Jacobs University Bremen

Lab Report 3 - Filters

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1. Introduction

The objective of the experiment is to show the behavior of simple passive RC circuits acting as filters, one low pass and one high pass, with a sinusoid of different frequencies as an input signal. We measure the properties of these circuits, and we analyze the result of the measurements and how they are represented.

A filter is a network used to select a frequency or range of frequencies while rejecting all others. Usually, they are constructed using active components like transistors or operational amplifiers, however we can also see similar behavior in passive networks of resistors, capacitors, and inductors. There are four general types. High Pass, Low Pass, Band Pass, and Notch Filters.

In our experiment, we analyze Lo-Pass and Band-Pass filters which are simple in nature, only using passive components such as inductors, capacitors, and resistors.

Furthermore, we will use the Bode plot to analyze the frequency response of these filters, and we will use the Nyquist plot to describe the parametric plot of the frequency response of the filters.

1.1 Lo-Pass

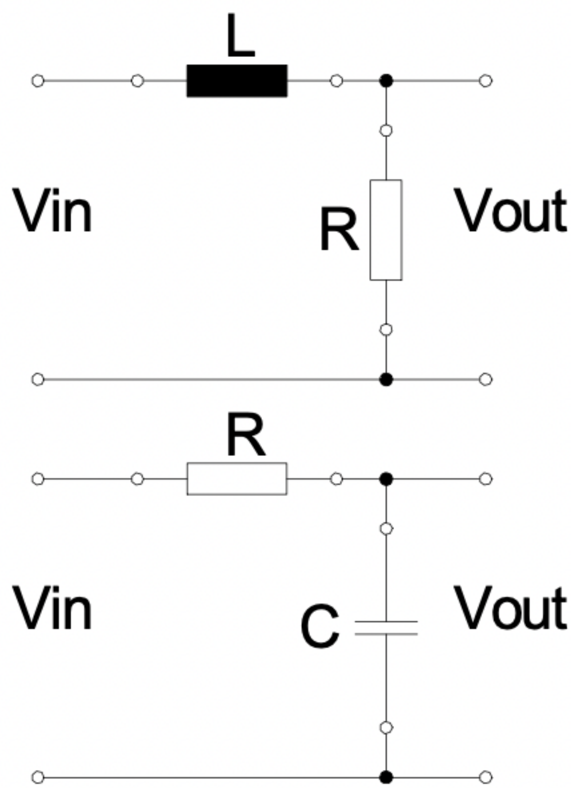

Figure 1.1: Low Pass Filter

A low pass filter allows low frequencies to pass, while it hinders higher frequencies. With high frequencies, the phase shift of the output signal becomes negative relative to the input signal.

It can be designed using a resistor and a capacitor, or an inductor and a resistor. In general, we can get the amplitude ratio of the output signal to the input signal by using the following equation:

For an RC circuit, which is what we design in the experiment:

$$\underline{A} = \frac{V_{out}}{V_{in}} = \frac{1}{1 + j\omega RC} \tag{1.1}$$

To obtain the cutoff frequency, we can use the following equation:

$$\underline{f_c} = \frac{1}{2\pi RC} \tag{1.2}$$

To obtain the amplitude and phase of the output signal, we use the following equations:

$$|\underline{A}| = \frac{1}{\sqrt{1 + (\omega RC)^2}} \text{ and } \phi = -\arctan(\omega RC) \tag{1.3}$$

1.2 Hi-Pass

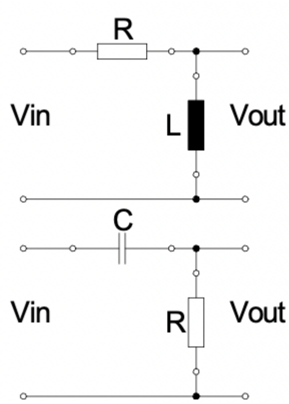

Figure 1.2: High Pass Filter

A high pass filter allows higher frequencies to pass through and attenuates lower frequencies. It can be designed using a resistor and a capacitor, or an inductor and a resistor.

The transfer function is described by:

$$\underline{A} = \frac{1}{1 + \frac{1}{j\omega RC}} \tag{1.4}$$

The amplitude is described by:

$$\underline{|A|} = \frac{1}{\sqrt{1 + \frac{1}{(\omega RC)^2}}} \tag{1.5}$$

And the phase is described by:

$$\phi = \arctan(\frac{1}{\omega RC}) \tag{1.6}$$

To find the cutoff frequency for both the low pass and high pass filter for an RC circuit, we use the following relation:

$$f_{-3dB} = \frac{1}{2\pi RC} \tag{1.7}$$

1.3 Band-Pass

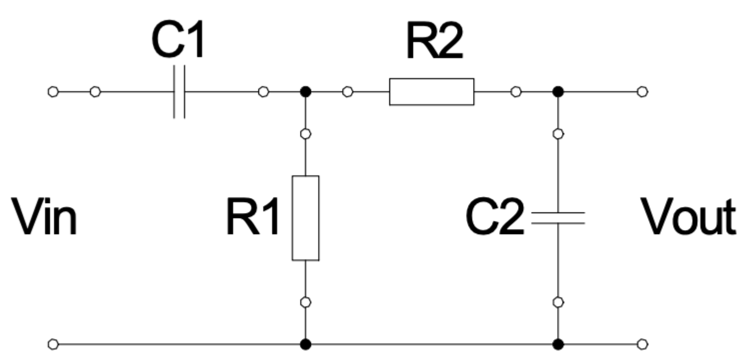

Figure 1.3: Band Pass Filter

A simple band pass filter is simply a combination of a high and low pass filter. It passes through frequencies within a certain range, and attenuates frequencies outside of it. The formula for the amplitude ratio is the same as the low pass filter, but we have to take into account the high pass filter as well. The formula for the cutoff frequency is also the same as the low pass, and high pass filter (1.2).

1.4 Obtaining the characteristic output of a filter

To describe the frequency response of these kinds of networks, we use the Bode plot. We plot the amplitude to frequencies over several decades, and the ratio is calculated. The unit of the result is **decibels**, or **dB**. **dB** is a logarithmic value used for such purposes. It is defined as:

$$A = 20 \cdot \log_{10} \frac{V_{out}}{V_{in}} \tag{1.8}$$

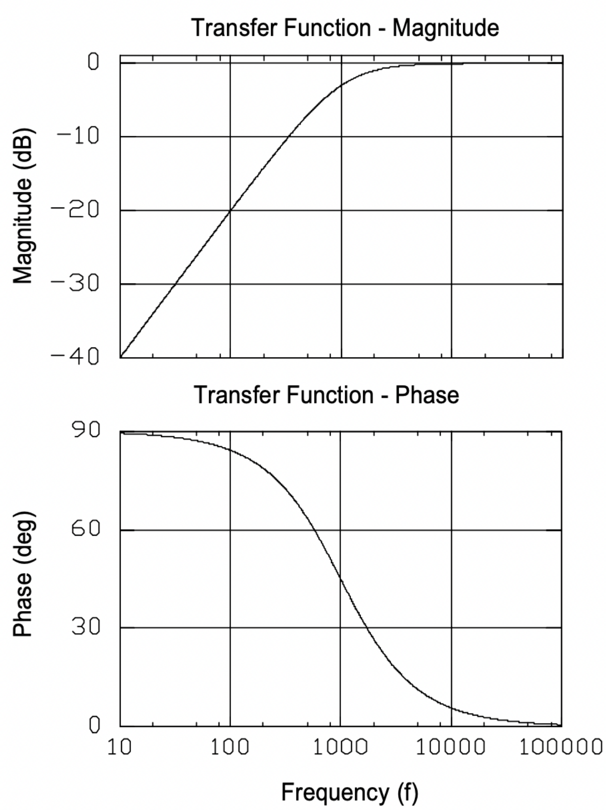

Figure 1.4: A bode plot, showing the magnitude plotted against the frequency, and the phase shift plotted against the frequency.

The other quantity measured is the phase shift ϕ between the input and output signal, taken relative to the input signal. When the output signal is ahead the input, the phase shift ϕ is positive, otherwise it is negative.

A **Nyquist plot** is a parametric plot of a frequency response. In Cartesian coordinates, the real part of the transfer function is plotted on the X axis. The imaginary part is plotted on the Y axis. The frequency is swept as a parameter, resulting in a plot per frequency.

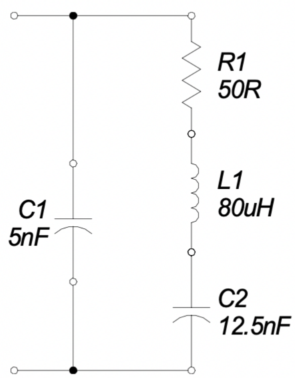

Figure 1.5: An RLC circuit with a capacitor in parallel.

