Constructor University Bremen

Lab Report 3: AM Modulation

Fall Semester 2024

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1. Prelab

1.1 Single frequency Amplitude Modulation

1. Describe the modulation index as a function of the envelope($A_{\max}$, $A_{\min}$) In order to obtain the modulation index as a function of the envelope, it is known that the maximum amplitude will be defined as:

$$A_{\max} = A_c(1+m) \tag{1.1}$$

And for the minimum amplitude, it will be defined as:

$$A_{\min} = A_c(1-m) \tag{1.2}$$

Where A_c is the carrier amplitude and m is the modulation index. By taking the ratio,

$$\frac{A_{\max}}{A_{\min}} = \frac{A_c(1+m)}{A_c(1-m)} = \frac{1+m}{1-m} \tag{1.3}$$

Then rearranging the equations above, the modulation index as a function of the envelope is obtained:

$$m = \frac{A_{\max} - A_{\min}}{A_{\max} + A_{\min}} \tag{1.4}$$

2. Derive an expression that describes the ratio of the total sideband power to the total power in the modulated wave delivered to a load resistor as expressed using the modulation index

$$P_{total} = P_{s,lower} + P_{s,upper} + P_c \tag{1.5}$$

Where $P_{s,lower}$ is the power of the lower sideband, $P_{s,upper}$ is the power of the upper sideband and P_c is the power of the carrier. The power of the lower sideband is given by:

$$P_{s,lower} = \frac{1}{R_L}(\frac{A_c}{\sqrt{2}}m)^2 = \frac{1}{R_L}\frac{A_c^2 m^2}{8} \tag{1.6}$$

The power of the upper sideband is also given by:

$$P_{s,upper} = P_{s,lower} \tag{1.7}$$

Therefore, it can be established that the total power is

$$P_{total} = \frac{1}{2R_L} A_c^2 (1 + \frac{m^2}{2}) \tag{1.8}$$

Knowing that

$$r_P = \frac{P_s}{P_{total}} = \frac{2\frac{A_c^2 m^2}{8}}{\frac{Ac^2(1+\frac{m^2}{2})}{2R_L}} = \frac{m^2}{2(1+\frac{m^2}{2})} = \frac{m^2}{2+m^2} \tag{1.9}$$

3. Calculate the ratio of sideband power to total power by knowing that the modulation index is 100%. In order to calcualte the ratio of the sideband power with an index of 100%, the formula obtained in the previous problem is used:

$$r_P = \frac{m^2}{2+m^2} = \frac{1^2}{2+1^2} = \frac{1}{3} \tag{1.10}$$

4. For a carrier:

$$V_c(t) = 5\cos(2000\pi t) \tag{1.11}$$

Modulated by a signal

$$V_m(t) = 2 + \cos(2000\pi t) \tag{1.12}$$

Find the ratio r_P: To find the ratio, first consider the RMS voltage of one sideband:

$$V_{s,RMS} = \sqrt{a_0^2 + \frac{a_1^2}{2}} = \sqrt{4 + \frac{1}{2}} = \frac{3}{\sqrt{2}} \tag{1.13}$$

The RMS voltage of the carrier is:

$$V_{c,RMS} = \frac{5}{\sqrt{2}} \tag{1.14}$$

The power of the carrier and the sideband are, respectively:

$$
\begin{aligned}
P_c &= \frac{1}{1\Omega}(\frac{5}{\sqrt{2}})^2 = \frac{25}{2}W \\
P_s &= 2 \cdot \left(\frac{1}{1\Omega} \left(\frac{3}{\sqrt{2}} \right)^2 \right) = 9W
\end{aligned}
\tag{1.15}
$$

The total power is then given by:

$$
P_{total} = \frac{25}{2}W + 9W = \frac{43}{2}W \tag{1.16}
$$

And the ratio can then be obtained as:

$$
r_P = \frac{P_s}{P_{total}} = \frac{9W}{\frac{43}{2}W} = \frac{18}{43} = 0.41 \tag{1.17}
$$

To maximize the ratio, the modulation index must be 100%, which can be obtained by tuning the amplitude of the modulating signal.

