Constructor University Bremen

Lab Report 3: AM Modulation

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1. Prelab

1.1 Single frequency Amplitude Modulation

1. Describe the modulation index as a function of the envelope($A_{\max}$, $A_{\min}$) In order to obtain the modulation index as a function of the envelope, it is known that the maximum amplitude will be defined as:

$$A_{\max} = A_c(1+m) \tag{1.1}$$

And for the minimum amplitude, it will be defined as:

$$A_{\min} = A_c(1-m) \tag{1.2}$$

Where A_c is the carrier amplitude and m is the modulation index. By taking the ratio,

$$\frac{A_{\max}}{A_{\min}} = \frac{A_c(1+m)}{A_c(1-m)} = \frac{1+m}{1-m} \tag{1.3}$$

Then rearranging the equations above, the modulation index as a function of the envelope is obtained:

$$m = \frac{A_{\max} - A_{\min}}{A_{\max} + A_{\min}} \tag{1.4}$$

2. Derive an expression that describes the ratio of the total sideband power to the total power in the modulated wave delivered to a load resistor as expressed using the modulation index

$$P_{total} = P_{s,lower} + P_{s,upper} + P_c \tag{1.5}$$

Where $P_{s,lower}$ is the power of the lower sideband, $P_{s,upper}$ is the power of the upper sideband and P_c is the power of the carrier. The power of the lower sideband is given by:

$$P_{s,lower} = \frac{1}{R_L}(\frac{A_c}{\sqrt{2}}m)^2 = \frac{1}{R_L}\frac{A_c^2 m^2}{8} \tag{1.6}$$

The power of the upper sideband is also given by:

$$P_{s,upper} = P_{s,lower} \tag{1.7}$$

Therefore, it can be established that the total power is

$$P_{total} = \frac{1}{2R_L} A_c^2 (1 + \frac{m^2}{2}) \tag{1.8}$$

Knowing that

$$r_P = \frac{P_s}{P_{total}} = \frac{2\frac{A_c^2 m^2}{8}}{\frac{Ac^2(1+\frac{m^2}{2})}{2R_L}} = \frac{m^2}{2(1+\frac{m^2}{2})} = \frac{m^2}{2+m^2} \tag{1.9}$$

3. Calculate the ratio of sideband power to total power by knowing that the modulation index is 100%. In order to calcualte the ratio of the sideband power with an index of 100%, the formula obtained in the previous problem is used:

$$r_P = \frac{m^2}{2+m^2} = \frac{1^2}{2+1^2} = \frac{1}{3} \tag{1.10}$$

4. For a carrier:

$$V_c(t) = 5\cos(2000\pi t) \tag{1.11}$$

Modulated by a signal

$$V_m(t) = 2 + \cos(2000\pi t) \tag{1.12}$$

Find the ratio r_P: To find the ratio, first consider the RMS voltage of one sideband:

$$V_{s,RMS} = \sqrt{a_0^2 + \frac{a_1^2}{2}} = \sqrt{4 + \frac{1}{2}} = \frac{3}{\sqrt{2}} \tag{1.13}$$

The RMS voltage of the carrier is:

$$V_{c,RMS} = \frac{5}{\sqrt{2}} \tag{1.14}$$

The power of the carrier and the sideband are, respectively:

$$
\begin{aligned}
P_c &= \frac{1}{1\Omega}(\frac{5}{\sqrt{2}})^2 = \frac{25}{2}W \\
P_s &= 2 \cdot \left(\frac{1}{1\Omega} \left(\frac{3}{\sqrt{2}} \right)^2 \right) = 9W
\end{aligned}
\tag{1.15}
$$

The total power is then given by:

$$
P_{total} = \frac{25}{2}W + 9W = \frac{43}{2}W \tag{1.16}
$$

And the ratio can then be obtained as:

$$
r_P = \frac{P_s}{P_{total}} = \frac{9W}{\frac{43}{2}W} = \frac{18}{43} = 0.41 \tag{1.17}
$$

To maximize the ratio, the modulation index must be 100%, which can be obtained by tuning the amplitude of the modulating signal.

