Barnett Waddingham Pensions Report

Group B3

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Chapter 1

Project Introduction

This report explores a new defined contribution pension scheme for a company to use, and investigates "an appropriate level of contribution" so the employee has enough money to live off during retirement. This includes developing a model "to project the total retirement at retirement age, and what level of pension this fund might secure".

Methods of contribution

There are various options available for paying into a pension and different options for how one may choose to take their pension. There are two types of pension. A defined contribution pension is built up overtime by an employee and employer making regular payments into it. Each employee of a company has a pension fund into which contributions are paid, and a defined percentage of the employee's salary is paid into this fund each month. The amount of pension usually depends of how much the employee's salary is, and how long they have worked for the company. Whereas a defined benefit pension is a workplace pension that pays a person a retirement income based on their salary.

Methods of withdrawal

There are several ways one can start taking their pension. A retiree may take the pension as a lump sum or take money directly from the pension fund and choose to leave the rest invested, which is known as drawdown. Although, it is important to consider for a cash withdrawal, the first 25% of the total pension will be tax free, the rest will be taxable. Alternatively, one may buy an annuity from an insurance company and swap their pension income for a guaranteed regular income. There is also the option of opting for a mixture of all three options[1]. For example, one may choose to take cash and income at different times depending on factors such as: age, health, whether one has financial dependants and the size of one's pension pot and other savings. Moreover, there are certain factors that need to be considered when taking retirement. For instance one may choose to boost their pension if the money they are going to receive is low, so they may pay more into it or push back the date they are going to retire. Next, it is important to clear debts as income will decrease when one retires. Employees may also receive incentives from employers to take early retirement to cut costs, so they may provide pension benefits every month. Moreover, an employee may choose to gradually move their money to lower risk investments, e.g. treasury bills and corporate bonds, as it protects them against damaging losses. Lastly, one is entitled to take a workplace pension before the state pension at age 66[2].

Report structure

In this report a basic model will be formed first. This will include certain assumptions in order to simplify the complexity of the problem. Then ideas from the basic model will be expanded, to create a more accurate and realistic advanced model. From here, deductions will be made as to what may be the best solution. Whereas the first model will only include drawdown, the advanced model will evaluate the other two withdrawal methods, complimented with the basic modelled drawdown.

Chapter 2

A Simple Model

2.1 The Basic Solution

2.1.1 Research and Goals

The aim of the model is to show how an adequate level of monthly contribution can build up a pension fund to give an adequate standard of living in retirement. First, contribution will be assumed before hand, but after a fitting level of contribution well be calculated.

2.1.2 Assumptions

As stated, to decrease the complexity of our first model the following variables will be assumed:

- There is no tax on withdrawal from the pension.
- The only withdrawal is 'drawdown'.
- Salary is fixed at £29600 per annum, as this is the average salary in the UK in 2020.[3]
- The retirement age is fixed at 66 years and the age of death is 81 years.
- Inflation is 3% over the period of the pension scheme.
- The interest on the pension is fixed at 7% per year.
- Markets will behave in a similar way in past as they will in the future.
- The standard of living does not change after retirement.
- The employee has at least 35 qualifying years on their National Insurance record and is therefore entitled to the full state pension of £175.20 per week.
- The employer contribution varies depending on the industry sector an employee works in, so for the simple model this has been taken as the average value of employer contribution across all sectors and is fixed at 4.5%.
- £250,000 is the total sum in the pension fund.
- The employee has worked for 39 years before retiring.

Reasoning for assumptions and effects of such

- The total sum of the pension should be around £250,000. This target value of pension value was an averaged figure from Unbiased.com, Pension Bee and Which.com. It allows for the standard of living not to decrease after retirement in contribution with the new state pension as will be shown.

- Inflation is set at 3% over the period of the pension scheme, so the real valued growth of the pension is just the percentage that the investment grew, in this case by minus 3% (The value for 3% is explained further in the advanced model). The total sum of the pension will not also have inflation applied to it as the cumulative pension value will be in real terms.
- The average working life for a person in the UK is 39 years, so it is assumed the employee works for 39 years and pays into the pension scheme for the whole time they work. As pension contributions are paid in monthly, the employee and employer pay into the pension scheme 468 times.
- For the simple model, cumulative pension funds are invested into a range of ventures which have an average nominal return of 7% per year. This value of 7% annual growth was an average from research from the following sources: Monevator, Superguide, Schroders, Thisismoney, Investorschronicle, and previous pension fund growth which can be analysed from indexes such as the IPE-Fundo. Market indexes were also taken into account. The S&P500 index has a historic annualized average return of around 10%.

