Special Relativity

Hamza Idris

1 Introduction

Many of you will be familiar with classical mechanics. This is a powerful area of mathematics and physics that describes the motion of objects, from projectiles to Formula One cars (Dyson 2020). Common and famous formulae include: $F = ma$, $E_k = \frac{1}{2}mv^2$ and $E_g = mgh$. Classical mechanics gives very accurate results when certain conditions are met. If these conditions are not met, then classical mechanics does not give accurate and reliable answers. For example when the size of an object is near or less than $10^{-9}m$, quantum mechanics is used instead of classical mechanics (Wikipedia 2021a). Some areas of quantum mechanics include: the uncertainty principle, discrete quantization and wave-particle duality (Wikipedia 2021b). One application of quantum mechanics is designing quantum computers, which can solve combinatorial problems such as the travelling salesman problem much quicker than ordinary computers (Lochhead 2019). While quantum mechanics is a fascinating and interesting area of physics, this report will be based on special relativity. In a nutshell, special relativity is used instead of classical mechanics when an object moves at speeds around or greater than $\frac{c}{10}ms^{-1}$, where c is the speed of light, which is $299,792,458ms^{-1}$ (Boston University 1998).

2 Frames Of Reference and Galilean Transformations

2.1 Frames of Reference

Suppose there is someone driving a car and there is an observer outside the car. Suppose the car is travelling at $30ms^{-1}$. How fast is the driver moving? Well it depends relative to what. They will be travelling at:

- $0ms^{-1}$ relative to the car
- $400ms^{-1}$ relative to the centre of the Earth because of the Earth spinning on its axis (Newman 2020)
- 30,000ms^{-1} relative to the Sun because of the Earth orbiting the Sun (Newman 2020)

This is why it is important to consider motion relative to another object. In this example, motion will be considered relative to a stationary observer on the surface of the Earth. So in the car example, the driver will be travelling at $30ms^{-1}$ relative to the observer on the Earth. An intriguing consequence of this is that the 'stationary' observer on Earth will be moving at $30ms^{-1}$ relative to the car and its driver.

Now that we have defined relative motion between two objects, we will now define frames of reference. This is a coordinate system for each of the two objects which describes each of the objects' position and velocity for given times.

In this report, unless otherwise stated, relative motion will always be between a moving object relative to a stationary observer on the surface of the Earth. Therefore there will be two frames of reference. One will be for an observer on the surface of the Earth and one will be for the moving object.

2.2 Galilean Transformations

Suppose there are two frames of reference travelling at constant velocities $v << c$. It is possible to define a mathematical relationship between the two frames of reference. Suppose that there are two frames of reference where one frame of reference is stationary (call this S) and the other frame of reference is moving at velocity u in the x-axis relative to the stationary frame (call this S'). Let's also suppose at time $t = 0s$ (original time) the two frames start at the same position, and the moving frame travels for t seconds. This can be viewed by the following diagrams:

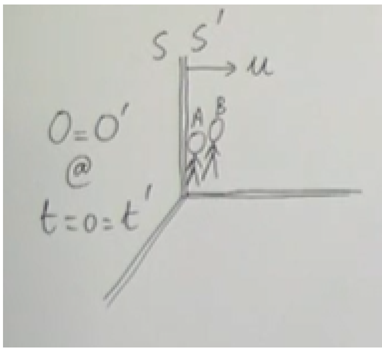

Figure 1: Both frames of reference at time $t = 0$ (Biezen 2015)

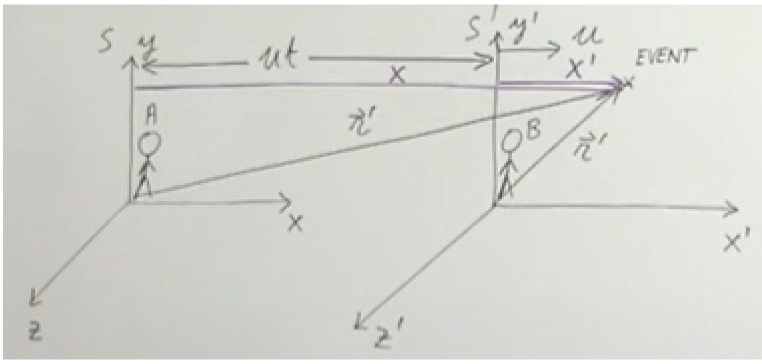

Figure 2: Both frames of reference at time t (Biezen 2015)

There is an event where we want to calculate the distance of it from the origin of S. This distance is x metres. The distance between the origin of S and S' is ut metres, because the frame S' is travelling at velocity u for t seconds. The distance from the event to the origin of S' is x' metres. Therefore we have $x = x' + ut$ (1). Differentiating this expression will lead to information about relative velocities and acceleration. We can see that:

- differentiating (1) once leads to $v = v' + u$ (2), where v is the velocity of the event relative to frame S and v' is the velocity of the event relative to the frame S'

- differentiating (1) twice leads to $a = a'$, where a is the acceleration of the event relative to frame S and a' is the acceleration of the event relative to frame S'

Given that the speeds are significantly less than the speed of light, it can be assumed that $t = t'$ (the time elapsed in both frames of reference are the same).