1.2 Amplitude Demodulation

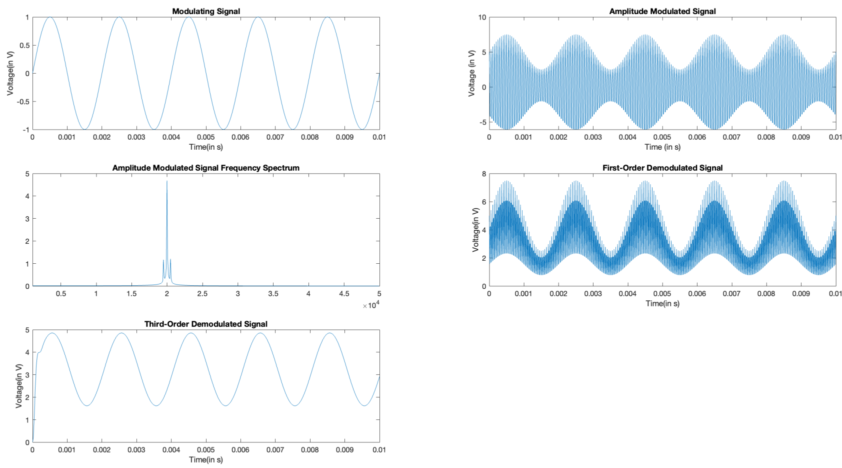

Figure 1.1: AM Demodulator

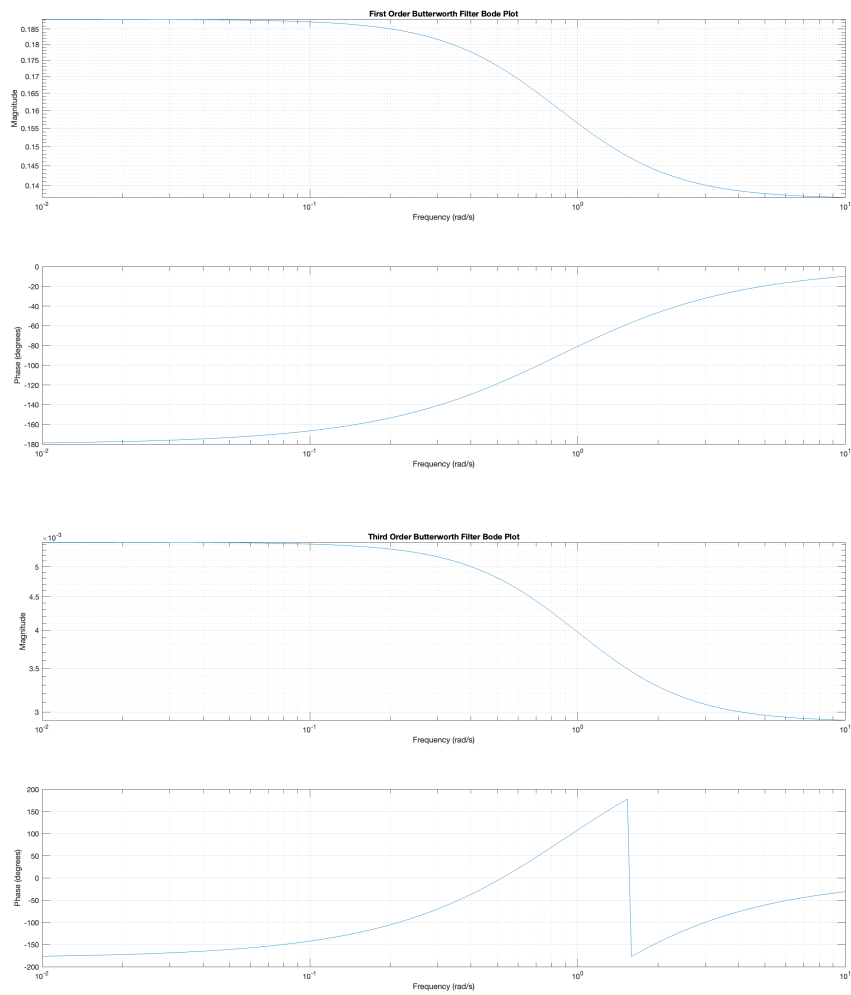

Figure 1.2: Bode plots of the butterworth filters

The code to plot these signals and the butterworth filters is shown in the appendix.