We wish to find how the complex conductance changes when ω changes, given that the impedance for the RLC series circuit is

$$\underline{Z_S} = R_1 + j(\omega L_1 - \frac{1}{\omega C_2})$$

and with the parallel capacitor the admittance is

$$\underline{Y_{all}} = j\omega C_1 + \frac{1}{\underline{Z_S}}$$

When varying ω, a Nyquist plot is generated. The right plot is $\frac{1}{Y_{all}}$

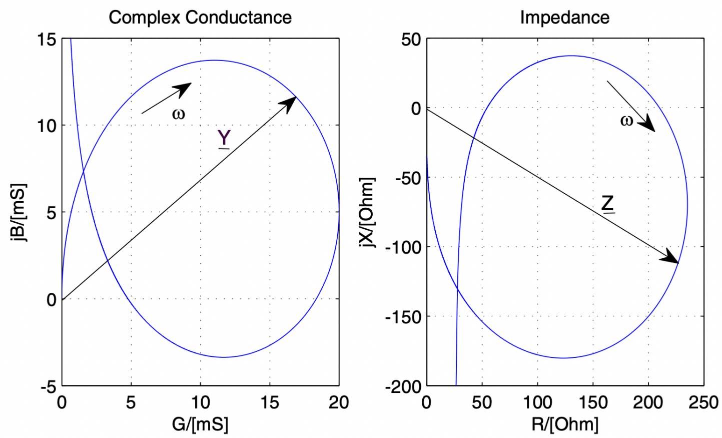

Figure 1.6: The nyquist plot of the RLC circuit.

2. Execution

Equipment: Oscilloscope, Function Generator, BNC-to-Kleps cable.

2.1 Part 1 : Lo-Pass

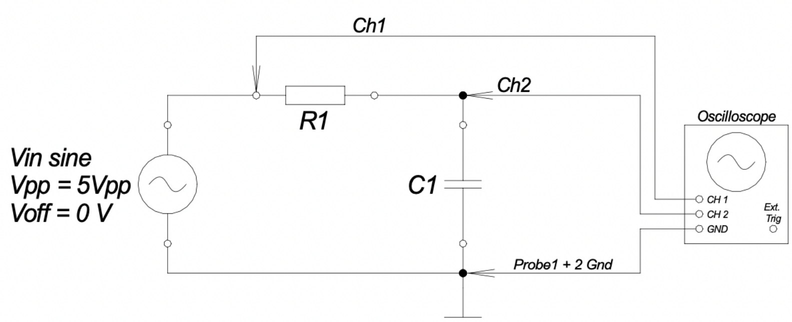

Figure 2.1: $C_1 = 1.5\text{nF}$ and $R_1 = 22\text{K}\Omega$

The signal generator is connected via the BNC-To-Kleps cable. The CH1 channel is used for the input, and the CH2 channel is used for the output. The frequency at the signal generator is then varied in steps of 1, 2, 5, 10 steps up to 100KHz.

Input Voltage(in V)	Output Voltage(in V)	Phase(in deg)	Frequency(in Hz)	Amplitude(in dB)
10.1	10	-0.36	50	-0.09
10.1	10	-0.36	100	-0.09
10.1	10	-2.59	200	-0.09
10.2	10	-6.49	500	-0.17
10.4	10.1	-11.8	1000	-0.25
10.9	10	-23	2000	-0.75
12.2	8	-47.4	5000	-3.67
11.8	5.04	-64	10000	-7.39
12.4	2.8	-75.3	20000	-12.93
11.8	1.14	-82.8	50000	-20.30
11.8	0.596	-84.6	100000	-25.93

Table 2.1: The effect of the frequency on the output amplitude is shown in the table above.

What we see is that the phase shift goes from positive to negative the further we increase the frequency, and that the output amplitude gets smaller and smaller. This is expected as we have built a low-pass filter.

2.2 Part 2 : Band-Pass

To determine the properties of a band-pass filter, we had two possible RC combinations. One combination would lead to a high pass filter, and the other would lead to a low pass filter.

We had two resistors, one of 82KΩ and one of 10.0KΩ. We also had two capacitors, one of 1.5nF and one of 100nF.

To determine the right combination, we used the formula for the cutoff frequency of a high and low pass filter:

$$f_c = \frac{1}{2\pi RC} \tag{2.1}$$

And plugging in the values for R=82KΩ and C=1.5nF, we get:

$$f_c = \frac{1}{2\pi \cdot 82K\Omega \cdot 1.5nF} = 12939.42\text{Hz} \tag{2.2}$$

Which is the low pass filter, as the cutoff frequency is the greatest out of all the other combinations.

And plugging in the values for R=10KΩ and C=100nF, we get:

$$f_c = \frac{1}{2\pi \cdot 10K\Omega \cdot 100nF} = 159.15\text{Hz} \tag{2.3}$$

Which is the high pass filter, as the cutoff frequency is the smallest out of all the other combinations.

We use a sine signal with a 5Vpp amplitude without an offset at the function generator. We vary the frequency of the generator from 50Hz all the way to 100kHz. We use the oscilloscope to measure the input and output amplitude, and we record the following values:

Input Voltage(in V)	**Output Voltage(in V)**	**Phase(in deg)**	**Frequency(in Hz)**	**Amplitude(in dB)**
10.4	2.92	73.4	50	-11.03
10.4	5.2	53	100	-6.02
10	7.68	38.8	200	-2.29
10	9.44	16.5	500	-0.50
10.4	10	4.32	1000	-0.34
10.9	10.5	-4.04	2000	-0.32
11.6	10.5	-19.4	5000	-0.87
11.6	9.12	-37.7	10000	-2.09
11.6	6.4	-55	20000	-5.17
11.6	2.92	-73.4	50000	-11.98
12.4	1.54	-80.5	100000	-18.12

Table 2.2: The effect of the frequency on the output amplitude is shown above. We see what we expect: Between certain frequencies the amplitude is greater, while when going as high as 100kHz or as low as 50Hz we notice a significant decline.