1.2 Amplitude Demodulation

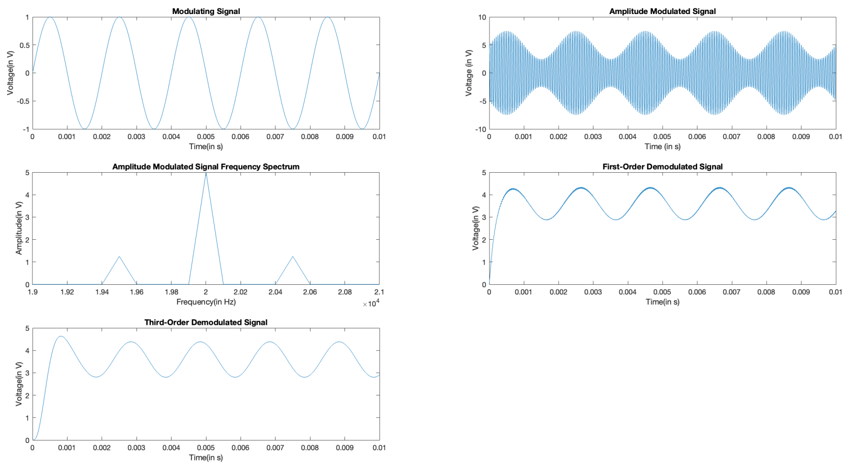

Figure 1.1: AM Demodulator

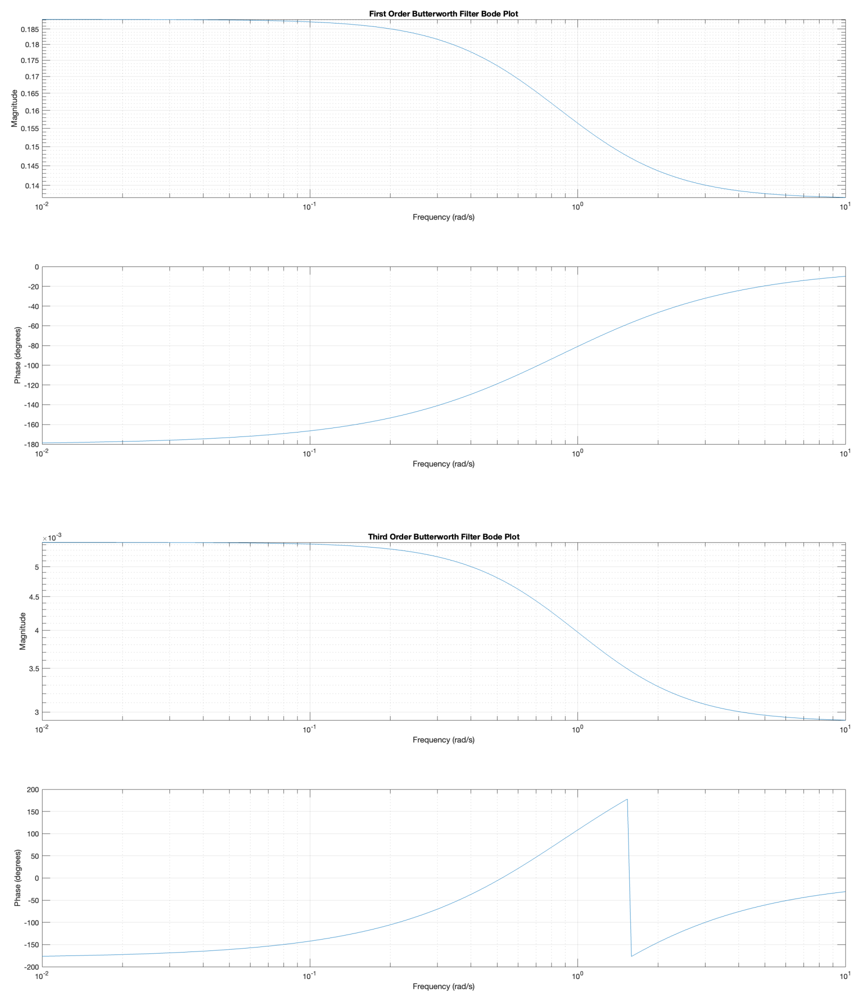

Figure 1.2: Bode plots of the butterworth filters

The code to plot these signals and the butterworth filters is shown in the appendix.

The higher the order of the filter, the better the de-modulated signal will be. However, the higher the order of the filter, the more expensive it will be to implement. Therefore, the order of the filter must be chosen carefully.

2. Introduction

The goal of this experiment is to study different techniques of AM modulation.

In the first part of the experiment, amplitude modulation will be investigated. The properties of double-sideband modulation and double sideband, supressed carrier modulation, single-sideband modulation and the frequency spectra will be investigated, where the oscilloscope will be used practically to view the effect that changing modulation parameters has on the modulated signal.

The reason signals are modulated is because of the attenuation that signals undergo when going through a medium like air, where low frequency signals are highly attenuated while high frequency signals are attenuated significantly less.

The second reason is because the size of the antenna required to receive a signal is inversely proportional to the frequency. The higher the frequency, the smaller the antenna can be when receiving the signal.

Finally, multiple signals must be transmitted simultaneously. Frequencies that matter to humans are between a few hertz to a few thousand hertz, so modulation is necessary as to not exhaust the available bandwidth.

2.1 Band-Limited Signal AM Modulation

For a sinusoidal carrier signal,

$$c(t) = A_c \cos(\omega_c t + \phi_c) \tag{2.1}$$

Where for convenience, $\phi_c = 0$. Consider a signal $x(t)$. The **amplitude modulated** signal can now be described by,

$$y(t) = (1 + kx(t)) \cdot c(t) = A_c \left[1 + kx(t)\right] cos(\omega_c t) \tag{2.2}$$

Where k is defined as the transmitter sensitivity. The expression $kx(t)$ must always be less than unity, otherwise the carrier signal becomes overmodulated and the signal is distorted.

