Constructor University Bremen

Lab Report 1: RLC Circuits - Transient Response

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1. Introduction

1.1 Objective

The objective of this experiment is to investigate the behavior of second-order systems, namely, the RLC circuit. During the lab, various RLC circuit configurations are built and theoretical results are compared with experimental results with the aid of MATLAB.

1.2 Introduction

Second order systems are very common, named due to the highest order of the differential equation describing the system. For an RLC circuit, we tend to use a second-order ordinary differential equation to describe the system, which is given by

$$a_2\frac{d^2y(t)}{dt^2} + a_1\frac{dy(t)}{dt} + a_0y(t) = x(t) \tag{1.1}$$

Where we define $y(t)$ as the output of the system, $x(t)$ as the input of the system, and a_2, a_1, and a_0 as system parameters.

However, in the context of the response of second-order systems we tend to use a more general form of this equation, given by

$$\frac{d^2y(t)}{dt^2} + 2\zeta\omega_n\frac{dy(t)}{dt} + \omega_n^2y(t) = K\omega_n^2x(t) \tag{1.2}$$

Where we now define the system parameters as:

- ζ - damping ratio
- ω_n - natural frequency
- K - gain of the system

For second order differential equations, we know that $y_t = y_p + y_h$ where y_p is the particular solution and y_h is the homogeneous solution.

To solve for the homogenous solution, we set the input to zero and solve the differential equation. The general solution for the homogenous solution is given by

$$y_h(t) = C_1 e^{\lambda_1 t} + C_2 e^{\lambda_2 t} \tag{1.3}$$

Where C_1 and C_2 are constants defined by initial conditions and λ_1 and λ_2 are the roots of the characteristic equation that are determined by the resistor, capacitor, and inductor values.

We know that depending on ζ and the undamped natural frequency ω_n we can classify the transient response of the system into three categories:

- Overdamped - $\zeta > 1$
- Critically damped - $\zeta = 1$
- Underdamped - $\zeta < 1$

Each of these categories has a different equation, which are given below:

1. **Overdamped**

   In the overdamped case, the response is the sum of two decaying exponentials, defined as

   $$y(t) = C_1 \exp\left(\left(-\zeta + \sqrt{\zeta^2 - 1}\right) w_n t\right) + C_2 \exp\left(\left(-\zeta - \sqrt{\zeta^2 - 1}\right) w_n t\right) \tag{1.4}$$

2. **Critically damped**

   In the case of critical damping, the system reaches steady state in the shortest amount of time, the equation given by the state below.

   $$y(t) = C_1 e^{\lambda_1 t} + C_2 t e^{\lambda_2 t} \tag{1.5}$$

3. **Underdamped**

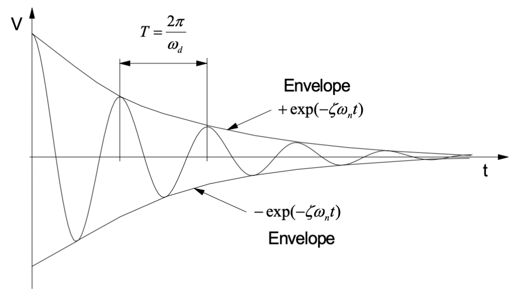


Figure 1.1: Underdamped Response

In the underdamped case, we can observe ringing in the response. This is due to the fact that the system is oscillating around the steady state value.

For the underdamped case, let's consider w_d as the damped natural frequency, given by $w_d = w_n\sqrt{(1-\zeta^2)}$. We can then write the equation as

$$y(t) = e^{-\zeta\omega_n t}\left(C_1 \cos\left(w_d t\right) + C_2 \sin\left(w_d t\right)\right) \tag{1.6}$$

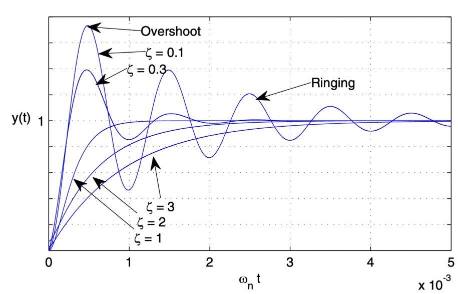


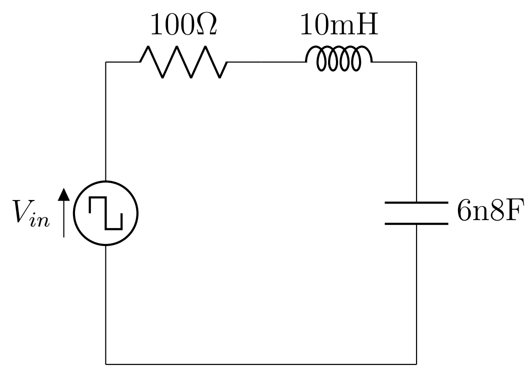
Figure 1.2: General Transient Response Diagram

Below is a figure that shows the cases of the transient response of a second order system according to the damping ratio ζ when provided with a step input.

Considering a forced solution is out of the scope of this lab, but in general, the output will usually be a weighted sum of the input signal $x(t)$ and its first and second derivatives.

2. Execution

- $V_{pp} = 1\text{V}$
- $V_{off} = 0.5\text{V}$
- $f = 100\text{Hz}$
- $R_i = 50\Omega$



1. The function generator was set to produce a 100Hz square wave with an amplitude of 0.5V and an offset of 0.5V. It was checked with the oscilloscope if the signal modulated between 0V and 1V.

2. Subsequently, the R-decade was set to 100Ω, and the oscilloscope was connected in parallel to the capacitor.

3. The damped frequency f_d was measured. To determine f_d, the period of the exponentially damped sinusoidal waveform was measured using the oscilloscope. A hardcopy was taken of one signal period and another focusing on the ringing phenomenon.