3. Evaluation

3.1 Part 1

Below are the results of the low pass filter. The measured frequencies and amplitudes are then plotted on a bode plot.

Input Voltage(in V)	Output Voltage(in V)	Phase(in deg)	Frequency(in Hz)	Amplitude(in dB)
10.1	10	-0.36	50	-0.09
10.1	10	-0.36	100	-0.09
10.1	10	-2.59	200	-0.09
10.2	10	-6.49	500	-0.17
10.4	10.1	-11.8	1000	-0.25
10.9	10	-23	2000	-0.75
12.2	8	-47.4	5000	-3.67
11.8	5.04	-64	10000	-7.39
12.4	2.8	-75.3	20000	-12.93
11.8	1.14	-82.8	50000	-20.30
11.8	0.596	-84.6	100000	-25.93

Table 3.1: The measured frequencies and amplitudes from the low pass filter.

The measured amplitude in decibels is found by using the relation:

$$|A_{dB}| = 20\log_{10}\left(\frac{V_{out}}{V_{in}}\right) \tag{3.1}$$

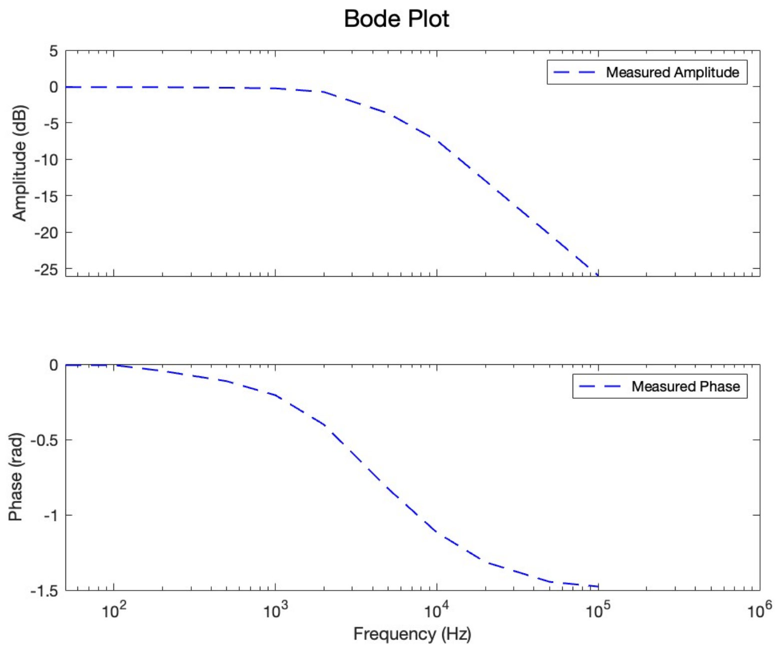

Figure 3.1: Bode plot of measured frequencies, amplitudes and phase shifts of the low pass filter.

To draw the bode plot of the calculated amplitudes and phase, we use the following formulas: $|\underline{A}| = \frac{1}{\sqrt{1+(\omega RC)^2}}$ and $\phi = -\arctan(\omega RC)$

Phase(in deg) - Calculated	**Frequency(in Hz)**	**Amplitude(in dB) - Calculated**
-0.59	50	-0.00
-1.19	100	-0.00
-2.37	200	-0.01
-5.92	500	-0.05
-11.71	1000	-0.18
-22.52	2000	-0.69
-46.03	5000	-3.17
-64.25	10000	-7.24
-76.44	20000	-12.60
-84.49	50000	-20.35
-87.24	100000	-26.34

Table 3.2: The calculated frequencies and amplitudes from the low pass filter using nominal values.