The higher the order of the filter, the better the de-modulated signal will be. However, the higher the order of the filter, the more expensive it will be to implement. Therefore, the order of the filter must be chosen carefully.

2. Introduction

The goal of this experiment is to study different techniques of AM modulation.

In the first part of the experiment, amplitude modulation will be investigated. The properties of double-sideband modulation and double sideband, supressed carrier modulation, single-sideband modulation and the frequency spectra will be investigated, where the oscilloscope will be used practically to view the effect that changing modulation parameters has on the modulated signal.

The reason signals are modulated is because of the attenuation that signals undergo when going through a medium like air, where low frequency signals are highly attenuated while high frequency signals are attenuated significantly less.

The second reason is because the size of the antenna required to receive a signal is inversely proportional to the frequency. The higher the frequency, the smaller the antenna can be when receiving the signal.

Finally, multiple signals must be transmitted simultaneously. Frequencies that matter to humans are between a few hertz to a few thousand hertz, so modulation is necessary as to not exhaust the available bandwidth.

2.1 Band-Limited Signal AM Modulation

For a sinusoidal carrier signal,

$$c(t) = A_c \cos(\omega_c t + \phi_c) \tag{2.1}$$

Where for convenience, $\phi_c = 0$. Consider a signal $x(t)$. The **amplitude modulated** signal can now be described by,

$$y(t) = (1 + kx(t)) \cdot c(t) = A_c \left[1 + kx(t)\right] cos(\omega_c t) \tag{2.2}$$

Where k is defined as the transmitter sensitivity. The expression $kx(t)$ must always be less than unity, otherwise the carrier signal becomes overmodulated and the signal is distorted.

The carrier signal's maximum frequency must also be greater than the maximum frequency of the modulating signal: $f_c \gg f_m$, otherwise an envelope may not be observed.

A band-limited signal is defined as any signal whose frequency spectrum goes to zero outside of a certain range, thereby

$$X(\omega) = 0 \quad \text{for} \quad |\omega| > \omega_m \tag{2.3}$$

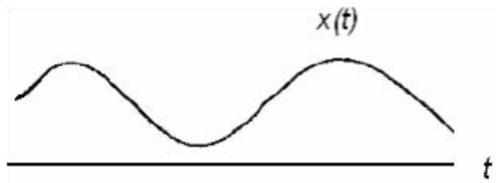

Figure 2.1: Time domain representation of a band-limited signal

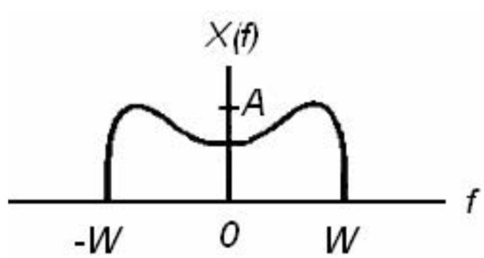

Figure 2.2: Frequency domain representation of a band-limited signal

And this signal, when modulated, gives the following figure:

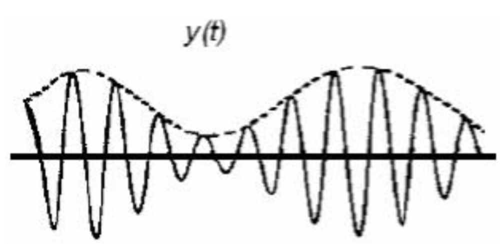

Figure 2.3: Time domain representation of the modulated signal

In the case when $x(t)$ is a sinusoid with a single frequency, the modulated signal can be represented as,

$$y(t) = A_c \left[1 + kA_m \cos(\omega_m t)\right] \cos(\omega_c t) \tag{2.4}$$

Where kA_m can be written as m, the modulation index. The modulation index must be kept below unity, as to not overmodulate the signal.

