Constructor University Bremen

Lab Report 1: RLC Circuits - Transient Response

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1. Introduction

1.1 Objective

The objective of this experiment is to investigate the behavior of second-order systems, namely, the RLC circuit. During the lab, various RLC circuit configurations are built and theoretical results are compared with experimental results with the aid of MATLAB.

1.2 Introduction

Second order systems are very common, named due to the highest order of the differential equation describing the system. For an RLC circuit, we tend to use a second-order ordinary differential equation to describe the system, which is given by

$$a_2\frac{d^2y(t)}{dt^2}+a_1\frac{dy(t)}{dt}+a_0y(t)=x(t) \tag{1.1}$$

Where we define $y(t)$ as the output of the system, $x(t)$ as the input of the system, and a_2, a_1, and a_0 as system parameters.

However, in the context of the response of second-order systems we tend to use a more general form of this equation, given by

$$\frac{d^2y(t)}{dt^2}+2\zeta\omega_n\frac{dy(t)}{dt}+\omega_n^2y(t)=K\omega_n^2x(t) \tag{1.2}$$

Where we now define the system parameters as:

- ζ - damping ratio
- ω_n - natural frequency
- K - gain of the system

For second order differential equations, we know that $y_t=y_p+y_h$ where y_p is the particular solution and y_h is the homogeneous solution.

To solve for the homogenous solution, we set the input to zero and solve the differential equation. The general solution for the homogenous solution is given by

$$y_h(t) = C_1 e^{\lambda_1 t} + C_2 e^{\lambda_2 t} \tag{1.3}$$

Where C_1 and C_2 are constants defined by initial conditions and λ_1 and λ_2 are the roots of the characteristic equation that are determined by the resistor, capacitor, and inductor values.

We know that depending on ζ and the undamped natural frequency ω_n we can classify the transient response of the system into three categories:

- Overdamped - $\zeta > 1$
- Critically damped - $\zeta = 1$
- Underdamped - $\zeta < 1$

Each of these categories has a different equation, which are given below:

1. **Overdamped**

   In the overdamped case, the response is the sum of two decaying exponentials, defined as

$$y(t) = C_1 \exp\left(\left(-\zeta + \sqrt{\zeta^2 - 1}\right) w_n t\right) + C_2 \exp\left(\left(-\zeta - \sqrt{\zeta^2 - 1}\right) w_n t\right) \tag{1.4}$$

2. **Critically damped**

   In the case of critical damping, the system reaches steady state in the shortest amount of time, the equation given by the state below.

$$y(t) = C_1 e^{\lambda_1 t} + C_2 t e^{\lambda_2 t} \tag{1.5}$$

3. **Underdamped**

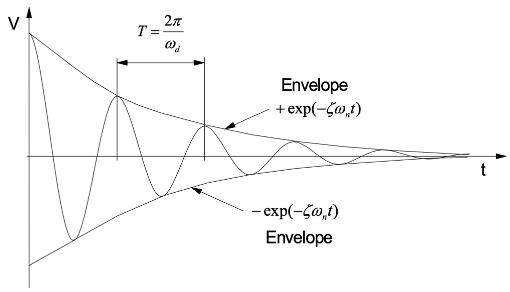


Figure 1.1: Underdamped Response

In the underdamped case, we can observe ringing in the response. This is due to the fact that the system is oscillating around the steady state value.

For the underdamped case, let's consider w_d as the damped natural frequency, given by $w_d = w_n\sqrt{(1-\zeta^2)}$. We can then write the equation as

$$y(t) = e^{-\zeta\omega_n t}\left(C_1\cos\left(w_d t\right) + C_2\sin\left(w_d t\right)\right) \tag{1.6}$$

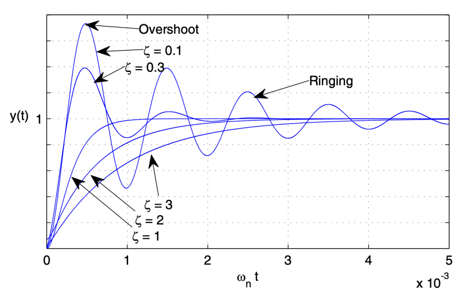


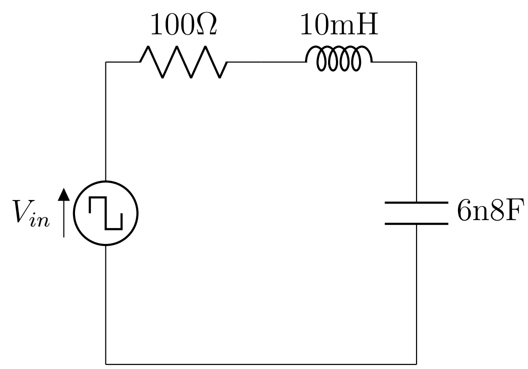
Figure 1.2: General Transient Response Diagram

Below is a figure that shows the cases of the transient response of a second order system according to the damping ratio ζ when provided with a step input.

Considering a forced solution is out of the scope of this lab, but in general, the output will usually be a weighted sum of the input signal $x(t)$ and its first and second derivatives.

2. Execution

- $V_{pp} = 1\text{V}$
- $V_{off} = 0.5\text{V}$
- $f = 100\text{Hz}$
- $R_i = 50\Omega$



1. The function generator was set to produce a 100Hz square wave with an amplitude of 0.5V and an offset of 0.5V. It was checked with the oscilloscope if the signal modulated between 0V and 1V.

2. Subsequently, the R-decade was set to 100Ω, and the oscilloscope was connected in parallel to the capacitor.

3. The damped frequency f_d was measured. To determine f_d, the time or frequency of the exponentially damped sinusoidal waveform was measured. A hardcopy was taken of one signal period and another focusing on the ringing phenomenon.

4. Afterward, the damped radian frequency ω_d was calculated. In this calculation, the internal resistance of the function generator was considered to be 50Ω. The calculated value was compared with the measured value from step (2). If they were consistent, the next steps were proceeded with.

5. The resistance required for the circuit to be critically damped was then calculated. The signal was displayed, and a hardcopy was taken.

6. To check if the practical signal was critically damped, the R-decade value was varied, and a hardcopy of the final result was taken.

7. Finally, the R-decade was set to $30\text{k}\Omega$, causing the circuit to be overdamped. The transient voltage across the capacitor was displayed, and a hardcopy was taken.

3. Evaluation

4. Conclusion

5. References

6. Appendix