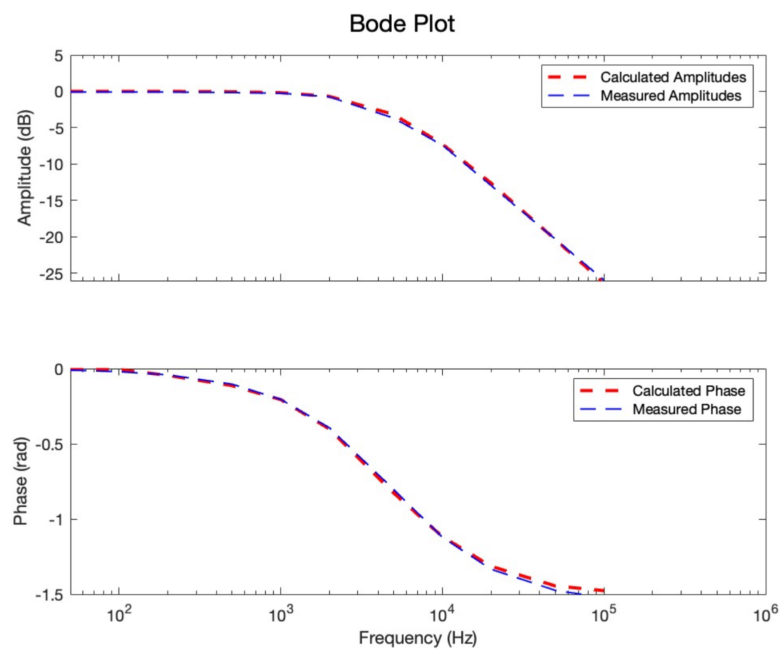

Figure 3.2: Bode plot of calculated amplitudes and phase shifts of the low pass filter using measured frequencies.

The calculated cutoff frequency can be obtained by the following relation for the low pass filter circuit:

$$f_{-3dB} = \frac{1}{2\pi RC} \tag{3.2}$$

Where the cutoff frequency becomes

$$f_{-3dB} = \frac{1}{2\pi(22 \cdot 10^3\Omega) \cdot (1 \cdot 10^{-9}F)} = 4822.8Hz \tag{3.3}$$

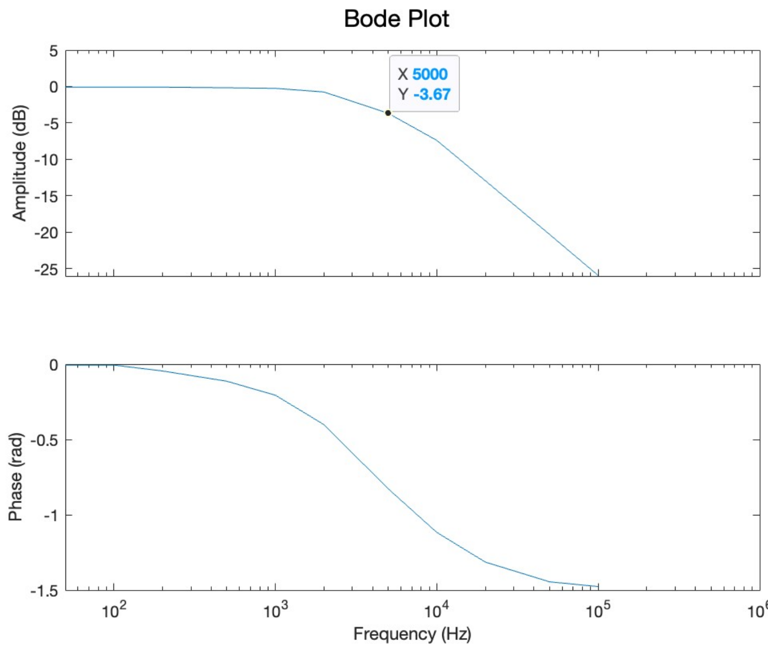

Figure 3.3: Bode plot of measured amplitudes and phase shifts of the low pass filter using measured frequencies.

Comparing against the cutoff frequency of the measured wave, we see that it is quite within range of the calculated cutoff frequency of 4822.8Hz.

To find the gradient of $|A|$ we can simply take the difference between two values. One at 100,000Hz and the other at 10,000Hz(one decade) and divide it by the logarithm of base 10 of the ratio of the frequencies(which in our case, will simply be one, as we have the unit of dB/decade).

Performing this operation, we find that:

$$\frac{\Delta|A|}{\Delta \log_{10} f} = \frac{-26.34 + 7.24}{\log_{10}(10^5/10^4)} = -19.1\text{dB/decade} \tag{3.4}$$

Which is relatively close to the nominal value of -20dB/decade that is expected.

Taking the limit of $|A| = \frac{1}{\sqrt{1+(\omega RC)^2}}$ and the limit of $\phi = -\tan^{-1}(\omega RC)$, we get the following table:

Case	Behaviour of f	Amplitude	Phase
$f \gg f_{-3dB}$	$f \to \infty$	0	$-90°$
$f = f_{-3dB}$	$f \to f_{-3dB}$	$\frac{1}{\sqrt{2}}$	$-45°$
$f \ll f_{-3dB}$	$f \to 0$	1	$0°$

Table 3.3: The behavior of the frequency depending on the case of f_{-3dB}

By this table we can infer that when the frequency is under that of the cutoff frequency, the ratio is one - the filter should ideally let the signal pass through without attenuation and any phase shift. We can also infer that when the frequency is that of the cut-off frequency, we get the amplitude and the phase of the corner frequency. Lastly, we can infer that when the frequency is way greater than the cut-off frequency, the amplitude is zero and the phase is $-90°$.

