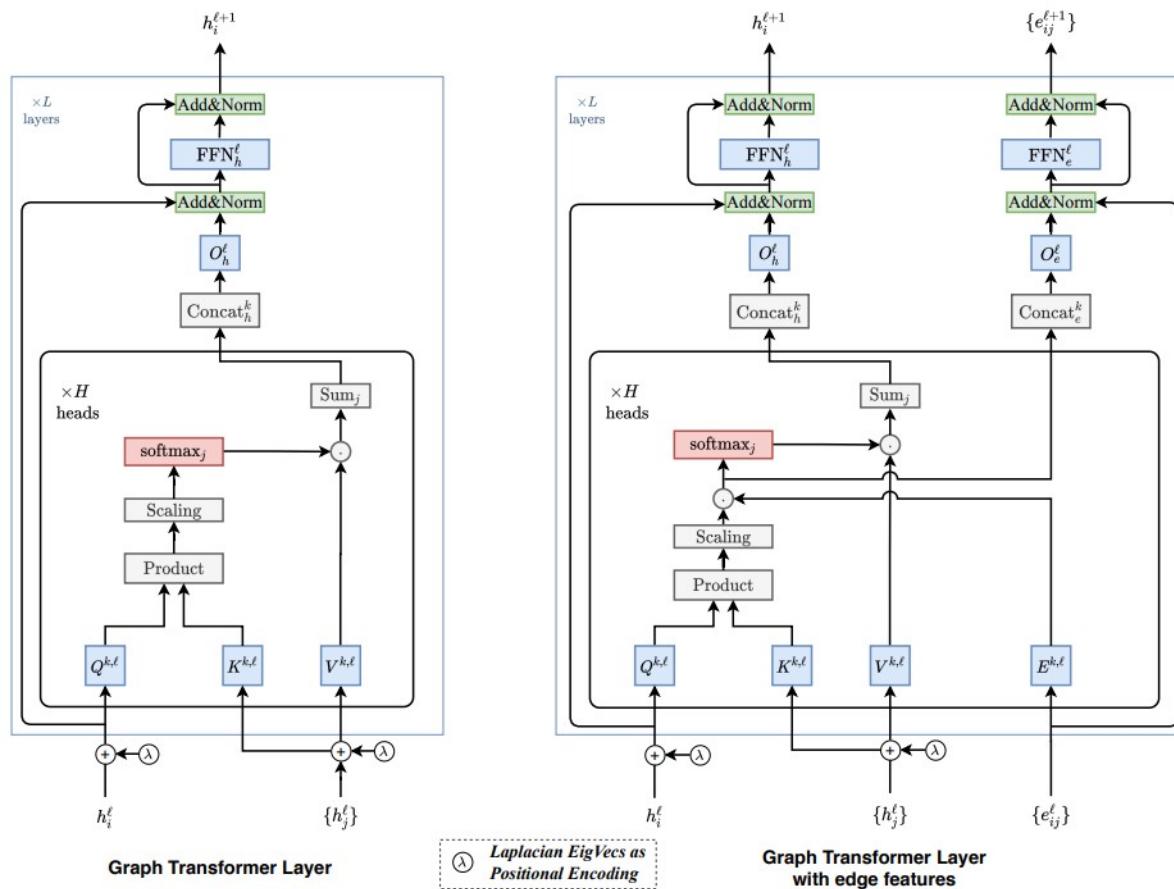


Corresponding edges in a graph



Supplementary Figure 7 | Triangular self-attention around starting node. Dimensions: r: residues, c: channels, h: heads

Graph Transformer Network



Dwivedi, Bresson A Generalization of Transformer Networks to Graphs 2020, <https://arxiv.org/abs/2012.09699>

Libraries

- Pytorch Geometric
- Deep Graph Library
- Graph Nets
- Spektral
- ...

- <https://logconference.org/>
- <https://ogb.stanford.edu/>

Tutorials

- https://antoniolonga.github.io/Pytorch_geometric_tutorials/
- <https://docs.dgl.ai/tutorials/blitz>



References

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 - Introduction to Graph Neural Networks (Introduction to Graph Neural Networks)
- Websites
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 - <https://neptune.ai/blog/graph-neural-network-and-some-of-gnn-applications>
 - <https://venturebeat.com/2021/10/13/what-are-graph-neural-networks-gnn/>
 - <https://theaisummer.com/graph-convolutional-networks/>
 - <https://towardsdatascience.com/node-embeddings-for-beginners-554ab1625d98>
- Articles
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