Group F

JUMP FROM SPACE

On the 14th October 2012, skydiver Felix Baumgartner embarked on a life changing journey into the stratosphere in a helium balloon. From his capsule, approximately 39 kilometres above sea level, Baumgartner then **jumped from space**, and in the process became the first human to break the sound barrier. The record-breaking free fall lasted for 4 minutes and 20 seconds before the parachute was deployed and Baumgartner descended safely back to Earth.

ASSUMPTIONS:

The ground-breaking jump can be modelled using mathematics, but first we start with some assumptions:

1. The combined mass of Felix Baumgartner and his suit is constant for the duration of the free fall.
2. Acceleration due to gravity is constant in this model, at a value of 9.75 ms^{-2}. This is an average of the values at the top and bottom of the jump, as gravity does not remain constant.
3. The Earth's atmosphere can be modelled in layers, with each layer having a unique, constant value of air density. The values for air density used were found using Figure 1.

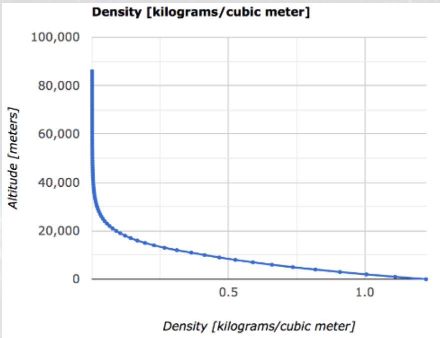


Figure 1: Graph of air density against altitude [1].

4. Baumgartner moves in a straight, vertical line for the entirety of the journey – there is no horizontal movement, and he does not spin during the jump.
5. The height of the Baumgartner at any point in the journey is measured from sea level.

We also must consider the effect of air resistance on Baumgartner as he falls. We assume Baumgartner travels headfirst for the entire journey, and thus model his cross-sectional surface area as an ellipse with the dimensions as illustrated in Figure 2.

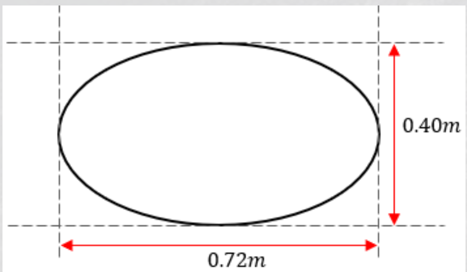


Figure 2: Model of Surface Area.

The cross-sectional surface area, A, of Baumgartner can hence be calculated as $0.36 \; x \; 0.20 \; x \; \pi \; = \; 0.072\pi \; = \; 0.23m^2$ (2.d.p).

METHODOLOGY:

From the outset this problem is very complicated due to the air resistance. Air resistance is dependent on k, where $k = \frac{\rho C_d A}{2}$ [2]. We can calculate a rough estimate of Baumgartner's surface area as before mentioned, which gives $0.23m^2$. Research into estimates for the drag coefficient, C_d, provides us with a reasonable value of 0.29 [4], which suggests that he is more streamlined than a sphere but less streamlined than an aeroplane wing.

Air resistance is a significant aspect of the journey and changes as he falls towards Earth. Taking this into account, we split his descent into 6 sections. The first section is $4000m$ vertically and all subsequent sections span $5000m$.

THE MODEL:

Downwards Force:

$F_g \; = \; mg$

Where:

- $F_g =$ The force of gravity, in Newtons.
- $m =$ The combined mass of Baumgartner and his suit, in kg [5].
- $g =$ The acceleration due to gravity $= 9.75ms^{-2}$.

Air Resistance:

$$F_d = -kv^2 \;\; \text{and} \;\; k = \frac{\rho C_d A}{2}$$

Where:

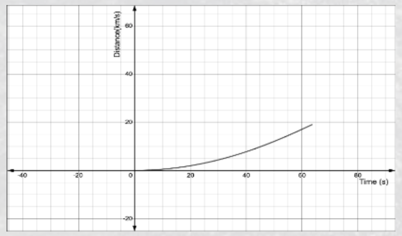
- $F_d =$ The drag force.
- $\rho =$ Air density, in kgm^{-3}.
- $C_d =$ The drag coefficient $= 0.29$
- $A =$ Surface area of affected body $= 0.23m^2$.
- $v =$ The speed of the falling body, in ms^{-1}.