3.2 Part 2

Below are the results of the band-pass filter. The measured frequencies and amplitudes are then plotted on a bode plot.

Input Voltage(in V)	**Output Voltage(in V)**	**Phase(in deg)**	**Frequency(in Hz)**	**Amplitude(in dB)**
10.4	2.92	73.4	50	-11.03
10.4	5.2	53	100	-6.02
10	7.68	38.8	200	-2.29
10	9.44	16.5	500	-0.50
10.4	10	4.32	1000	-0.34
10.9	10.5	-4.04	2000	-0.32
11.6	10.5	-19.4	5000	-0.87
11.6	9.12	-37.7	10000	-2.09
11.6	6.4	-55	20000	-5.17
11.6	2.92	-73.4	50000	-11.98
12.4	1.54	-80.5	100000	-18.12

Table 3.4: The measured input voltage, output voltage, phase, and amplitude ratio in dB

The measured amplitude in decibels is found by using the relation:

$$|A_{dB}| = 20\log_{10}\left(\frac{V_{out}}{V_{in}}\right) \tag{3.5}$$

We then plot the frequency response on the bode plot as shown below:

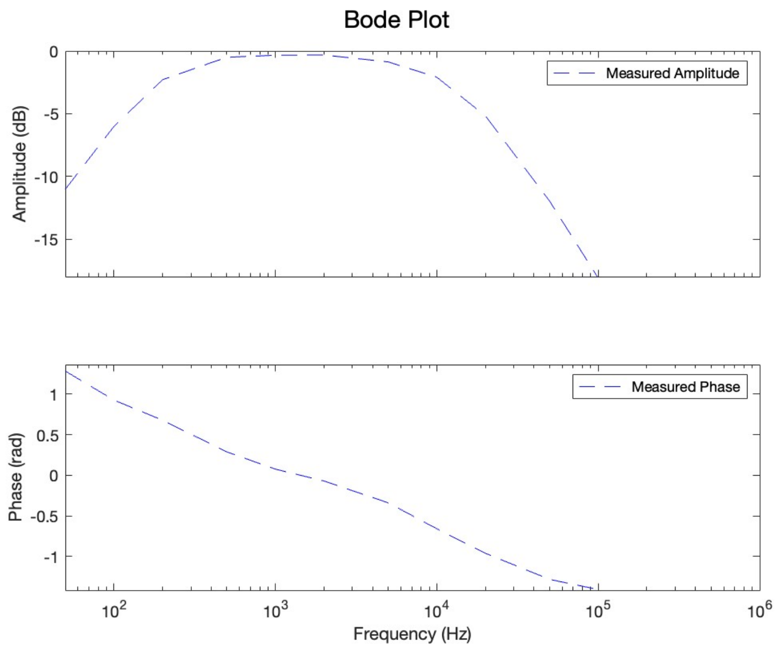

Figure 3.4: Bode plot of measured frequencies, amplitudes and phase shifts of the band pass filter.

Furthermore, to obtain the calculated amplitude and phase of the bandpass filter, one must use the appropriate formulas:

$$|A_{lo}| = \frac{1}{\sqrt{1+(\omega RC)^2}} \text{ and } \phi_{lo} = -\arctan(\omega RC) \text{ for the low pass filter, and}$$

$$|A_{hi}| = \frac{1}{\sqrt{1+1/(\omega RC)^2}} \text{ and } \phi_{hi} = \arctan\left(\frac{1}{\omega RC}\right) \text{ for the high pass filter}$$

To combine these results, the low and high amplitudes are multiplied, and the phases are added.

$|A_{total}| = |A_{hi}| * |A_{lo}|$ and $\phi_{total} = \phi_{hi} + \phi_{lo}$

Phase(in deg)	Frequency(in Hz)	Amplitude(in dB)
72.34	50	-10.47
57.42	100	-5.48
37.63	200	-2.13
15.44	500	-0.43
4.62	1000	-0.13
-4.24	2000	-0.13
-19.30	5000	-0.61
-36.79	10000	-2.03
-56.64	20000	-5.30
-75.31	50000	-12.02
-82.54	100000	-17.83

Table 3.5: The table of calculated phase and amplitudes from the nominal values, using the formulas above.

Using this methodology, we can compose a graph of calculated amplitude and phase values and compare it to our measured values.