The Galilean Transformation is accurate at speeds significantly less than the speed of light. However as the velocities approach $\frac{c}{10}ms^{-1}$ this model breaks down and Lorentz transformations are needed, which will be touched upon later in this report.

3 Postulates of Special Relativity and the Michelson-Morley Experiment

3.1 Postulates of Special Relativity

Special Relativity was discovered by Albert Einstein in 1905 and was derived by two key principles. These principles are known as the postulates of special relativity and state:

1. The laws of physics are the same in all inertial frames

2. The speed of light c is a constant, independent of relative motion of the source

The first postulate states that if there are frames of reference that are not accelerating, then measurements from one frame of reference, such as position, velocity and time can be converted from one frame of reference to another frame of reference. For example, if we again consider the frames of reference of a stationary observer on Earth and a car moving at a constant speed of 40 ms^{-1} relative to the observer, because both frames of reference are not accelerating, we can convert measurements between the frames of reference. The conversion is done either using Galilean transformations or Lorentz transformation.

The second postulate is best explained through examples. Consider the following image

Figure 3: Moving car and ball example (GeeklyEDU 2021)

The observer is stationary, the car is moving at $10ms^{-1}$ relative to the observer, and the ball is moving at $3ms^{-1}$ relative to the car and in the same direction as the car. By equation (2) from the previous section, the relative velocity of the ball with respect to the observer is $10 + 3 = 13ms^{-1}$.

Now consider the following image.

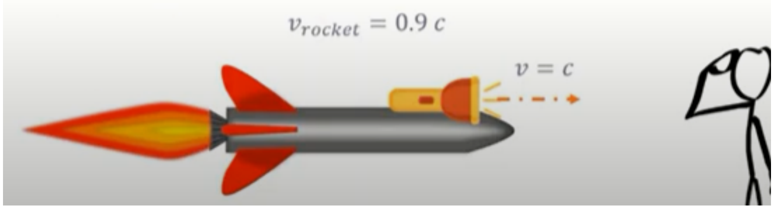

Figure 4: Rocket and light example (GeeklyEDU 2021)

The observer is stationary, the rocket is moving at $0.9c\ ms^{-1}$ relative to the observer and the light ray is moving at $c\ ms^{-1}$ relative to the rocket and in the same direction as the rocket.

The relative velocity of the light ray with respect to the observer is $c\ ms^{-1}$, even though $0.9c + c = 1.9c\ ms^{-1}$. This means the observer will see the light ray at speed $c\ ms^{-1}$, even though calculating relative velocities by Galilean and Lorentz transformations will give different answers.

3.2 The Michelson-Morley Experiment

The second postulate can be proved by the Michelson-Morley experiment. The following diagram shows the theoretical ideas for the Michelson-Morley experiment.

Figure 5: Speed of Light Relative to Earth's Motion

The relative velocity of the light ray from a stationary observer will be considered. The Earth moves around 180000 kmh^{-1} around the Sun. The speed of light is $c\ ms^{-1}$ relative to the moving Earth. Therefore from the stationary observer's viewpoint the speed of the light will either be

- $c\ ms^{-1}$ + $180000 kmh^{-1}$ if the light travels in the same direction as the Earth
- $c\ ms^{-1}$ - $180000 kmh^{-1}$ if the light travels in the opposite direction as the Earth

This leads on to the Michelson-Morley experiment. This was conducted by Albert Michelson and Edward Morley in 1905 and the setup of the experiment can be viewed in figure 6.

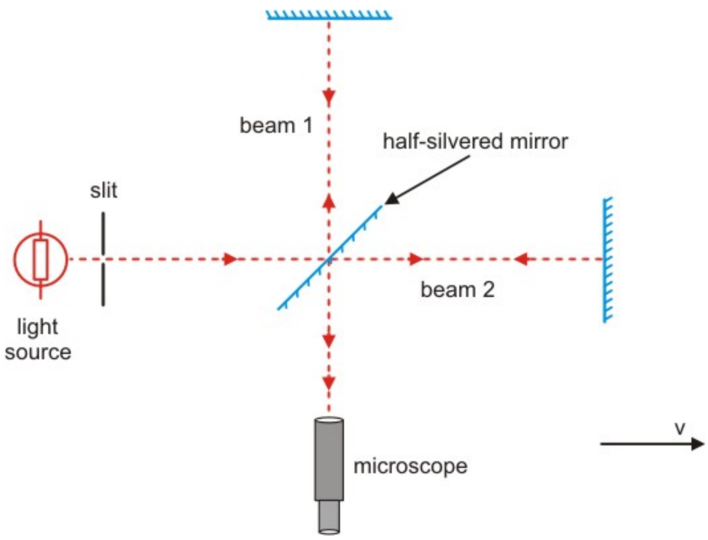

Figure 6: Michelson-Morley Experiment (Saburchill)