2.1.3 Solution

For the simple model, the monetary contribution size of the employer and employee combined (a) is fixed because the salary is fixed and contributions are made monthly. The contribution number increases with each contribution, and monthly interest (x) gained on contributions are calculated with each contribution. The new cumulative value of the pension is the previous cumulative pension value plus the current contribution size multiplied by the monthly interest. Hence as contribution number increases cumulative pension value increases to, as can be seen:

Contribution number	Contribution size	Cumulative pension value
1	a	ax
2	a	$(a+(ax))x$
3	a	$(a+(a+(ax))x)x$
4	a	$(a+(a+(a+(ax))x)x)x$
5	a	$ax+ax^2+ax^3+ax^4+ax^5$
$\vdots$	$\vdots$	$\vdots$
468	a	$\sum_{n=1}^{468} ax^n$

From the table it is evident that the value of the pension fund has a geometric progression, and at the assumed retirement age will be valued at $\sum_{n=1}^{468} ax^n$. This is a function of time represented by: contribution number in months, a monetary contribution size and some monthly interest. With an assumed real constant yearly interest on contributions of 4% the monthly interest is roughly 0.3% as calculated below:

Since yearly interest rates are presumed to be a constant real 4% growth, monthly compound interest (x)

$$x^{12} = 1.04$$

$$x = 1.0032737(5.\text{s.f})$$

By equating this end formula, the sum of a geometric progression with the desired pension value after new government state pension is removed, a value for contribution size (a) can be derived:

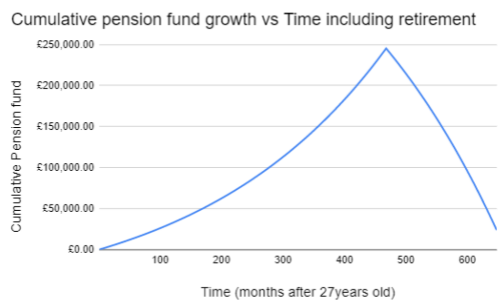
$$\begin{aligned}\sum_{n=1}^{468} ax^n = a(\frac{1-x^n}{1-x}) &= 250,000 \\ a =& \frac{(250,000)(1-x)}{(1-x^n)} \\ =& \frac{(250,000)(1-1.0032737)}{(1-1.0032737^{468})} \\ =& 226.32(5.\text{s.f})\end{aligned} \tag{2.1}$$

Monthly contribution (a) has components: employer contribution and employee contribution. Since employer contribution is 4.5% of the salary[4], it can be calculated to be £111. Employee monthly contribution can therefore be calculated to be $226.32 - 111$ which equals 115, to the nearest pound. This is 4.66% of monthly wages. However this is rounded down to 4.5% for simplicity and to match employer contribution.

Accumulating values of monthly contribution, contribution number and monthly interest, a discrete graph can be plotted of cumulative pension value vs time since the first contribution. The model is based on a geometric progression and as such, the graph follows the partial sums shown above. The effect of rounding employee contribution down to 4.5% to match employer contribution is that after 39 years the fund amasses a total of £245,234.29, rather than the £250,000 originally targeted. However since this value was calculated by averaging other pension funds, standards of living do not drop after retirement and using the assumption of the employee living to 81 years they do not run out of money after 15 years in retirement.

Pension Drawdown Withdrawal Method

With pension drawdown, the fund can be withdrawn as one lump sum upon retirement, taken as regular monthly or annual payments, or flexibly as and when one wants. Here, in the simple model, we assumed the standard of living is consistent and does not decrease upon retirement, so the pension fund here is being taken out in equal proportions over the 15 years of retirement. This means each month £2,466.67 is withdrawn from the pension pot as this is the monthly wage prior to retiring.



The graph to the left shows the discrete rapid decrease of pension value after the long growth before retirement. The gradient of the line showing the withdrawal is not straight. This is because the retiree continues to gain returns as the remaining funds stay invested. Taking the pension fund as a lump sum upon retirement would forfeit these gains, so drawdown maximises the value of the pension and so the standard of living in this case.

2.1.4 Evaluation of the Simple Model

The annual pension growth used in the simple model was assumed to be 7% nominal and 4% real growth. In reality growth inconsistent markets are not stable and fluctuate in the short term. This can be seen throughout the last century with notable examples of market declines such as the United States bear market of 2007–2009, where the Dow Jones Industrial Average, Nasdaq Composite and S&P 500 all suffered declines of over 50%[5]. This means that due to the nature of compound interest, although the average growth of the market may grow at the assumed rate 7% per year, the actual long term growth may not average ss each year the market had grown 7%.