The frequency spectrum of this function contains deltas, as given by:

$$\begin{aligned} Y(f) = \frac{A_c}{2}\left[\delta(f - f_c) + \delta(f + f_c)\right] \\ + \frac{mA_c}{4}\left[\delta(f - f_m - f_c) + \delta(f + f_m + f_c)\right] \\ + \frac{mA_c}{4}\left[\delta(f - f_m + f_c) + \delta(f + f_m - f_c)\right] \end{aligned} \tag{2.5}$$

2.2 Demodulation Techniques

For a double-sideband supressed carrier signal, multiplying the modulated signal with the carrier signal will give the original signal back, albeit with a smaller amplitude,

$$\begin{aligned} y(t) &= x(t)\cos(\omega_c t) \cdot \cos(\omega_c t) \\ &= x(t)\cos^2(\omega_c t) \\ &= \frac{1}{2}x(t) + \frac{1}{2}x(t)\cos(2\omega_c t) \end{aligned} \tag{2.6}$$

Which is obtained using the trigonometric identity,

$$\cos(\omega_c t)^2 = \frac{1}{2} + \frac{1}{2}\cos(2\omega_c t) \tag{2.7}$$

By applying a low pass filter to the signal, the $\frac{1}{2}x(t)$ term can be kept, while removing the sinusoidal term, recovering the original signal, although with a smaller amplitude.

This is, however, highly ideal. This requires that there is no phase difference between the carrier and the demodulating signal, besides the fact that they must have the same frequency. This is difficult to implement in practice, so another method that can be used is asynchronous detection.

In **asynchronous** detection, the signal is shifted upward by a DC component, such that:

$$x_c(t) = x(t) + C \tag{2.8}$$

The signal's fourier transform will then have another delta, which represents an inefficiency in the power draw, however, a simple envelope detector can then be used to recover the original signal.

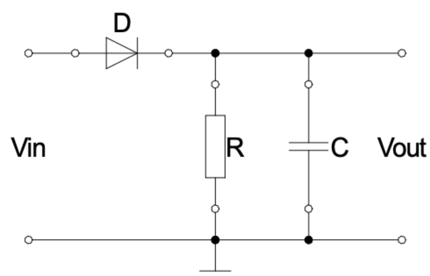

Figure 2.4: Envelope detector

The envelope detector works by the diode allowing only positive voltages to pass through, and the capacitor and resistor are responsible for extracting the shape or the envelope of the signal.

3. Execution

3.1 Problem 1: AM Modulated Signals in Time Domain

The AM signal is generated by the function generator, where a sinusoid with a modulation index of 70%, a carrier frequency of 20kHz, and a voltage of $10V_{pp}$ is generated. The frequency and amplitude properties of the signal are then measured.