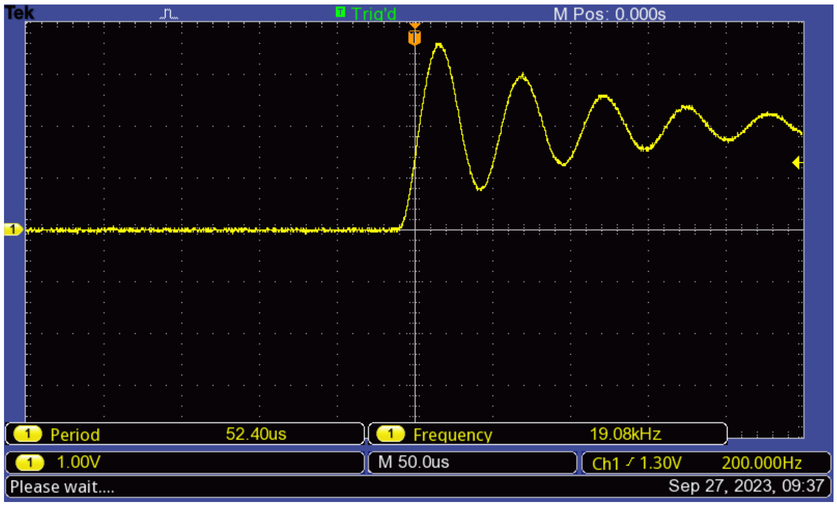


Figure 2.1: Ringing phenomenon

According to the figure above, the period of the ringing phenomenon is $52\mu s$.

4. Afterward, the damped radian frequency ω_d was found by knowing that the period of the damped sinusoidal waveform is $f_d = \frac{1}{T}$, where $T = 52\mu s$, the period of the ringing phenomenon. Therefore we find that the damped radian frequency ω_d is

$$\begin{aligned} f_d = \frac{1}{T} = \frac{1}{52\mu s} = 1.923 \cdot 10^4 \text{ Hz} \\ \omega_d = 2\pi f_d = 1.208 \cdot 10^5 \text{ rad/s} \end{aligned} \tag{2.1}$$

Very close to the nominal values of

$$\begin{aligned} \omega_d = \frac{1}{\sqrt{LC}} = 1.21 \cdot 10^5 \text{ rad/s} \\ f_d = \frac{\omega_d}{2\pi} = 1.930 \cdot 10^4 \text{ Hz} \end{aligned} \tag{2.2}$$

5. The resistance required for the circuit to be critically damped was then calculated.

$$R = \frac{2\zeta}{\sqrt{C/L}} \implies R_{optimal} = 2\frac{1}{\sqrt{C/L}} - 50\Omega = 2375\Omega = 2.375\text{k}\Omega \tag{2.3}$$

Where the 50Ω is subtracted to account for the internal resistance of the function generator.

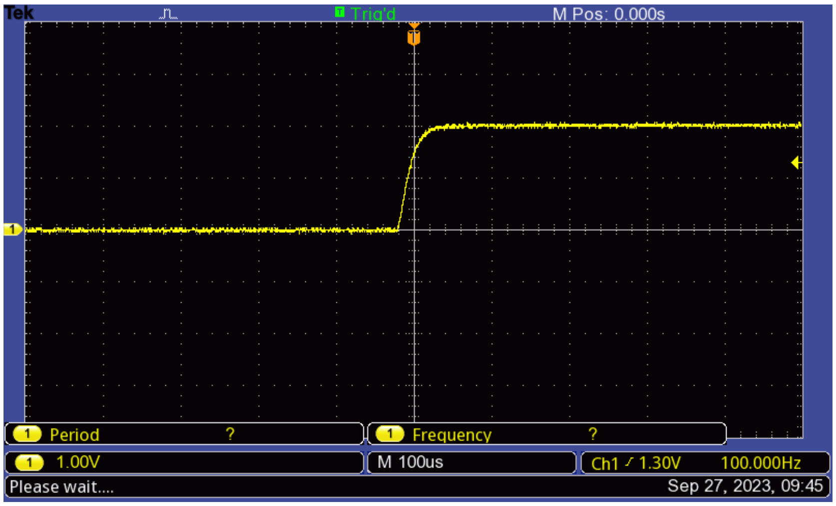


Figure 2.2: Critically damped signal using the nominal resistance value of 2.375kΩ

However, we find that the optimal resistance is not very close to the nominal value. By playing around with the R-decade, we find that the optimal resistance is

$$R_{optimal} = 1905\Omega = 1.905\text{k}\Omega \tag{2.4}$$

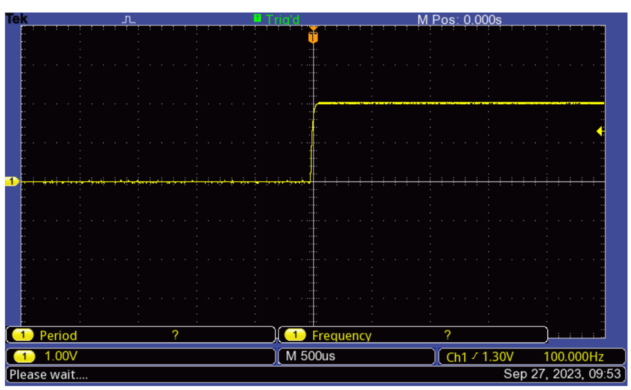


Figure 2.3: Critically damped signal using the tuned resistance value of 1.905kΩ

6. Finally, the R-decade was set to 30kΩ, causing the circuit to be over-damped. The transient voltage across the capacitor was displayed, and a hardcopy was taken.