The simple model used above assumes a total sum of £250,000. Although as previously explained, the reasoning for the assumption is this value is not the upmost optimum value of pension sum. This is because at the assumed age of death of 81 years there is still £23,909.35 left in the pension fund. In reality, this could be a perceived positive as 81 years of age is an average age of death and having pension pot left over allows for people who outlive this average, and for those who don't, to have money to give as inheritance. However, an optimum value of the total pension sum, which would run to zero exactly after 15 years of retirement, could be calculated by allowing the pension total sum to be a variable. Calculating the total usage with time, redrawing from the total sum for the 15 years of retirement and letting the total sum equal this value. Then working back once again to calculate the employee contribution. This will be looked at briefly in the following subsection.

Non-Fixed Total Pension Sum

Assuming all previous assumptions except no longer fixing total pension sum to be exactly £250,000, a further model can be built.
Let T equal the new total pension sum. The monthly withdrawal is still £1707 (denoted as a below). This is from a new state pension of £759 per month entering the pension and £2467 exiting the pension each month for living costs. Each month, until the end of the pension, real interest on T is being earned of roughly 0.3% monthly (denoted as i below) as explained in the simple model. A new table, like the one in the simple model solution, can be made, where the withdrawal a each month is removed from the cumulative pension value while a monthly interest of i is added.

Month's after retirement	Withdrawal Size	Cumulative pension value
1	a	$Ti - a$
2	a	$(Ti - a)i - a$
3	a	$(Ti - a)i - a)i - a$
4	a	$Ti^4 - ai^3 - ai^2 - ai - a$
$\vdots$	$\vdots$	$\vdots$
180	a	$Ti^{180} - \sum_{n=1}^{180} ai^n$

After 180 months the cumulative pension value is now assumed to be £0 and a value of x can be calculated:

$$Ti^{180} - \sum_{n=1}^{180} ai^n = 0$$

$$Ti^{180} = \sum_{n=1}^{180} 1707 \cdot 0.0032737^n$$

$$T = 232717.68$$

Using the methods in the simple method to calculate an employee contribution with employer contribution of 4.5% to reach the total pension sum of £232717.68, the employee contribution per month would be £99, which is 4% of monthly wage (2.s.f).

This contribution is less than 4.5% in the first simple model, this benefits the employee more. It ensures they keep a greater percentage of their wages which should ultimately increase their living standards during their working lifetime, regardless of salary level. In this model the total pension fund reaches zero at the assumed age of death of 81. This is the most efficient form of contribution as no money contributed in working life was not used in retirement.

Chapter 3

Advanced Model

3.1 The Complex Solution

3.1.1 Variable Introduction

In the basic model many assumptions were made. As described in the evaluation, these assumptions are not realistic and vary in the real world from person to person. In order for an employee and an employer to pay in adequate payments into the pension fund, it is necessary to remove some of these assumptions and introduce new variables in order to refine and improve the model.

Salary increase

In the complex model, a person's salary is no longer fixed throughout their working age. In reality wages fluctuate with age. Figure 9 below shows the change in men's and women's earnings from the ages of 16 to 65 based on 2018 data from the Office for National Statistics. The retirement age was increased to 66 by 2020. Combining these two sets of data, a third line can be found where there is no gender bias, formulated from an equation that shows how salary changes with age.