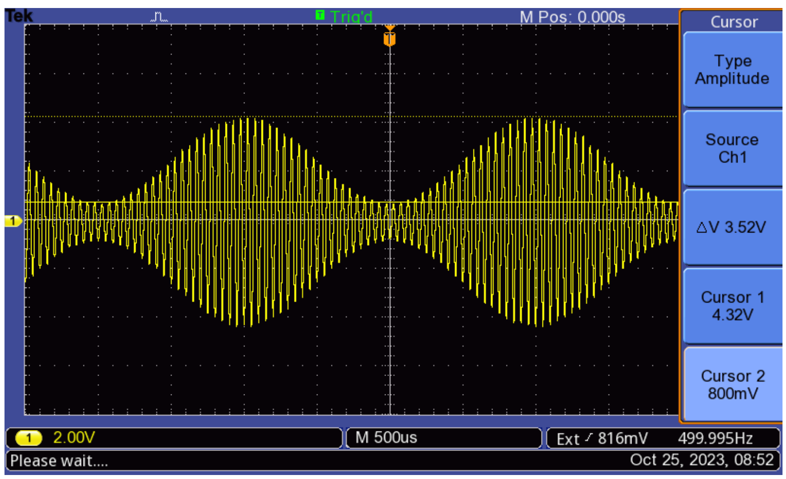

Figure 3.1: 70% modulation index amplitude, shown at 3.52V

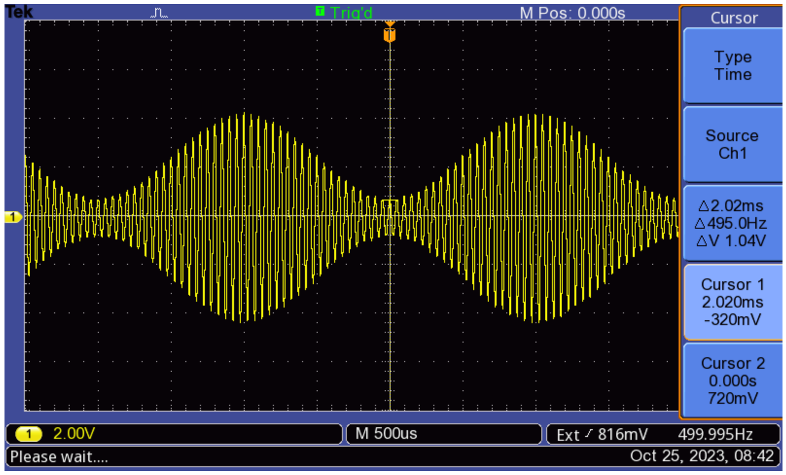

Figure 3.2: 70% modulation index frequency, shown at 495Hz

The modulation index is then changed to 50%.

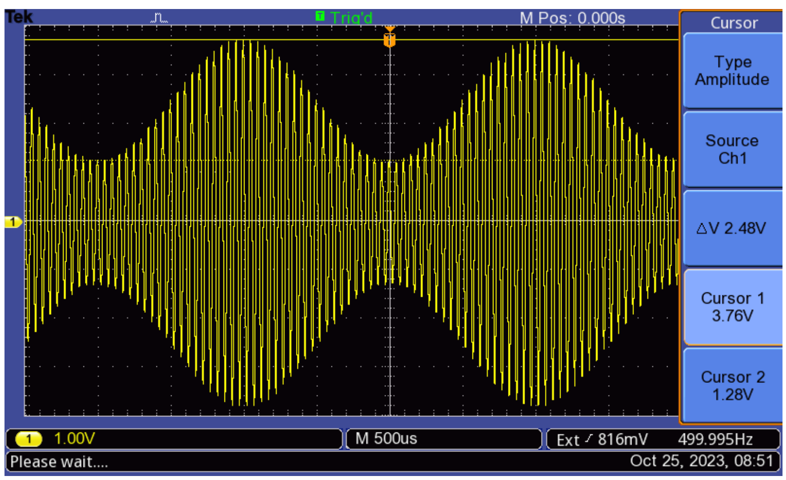

Figure 3.3: 50% modulation index amplitude, shown at 2.48V

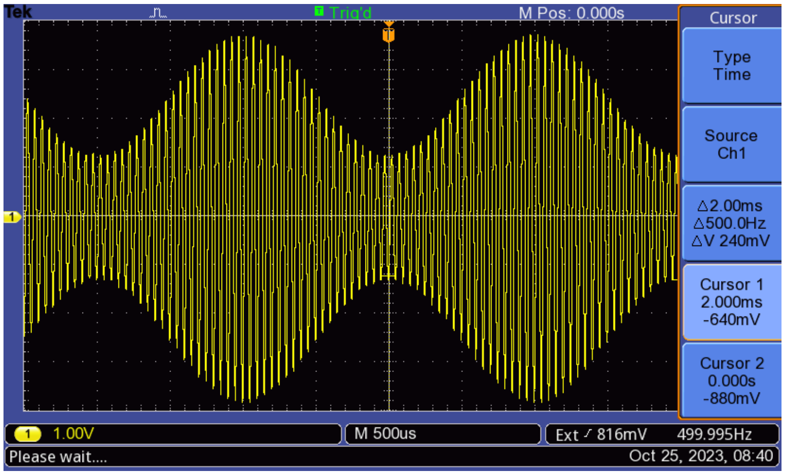

Figure 3.4: 50% modulation index frequency, shown at 500Hz

The modulation index is then changed to 120% to show an overmodulated signal.