All the equipment grouped together is known as a Michelson Interferometer. The light would shine from the light source to the half-transparent mirror angled at 45 degrees. Half of the light would go up and half of the light would go right. The light would hit the two mirrors, reflect back to the half-transparent mirror and then reach the screen. On the screen the light patterns would be analysed. This experiment was then done by rotating the interferometer 90 degrees each time. By rotating the interferometer 90 degrees, the apparatus was in a different location relative to the movement of the Earth. Therefore it was expected that the speed of light relative to the stationary observer would be different, hence the light pattern on the screen should have been different. However Michelson and Morley discovered the same pattern on the screen each time, and hence concluded the speed of light is not relative to the motion of the Earth, and hence constant for all observers (Britannica 2006).

4 Lorentz Factor

Later in this report, the Lorentz Factor will be used a lot. Therefore, it will be useful to analyse it. The Lorentz Factor will be denoted by γ and is equal to $\frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$, where v is velocity of an object relative to an observer or object. The graph of the Lorentz Factor is given below.

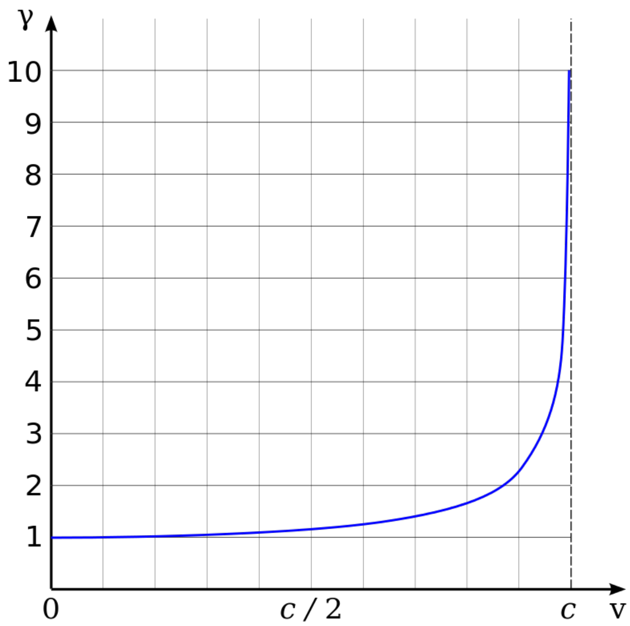

Figure 7: Lorentz Factor graph (Wikimedia 2007)

From the graph and expression for γ we can observe that:

- the domain of the Lorentz Factor is [0,c)
- the range of the Lorentz Factor is $[1, \infty)$
- $\lim_{x \to c} \gamma = \infty$
- if $v = c$ then $\sqrt{1 - \frac{v^2}{c^2}} = 0$ and $\frac{1}{0}$ is not defined
- if $v > c$ then $1 - \frac{v^2}{c^2} < 0$, and this will involve complex numbers, but these are not considered in the Lorentz Factor

- for $v > 0.85c$ the derivative of the Lorentz Factor will be significantly higher, because at these speeds the graph gets steeper
- differentiating γ with respect to v leads to $\frac{d\gamma}{dv} = \frac{v}{c^2}\frac{1}{(1-\frac{v^2}{c^2})^{\frac{3}{2}}}$

5 Time Dilation

5.1 Introduction to Time Dilation

At speeds respectable to $c\ ms^{-1}$, it becomes noticeable that the time an event takes is different according to which frame of reference you are in. For example suppose that there is a stationary observer on the Earth and a stationary person on a spaceship which moves at a constant speed respectable to $c\ ms^{-1}$. Lets suppose that the person on the spaceship shines a torch for t seconds in their frame of reference. Then the stationary observer on Earth will see the torch being shone for longer than t seconds. This is due to effects caused by time dilation. As a result of this phenomena, one famous phrase associated with this is that a moving clock runs more slowly than a stationary clock. Time dilation can be caused by two factors: gravitational fields and velocity. In this section we will focus on velocity. Suppose there is a stationary observer on Earth and suppose there is an object which moves at constant speed vms^{-1} relative to the stationary observer on Earth. Suppose from the the moving object's frame of reference that an event happens in time t_0 seconds. Then from the stationary observer's frame of reference the event happens in time t seconds, where t can be calculated by $t = \frac{t_0}{\sqrt{1-\frac{v^2}{c^2}}}$. It is important to note:

1. for time dilation to work, the stationary observer and moving object have to be in an inertial frame of reference (i.e. not accelerating)
2. if there is a stationary observer in a moving object, then t_0 will be same for the stationary observer in moving object and the moving object itself, and v is still the velocity of the moving object relative to a stationary observer on Earth

Lets consider an example. Suppose observer A is travelling on a train, which travels at a constant speed of $0.8c\ ms^{-1}$. Observer A switches on a torch for 3 seconds in their frame of reference. If observer B is standing stationary on a platform, how long will the torch be on from their point of reference? Here, we have $t_0 = 3s$ and $v = 0.8c\ ms^{-1}$. Therefore we have:

$$t = \frac{t_0}{\sqrt{1-\frac{v^2}{c^2}}}$$
$$t = \frac{3}{\sqrt{1-\frac{(0.8c)^2}{c^2}}}$$
$$t = 5$$

Therefore the observer on the Earth will see the torch on for 5 seconds.

5.2 Theoretical Proof of Time Dilation

For this derivation let us assume there is a stationary observer on Earth, and let there be a spaceship which travels at speed vms^{-1} relative to the stationary observer on Earth. On the spaceship assume there is an stationary observer and two mirrors shown in Figure 8.

Suppose light is shone from the bottom mirror to the top mirror, and we want to calculate the time taken t_0 for light to go from the bottom mirror to the top mirror once, from the observer on the spaceship's frame of reference. The distance the light travels is d metres, and the light travels at speed $c\ ms^{-1}$. Therefore $t_0 = \frac{d}{c}$ seconds.

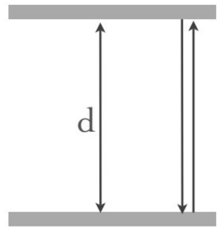

Figure 8: Mirror experiment from the observer on the spaceship reference (Metaphysikk)

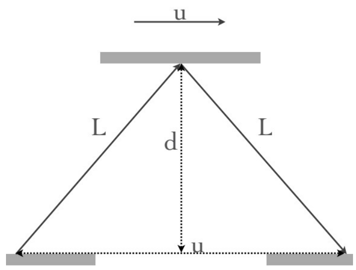

Figure 9: Mirror experiment from the observer on the Earth reference (Metaphysikk)

Now let us consider this event from the observer on the Earth's frame of reference. Let L be the length of the path the light takes from hitting the bottom mirror to the top mirror from the observer on the Earth's frame of reference in metres. Let t be the time taken for the light to travel from bottom mirror to top mirror from the observer on the Earth's frame of reference in seconds. Since the spaceship is travelling at speed ums^{-1} relative to the Earth observer, from the observer on the Earth's frame of reference, the light will travel a distance of $utms^{-1}$ horizontally and dms^{-1} vertically before hitting the top mirror. Therefore from Pythagoras' Theorem this leads to $L^2 = d^2 + (ut)^2$. Also, from the Michelson-Morley experiment, the observer on the Earth will still observe light to travel at $c\ ms^{-1}$, so $L = ct$ metres. From the observer on the spaceships frame of reference we know that $d = ct_0$ metres. Substituting this leads to $(ct)^2 = (ct_0)^2 + (ut)^2$. By mathematical manipulation this leads to:

$$c^2t^2 = c^2t_0^2 + u^2t^2$$
$$c^2t^2 - u^2t^2 = c^2t_0^2$$
$$t^2 - \frac{u^2}{c^2}t^2 = t_0^2$$
$$t^2(1 - \frac{u^2}{c^2}) = t_0^2$$
$$t^2 = \frac{t_0^2}{1-\frac{u^2}{c^2}}$$
$$t = \frac{t_0}{\sqrt{1-\frac{u^2}{c^2}}}$$

5.3 Experimental Proof For Time Dilation

Cosmic Rays are high energy radiation particles that are formed in the Solar System. When they reach the Earth's atmosphere, they collide with the particles in the Earth's atmosphere to form a range of particles. One of the particles formed is a muon particle. Muon particles are very unstable and quickly decay into more stable particles such as electrons and neutrinos. If there is a sample of a particular type of unstable particle, then the half-life is the time taken for half of the sample of that particular type of unstable particle to decay. Half-life is denoted by $t_{\frac{1}{2}}$. The half-life of a muon particle is 1.5μs. Since the muon particles are travelling at relativistic speeds, the number of muon particles that are observed in a laboratory using classical mechanics is considerably less than the number of muon particles that are actually present.