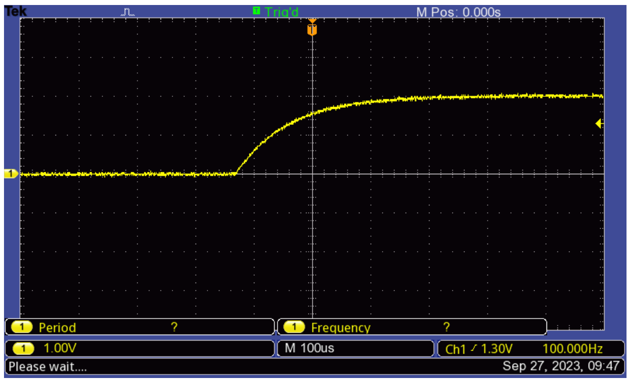
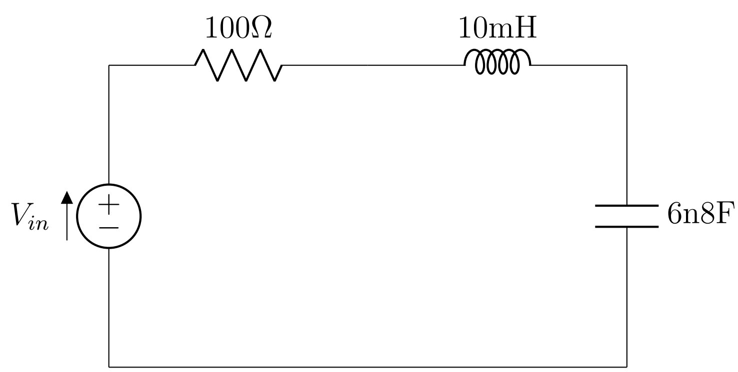


Figure 2.4: Over-damped signal

3. Evaluation

3.0.1 Series RLC Underdamped and Critically Damped Behaviour



The second order differential equation for this RLC series circuit is solved by remembering that

$$i = i_C = C\frac{dV_c}{d_t} \tag{3.1}$$

Taking this into account, our mesh will be defined as follows:

$$V_{in} = V_R + V_C + V_L \tag{3.2}$$

Where each component is defined by their respective relations:

$$\begin{aligned} V_L &= L\frac{di}{dt} \\ V_R &= iR \\ i_C &= C\frac{dV_C}{dt} \end{aligned} \tag{3.3}$$

Substituting these relations into the mesh equation, the following is obtained:

$$V_{in} = LC\frac{d^2V_C}{dt^2} + RC\frac{dV_C}{dt} + V_C \tag{3.4}$$

Which is a second-order differential equation. The following constants can now be defined:

$$a_2 = LC, \quad a_1 = RC, \quad a_0 = 1 \tag{3.5}$$

Subsequently, for the proper form of the differential equation the following are defined:

$$\zeta = \frac{a_1}{2\sqrt{a_0 a_2}}, \quad \omega_n = \sqrt{\frac{a_0}{a_2}} \tag{3.6}$$

Using **MATLAB**, the behaviour of the circuit can be verified.

```
% For the case where R = 100Ohm, C = 6.8nF and L=10mH
R = 100;
C = 6.8E-9;
L = 10E-3;

zeta = R/2 * sqrt(C/L); % approximately 0.0412, so underdamped
w_n = 1/sqrt(L*C);
w_d = w_n * sqrt(1 - zeta^2);
```

It is found that

$$\zeta = 0.0412, \quad \omega_n = 1.2127 \times 10^5 \text{rad/s} \quad \omega_d = 1.2116 \times 10^5 \text{rad/s} \tag{3.7}$$

Which indicates that the circuit is underdamped. The initial conditions can be identified using the total response of the circuit, given by:

$$y_t(t) = y_h(t) + y_f(t) \tag{3.8}$$

Where $y_h(t)$ is the homogeneous response and $y_f(t)$ is the forced response.

$$y(t) = e^{-\zeta\omega_n t}\left(C_1 \cos(\omega_d t) + C_2 \sin(\omega_d t)\right) + V_{in} \tag{3.9}$$

Where V_{in} in this case is simply 1V. At at $t = 0$, the voltage over the capacitor is 0V.

$$y(0) = C_1 + V_{in} \implies y(0) = C_1 + V_{in} \implies C_1 = -V_{in} = -1V. \tag{3.10}$$

Consider that the change over the capacitor is also 0V at immediately $t = 0$, it follows that for:

$$\frac{dy}{dt} = e^{-\zeta\omega_n t}\left(C_2\omega_d\cos(\omega_d t) - C_1\omega_d\sin(\omega_d t)\right) - \omega_n\zeta e^{-\zeta\omega_n t}\left(C_1\cos(\omega_d t) + C_2\sin(\omega_d t)\right) \tag{3.11}$$

Evaluated at $t = 0$, it is obtained that:

$$\begin{aligned} \frac{dy}{dt}(0) &= C_2\omega_d - \omega_n\zeta C_1 \\ 0 &= C_2\omega_d - \omega_n\zeta C_1 \\ C_2 &= -\frac{\omega_n}{\omega_d}\zeta \end{aligned} \tag{3.12}$$

Which, using **MATLAB**, leads that $C_2 = -0.0413$.

```
C1 = -1;
C2 = -(w_n/w_d) * zeta; % -0.0413
```

Plotting the data using **MATLAB** the following for the voltage over the capacitor is obtained:

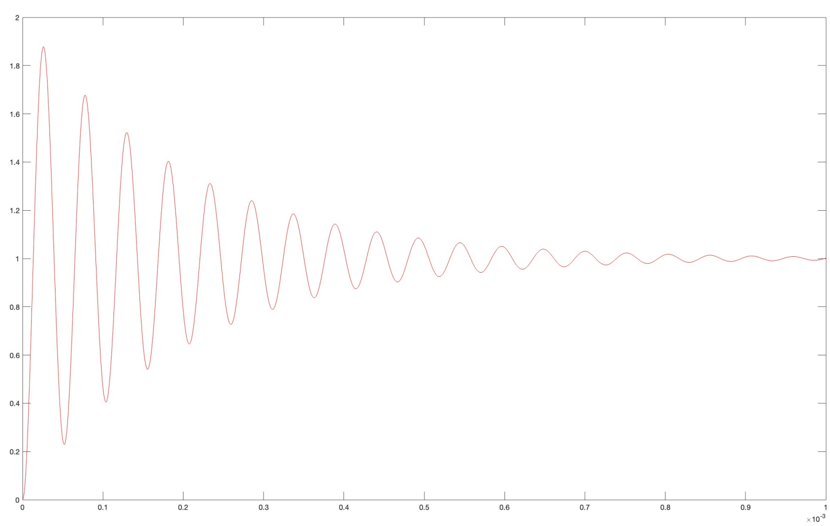


Figure 3.1: Voltage over the capacitor

Showing that the circuit is indeed underdamped.

In order to get **critically damped** behaviour, $\zeta = 1$. The following relation to find the optimal resistance to make ζ go to 1.

$$\begin{gathered}\zeta = \frac{R}{2}\sqrt{C/L} \\ \text{Knowing that } \zeta = 1 \\ R = \frac{2}{\sqrt{C/L}}\end{gathered} \tag{3.13}$$

For the critically damped circuit, the initial conditions C_1 and C_2 can be found by knowing that the general solution for a critically damped circuit is given

$$y(t) = e^{-\zeta\omega_n t}\left(C_1 + C_2 t\right) + V_{in} \tag{3.14}$$

Where V_{in} is the particular solution to the differential equation, because at steady state, the voltage over the capacitor is equal to the voltage of the voltage source.

Knowing that V_{in} is 1V, the initial condition C_1 straightforwardly can be found:

$$y(0) = C_1 + V_{in} \implies C_1 = -V_{in} = -1V. \tag{3.15}$$

Finding C_1, C_2 can be found by taking the derivative of the general solution for this critically damped circuit:

$$\frac{dy}{dt} = -\zeta\omega_n C_1 e^{-\zeta\omega_n t} - \zeta w_n C_2 t e^{-\zeta\omega_n t} + C_2 e^{-\zeta\omega_n t} \tag{3.16}$$

Evaluating this at $t = 0$, and knowing $\zeta = 1$, it is obtained that:

$$0 = -\zeta\omega_n C_1 + C_2 \implies C_2 = \zeta\omega_n C_1 \implies C_2 = -\omega_n \tag{3.17}$$

Subsequently, arrive at the conclusion that $C_2 = -1.2127 \times 10^5$. Plotting the data using **MATLAB** the following for the voltage over the capacitor in both the critically damped and underdamped cases:

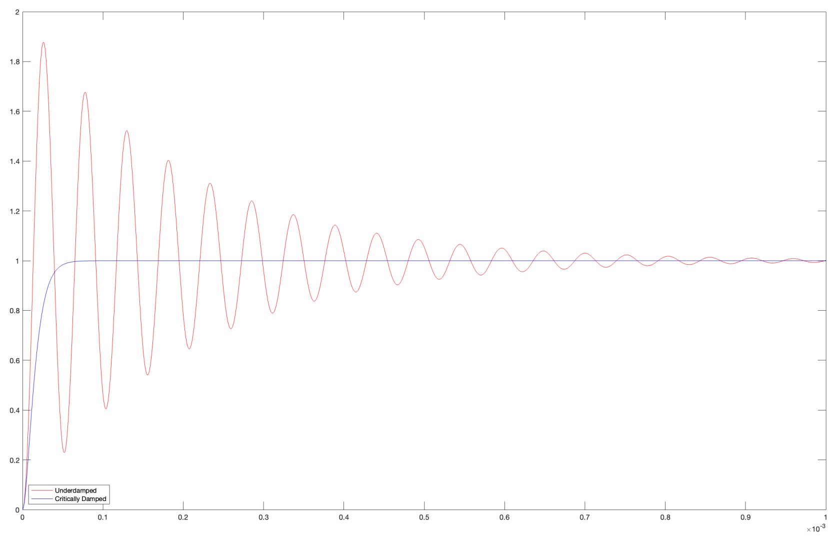


Figure 3.2: Voltage over the capacitor for both underdamped and critically damped cases

The **MATLAB** code used to generate both of these plots is provided below:

```
%% Part 1:
clear

% For the case where R = 100Ohm, C = 6.8nF and L=10mH
R = 100;
C = 6.8E-9;
L = 10E-3;

zeta = R/2 * sqrt(C/L); % approximately 0.0412, so underdamped
w_n = 1/sqrt(L*C);
w_d = w_n * sqrt(1 - zeta^2);

% We know that the circuit is underdamped because zeta < 1

% We obtained C1 = -1, C2 = -(w_n/w_d) * zeta
C1 = -1;
C2 = -(w_n/w_d) * zeta;

t = 0:1E-6:1E-3;
y = exp(-zeta * w_n .* t) .* (C1*cos(w_d.*t) + C2 * sin(w_d.*t)) + 1;
plot(t, y, 'red');

hold on

%% Critically damped

% For the critically damped case:
R = 2 * (1 / sqrt(C/L));
zeta = R/2 * sqrt(C/L);
w_n = 1/sqrt(L*C);

% We obtained that C1 = -1, and C2 = -w_n.
C2 = -w_n;
C1 = -1;
y = (C1*exp(-zeta * w_n .* t) + C2.*t.*exp(-zeta * w_n .* t)) + 1;
plot(t, y, 'blue');

legend({'Underdamped','Critically Damped'},'Location','southwest')
```