The carrier signal's maximum frequency must also be greater than the maximum frequency of the modulating signal: $f_c \gg f_m$, otherwise an envelope may not be observed.

A band-limited signal is defined as any signal whose frequency spectrum goes to zero outside of a certain range, thereby

$$X(\omega) = 0 \quad \text{for} \quad |\omega| > \omega_m \tag{2.3}$$

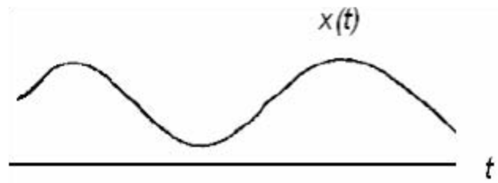

Figure 2.1: Time domain representation of a band-limited signal

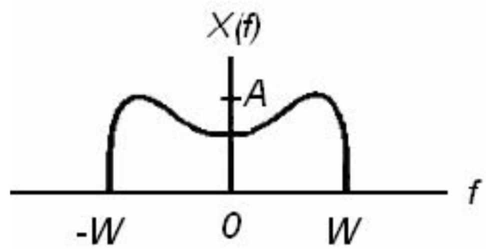

Figure 2.2: Frequency domain representation of a band-limited signal

And this signal, when modulated, gives the following figure:

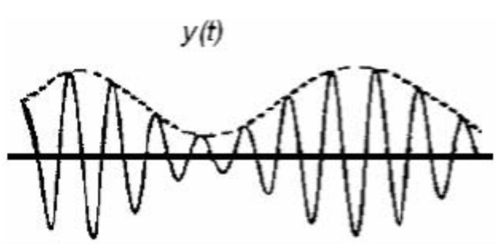

Figure 2.3: Time domain representation of the modulated signal

In the case when $x(t)$ is a sinusoid with a single frequency, the modulated signal can be represented as,

$$y(t) = A_c \left[1 + kA_m \cos(\omega_m t)\right] \cos(\omega_c t) \tag{2.4}$$

Where kA_m can be written as m, the modulation index. The modulation index must be kept below unity, as to not overmodulate the signal.

The frequency spectrum of this function contains deltas, as given by:

$$\begin{aligned} Y(f) = \frac{A_c}{2}\left[\delta(f - f_c) + \delta(f + f_c)\right] \\ + \frac{mA_c}{4}\left[\delta(f - f_m - f_c) + \delta(f + f_m + f_c)\right] \\ + \frac{mA_c}{4}\left[\delta(f - f_m + f_c) + \delta(f + f_m - f_c)\right] \end{aligned} \tag{2.5}$$

2.2 Demodulation Techniques

For a double-sideband supressed carrier signal, multiplying the modulated signal with the carrier signal will give the original signal back, albeit with a smaller amplitude,

$$\begin{aligned} y(t) &= x(t)\cos(\omega_c t) \cdot \cos(\omega_c t) \\ &= x(t)\cos^2(\omega_c t) \\ &= \frac{1}{2}x(t) + \frac{1}{2}x(t)\cos(2\omega_c t) \end{aligned} \tag{2.6}$$

Which is obtained using the trigonometric identity,

$$\cos(\omega_c t)^2 = \frac{1}{2} + \frac{1}{2}\cos(2\omega_c t) \tag{2.7}$$

By applying a low pass filter to the signal, the $\frac{1}{2}x(t)$ term can be kept, while removing the sinusoidal term, recovering the original signal, although with a smaller amplitude.

This is, however, highly ideal. This requires that there is no phase difference between the carrier and the demodulating signal, besides the fact that they must have the same frequency. This is difficult to implement in practice, so another method that can be used is asynchronous detection.

In **asynchronous** detection, the signal is shifted upward by a DC component, such that:

$$x_c(t) = x(t) + C \tag{2.8}$$

The signal's fourier transform will then have another delta, which represents an inefficiency in the power draw, however, a simple envelope detector can then be used to recover the original signal.

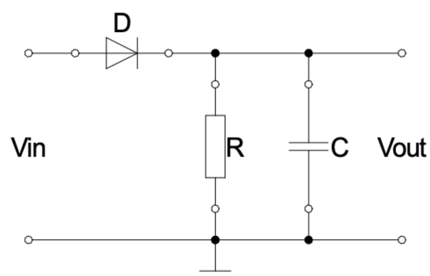

Figure 2.4: Envelope detector

The envelope detector works by the diode allowing only positive voltages to pass through, and the capacitor and resistor are responsible for extracting the shape or the envelope of the signal.

3. Execution

3.1 Problem 1: AM Modulated Signals in Time Domain

The AM signal is generated by the function generator, where a sinusoid with a modulation index of 70%, a carrier frequency of 20kHz, and a voltage of $10V_{pp}$ is generated. The frequency and amplitude properties of the signal are then measured.