Let's consider an example. Suppose there are two detectors (detector 1 and detector 2) that are $2km$ apart. Suppose a sample of muon particles starts at detector 1 and travels at a speed of $0.99c\ ms^{-1}$. What percentage of the original sample does detector 2 detect as being muons?

We will first consider solving this problem using classical mechanics. The time taken for the muon sample to travel between the two detectors is $\frac{2000}{0.99*3*10^8} \approx 6.73\mu s$. The number of half lives during this time is approximately $\frac{6.73}{1.5} \approx 4.5$. Therefore the percentage of the original sample that now consists of muon particles is approximately $100*(\frac{1}{2})^{4.5} \approx 4.5\%$.

We will now consider solving this problem using time dilation. There are several ways to approach this. We will consider two ways. The first is as follows. We are told that the half life of muon particles is 1.5μs. This is from the muons in the laboratory's frame of reference. A scientist in the laboratory will measure the half-life as being $\frac{1.5}{\sqrt{1-0.99^2}} \approx 10.63\mu s$. Again the sample takes 6.73μs to travel between the two detectors. Therefore from the scientist's frame of reference, the number of half lives during this time is approximately $\frac{6.73}{10.63} \approx 0.63$. Therefore the percentage of the original sample that is now consists of muon particles is approximately $100*(\frac{1}{2})^{0.63} \approx 64.5\%$

The second way is as follows. We are told that the distance between the two observers is $2km$. This is from the scientist's frame of reference (see length contraction section for more information). We are also told that the muons are travelling at $0.99c\ ms^{-1}$ relative to the scientist. Therefore from the scientist's frame of reference the time observed is $\frac{2000}{0.99*3*10^8} \approx 6.66\mu s$. Therefore from the muon's frame of reference the time observed is $t_0 = 6.6\sqrt{1-0.99c^2} \approx 94.04\mu s$. Therefore the number of half lives during this time is $\frac{94.04}{1.5}$, which is approximately 0.63, which matches the number of half lives from the previous approach.

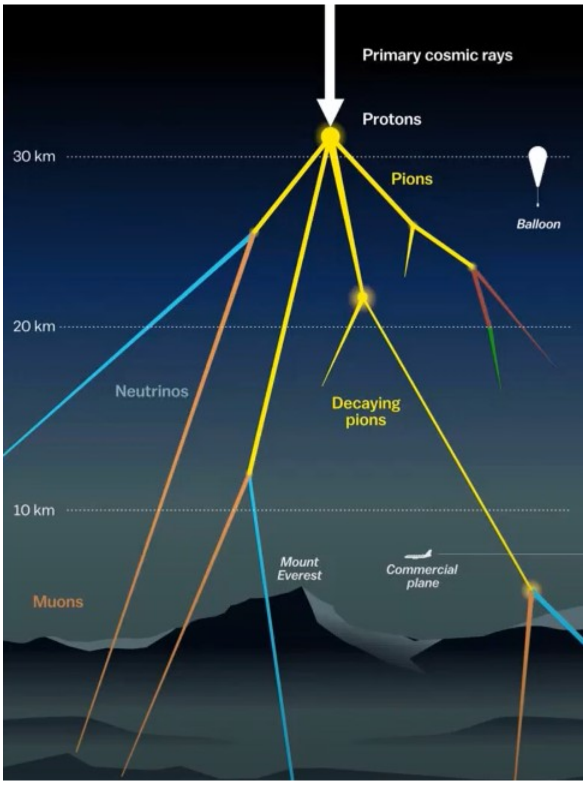

Figure 10: Muon decay experiment (Ark Physics 2021)

6 Length Contraction

6.1 Introduction to Length Contraction

In the previous section we discussed that at speeds respectable to c, it becomes noticeable that the time an event takes is different according to which frame of reference you are in. Another phenomenon is length contraction. This states that when an object is travelling at speed v relative to observer A, observer A will see the length of the object to be contracted, compared to observer B who is stationary relative to the object. It is important to note that the object only appears to be shortened in the direction of the motion. If l_0 is the proper length of the object (i.e. the length of the object measured by a stationary observer relative to the object) in metres, then when the object is moving at speed vms^{-1} relative to observer C, observer C will measure the object to be $l = l_0\sqrt{1-\frac{v^2}{c^2}}$ metres.