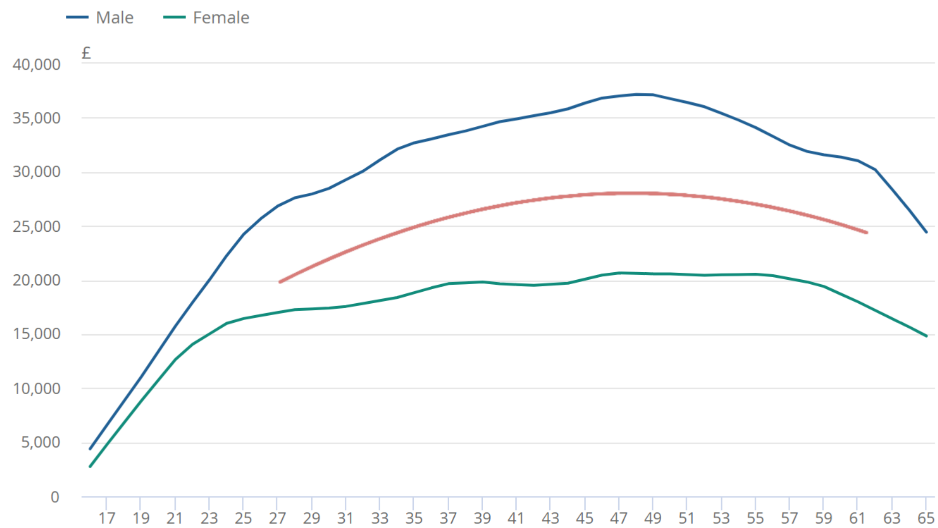


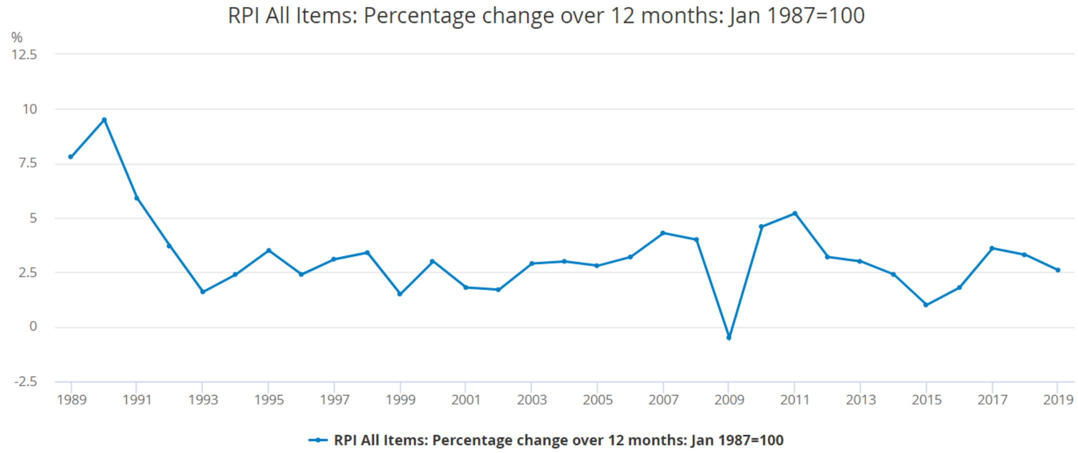
Figure 9 showing change in average wage with age[6]

At first glance, Figure 9 suggest the relationship could be modelled as a form of $y = -x^2$ where x is the age in years of employee and y is the earnings in pound sterling. Upon further investigation, due to the position of the graph, it would be sensible to model this as a $y = -a(x-b)^2 + c$ equation, where a, b and c are all real constants. Both men and women's earnings peaked at 48 years old, therefore $b = 48$. Taking a mean of the peak earnings for men and women we conclude $c = 28,895$, hence the line will have a maxima at $(48, 28895)$. To find the value of a, take a mean of the minimum earnings of both genders. As we know this occurs at $x = 16$, we can use our equation to solve for a giving us

$a = 24.74$ to to 4 significant figures. Hence the equation showing the relationship between age and earnings is given by $y = -24.74(x - 48)^2 + 28895$. This is represented as the red curve. The domain of the red curve spans from 27 to 62, thus representing the working life of the employee from age 27 years to 62 years assuming retirement at 62 and not receiving full state pension for 4 years after. The domain can easily be extrapolated to include x up to 66 to receive full state pension straight away after retiring and working for 39 years, such as in the simple model.

Inflation

Inflation is the rate at which the prices for goods and services increase. In terms of a pension fund, inflation affects how much the money that is invested in stocks, shares or bonds, grows by each year. For example if inflation was 2% and the pension also grew by 2% a year then the value of the pension stays the same. Therefore the pension fund should be invested in stocks or shares that have a higher return than the rate of inflation.



[7]

This graph shows the average yearly inflation from 1989 to 2019 which is the Office for National Statistics (ONS) data. Using these values, the average inflation over the last 30 years is around 3%. Therefore, the future inflation for the time that the employee pays into a pension is to be this value of 3%. So when it comes to looking at investment opportunities for the pension funds, the returns on the money invested in stocks should be higher than 3%. Otherwise the value of the pension pot will decrease over time, so the employee would have to pay more than £250,000 in this case.