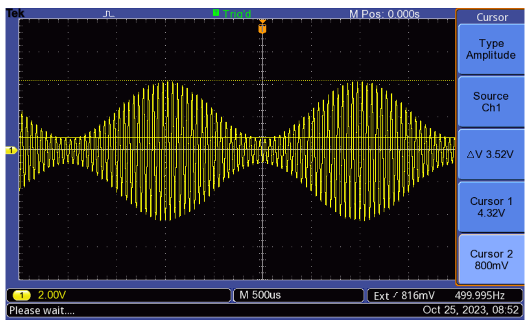

Figure 3.1: 70% modulation index amplitude, shown at 4.32V

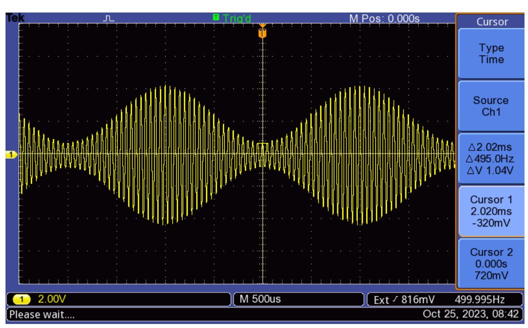

Figure 3.2: 70% modulation index frequency, shown at 495Hz

The modulation index is then changed to 50%.

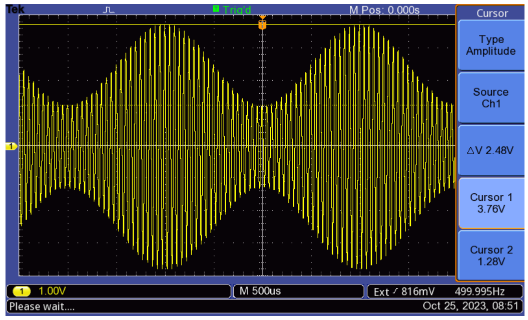

Figure 3.3: 50% modulation index amplitude, shown at 3.76V

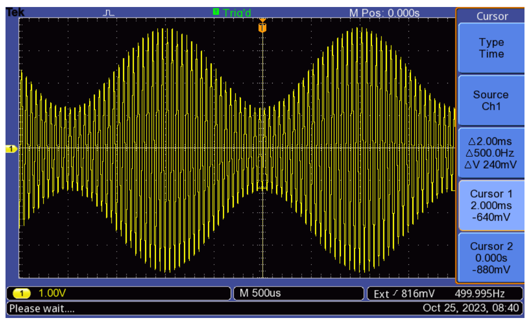

Figure 3.4: 50% modulation index frequency, shown at 500Hz

The modulation index is then changed to 120% to show an overmodulated signal.

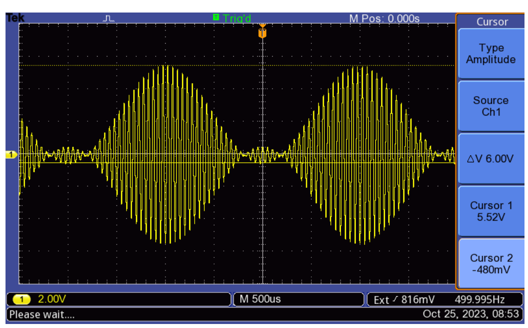

Figure 3.5: 120% modulation index amplitude, shown at 5.52V

3.2 Problem 2: AM Modulated Signals in Frequency Domain

The same setup is used, with the AM modulation index now once more set to 70%.

The amplitude modulated signal's frequency is then measured via the use of the oscilloscope's FFT function.

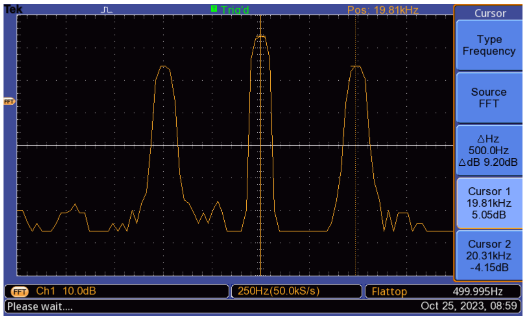

Figure 3.6: 70% modulation index FFT, with the peaks at 20kHz and 19.81kHz

3.3 Problem 3: Demodulation of a message signal

The following circuit is assembled on the breadboard,

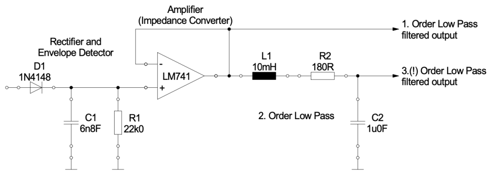

Figure 3.7: Demodulation circuit

The following settings are used for the oscilloscope:

- Signal Shape = Sine
- Modulation = AM
- Carrier Frequency = 20KHz
- Carrier Amplitude = 10Vpp
- Modulation Frequency = 500Hz
- Modulation Index = 50%

The AM modulated signal is displayed with the first-order filter output.