Let's consider an example. Suppose there there are two observers, observer A and observer B. Observer A is on a train which is travelling at $0.8c\ ms^{-1}$ relative to observer B, because observer B is stationary on the platform. Observer A measures the length of the carriage to be $15m$. Determine how long observer B measures the carriage to be. It is important to note that since observer A is at rest relative to the carriage, $15m$ is the proper length of the carriage, so $l_0 = 15$ and $v = 0.8c$. Therefore we have:

$$l = l_0\sqrt{1-\frac{v^2}{c^2}}$$
$$l = 15\sqrt{1-\frac{(0.8c)^2}{c^2}}$$
$$l = 9$$

Therefore the observer on the Earth measures the length of the carriage to be 9 metres.

Time dilation and length contraction go hand in hand, in the sense that given certain information it can be possible to solve a problem by using time dilation or length contraction. Hence we omit a proof for length contraction.

6.2 Experimental Proof For Length Contraction

Let's consider the same example from the time dilation section, i.e. the muon decay example. From the muon's frame of reference, the detector will be moving at $0.99c\ ms^{-1}$ relative to itself. Therefore the muons will measure the length between the detectors to be approximately $2000\sqrt{1-\frac{(0.99c)^2}{c^2}} = 282m$. The time it takes the muons to travel between the two detectors from their frame of reference is $\frac{282}{0.99*3*10^8} \approx 0.95\mu s$. The number of half-lives is approximately $\frac{0.95}{1.5} \approx 0.63$, which is the same answer we get if we use time dilation. (Ark Physics 2021)

7 Relativistic Mass and Energy

7.1 Relativistic Mass

We have seen that at speeds comparable to the speed of light, time dilation and length contraction become noticeable. Another phenomenon that becomes noticeable is that the mass of an object increases. Suppose an object B is moving at speed vms^{-1} relative to observer A. Then observer A will measure the mass of the object to be $m = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}kg$, where m_0 is the mass measured by observer A when object B is at rest relative to observer A (i.e. its relative rest mass). This can be proved by considering conservation of momentum.

7.2 Introduction to Relativistic Energy and Derivation of Classical Mechanics Formula

Albert Einstein famously proved that the total energy an object has due to its mass is $E = mc^2$. This is known as the mass-energy equivalence and is one of the most famous equations in physics and science. One use of this formula is to calculate the energy released in nuclear reactions because some of the matter will be converted into heat energy, which can be used to heat water into steam. This formula is an excellent approximation for classical mechanics, so we will now slightly formalise this for our use in special relativity. Suppose object B is moving at speed v relative to observer A. When object B is stationary relative to observer A, its total energy due to mass relative to A is $\frac{m_0c^2}{\sqrt{1-\frac{0^2}{c^2}}} = m_0c^2$. This is known as the rest energy. When object B is moving at speed v relative to observer A, its total energy due to mass relative to A is now $\frac{m_0c^2}{\sqrt{1-\frac{v^2}{c^2}}}$. Because the only variable that has changed between the two expressions is velocity, we must have kinetic energy (E_k) is equal to total energy - rest energy. This leads to:

$$E_k = \frac{m_0c^2}{\sqrt{1-\frac{v^2}{c^2}}} - m_0c^2$$
$$E_k = m_0c^2(\frac{1}{\sqrt{1-\frac{v^2}{c^2}}} - 1) \ (3)$$

We will consider the Taylor series of the Lorentz Factor. We have:

$$\gamma = 1 + \frac{1}{2}\frac{v^2}{c^2} + \frac{3}{8}\frac{v^4}{c^4} + \frac{5}{16}\frac{v^6}{c^6} + \frac{35}{128}\frac{v^8}{c^8} + \frac{63}{256}\frac{v^{10}}{c^{10}} + ... \text{ (Liquisearch) } (4)$$

Substituting formula (4) into formula (3) leads to:

$$E_k = m_0c^2(1 + \frac{1}{2}\frac{v^2}{c^2} + \frac{3}{8}\frac{v^4}{c^4} + \frac{5}{16}\frac{v^6}{c^6} + \frac{35}{128}\frac{v^8}{c^8} + \frac{63}{256}\frac{v^{10}}{c^{10}} + ... - 1)$$
$$E_k = m_0c^2(\frac{1}{2}\frac{v^2}{c^2} + \frac{3}{8}\frac{v^4}{c^4} + \frac{5}{16}\frac{v^6}{c^6} + \frac{35}{128}\frac{v^8}{c^8} + \frac{63}{256}\frac{v^{10}}{c^{10}} + ...)$$
$$E_k = m_0(\frac{1}{2}v^2 + \frac{3}{8}\frac{v^4}{c^2} + \frac{5}{16}\frac{v^6}{c^4} + \frac{35}{128}\frac{v^8}{c^6} + \frac{63}{256}\frac{v^{10}}{c^8} + ...)$$
$$E_k = \frac{1}{2}m_0v^2 + m_0(\frac{3}{8}\frac{v^4}{c^2} + \frac{5}{16}\frac{v^6}{c^4} + \frac{35}{128}\frac{v^8}{c^6} + \frac{63}{256}\frac{v^{10}}{c^8} + ...)$$