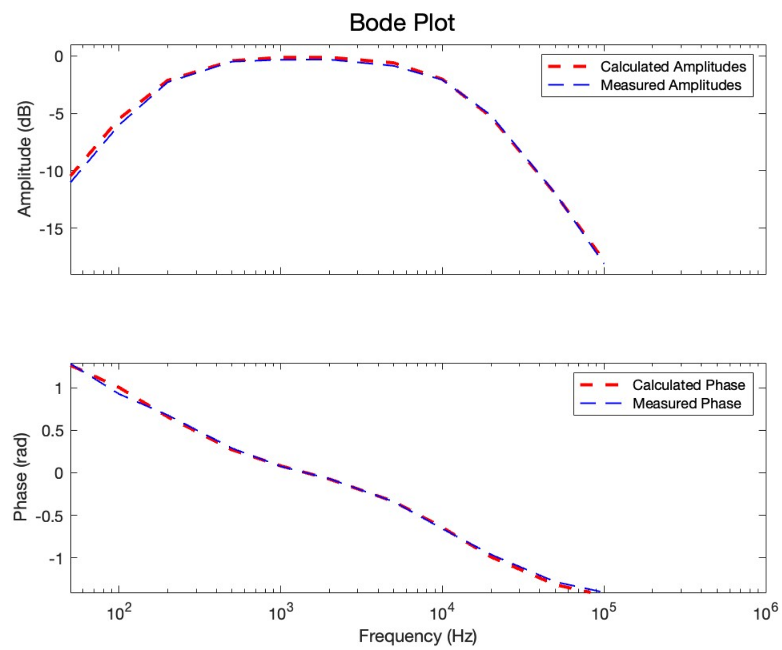

Figure 3.5: Bode plot of calculated phase and amplitude vs the bode plot of the measured phase and amplitude.

We see that the calculated values are very close, almost superimposed. This means that our measured values are as expected, and very close to the nominal amplitude and phase.

To calculate the lower and upper cut-off frequencies, we have to treat each component of the band-pass filter(high and low pass) independently.

We find that for the high-pass component's frequency:

$$f_{-3dB} = \frac{1}{2\pi \cdot 10^4 \cdot 10^{-7}} = 159.15\text{Hz} \tag{3.6}$$

and for the low-pass component's frequency:

$$f_{-3dB} = \frac{1}{2\pi \cdot 8200 \cdot 1.5 \cdot 10^{-9}} = 12939\text{Hz} \tag{3.7}$$

The center frequency is then defined as the root of the multiple of both components' frequencies, so:

$$f_c = \sqrt{f_{c,lo} \cdot f_{c,hi}} = \sqrt{12939 \cdot 159.15} = 1435\text{Hz} \tag{3.8}$$

The bandwidth is simply the difference between the components, so

$$B = f_{c,lo} - f_{c,hi} = 12939 - 159.15 = 12779.85\text{Hz} \tag{3.9}$$

The phase shift, ϕ, at the cutoff frequencies is calculated for their respective component:

$$\phi_{hi} = \arctan\left(\frac{1}{2\pi \cdot 12799.85 \cdot 8200 \cdot 1.5 \cdot 10^{-9}}\right) = 45^\circ \tag{3.10}$$

And for the low pass component:

$$\phi_{hi} = -\arctan\left(2\pi \cdot 159.15 \cdot 10^4 \cdot 10^{-7}\right) = -45^\circ \tag{3.11}$$

So the total phase shift, by summation:

$$\phi_{total} = \phi_{hi} + \phi_{lo} = 45^\circ - 45^\circ = 0^\circ \tag{3.12}$$

The results of the measurements are very close to the calculations. The only error that we encounter is from the measurements of the oscilloscope, and due to the passive components used not being completely ideal.

To obtain the Nyquist plot of the band-pass filter, we multiply the transfer functions of the low-pass and high-pass components, and then multiply by the input voltage.

$$U_R(\omega) = (H_{lo}(j\omega) \cdot H_{hi}(j\omega)) \cdot V_{in} \tag{3.13}$$

$$U_R(\omega) = \frac{1}{(C_1 \cdot R_1 \cdot j\omega + 1)\left(\frac{1}{C_2 \cdot R_2 \cdot j\omega} + 1\right)} V_{in} \tag{3.14}$$

Where we have established that $\omega = 2\pi f$.
The nyquist plot is then drawn using MATLAB.