3.2 Complex Model

Building upon the simple model, the accumulation of the pension fund in the complex model will be derived from the contributions on each month of the employee's working life and the interest earned between these months. Much of the assumptions follow through from the simple model to here and so will not be restated. The three (largest) assumptions which change are as follows:

- Salary is not fixed with age and therefore monthly contribution is also not fixed.
- Retirement age is 62 inline with the average retirement age in the UK.[8]
- Employer matches contribution.

As explained above, an employee's salary with age can be modeled by the curve $y = -24.74(x-48)^2 + 28895$ with x bounded by 62 above and 27 below. To translate that to the model, let $x = (n+324)/12$. The variable x still represents the age of the employee in years. This makes sense because there has been a translation of plus 324 months onto x. This is the amount of months in twenty seven years. The substitute for x is then divided by 12 to revert back to years. Now when n increases, x increases by one twelfth of a year starting at twenty seven. E.g. when $n = 0$, $x = 27$, whereas after a year

$n = 12$ and $x = 28$. Substituting the new value of x into the equation and dividing by twelve to find a monthly salary gives the "wage/age function" $MW(n)$ as follows:

$$MW(x) = ((-24.74(((x) - 48)^2 + 28895))$$

$$MW(n) = (-24.74(((n + 324)/12) - 48)^2 + 28895)/12$$

From the wage/age function, the total contribution to the pension can be found by multiplying the equation above to the percentage of the wage contributed monthly (c), and multiplying this by two to represent the employer matching contributions. A contribution function:

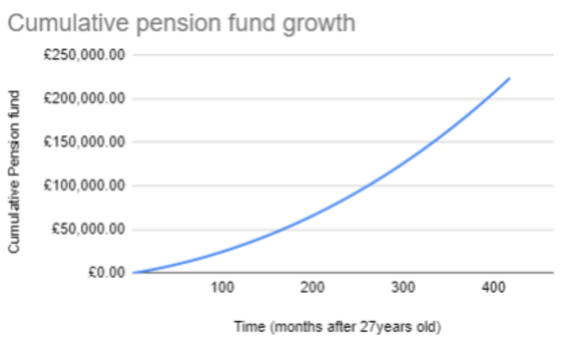
$$f(n) = 2MW(n) \cdot c$$

Much like the tables summarising the growth of the simple models, a complex table can be built. This table spans from $n = 1$ to $n = 420$ as the employee ages from 27 to 62. The contribution size is dependant on the age of the employee, which is a function f, of n as shown above. The cumulative pension value is the sum of the previous pension value summed with the next contribution size all multiplied by the real monthly interest (x) gained over this period.

Contribution number (n)	Contribution size	Cumulative pension value
1	$f(1)$	$f(1)x$
2	$f(2)$	$(f(2) + f(1)x)x$
3	$f(3)$	$(f(3) + (f(2) + f(1)x)x)x$
4	$f(4)$	$(f(4) + (f(3) + (f(2) + f(1)x)x)x)x$
5	$f(5)$	$f(5)x + f(4)x^2 + f(3)x^3 + f(2)x^4 + f(1)x^5$
$\vdots$	$\vdots$	$\vdots$
420	$f(420)$	$\sum_{n=1}^{420} f(420 - n)x^n$

From the table after 420 months, the cumulative pension value is $\sum_{n=1}^{420} f(420 - n)x^n$. Like the simple model, the interest is roughly 0.3%. In the UK pension contribution percentage varies between 3% and 9.5%[8]. Using either of these extremes in the model outputted a difference in cumulative pension value of over £240,000. As explained in more detail later, to ensure a sustainable quality of life before and after retirement a matched percentage of 6.06% was chosen, so $c = 0.0606$. Inputting these values the total pension fund is:

$$\sum_{n=1}^{420} f(420 - n) \cdot 1.0032737^n = 225,557.20$$



To the left is the graph of Cumulative Pension Fund and Time rising from £0 to £225,557.20. Much like the simple model the graph shows the sum of the discrete partial sums in the graph. However unlike the simple model the contributions are not equal, and therefore the growth fluctuates with time.

3.3 Withdrawal Methods

There are two withdrawal methods this model will consider. These are drawdown and annuities.

3.3.1 Annuity

Life-term annuity
What is the expected present value of a pension for a person living between ages 62 to 81?
Assumptions:

- Pension is paid annually at the start of each year.
- Pension increases annually in line with inflation - assumed to be 3% per annum.
- Intial value of pension is £P per annum.
- Investments will achieve nominal interests of 7% pa [discount rate is $v = 1/(1 + 0.07)$].
- No pension paid to spouse.
- Member is currently age x.