Compared to the results obtained in the laboratory, it is observed that the results are very similar, with the only difference being that the voltage over the capacitor in the laboratory when the resistance is at its optimum theoretical value of 2375Ω, where ζ becomes 1 and thereby gives us critical damping, gives us a slightly more over-damped response than playing around with the resistance in the R-Decade, which got us a resistance of around 1905Ω. This is because of the internal resistance of the R-Decade, which is not taken into account in the theoretical calculations. Furthermore, the theoretical calculations do not take into account the resistance of the wires, the resistance of the capacitor, nor the resistance of the inductor, which all contribute to the overall resistance of the circuit. This is why the theoretical calculations do not match the laboratory results exactly.

3.0.2 Circuit Problem

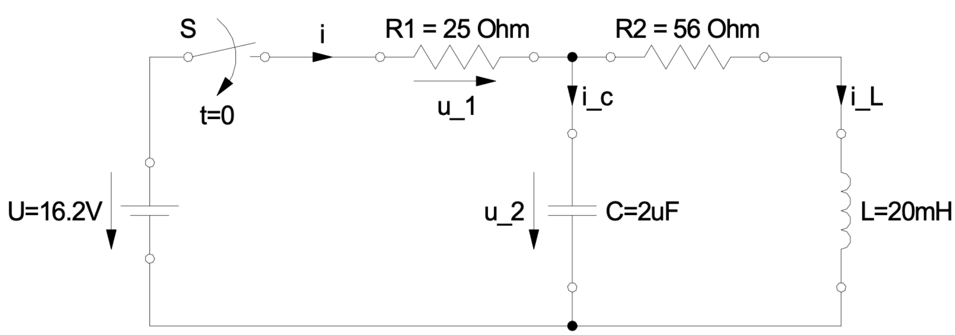


Figure 3.3: The circuit that must be solved.

To find the current over the inductor i_L, first two mesh equations are written:

For the first mesh,

$$V_{in} = R_1 i_1(t) + \frac{1}{C}\int i_1(t) - i_2(t)dt \tag{3.18}$$

For the second mesh,

$$R_2 i_2(t) + L\frac{di_2(t)}{dt} + \frac{1}{C}\int i_2(t) - i_1(t)dt = 0 \tag{3.19}$$

Relations to find $i_2 = i_L$ are obtained. Starting with the second mesh, multiplying both sides by C and by knowing that the integral $\int i_2(t) - i_1(t)dt$ can be separated by linearity, it is found that:

$$\int i_1(t)dt = R_2 C i_2(t) + LC\frac{di_2(t)}{dt} + \int i_2(t)dt \tag{3.20}$$

Multiplying both sides by C in the first mesh and plugging the found relation, it is found that:

$$R_1 C i_1 + R_2 C i_2 + LC\frac{di_2(t)}{dt} + \int i_2(t)dt - \int i_2(t)dt = CV_{in} \tag{3.21}$$

Taking the derivative of equation 3.20, it is obtained:

$$i_1(t) = R_2 C\frac{di_2(t)}{dt} + LC\frac{d^2 i_2(t)}{dt^2} + i_2(t) \tag{3.22}$$

This setup allows finding the full differential equation for $i_2 = i_L$.

First, equation 3.21 is divided by C on both sides, and equation 3.22 can be substituted into equation 3.21, which leads to the following:

$$R_1\left(R_2 C\frac{di_2(t)}{dt} + LC\frac{d^2 i_2(t)}{dt^2} + i_2(t)\right) + R_2 i_2(t) + L\frac{di_2(t)}{dt} = V_{in} \tag{3.23}$$

Normalizing into the standard form of a second order differential equation, the following is obtained:

$$(R_1 LC)\frac{d^2 i_2(t)}{dt^2} + (R_1 R_2 C + L)\frac{di_2(t)}{dt} + (R_1 + R_2)i_2(t) = V_{in} \tag{3.24}$$

Where the following constants can now be denoted:

$$a_2 = R_1 LC, \quad a_1 = R_1 R_2 C + L, \quad a_0 = R_1 + R_2 \tag{3.25}$$

This information can be used to find the damping ratio of the circuit ζ, which can be used to find the behaviour of the circuit. By **MATLAB**, it is found that $\zeta = 1.266$, which means the circuit is **overdamped**. Furthermore, it is found that $\omega_n = 9000\text{rad/s}$

The **MATLAB** code used to find the damping ratio, gain, and the natural frequency is provided below:

```
R1 = 25;
R2 = 56;
L = 20E-3;
C = 2E-6;
Vin = 16.2;

a2 = R1*L*C;
a1 = R1*R2*C + L;
a0 = R1+R2;

w_n = sqrt(a0 / a2);
zeta = a1 / (2*sqrt(a0 * a2));
K = 1/a0;
```

The complete response of the circuit is given by:

$$y(t) = C_1 e^{\left(-\zeta+\sqrt{\zeta^2-1}\right)\omega_n t} + C_2 e^{\left(-\zeta-\sqrt{\zeta^2-1}\right)\omega_n t} + i_{Lp}(t) \tag{3.26}$$

Where i_{Lp} is the particular solution to the differential equation, which is given by:

$$i_{Lp} = \frac{V_{in}}{R_1 + R_2} = 0.2\text{A} \tag{3.27}$$

Because the inductor at the steady state becomes a wire. It must also be noted that for $t = 0$:

$$i_L(0) = 0, \quad \frac{di_L}{dt}(0) = 0 \tag{3.28}$$

Because in the transient state, the inductor is an open-circuit to sudden changes of current.