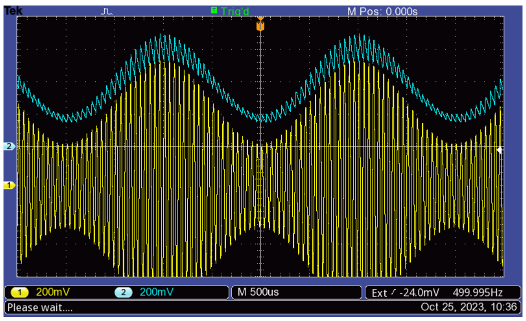

Figure 3.8: First order filter output

The AM modulated signal is then displayed with the third-order filter output.

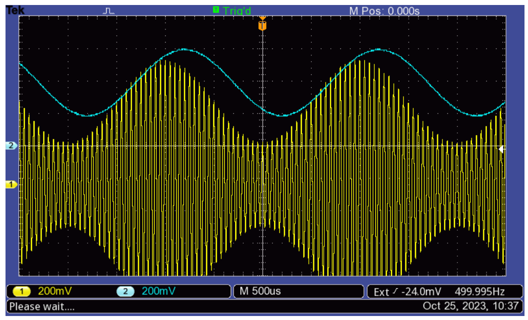

Figure 3.9: Third order filter output

The amplitude of the signal produced by the third-order filter is then measured.

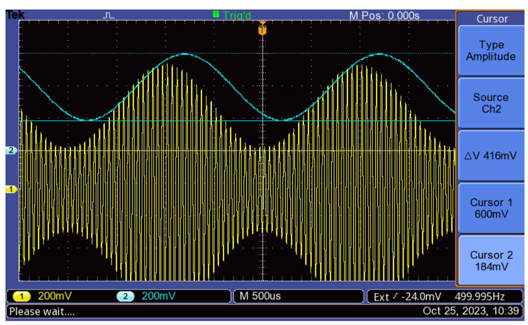

Figure 3.10: Third order filter amplitude, shown at 600mV

Furthermore, the FFT of the third-order filter output is taken, where the 20KHz peak is not present anymore because of the filter.

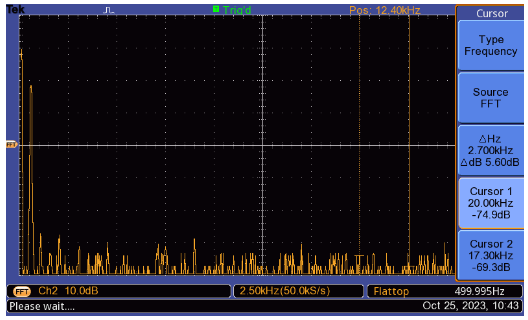

Figure 3.11: Third order filter FFT, without the 20KHz peak

4. Evaluation

4.1 Part 1

1. Based on the experimental observations where when the modulation index was increased the amplitude of the frequency components was increased, and the mathematical theory, it is observed that the amplitude of the relative frequency components grows in relation to the modulation index.

   Mathematically, it is known that the modulation index for an AM signal is given by:

   $$m = kA_m \tag{4.1}$$

   Where k is the transmitter sensitivity, and A_m is the modulating signal amplitude, meaning that the modulation amplitude is proportional to the modulation index, while inversely proportional to the transmitter sensitivity.

2. The modulation index can be computed from the equation found in the prelab, where the modulation index can be found using the maximum amplitude and the minimum amplitude.

   $$m = \frac{A_{max} - A_{min}}{A_{max} + A_{min}} \tag{4.2}$$

   For 70% modulation at the function generator:

   $$m = \frac{4.32V - 0.8V}{4.32V + 0.8V} \simeq 0.68 \simeq 68\% \tag{4.3}$$

   For 50% modulation:

   $$m = \frac{3.76V - 1.28V}{3.76V + 1.28V} \approx 0.49 \approx 49\% \tag{4.4}$$

   For 120% modulation:

   $$m = \frac{5.52V + 0.48V}{5.52V - 0.48V} \approx 1.19 \approx 119\% \tag{4.5}$$

3. The disadvantages of using a modulation index greater than 100% lie in the distortion of the signal, as the carrier wave is not able to fully represent the modulating signal, therefore the envelope of the signal is distorted, making the signal unrecoverable.

4.2 Part 2

1. The spectrum in the oscilloscope for the 70% modulation is shown to be 20KHz for the carrier wave, and 19.81KHz for the modulating wave. When plotting the FFT of the signal in theory using MATLAB, similar results are observed:

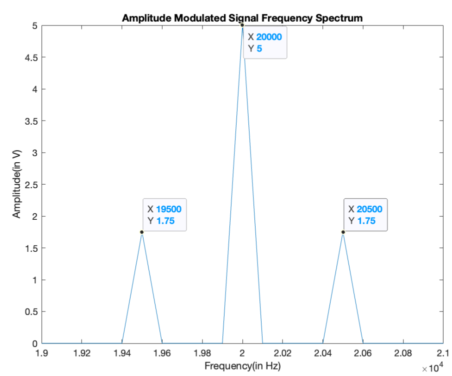

Figure 4.1: Theoretical 70% modulation index FFT, with the peaks at 19.5KHz, 20KHz, and 20.5KHz respectively

2. The function generator clearly generates a double-sideband signal, without suppressing the carrier as the carrier is observed in the fourier transform of the signal in the oscilloscope.