Notice that for $v << c$ we have $\frac{v^{2n+2}}{c^{2n}} \approx 0$ for all n ϵ N. Hence the above can be approximated by:

$$E_k \approx \frac{1}{2}m_0v^2 + m_0(\frac{3}{8} * 0 + \frac{5}{16} * 0 + \frac{35}{128} * 0 + \frac{63}{256} * 0 + ...)$$
$$E_k \approx \frac{1}{2}m_0v^2$$

Hence this is the reason that for classical mechanics we calculate kinetic energy by $\frac{1}{2}mv^2$.

7.3 Experimental Proof for Relativistic Energy

There have been lots of experiments to prove relativistic energy. One of the most famous was by the physicist William Bertozzi. The setup of the experiment can be seen in the diagram below.

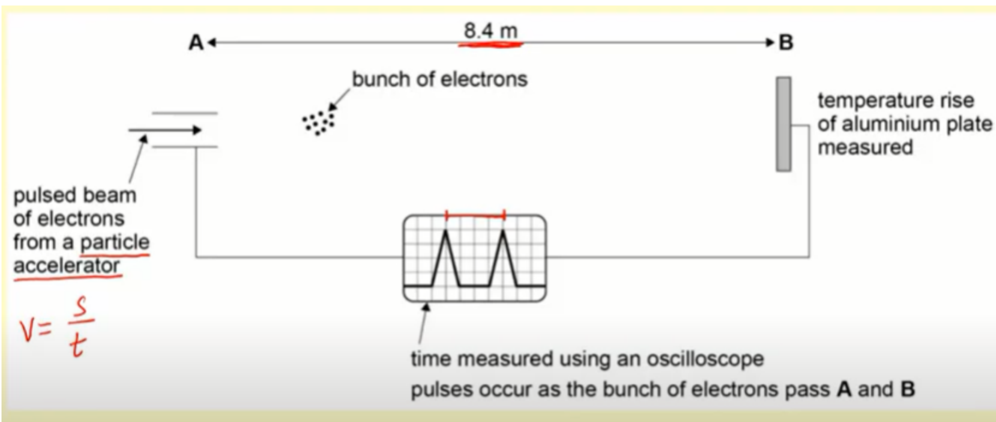

Figure 11: Bertozzi's Experiment Setup (Ark Physics 2021)

Originally the pulse of electrons are accelerated by a particle accelerator, before entering A. As soon as the electrons reach A, the electrons are travelling at a constant velocity. The electrons travel a distance of $8.4m$ before reaching B. The time the electron takes to travel between A and B (let's call this t) can be calculated by observing readings on oscilloscope. Hence the velocity of the electron can be calculated by $\frac{8.4}{t}$. By conservation of energy, when each electron hits the aluminum plate at B, its kinetic energy is converted into heat energy, by the formula $E_h = md\Delta e$, where d is the specific heat capacity of aluminum, and Δe is the change in temperature. Therefore, if n electrons are fired simultaneously, the kinetic energy of each electron is equal to $\frac{md\Delta e_{total}}{n}$. The experiment was repeated for different velocities, by adjusting the strength of electric field. A graph of velocity of against kinetic energy was drawn, and it followed that as the speed of the electron is entering relativistic speeds, the kinetic energy did not follow $E_K = \frac{1}{2}mv^2$ but instead formula (3).

8 Lorentz Transformations

Now that we have defined time dilation and length contraction, we can extend our knowledge of Galilean transformations, to speeds respectable to $c\ ms^{-1}$. These are known as Lorentz transformations.

Suppose that there are two frames of reference where one frame is stationary (call this S) and the other frame of reference is moving at velocity ums^{-1} in the x-axis relative to the stationary frame (call this S'). Let's also suppose at the time $t = 0s$ that the two frames start at the same position, and the moving frame travels for t seconds in the frame of reference of S. This can be viewed by the following diagrams:

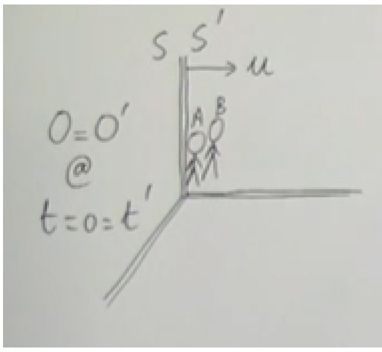

Figure 12: Both frames of reference at time t = 0 (Biezen 2015)