Solving for the initial conditions:

$$\begin{aligned} y(0) &= C_1 + C_2 + i_{Lp} \\ 0 &= C_1 + C_2 + i_{Lp} \\ C_1 &= -C_2 - i_{Lp} \end{aligned} \tag{3.29}$$

Solving for C_2:

$$
\begin{gathered}
\frac{di_L}{dt}(0) = \frac{d}{dt}(C_1 e^{\left(-\zeta+\sqrt{\zeta^2-1}\right)\omega_n t} + C_2 e^{\left(-\zeta-\sqrt{\zeta^2-1}\right)\omega_n t} + i_{Lp}(t)) \\
0 = ((-\zeta + \sqrt{\zeta^2-1})\omega_n) C_1 e^{\left(-\zeta+\sqrt{\zeta^2-1}\right)\omega_n t} + ((-\zeta - \sqrt{\zeta^2-1})\omega_n) C_2 e^{(-\zeta-\sqrt{\zeta^2-1})\omega_n t} \\
0 = \omega_n \left((-\zeta + \sqrt{-\zeta^2-1})(-C_2 - i_{Lp}) + (-\zeta - \sqrt{-\zeta^2-1})C_2\right) \\
0 = -2C_2\sqrt{\zeta^2-1} + \zeta i_{Lp} - i_{Lp}\sqrt{\zeta^2-1} \\
C_2 = \frac{i_{Lp}(\zeta - \sqrt{\zeta^2-1})}{2\sqrt{\zeta^2-1}} \\
C_2 = 0.0629 \\
C_1 = -C_2 - i_{Lp} = -0.2629
\end{gathered}
\tag{3.30}
$$

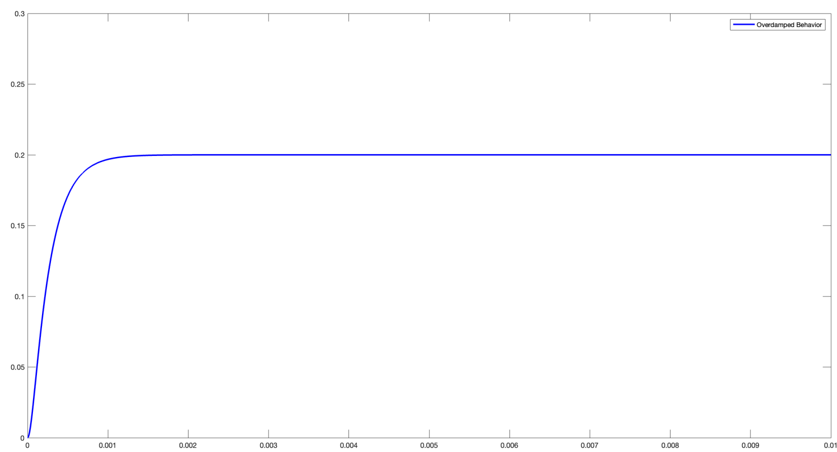


Figure 3.4: The current over the inductor.

Using C_1, C_2 and i_{Lp}, the full response of the circuit is plotted using **MATLAB** with the code provided below.

```
R1 = 25;
R2 = 56;
L = 20E-3;
C = 2E-6;
Vin = 16.2;

a2 = R1*L*C;
a1 = R1*R2*C + L;
a0 = R1+R2;

w_n = sqrt(a0 / a2);
zeta = a1 / (2*sqrt(a0 * a2));
K = 1/a0;

iL_p = Vin / (R1 + R2);
C2 = iL_p * (zeta - sqrt(zeta^2 - 1)) / (2 * sqrt(zeta^2 - 1));
C1 = -C2 - iL_p;

% The current over the inductor.
t = 0:1E-6:1E-2;
% It is overdamped.
y = C1 .* exp((-zeta + sqrt(zeta^2 - 1)) .* t .* w_n) + ...
        C2 .* exp((-zeta - sqrt(zeta^2 - 1)) .* t .* w_n) + iL_p;

plot(t, y, "blue", "LineWidth", 2);
ylim([0, 0.3])
legend("Overdamped Behavior")
```

4. Conclusion

In this experiment, the transient response of the RLC circuit to various different configurations of resistance was explored.

The oscilloscope, in general, showed what is expected of the circuit. The transient voltage of the capacitor was observed to show the ringing phenomenon characteristic of underdamped systems due to the low resistance of 100Ω.

When the resistance was tuned to close to the optimal nominal values for a critically damped response, the transient response was observed to be slightly overdamped. This is because of the R-Decade having its own internal resistance, as well as the internal resistance of the non-ideal capacitor.

The resistance was then tuned until critical damping behaviour was observed. The value has a slight error from the nominal value, for reasons discussed above. Afterward, the resistance was increased to $30k\Omega$ to view the effect of overdamping, where the result was shown in the oscilloscope, as expected.

To solve the evaluation circuits, second order differential equations were used. The damping behaviour was generally determined by solving for the damping ratio, ζ, and the general solution to the circuit was then solved by finding the initial conditions C_1 and C_2, and by finding the particular solution.