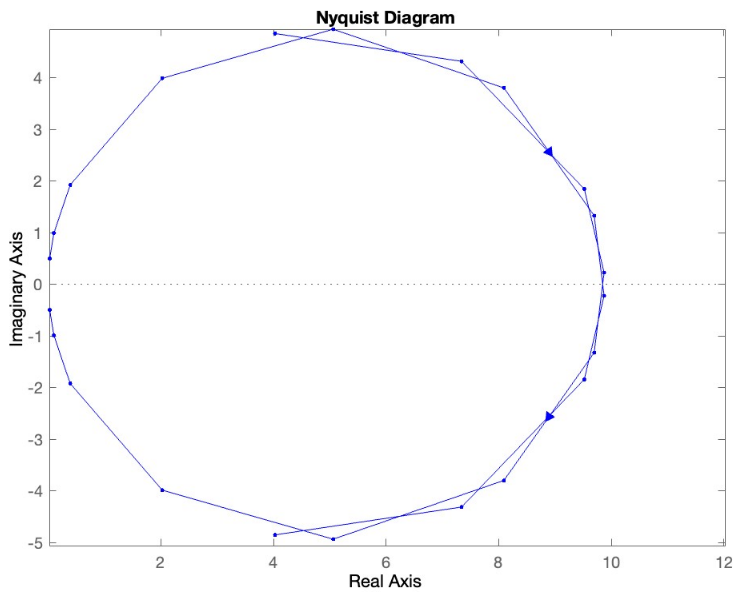

Figure 3.6: Nyquist plot of the bandpass filter, composed of the transfer function of the high pass filter and the low pass filter. Found using MATLAB's nyquist function with the combined transfer functions of the low and high pass filter, with the frequencies we have tested ranging from 50Hz to 100kHz.

4. Conclusion

In conclusion, we examined the bode plot, and thereby frequency response of a low pass filter and a band-pass filter. The low pass filter was measured in the lab using the oscilloscope, using a simple passive component-based RC circuit.

To analyze the band-pass filter, we also included a nyquist plot. The calculated values were very close to the measured values, and the only error source are the passive elements themselves not being entirely ideal, and the oscilloscope's measurements. The values were as expected, and the cut-off frequencies lined up nicely with what we measured.

Furthermore, we learned how the product of transfer functions can be used to identify more complex filters, such as the band-pass filter, which while simple in nature is very useful in proving the usefulness of transfer functions.

We found that taking the limit of the low pass filter as the frequency is greater than the cut-off frequency, the amplitude tends to zero, and the phase to -90 degrees, which was observed in our plots as well. We also found that taking the limit of the low pass filter as the frequency is just at the cut-off frequency, we get a phase shift of about -45 degrees, and as the frequency tends to zero, the amplitude is maximized and the phase is 0 degrees.

The band-pass filter was also built and it was shown to only let a range of frequencies pass through, where an increasing frequency around the center frequency range(about 1000-2000Hz) would result in a larger amplitude, and a decreasing frequency or a larger one would result in a smaller amplitude.

5. References

- Gen EE 2 - CH-211-B Manual
- Google Sheets
- MATLAB
- CH-211-A Lecture 2 - Frequency Response Analysis

6. Appendix

The data gathered for experiment 6 is shown below.

R1: 10k Ohms			
V_in(in V)	V_out(in V)	Frequency(in Hz)	Phase(in deg)
0.536	0.532	1000	-180
0.568	0.56	2000	177
0.616	0.608	5000	178
0.624	0.616	10000	178
0.608	0.612	20000	175
0.612	0.616	50000	172
0.62	0.6	100000	165
0.612	0.576	200000	150
0.612	0.416	500000	111
0.612	0.236	1000000	75.4
0.612	0.116	2000000	41.3
0.608	0.044	5000000	143

R1: 22k Ohms			
V_in(in V)	V_out(in V)	Frequency(in Hz)	Phase(in deg)
0.536	2.36	1000	-179
0.568	2.5	2000	178
0.596	2.62	5000	177
0.604	2.68	10000	175
0.608	2.68	20000	170
0.612	2.58	50000	160
0.616	2.16	100000	135
0.624	1.3	200000	109
0.624	0.58	500000	95.1
0.632	0.32	1000000	84.9
0.616	0.18	2000000	48.2
0.616	0.052	5000000	134

R1: 1k Ohms			
V_in(in V)	V_out(in V)	Frequency(in Hz)	Phase(in deg)
0.52	5.08	1000	177
0.556	5.36	2000	176
0.592	5.68	5000	175
0.596	5.72	10000	171
0.6	5.64	20000	163
0.628	4.52	50000	137
0.612	2.68	100000	115
0.62	1.44	200000	99.9
0.616	0.64	500000	86.1
0.62	0.272	1000000	66
0.636	0.132	2000000	41.5
0.608	0.046	5000000	163

Part 2:

Square Wave Integrating Amplifier			
V_in(in V)	V_out(in V)	Frequency(in Hz)	Phase(in deg)
1.01	2.56	100	92.3

Sine Wave Integrating Amplifier			
V_in(in V)	V_out(in V)	Frequency(in Hz)	Phase(in deg)
1.04	16.6	10	97
1.02	8.32	20	93.6
1.02	3.36	50	92.2
1.02	1.7	100	91.1
1.02	0.86	200	89.7
1.02	0.344	500	88.2
1.02	0.18	1000	95

Part 3:

V_out(in V)	-1.22
V+(in V)	4.961
V-(in V)	5.087
V_AM(in V)	0.1264