The expected present value of a pension for life is calculated by the formula below. The pension is paid annually at the start of each year, so the first payment happens right away. Therefore, the initial value of the pension P per annum is multiplied by 1 (being the first year) plus the sum of the probabilities that the person aged x has survived n years after the initial payment, which is then multiplied by v the discount rate (referring to the time value of money) applied to the payment for the year after. This is also multiplied by $(1 + \alpha)^k$ describing the fact that the pension increases by α% in line with inflation each year.

$$EPV = P \times (1 + \sum_{k=1}^{n} {}_kp_x v^k \, (1 + \alpha)^k)$$

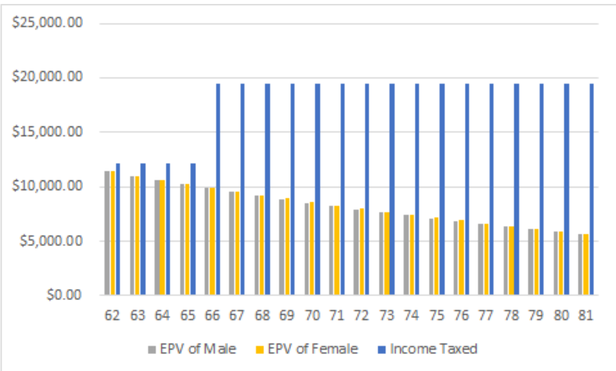


Figure 3.1: Part pension fund and supplementary tax free lump sum

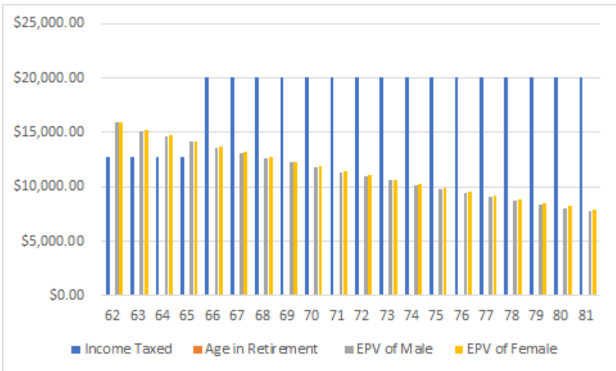


Figure 3.2: Total pension fund tax free lump sum not taken

The blue bar in Figure 3.1 represents the income you would receive if you took the total pension fund of £225557.20 and used 75% of it (£169167.90) to buy an annuity which will give you a fixed payment of £11491.74 pre-tax for 19 years from the age that you retire, 62, to the age that you die, 81. The 25% tax free lump sum of £56389.30 which is used to supplement the annuity and also the full state pension of £175.20 per week pre-tax (assuming the individual has full national contribution years) which a person is eligible for from the age of 66, which is why there is a jump in the graph from 66: the person does not receive the state pension from 62 to 66 and then from 66 onwards until the recipient is dead. The grey and yellow bars represent how the expected present value (EPV) of that fixed annual annuity income £11491.74 changes with time for a male and female respectively. I.e. if I give you £11491.74 from 5 years in the future, this would only be worth £9501.91 (Male) and £9566.78 (Female) in today's money because that £11491.74, 5 years from now is not as valuable as if I had that same amount today. This is due to factors such as: inflation, me using that money to make an investment (and get a return), and also the probability that I will still be alive then[9]. The total EPV from Figure 3.1 for a male is £164721.09 and for a female is £166013.96. This is +0.78% higher than the males, which is a result of the fact that females statistically have a slightly higher life expectancy than their male counterparts of the same age.

The blue bar in Figure 3.2 represents the income you would receive if you took the total pension fund of £225557.20 and purchased an annuity without taking a tax free lump sum. The fixed income you would receive would be £15885.58 pre-tax and the full state pension of £175.20 per week pre-tax. The grey and yellow bars represent how the EPV of the fixed annual annuity income of £15885.58 for a male and female respectively change with time. The total EPV for a male is £227524.86 and £229488.96 for a female. The reason the grey and yellow bars are higher for Figure 3.2 is because the whole pension fund is used for the annuity and so the tax free lump sum that was excluded from the annuity in Figure 3.1 (which does not affect the EPV) is now included in the annuity (so the tax free lump sum affects the EPV as well). EPV is only dependent on the money used to purchase the annuity and fixed payments received, and not the state pension nor the tax-free lump sum that was not used to buy the annuity.