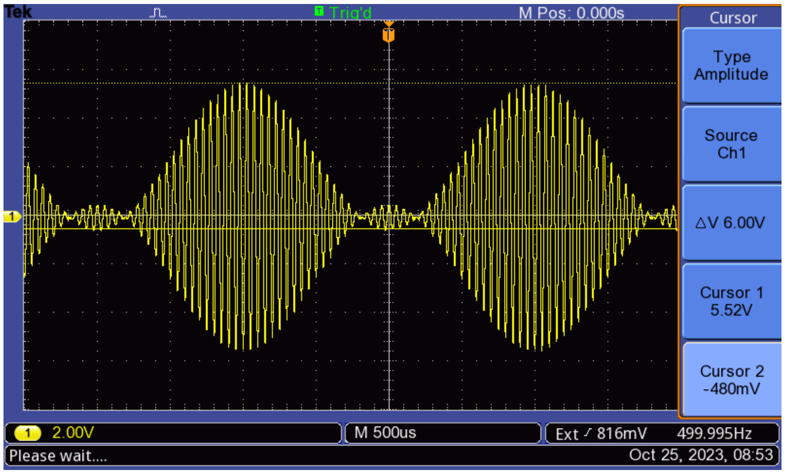

Figure 3.5: 120% modulation index amplitude, shown at 6V

3.2 Problem 2: AM Modulated Signals in Frequency Domain

The same setup is used, with the AM modulation index now once more set to 70%.

The amplitude modulated signal's frequency is then measured via the use of the oscilloscope's FFT function.

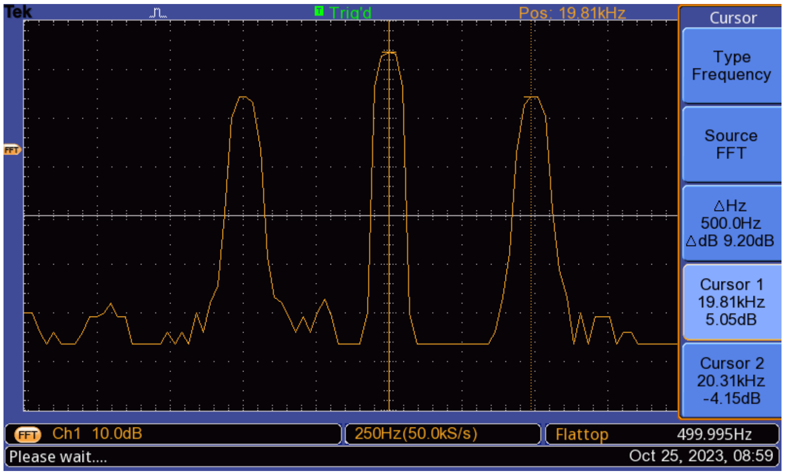

Figure 3.6: 70% modulation index FFT, with the peaks at 20kHz and 19.81kHz

3.3 Problem 3: Demodulation of a message signal

The following circuit is assembled on the breadboard,

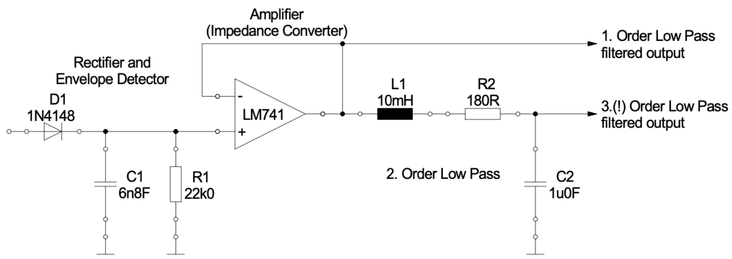

Figure 3.7: Demodulation circuit

The following settings are used for the oscilloscope:

- Signal Shape = Sine
- Modulation = AM
- Carrier Frequency = 20KHz
- Carrier Amplitude = 10Vpp
- Modulation Frequency = 500Hz
- Modulation Index = 50%

The AM modulated signal is displayed with the first-order filter output.

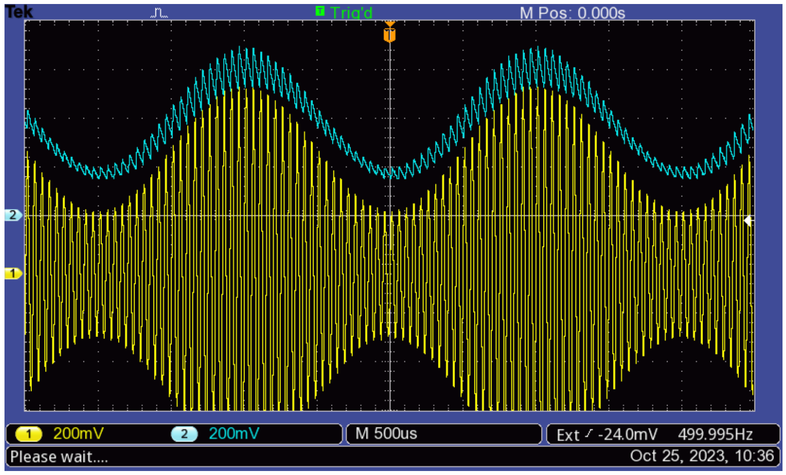

Figure 3.8: First order filter output

The AM modulated signal is then displayed with the third-order filter output.