5. References

1. **MATLAB** Documentation
2. Signals and Systems Lab Manual

6. Appendix

6.1 MATLAB Code for RLC Transient Response

```
%% Part 1:
clear

% For the case where R = 100Ohm, C = 6.8nF and L=10mH
R = 100;
C = 6.8E-9;
L = 10E-3;

zeta = R/2 * sqrt(C/L); % approximately 0.0412, so underdamped
w_n = 1/sqrt(L*C);
w_d = w_n * sqrt(1 - zeta^2);

% We know that the circuit is underdamped because zeta < 1

% We obtained C1 = -1, C2 = -(w_n/w_d) * zeta
C1 = -1;
C2 = -(w_n/w_d) * zeta;

t = 0:1E-6:1E-3;
y = exp(-zeta * w_n .* t) .* (C1*cos(w_d.*t) + C2 * sin(w_d.*t)) + 1;
plot(t, y, 'red');

hold on

%% Critically damped

% For the critically damped case:
R = 2 * (1 / sqrt(C/L));
zeta = R/2 * sqrt(C/L);
```

```
w_n = 1/sqrt(L*C);

% We obtained that C1 = -1, and C2 = -w_n.
C2 = -w_n;
C1 = -1;
y = (C1*exp(-zeta * w_n .* t) + C2.*t.*exp(-zeta * w_n .* t)) + 1;
plot(t, y, 'blue');

legend({'Underdamped','Critically Damped'},'Location','southwest')

%% Part 3:
clear
close all

R1 = 25;
R2 = 56;
L = 20E-3;
C = 2E-6;
Vin = 16.2;

a2 = R1*L*C;
a1 = R1*R2*C + L;
a0 = R1+R2;

w_n = sqrt(a0 / a2);
zeta = a1 / (2*sqrt(a0 * a2));
K = 1/a0;

iL_p = Vin / (R1 + R2);
C2 = iL_p * (zeta - sqrt(zeta^2 - 1)) / (2 * sqrt(zeta^2 - 1));
C1 = -C2 - iL_p;

% The current over the inductor.
t = 0:1E-6:1E-2;
% It is overdamped.
y = C1 .* exp((-zeta + sqrt(zeta^2 - 1)) .* t .* w_n) + ...
        C2 .* exp((-zeta - sqrt(zeta^2 - 1)) .* t .* w_n) + iL_p;
```

```
plot(t, y, "blue", "LineWidth", 2);
ylim([0, 0.3])
legend("Overdamped Behavior")
```

6.2 Prelab - RLC Frequency Response

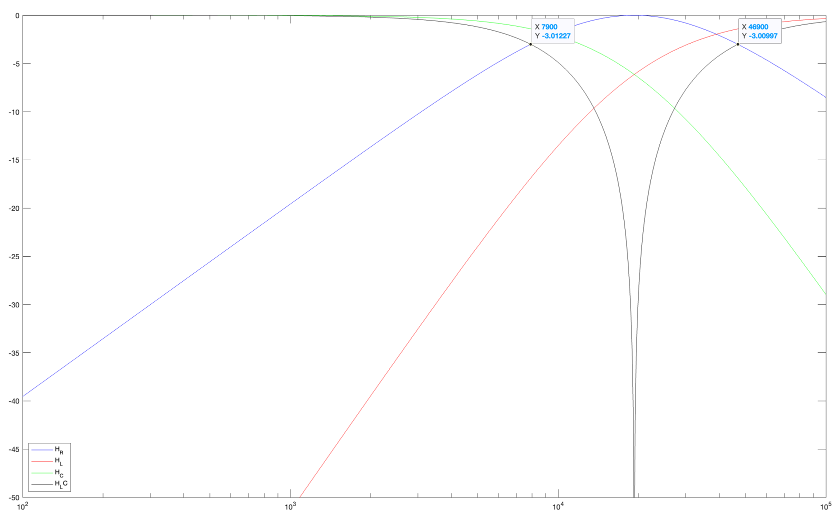


Figure 6.1: RLC Series Voltages taken over different components

From the plot, we find that the corner frequencies are given as

$$\omega_1 = 7.9E3 \text{ rad/s and } \omega_2 = 4.69E4 \text{ rad/s} \tag{6.1}$$

We also find that the bandwidth is, in the series RLC circuit:

Calculated: $3.9E4$
Plotted: $3.9E4$

The quality factor is calculated to be(by knowing that $Q = \frac{X_0}{R} = \frac{\sqrt{\frac{L}{C}}}{R}$)

$$Q = 0.49346$$

We find that the voltage taken over the resistor makes the circuit act as a **bandpass** filter, the voltage taken over the inductor makes the circuit act as a **high-pass** filter, the voltage taken over the capacitor makes the circuit act as a **low-pass** filter, and the voltage taken over the inductor and the capacitor makes the circuit act as a **band-stop** filter. The bode magnitude of the RLC series circuit plot and the MATLAB code used to obtain the corner frequencies and create the plot is given below:

```
R = 390; % in ohm
C = 270E-9; % in F
L = 10E-3; % in H

w = 100:100:100E3;
jw = 1i*w;

% Voltage over the resistor
H_r = (jw .* (R .* C)) ./ ((jw.^2 .* L .* C) + (jw .* R .* C) + 1);

semilogx(w, 20 * log10(abs(H_r)), "blue");

hold on

% Voltage over the inductor
H_l = (jw.^2 .* L .* C) ./ ((jw.^2 .* L .* C) + (jw .* R .* C) + 1);
semilogx(w, 20 * log10(abs(H_l)), "red");

% Voltage over the capacitor
H_c = 1./((jw.^2 .* L .* C) + (jw .* R .* C) + 1);
semilogx(w, 20 * log10(abs(H_c)), "green");

% Voltage over the inductor and the capacitor
H_lc = ((jw.^2 .* L .* C) + 1) ./ ((jw.^2 .* L .* C) + jw .* R .*C + 1);
semilogx(w, 20 * log10(abs(H_lc)), "black");
ylim([-50, 0]);

legend("H_R", "H_L", "H_C", "H_LC", "Location", "southwest")

% Calculated bandwidth and quality factor:
```

```
B_calculated = R/L;
w_0 = 1/sqrt(L*C);
X_0 = sqrt(L/C);
Q_s = X_0/R;

% Obtained from the plot, respectively at their 11dB cutoff
% points: -20log10(1/sqrt(2) * 5) and 20log10(1/sqrt(2)*5)
w_2 = 4.69E4;
w_1 = 7.9E3;

B_plot = w_2 - w_1;

disp("Plot Bandwidth: " + (w_2 - w_1));

disp("Calculated Quality Factor: " + Q_s);
disp("Calculated Bandwidth: " + B_calculated);
```