3. When changing the carrier frequency of the signal, the spacing of the sidebands changes to move accordingly, as the modulating signal is convolved with the carrier signal.

4. Changing the message frequency of the signal changes the width of the spectrum of the signal, as the modulating signal is convolved with the carrier signal and the width of the sidebands changes.

5. The modulation index in the frequency spectrum can be found by taking the ratio between one of the sideband peaks amplitudes and the carrier peak amplitudes, so

$$m = \frac{4.15dB}{5.05dB} \simeq 0.82 \simeq 82\% \tag{4.6}$$

This implies that the FFT of the signal as measured on the oscilloscope is not a perfect representation of the signal, as the modulation index is not 70% as expected.

4.3 Part 3

1. The first order and the third order message signal as displayed in the oscilloscope show that the envelope of the signal is tracked better by the third order filter, with the first order filter not being able to track the amplitude peaks in order to track the envelope of the signal in the same manner.

2. The first order filter is quite similar to what is observed in MATLAB, with the first order filter not being able to track the envelope of the signal with the same accuracy as the third order filter. The differences between the measurement and the simulation are due to the fact that the simulation is done using an ideal filter, while the measurement is done using a real filter, which has a non-ideal frequency response.

5. Conclusion

The behavior of AM modulation was explored via the use of the function generator's ability to generate AM signals, and the behaviour was observed using the oscilloscope. Furthermore, a slope detection circuit was assembled on the breadboard in order to demodulate the AM signal. The difference between first and third order slope detection was observed. The FFT of the oscilloscope's resolution, in part, causes the error in trying to observe the frequency peaks and finding the true modulation index in the case of 70% modulation provided by the function generator, where it was obtained from the determined frequency that it was 82%. Artifacts in the de-modulated signal are due to the internal resistance of the inductors, capacitors, and also due to the other components not being ideal. The wires have their own resistance as well. Overmodulation was also observed, and the effect it has on the envelope of the signal.

6. References

- Signals and Systems Lab Manual
- Oscilloscope Manual
- Function Generator Manual
- MATLAB documentation

7. Appendix

7.1 Prelab Code:

```
%% Prelab:
close all
clear all

% Prelab Problem 2:
Fs = 1e6;
t = 0:1/Fs:0.01 - 1/Fs; % 5 seconds
f = 20E3; % 20 Khz

A_c = 5;

f_m = 5E2;
x = sin(2*pi*f_m*t);

f_c = 20E3;
y = A_c*(1+0.5*x).*cos(2*pi*f_c*t);

subplot(3, 2, 1);
plot(t, x);
title('Modulating Signal');
xlabel('Time(in s)');
ylabel('Voltage(in V)');

subplot(3, 2, 2);
plot(t, y);
title('Amplitude Modulated Signal');
xlabel('Time (in s)');
ylabel('Voltage (in V)');

% Frequency spectrum
N = length(y);
```

```
Y = fft(y);

spectrum = abs(Y/N);
spectrum_single = spectrum(1:N/2+1);
spectrum_single(1:end-1) = 2*spectrum_single(1:end-1);

F = Fs * (0:(N/2)) / N;
subplot(3, 2, 3);
plot(F, spectrum_single);
xlim([1.9E4, 2.1E4]);
title('Amplitude Modulated Signal Frequency Spectrum');
xlabel("Frequency(in Hz)")
ylabel("Amplitude(in V)")

frequencies = logspace(3, 5, 100); % 100 Hz to 100 KHz
Wn = 1000/(Fs/2); % Normalized cutoff frequency

[b1, a1] = butter(1, Wn); % Butterworth filter of first order
[b3, a3] = butter(3, Wn); % Butterworth filter of third order

% Rectify the signal

% We can use an envelope detector for this, the simplest analogue to a
% diode would be the absolute value of the signal, so that's what will be
% used

% (Ac + abs(Ac)) / 2;
% Or -0.5 for the diode effect
rectified = (A_c + abs(y)) / 2 - 0.5;
filtered = filter(b1, a1, rectified);

% figure;
subplot(3, 2, 4);
plot(t, filtered);
title('First-Order Demodulated Signal');
xlabel('Time(in s)');
ylabel('Voltage(in V)');
```

```
filtered3 = filter(b3, a3, rectified);
subplot(3, 2, 5);
plot(t, filtered3);
title('Third-Order Demodulated Signal');
xlabel('Time(in s)');
ylabel('Voltage(in V)');

% Bode plot for the filters
figure;
freqs(b1, a1);
title("First Order Butterworth Filter Bode Plot");

figure;
freqs(b3, a3);
title("Third Order Butterworth Filter Bode Plot");

%% Evaluation: Part 2
close all
clc

w_c = 20E3 * 2 * pi;
w_m = 5E2 * 2 * pi;
c = 5*sin(w_c*t);
y = (1 + 0.70*cos(w_m * t)) .* c;

N = length(y);
Y = fft(y);

spectrum = abs(Y/N);
spectrum_single = spectrum(1:N/2+1);
spectrum_single(1:end-1) = 2*spectrum_single(1:end-1);

F = Fs * (0:(N/2)) / N;
plot(F, 20*log10(spectrum_single / sqrt(2)));
xlim([1.9E4, 2.1E4]);
title('Amplitude Modulated Signal Frequency Spectrum');
xlabel("Frequency(in Hz)")
ylabel("Amplitude(in V)")
```

