Constructor University Bremen

Lab Report 2: The Fourier Series and The Fourier Transform

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1. Prelab

1.1 Problem 1: Decibels

1. Given $x(t) = 5\cos(2\pi 1000t)$
   - What is the amplitude and the V_{pp} of the signal?
     The amplitude of the signal is $5V$ and the V_{pp} is $2 \cdot 5V = 10V$.
   - What is the V_{rms} of the signal?
     The V_{rms} of a sinusoidal signal is given by
     $$V_{rms} = \frac{V_A}{2\sqrt{2}} = \frac{5V}{\sqrt{2}} = 3.535V \tag{1.1}$$
   - What is the amplitude of the spectral peak in dBVrms? The amplitude of the spectral peak in dBVrms is given by the following relation:
     $$A_{dBV_{rms}} = 20\log_{10}(V_{rms}) = 20\log_{10}(3.535V) = 10.9dBV_{rms} \tag{1.2}$$
2. Given a square wave of $1V_{pp}$ the voltage changes between -0.5V to 0.5V
   - What is the signal amplitude in V_{rms}? To find the RMS value of a square wave, the following relation is used:
     $$V_{rms} = \sqrt{\frac{1}{T}\left[\int_0^T f(t)^2 dt\right]} \tag{1.3}$$
     The square wave can be specified as a piecewise function:
     $$f(t) = \begin{cases} -0.5 & \text{if } 0 \le t \le \frac{T}{2} \\ 0.5 & \text{if } \frac{T}{2} \le t \le T \end{cases} \tag{1.4}$$

Performing the integral over the function,

$$\begin{aligned}
V_{rms} &= \sqrt{\frac{1}{T}\left[\int_0^{T/2}(-0.5)^2 dt + \int_{T/2}^{T}(0.5)^2 dt\right]} \\
&= \sqrt{0.25 \cdot \frac{1}{T}\left(\frac{T}{2} + T - \frac{T}{2}\right)} \\
&= \sqrt{0.25}V \\
&= 0.5V
\end{aligned} \tag{1.5}$$

- What is the amplitude in dBVrms?

$$V_{dB_{rms}} = 20\log_{10}(V_{rms}) = 20\log_{10}(0.5V) = -6.02dBV_{rms} \tag{1.6}$$

1.2 Problem 2: Determination of Fourier Series Coefficients

1. Determine the Fourier series coefficients up to the 5th harmonic of the function $f(t) = 4t^2$

   To determine the fourier series coefficients, first recognize that the function is an even function by the very fact that it is a polynomial of an even degree. Knowing this, the relations used to compute the fourier series coefficients are:

$$\begin{aligned}
a_n &= \frac{4}{T}\int_{-T}^{T} t^2 \cos(n\omega_0 t)dt \\
a_0 &= \frac{1}{T}\int_0^T t^2 dt \\
b_n &= 0
\end{aligned} \tag{1.7}$$

   Where it is known that $T = 1$ due to the interval being given as $[-0.5, 0.5]$. Furthermore, $\omega_0 = \frac{2\pi}{T} = 2\pi$. By using the help of an integration table, the integral for a_n is

$$a_n = 4 \cdot \left[\frac{2t\cos(n\omega_0 t)}{4\pi^2 n^2} + \frac{n^2 4\pi^2 - 2}{8\pi^3 n^3} sin(n\omega_0 t)\right]_{-T}^{T} \tag{1.8}$$

Evaluating this integral:

$$a_n = 4 \cdot \left(\frac{4 \cdot (-1)^n}{4\pi^2 n^2} \right) = \frac{4 \cdot (-1)^n}{\pi^2 n^2} \tag{1.9}$$

For a_0, the integral is straightforward:

$$a_0 = \frac{1}{T} \int_0^T t^2 dt = \left[\frac{t^3}{3} \right]_0^T = \frac{1}{3} \tag{1.10}$$

So the first five coefficients are:

$$a_n = \left\{ \frac{-4}{\pi^2}, \frac{4}{16\pi^2}, \frac{-4}{36\pi^2}, \frac{4}{64\pi^2}, \frac{-4}{100\pi^2} \right\} \tag{1.11}$$

2. Use MATLAB to plot the original function and the inverse Fourier transform. Put both graphs into the same diagram.

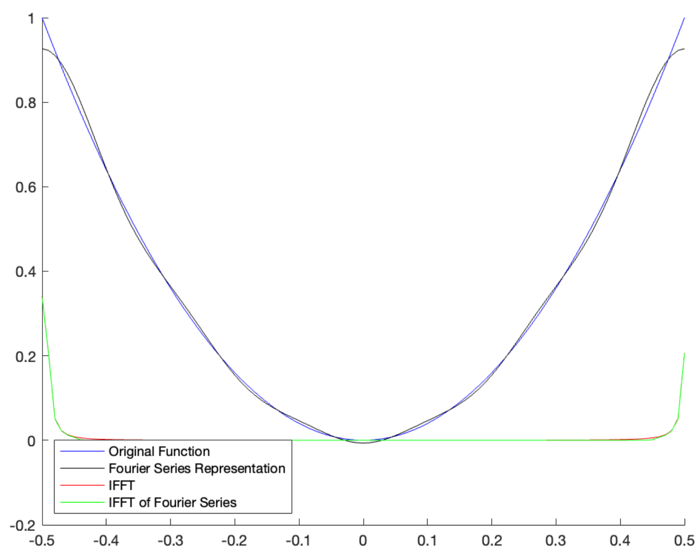

Figure 1.1: The plot of the original function, its inverse fourier transform, the fourier series representation up to the fifth harmonic, and the representation's inverse fourier transform