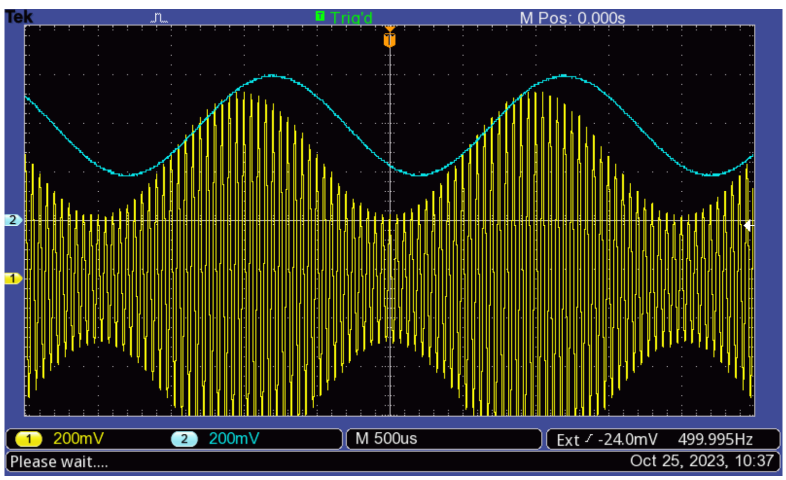

Figure 3.9: Third order filter output

The amplitude of the signal produced by the third-order filter is then measured.

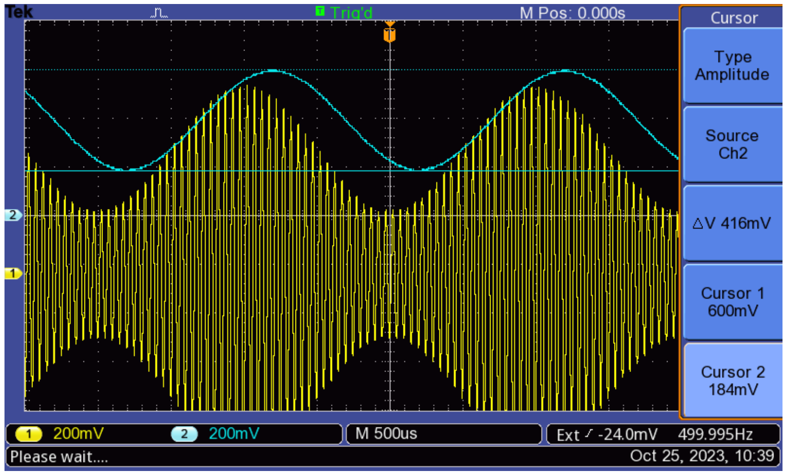

Figure 3.10: Third order filter amplitude, shown at 416mV

Furthermore, the FFT of the third-order filter output is taken, where the 20KHz and 500Hz peaks are shown.