7.2 Prelab 6

7.2.1 Problem 1

Knowing that the change of frequency in FM is defined as,

$$\frac{d\theta}{dt} = \omega_c + k_f m(t) = w_c + k_f 4\cos(8000\pi t) \tag{7.1}$$

And knowing that cosine can, at most, be 1, and at the minimum be -1, we can define the frequency deviation to be(for $f - 4k_f, f + 4k_f$):

$$\Delta f = 4k_f = 4 \cdot 10^4\text{Hz} = 40\text{KHz} \tag{7.2}$$

And knowing that the modulation index is defined as:

$$m = \frac{\Delta\omega}{\omega_m} = \frac{2\pi 40\text{KHz}}{8000\pi\text{Hz}} = \frac{2\pi 40000\text{Hz}}{8000\pi\text{Hz}} = 10 \tag{7.3}$$

Therefore, the signal is overmodulated.

7.2.2 Problem 2

The tables are provided below. The plot is provided as well. There are more peaks whose amplitudes are under -40dB, but they are not shown in the plot, nor in the table. The code used to generate the plot and the table is provided further down in the appendix.

Carlston's rule is used to obtain the bandwidth of the signal.

$$B_T \simeq 2f_m(\beta + 1) \tag{7.4}$$

Where β will be referred to as m in this calculation,

- For $m = 0.2$, it is found that the bandwidth is 12KHz.
- For $m = 1$, it is found that the bandwidth is 20KHz.
- For $m = 2$, it is found that the bandwidth is 30KHz.

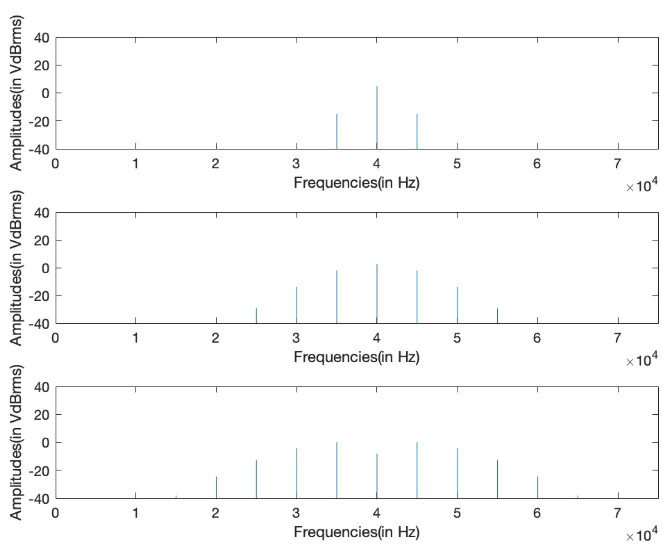

Figure 7.1: Frequency spectra for m=0.2, m=1, m=2

	Frequency(in Hz)	Amplitude(in VdBRms)
m=0.2	3501	-15
	4000	5
	4501	-15
	Frequency(in Hz)	**Amplitude(in VdBRms)**
m=1	4001	3
	4501	-2
	5001	-14
	5501	-29
	Frequency(in Hz)	**Amplitude(in VdBRms)**
m=2	1501	-38
	2001	-24
	2501	-13
	3001	-4
	3501	0
	4001	-8
	4501	0
	5001	-4
	5501	-13
	6001	-24
	6501	-38

The decibel values in the table are rounded to the nearest integer.

7.2.3 Problem 3

The circuit turns an FM signal into an AM signal, and then uses a slope detector in order to follow the envelope of the signal. The circuit is re-created using LTSpice to find the bode plot of the circuit. The bode plot and the circuit in LTSpice is provided below. The circuit's behaviour seems to be a bandpass filter.