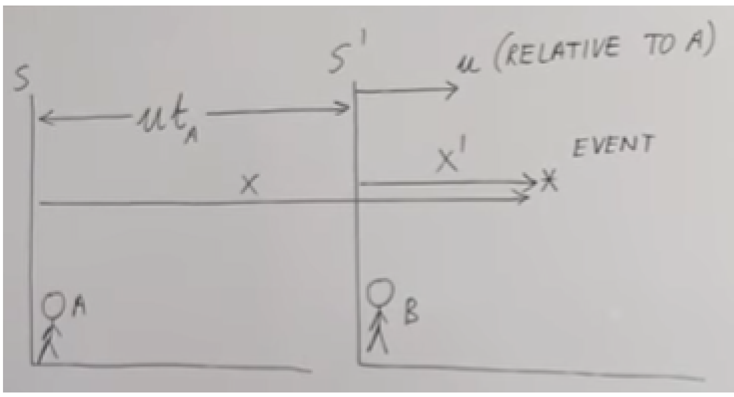

Figure 13: Both frames of reference at time t (Biezen 2015)

There is an event where we want to calculate the distance of it from the origin of S. This distance is x metres, which is from the the stationary observer's frame of reference. Notice that x is equal to the distance travelled by S' plus the distance of the event from the origin of S'. S' travels for t seconds as measured by S'. The distance from the origin to the event is x' metres as measured by S'. Therefore this distance is $x'\sqrt{1-\frac{v^2}{c^2}}$ metres as measured by S. Therefore the distance from the origin of S to the event from observer A's frame of reference is $x = ut + x'\sqrt{1-\frac{v^2}{c^2}}$ metres (5). Rearranging this leads to $x' = \frac{x-ut}{\sqrt{1-\frac{v^2}{c^2}}}$.

We will now consider the Lorentz transformation of time. Let t be the time an event occurs in seconds in the frame of reference of S, while t' be the time an event occurs in seconds in the frame of reference of S'. Let both S and S' start at the origin. We assume that S' is moving at velocity ums^{-1} relative to S. From the frame of reference of the observer on S', it seems that S is moving at velocity $-ums^{-1}$ relative to S'. By similar concepts from above, this also leads to $x' = -ut + x\sqrt{1-\frac{v^2}{c^2}}$ (6). By combining formulae (5) and (6), this leads to $t = t'\sqrt{1-\frac{v^2}{c^2}} + \frac{xu}{c^2}$. Rearranging this formula leads to $t' = \frac{t-\frac{xu}{c^2}}{\sqrt{1-\frac{v^2}{c^2}}}$. (Biezen 2015). It is important to note that the formula $\frac{t_0}{\sqrt{1-\frac{v^2}{c^2}}}$ describes how long time is observed in a specific frame of reference, while this formula describes at what specific time an event occurs by converting between different frames of reference.

We will finally consider relative velocity. Suppose that frame S' observes an event happening at relative velocity $v'ms^{-1}$, S at vms^{-1} and S' moves at velocity ums^{-1} relative to S. We would like to link these three velocities. The relative velocity observed by S' can be calculated by $v' = \frac{dx'}{dt'}$. From the previous paragraphs we know that:

- $x' = \gamma(x - ut) \implies$ dx' $= \gamma(dx - udt)$
- $t' = \gamma(t - \frac{xu}{c^2}) \implies$ dt' $= \gamma(dt - \frac{xu}{c^2})$ (Biezen 2015)

Therefore substituting these into v' leads to:

$$v' = \frac{dx'}{dt'}$$
$$v' = \frac{\gamma(dx-udt)}{\gamma(dt-\frac{xu}{c^2})}$$
$$v' = \frac{dx-udt}{dt-\frac{xu}{c^2}}$$
$$v' = \frac{\frac{1}{dt}(dx-udt)}{\frac{1}{dt}(dt-\frac{xu}{c^2})}$$
$$v' = \frac{\frac{dx}{dt}-u}{1-\frac{\frac{dx}{du}u}{c^2}}$$
$$v' = \frac{v-u}{1-\frac{vu}{c^2}}$$

9 Conclusion

To conclude, special relativity is a really powerful tool when objects move at speeds respectable to c ms^{-1}. We have seen that in theory that on object cannot travel faster than c. However, scientists having been doing research and have discovered two concepts that could break this rule. Firstly, there are tachyons, which are hypothetical particles which travel at speeds faster than the speed of light. Also, at certain points in the universe, the rate of expansion of the universe can be greater than the speed of light.

We have seen that relativistic speeds are one factor that cause time dilation. Another factor that causes time dilation are gravitational fields. Depending on where you are in a gravitational field, time will appear different to several observers. This explains why the Earth's core is 2.5

years younger than its crust. This is part of an area of physics called general relativity, which expands on special relativity, and explains why lights bends towards the Earth, as it travels near it.

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