In conclusion, taking into account both options, you would receive a fixed income post tax of £19449.57 (including state pension) if you were to choose the option summarized graphically in Figure 3.1 and a fixed income of £19996.78 (including state pension) if you were to choose the option summarized graphically in Figure 3.2, which would mean that instead of taking the tax-free lump sum you would use the whole pension pot to buy an annuity which would yield a +2.74% increase in annual income.

3.3.2 Drawdown

The other form of withdrawal from the cumulative pension value after retirement is "drawdown". Drawdown was the sole withdrawal method in the simple model and so the explanation of such will not be repeated. The difference between the drawdown methods in the two models is that the value of the pension fund is different, and the employee is assumed to retire at 62 in line with the national average. Since the state pension is only accessible from 66 years old, the retiree only has to rely on their acquired pension for those 4 years which is the pension in this model. This means, to ensure the same standards of living after retirement to before, more money has to be withdrawn monthly before the retiree receives the state pension. The employee retires making £1,954.83 a month at 62 and so until the age of 66 they withdraw this amount from the pension. This decreases to £1,195.63 when they receive the new state pension of £759.20, which when summed with this value is still the £1,954.83 monthly.

When graphed, a steeper section across the four years of no state pension occurs straight after the retirement starts. Although initially interest is being made on the pension pot here of around £700 monthly, with £1,954.83 being withdrawn the deficit is large and the decrease is fast, so the curve is steep. When state pension is granted at 66 years old, the gradient increases as the curve is not as steep. This is because although the interest is less at around £500 due to having less money in the total pension pot, the amount withdrawn monthly is also less at £1,195.63. This is roughly a 40% decrease in withdrawal with only a 25% decrease in interest.

Only now can the reason for the 6.06% contribution be explained: as can be seen above, the graph approaches £0, at 81 years old the retiree has £31.94 left in their pension pot. The level of contribution is 6.06%. This is the smallest monthly contribution that ensures in retirement, the pension pot doesn't fall below £0 in the expected life time while the standard of life is maintained.

3.4 Evaluation of the Complex Model

Some people contribute to their pensions before the age of 27 and some people work till after 62. In this case, monthly contributions for the employee may be less as they would pay into a pension more than 420 times. On the other hand, the employee may make the same contributions but would save more money into a pension, which means they have more money saved up for retirement. Since the men's and women's mean wages were averaged the curve we used falls into a "no mans land", where in reality not many people have that wage/age curve. Maybe it is better to do two different models for men and women or to consider couples. Factoring in a high curve for the men and a lower curve for the women instead of making our own curve. This method can be repeated for same-sex relationships.

Another point of evaluation is of the growth model used. Presuming a constant yearly real growth of 4% may not be representative of real market conditions. Although 4% seemed like a reasonable level of growth, this does not consider trading fees or years where the markets did not see such growth, such as in a recession. The model relies on compounded interest to dramatically increase the contributed funds to a level which can be viable to live off in retirement. Another down fall of the growth model assumes employees will continue to work without a break between 27 and 66. A lot of people tend to see breaks in their working life, and this is something that is not accounted for. Taking a gap from working would make contributions 0 for that time period, and thus the overall cumulative pension would be less as no compound interest is made through investing that money. This is called "opportunity cost" and is the hidden disadvantage of taking a gap from contributing to the pension fund.

In such retirement, although a goal of the model, most retirees accept a decrease in salary. The target of keeping the salary the same after retirement as before is potentially unrealistic. The state pension for example is considerably less then the average salary before retirement. If the government expect retirees to live on such little then maybe trying to keep the same level of salary after retirement was to high of an expectation even if both models achieve this.

Chapter 4

Deductions

4.1 Conclusion

Contributions

From previous research, the contribution percentage needed to be below 9.5%[10] and the lower the value, the better, because a lower value indicates more of the salary is taken home by the employee which supports a higher standard of living. In the simple model 4% and 4.5% were calculated as appropriate levels of contribution. In the complex model men's and women's wages were combined to make a curve which represented the "average" person. In reality this is not a good decision. This is because since the average person does not exist in the real world, our curve fell into the "gender wage gap" and meant the model employee earned more than the "average" woman, but less than that of the "average" man. This result is due to a larger contribution percentage of over 6%. Although, as explained, this is not a considerably unreasonable value, it is larger than both the simple models.