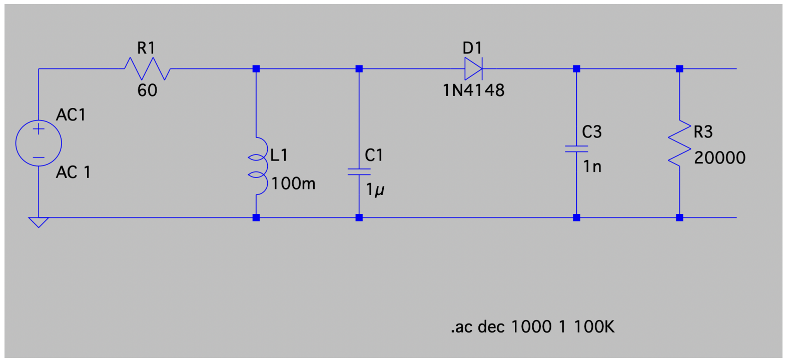

Figure 7.2: Circuit in LTSpice

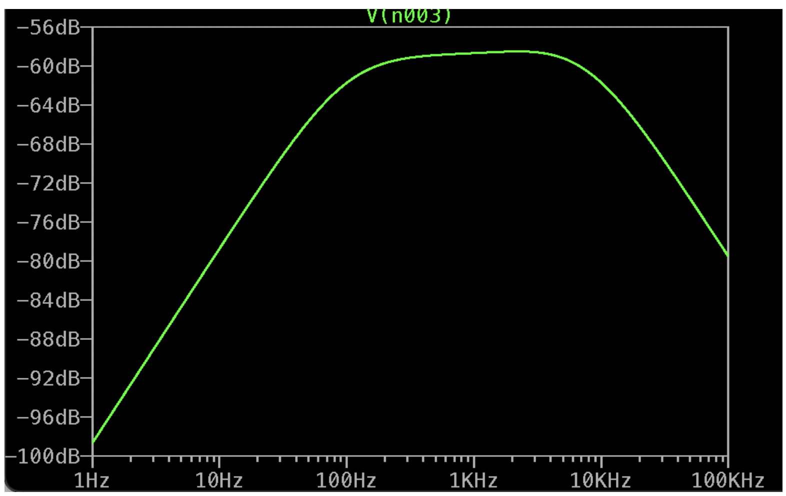

Figure 7.3: Bode plot of the circuit

The code for the prelab is provided.

```
%% Prelab 6:

Fs = 1E6;
t = 0:1/Fs:0.1-(1/Fs);
w_c = 40E3 *2*pi;
w_m = 5E3*2*pi;

% For m = 0.2
```

```
m = 0.2;
y = 2.5.*cos(w_c.*t + m.*sin(w_m.*t));

% subplot(3, 1, 1);
% plot(t, y);

N = length(y);
Y = fft(y);

spectrum = abs(Y/N);
spectrum_single = spectrum(1:N/2+1);
spectrum_single(1:end-1) = 2*spectrum_single(1:end-1);

F = Fs * (0:(N/2)) / N;
subplot(3, 1, 1);

BT = w_m/pi*(0.2+1);
disp("Bandwidth for m=0.2");
disp(BT);

m_02_dB = 20*log10(spectrum_single / sqrt(2));
disp("For m=0.2, peaks are");
display_peaks(m_02_dB);

plot(F, m_02_dB);
xlabel("Frequencies(in Hz)");
ylabel("Amplitudes(in VdBrms)");
xlim([0, 75000]);
ylim([-40, 40]);

% For m = 1
m = 1;
y = 2.5.*cos(w_c.*t + m.*sin(w_m.*t));

N = length(y);
Y = fft(y);

spectrum = abs(Y/N);
```

```
spectrum_single = spectrum(1:N/2+1);
spectrum_single(1:end-1) = 2*spectrum_single(1:end-1);

F = Fs * (0:(N/2)) / N;
subplot(3, 1, 2);

m_1_dB = 20*log10(spectrum_single / sqrt(2));

BT = w_m/pi*(1+1);
disp("Bandwidth for m=1");
disp(BT);
disp("For m=1, peaks are");
display_peaks(m_1_dB);

plot(F, m_1_dB);
xlabel("Frequencies(in Hz)");
ylabel("Amplitudes(in VdBrms)");
xlim([0, 75000]);
ylim([-40, 40]);

% For m = 2
m = 2;
y = 2.5.*cos(w_c.*t + m.*sin(w_m.*t));

N = length(y);
Y = fft(y);

% BT ~=2fm(B + 1)
BT = w_m/pi*(2+1);
disp("Bandwidth for m=2");
disp(BT);

spectrum = abs(Y/N);
spectrum_single = spectrum(1:N/2+1);
spectrum_single(1:end-1) = 2*spectrum_single(1:end-1);

F = Fs * (0:(N/2)) / N;
m_2_dB = 20*log10(spectrum_single / sqrt(2));
```

```
% Peaks:
disp("For m=2, peaks are:");
display_peaks(m_2_dB);

subplot(3, 1, 3);
plot(F, m_2_dB);
xlabel("Frequencies(in Hz)");
ylabel("Amplitudes(in VdBrms)");
xlim([0, 75000]);
ylim([-40, 40]);

function [] = display_peaks(m_dB)
    frequencies = find(m_dB > -40); % more than -40dB
    decibels = m_dB(frequencies);
    for i = 1:length(frequencies)
        fprintf("Frequency: %d Hz, Decibel: %f dB\n", frequencies(i), round(decibe
    end
end
```