NOTE: k is negative because the drag force opposes motion.

We also generally define:

- $x =$ Height of Baumgartner above sea level, in metres.
- $t =$ Time from start of jump, in seconds.

Figure 3: Distance against time.



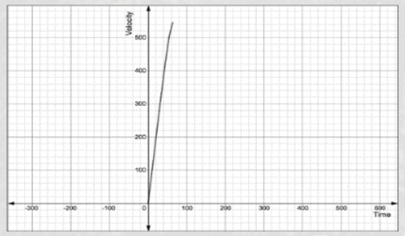


Figure 4: Speed against time.

Resultant Force:

To model his speed throughout the journey, we use Newton's 2nd Law. We can find a resultant force by combining the drag force and the weight force as follows:

$$m\frac{dv}{dt} = mg - kv^2$$

Solving this differential equation then produces an equation for v in terms of t

$$v = \frac{\sqrt{g}\sqrt{m}\tanh\left(\frac{c_1\sqrt{g}\sqrt{k}m + t\sqrt{g}\sqrt{k}}{\sqrt{m}}\right)}{\sqrt{k}}$$

where c_1 is a constant of integration.

Integrating this equation for speed then leads to an equation for distance with respect to time:

$$x = \frac{m\log\left[\cosh\left(\frac{\sqrt{g}\sqrt{k}(c_1 m + t)}{\sqrt{m}}\right)\right]}{k} + c_2$$

where log is the natural logarithm and c_2 is a second constant of integration.

This equation can be plotted, as shown in Figure 3.

This equation can also be rearranged to find the time at which Baumgartner reaches a certain distance

$$t = \frac{\sqrt{m}\cosh^{-1}\left(e^{\frac{(x-c_2)k}{m}}\right)}{\sqrt{g}\sqrt{k}} - \; c_1 m$$

c_1 can be calculated using the equation for speed when $v = 0$ and $t = 0$. From this, c_2 can then be calculated using the equation for distance. After finding these two values, the time at which Baumgartner reaches the end of the first section of the fall can be calculated.

Using these constants, we can then plot our equation for speed against time, as shown in Figure 4.

ISSUES:

The model breaks after the fourth section of the descent. tanh( ) becomes undefined meaning that a c_1 value cannot be calculated.

There are several reasons for this. Firstly, the change in air density over each 5km section is still large, and so mean values are potentially inaccurate.

Secondly, the estimated drag coefficient could be inaccurate. The drag coefficient is determined experimentally and so trying to predict the value is imprecise.

Finally, we have modelled his surface area as a 2-D shape. Modelling him as a 3-D shape could lead to a different surface area and hence a more accurate model.

All these problems can be attributed to the inaccuracy in the k value. The k value being inaccurate is the likely cause of the model breaking.

PREDICTIONS:

Our model predicts that Baumgartner will reach the speed of sound at 35.57 seconds into the journey.

He reaches a speed of $545.841ms^{-1}$ before the model breaks.

VALIDITY:

There are clear limits to the validity of the model due to it breaking down. As a result of this, the model does not show a reduction in speed at any point which is undoubtedly incorrect.

Comparing the predictions made from our model to the data released by Red Bull [6] also highlights the inaccuracy of the model. Baumgartner reached a maximum speed of $377.11ms^{-1}$ which is almost $200ms^{-1}$ slower than our model predicts.

In the actual jump he reaches the speed of sound after approximately 40 seconds, whereas our model predicted that this would occur at 35.57 seconds.

BIBLIOGRAPHY: [1] *Standard Atmosphere Calculator*. Available: www.digitaldutch.com/atmoscalc/graphs.htm. [2] Matthew West. (2015). *Dynamics*. Available: http://dynref.engr.illinois.edu/afp.html. [3] Wolfram Alpha www.wolframalpha.com [4] Journal of Engineering. (2017). *Terminal Velocity in Skydiving*. Available: www.longdom.org/open-access/buoyancy-explains-terminal-velocity-in-skydiving-2168-9792-1000189.pdf. [5] Live Science. (2012). *The Physics of the First Ever Supersonic Skydive*. Available: www.google.co.uk/amp/s/www.livescience.com/amp/23710-physics-supersonic-skydive.html. [6] Red Bull Stratos. (2013). *Mission Data*. Available: www.redbull.com/gb-en/red-bull-stratos-release-mission-data.