The code used:

```
t = -0.5:0.01:0.5;
f = 4*t.^2;
f_fs = 1/3;

% Fourier series representation of the function
for i=1:5
    % w_0 = 2pi
    % T = 1
    a_i = (4*(-1)^i)/(pi^2*i^2);
    f_fs = f_fs + (a_i .* cos(i.*2.*pi.*t));
end
hold on

plot(t, f, "blue");
plot(t, f_fs, "black");
xlim([-0.5, 0.5]);

plot(t, abs(ifft(f)), "red");
plot(t, abs(ifft(f_fs)), "green");

legend({"Original Function", ...
    "Fourier Series Representation", ...
    "IFFT", ...
    "IFFT of Fourier Series" ...
    }, "Location", "southwest")
```

1.3 Problem 3: FFT of a Rectangular Wave

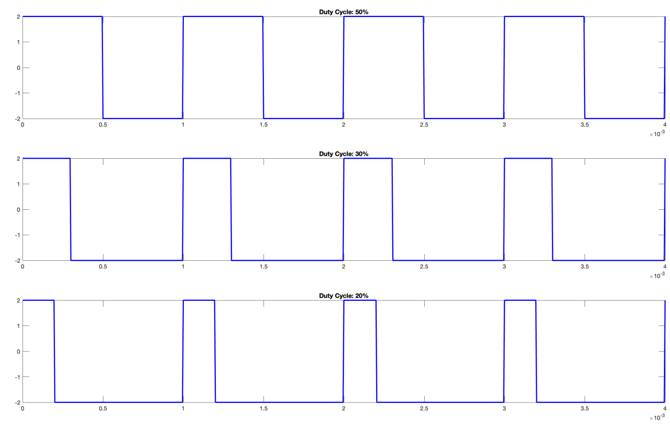

Figure 1.2: A figure showing the different rectangular waves based on their duty cycles. Plotted using MATLAB.

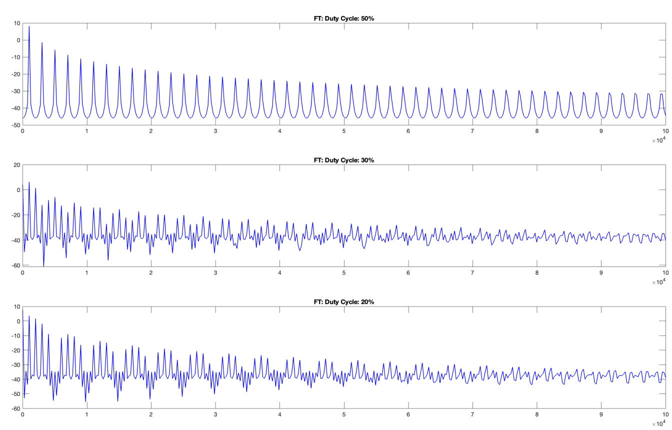

Figure 1.3: A figure showing the different rectangular waves' spectra based on their duty cycles. Plotted using MATLAB.

The code used to plot the figures:

```
%% Time plots

[t_fifty, signal_fifty, f_fifty, y_fifty] = get_square(50);
[t_thirty, signal_thirty, f_thirty, y_thirty] = get_square(30);
[t_twenty, signal_twenty, f_twenty, y_twenty] = get_square(20);

subplot(3, 1, 1);
plot(t_fifty, signal_fifty, "blue", "LineWidth", 2);
title("Duty Cycle: 50%");

subplot(3, 1, 2);
plot(t_thirty, signal_thirty, "blue", "LineWidth", 2);
title("Duty Cycle: 30%");

subplot(3, 1, 3);
plot(t_twenty, signal_twenty, "blue", "LineWidth", 2);
title("Duty Cycle: 20%");

%% Frequency plots
subplot(3, 1, 1);
plot(f_fifty, y_fifty, "blue", "LineWidth", 1);
title("FT: Duty Cycle: 50%");


subplot(3, 1, 2);
plot(f_thirty, y_thirty, "blue", "LineWidth", 1);
title("FT: Duty Cycle: 30%");


subplot(3, 1, 3);
plot(f_twenty, y_twenty, "blue", "LineWidth", 1);
title("FT: Duty Cycle: 20%");
```

Where the function `get_square` is defined as:

```
function [t, signal, f, y_single_db] = get_square(duty_cycle)
    period = 1e-3;
    Fs = 200e3;
    frequency = 1/period;
    Vpp = 2;
    duration = period * 4;

    % The signal
    t = 0:1/Fs:duration;
    signal = Vpp*square((2*pi*frequency)*t, duty_cycle);
    plot(t, signal, "blue", "LineWidth", 2);

    % The fourier transform of the signal
    rms_value = sqrt(mean(signal.^2));
    N = length(signal); % The length of the signal

    y = fft(signal, N);
    y_mag = 2*(abs(y)/N); % Magnitudes of y

    y_single = y_mag(1:floor(N/2)) * 2;
    f_nyquist = Fs / 2;

    y_single_db = 20*log10(y_single / rms_value);
    f = linspace(0, f_nyquist, length(y_single));
end
```

2. Introduction

3. Execution

4. Evaluation

5. Conclusion

6. References

7. Appendix