Method of withdrawal

With drawdown, assuming one lives for the average number of years, our simple and complex models both provide sufficient funds for a retiree's standard of living not to fall in retirement. This may be attractive to an employee who values this standard. However, this comes with a risk that the personal pension fund will run out once a person lives past the average expected age of death, currently 81 years of age, hence there is a risk a person's living standards would drop significantly as they are forced to survive on the state pension alone. With buying an annuity, a retiree is guaranteed a pension for life. This reduces the risk of living standards falling as much, but comes with the disadvantage of smaller payments to the retiree compared to the drawdown method. This risk of running out of funds in drawdown may be combated by continuing to invest the pension in retirement which cannot be done with annuities.

Alternatively, drawdown has the advantage of flexibility. If a retiree can freely move money into different funds then the amount withdrawn each month can vary as much as they like. This potentially may be more convenient as it reflects real life more because expenses change year to year. Annuity does not give this freedom and more saving and budgeting may be required. Another advantage of drawdown is it maximises quality of life before and after retirement. Contributions can be made in a way such that no money is saved that is not withdrawn, whereas in annuities the amount you get in a year is going to be less as it is guaranteed for life. Within the drawdown models, the idea of taking 25% of the pension pot tax-free was not explored. This is because not taking the 25% means that the level of contribution required to maintain standards of living in retirement is less. If 25% was removed from the pension pot, to keep the same level of withdrawal closer to 8%, contribution would be required. This decreases standard of living before retirement which is something that should be maximised by trying to keep contribution levels as low as possible.

Taking all factors into consideration, it is recommended to withdraw 25% of the pension fund upon retirement and use the remainder of it to purchase an annuity. It is then recommended to reinvest the lump sum into funds. This is because the lump sum allows the retiree to have an extra income from dividends, likely more than they would receive using the money to put towards an annuity, and gives the pensioner the option of flexible drawdown for unexpected expenses in retirement. Along with this safety, purchasing an annuity gives the security of an income throughout retirement. This combination of withdrawal methods recaps the benefits of both, but most importantly gives security and piece of mind that income will always be received.

4.2 Further Analysis

Given a larger scope, or more pages to the report, there were a selection of topics that would be interesting to analyse moving forwards from this document:

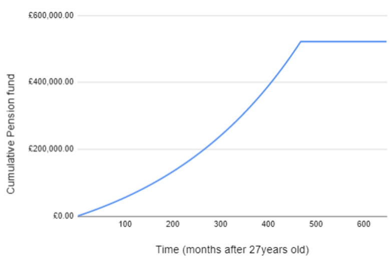
- Differentiating the cumulative pension growth curve with respect to age.

Doing so would show the rate of growth of the pension at different ages of the employee. From here it would be possible to see where the highest growth was. It would be interesting to know whether the highest pension growth was when contribution was highest or towards the end of the employees working life where compound interest allows for a higher rate of growth. Varying salary and contribution percentage or employer contribution would make contributions different and therefore growth would change at that point. This research may lead into finding the best time in someone's life to invest into their pension. Investing a lot when they are young and sacrificing a larger proportion of their salary but gaining from compound interest and time, or investing more later in life when wages are higher and quality of life is not effected as drastically.

- Investigating withdrawing pension at different rates within drawdown.

Within both of the drawdown models a fixed salary was taken after retirement. Calculated to ensure standards of living do not change. It would be fascinating to see the effect withdrawing up to 25% of the pension pot would have on the retirement salary. Since, in the two models, the aim was to finish the pensions with £0, withdrawing a lump sum first would allow for a inheritance to be given to loved ones, or possibly to buy luxury, once in a lifetime goods or holidays.

- Calculating a level of contribution where pension funds does not decrease in retirement.



Due to interest gained on the pension pot when in retirement in drawdown, it is possible that the pension pot does not decrease, but can stay level or even increase. For the simple model this occurs at a matched contribution of roughly 9.6%. This is higher than any pension scheme easily accessible and above the 9.5% hard limit we set ourselves. It would be interesting to calculate how fast this equilibrium can be achieved in order to reach financial independence. This investigation may conclude the best way to retire young by contributing at different rates.

- Annuities where the person doesn't pass away at 81.

Across all of the models, ages had to be presumed when things happened in peoples lives: age of retirement, age of death, age of first contribution. It would be intriguing to stop assuming ages and model what would happen to the pensions with this change. For annuities specifically, the probability of living as age increases decreases to 0. Taking the limit of age as the retiree ages indefinitely would lead to a lower annuity year payout as, unlike we have assumed, people live past 81!

- Multiple growth models for different genders and relationships.

As previously stated, our growth model for women and men merged together. Although this is an accurate average of the UK population, maybe it was not the best solution for modelling wages. Using multiple models for different genders or considering models made of people in relationships may have been more representative.

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