6.2.1 Execution and Experiment Data

A series RLC resonant circuit was implemented with $R = 390\Omega, C = 270nF, L = 10mH$. The circuit was driven by a function generator in log sweep mode at $5V_{pp}$ without an offset, with the frequency being varied every 500ms from 100Hz to 100kHz.

For V_R, V_C, V_L, V_{CL} the following hardcopies are shown:

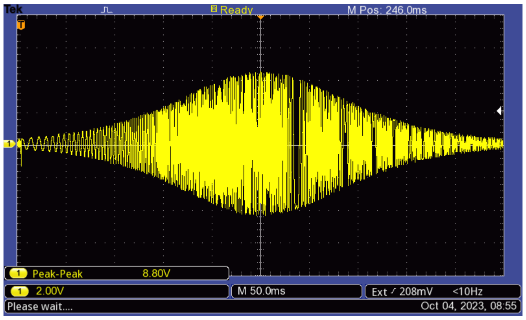


Figure 6.2: Voltage over the resistor showing band-pass behavior

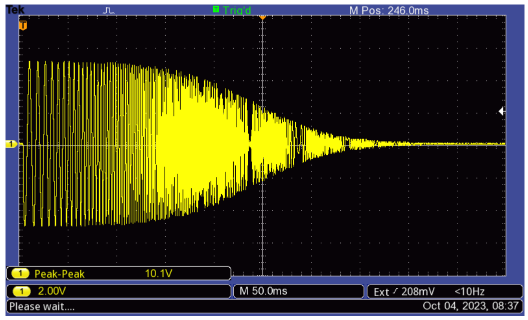


Figure 6.3: Voltage over the capacitor showing low-pass behavior

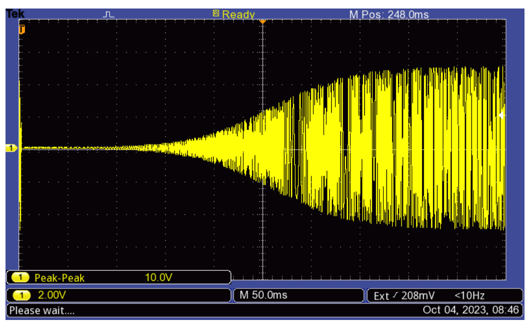


Figure 6.4: Voltage over the inductor showing high-pass behavior

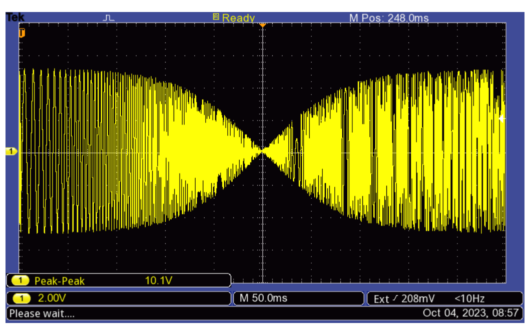


Figure 6.5: Voltage over the capacitor and inductor showing band-stop behavior

For the next part, the circuit is configured to be band-pass once more. It was observed that the Lissajou figure could be used to determine whether the frequency provided is above or below the resonant frequency. If the Lissajou figure is linear, then the frequency is the resonant frequency. It was found that the frequency at which the Lissajou figure turns linear is around 3100Hz.

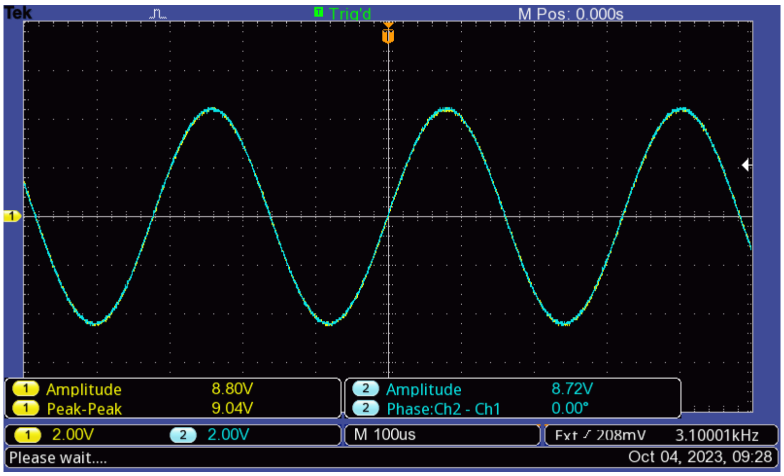


Figure 6.6: The phase difference between the input and the output voltage over the resistor is shown to be 0deg at the resonant frequency

The corner-frequencies were found to be 1290Hz and 7640Hz respectively, very close to nominal frequencies when compared: $f_1 \times 2\pi = 8105$rad/s and $f_2 \times 2\pi = 48000$rad/s.