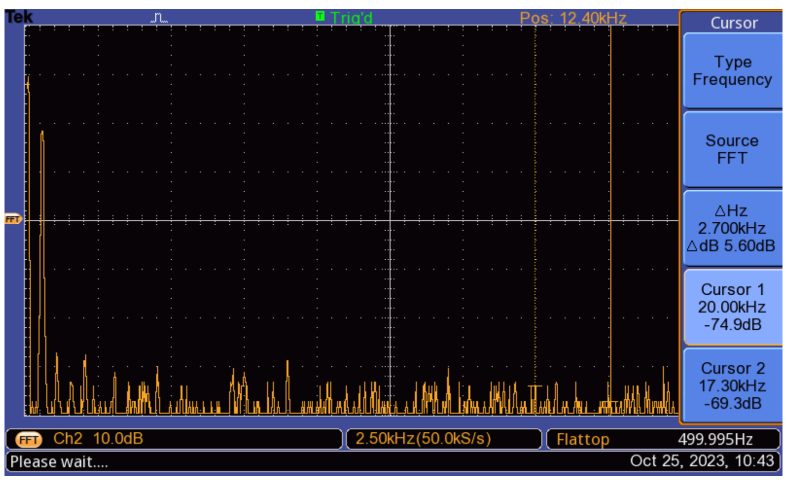

Figure 3.11: Third order filter FFT, with the 20KHz peak

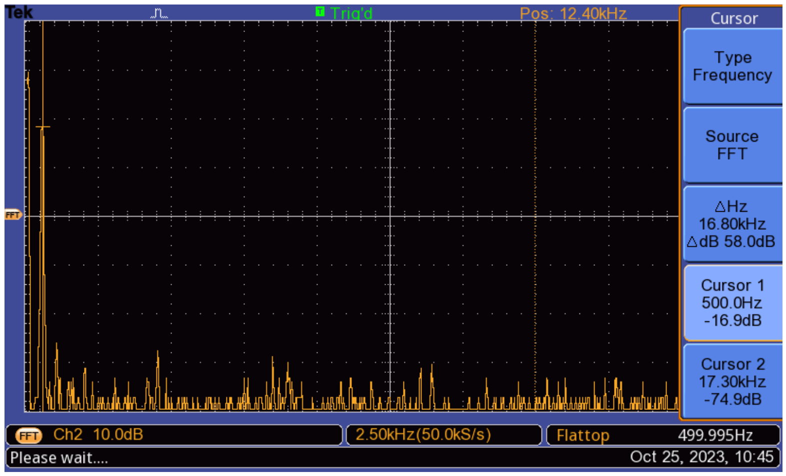

Figure 3.12: Third order filter FFT, with the 500Hz peak

4. Evaluation

5. Conclusion

6. References

7. Appendix

7.1 Prelab Code:

```
%% Prelab:

% Prelab Problem 2:
Fs = 1e5;
t = 0:1/Fs:0.01;
f = 20E3; % 20 Khz

A_c = 5;

f_m = 5E2;
x = sin(2*pi*f_m*t);

f_c = 20E3;
y = A_c*(1+0.5*x).*cos(2*pi*f_c*t);

subplot(3, 2, 1);
plot(t, x);
title('Modulating Signal');
xlabel('Time(in s)');
ylabel('Voltage(in V)');

subplot(3, 2, 2);
plot(t, y);
title('Amplitude Modulated Signal');
xlabel('Time (in s)');
ylabel('Voltage (in V)');

% Frequency spectrum

N = length(y);
Y = fft(y);
```

```
spectrum = abs(Y/N);
spectrum_single = spectrum(1:N/2+1);
spectrum_single(1:end-1) = 2*spectrum_single(1:end-1);

F = Fs * (0:(N/2)) / N;
subplot(3, 2, 3);
plot(F, spectrum_single);
xlim([1E3, 5E4]);
title('Amplitude Modulated Signal Frequency Spectrum');

frequencies = logspace(3, 5, 100); % 100 Hz to 100 KHz
Wn = 1000/(f/2); % Normalized cutoff frequency

[b1, a1] = butter(1,Wn); % Butterworth filter of first order
[b3, a3] = butter(3, Wn); % Butterworth filter of third order

% Rectify the signal

% We can use an envelope detector for this, the simplest analogue to a
% diode would be the absolute value of the signal, so that's what will be
% used

rectified = abs(y);
filtered = filter(b1, a1, rectified);

% figure;
subplot(3, 2, 4);
plot(t, rectified);
title('First-Order Demodulated Signal');
xlabel('Time(in s)');
ylabel('Voltage(in V)');

filtered3 = filter(b3, a3, rectified);
subplot(3, 2, 5);
plot(t, filtered3);
title('Third-Order Demodulated Signal');
xlabel('Time(in s)');
```

```
ylabel('Voltage(in V)');

% Bode plot for the filters
figure;
freqs(b1, a1);
title("First Order Butterworth Filter Bode Plot");

figure;
freqs(b3, a3);
title("Third Order Butterworth Filter Bode Plot");
```