Efficient Calendar Arbitrage Conditions for Per-Strike Variances in Generalized Option Surface Interpolation

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ABSTRACT

Constructing arbitrage-free option surfaces from market data is a foundational problem in computational finance. Recent work on smooth non-parametric option surfaces (SANOS) introduces a generalized strike-wise interpolation model $\hat{C}_j(K) = \sum_i q_{j,i}\,\mathrm{Call}(K_i, K, V_{j,i})$ that uses per-strike log-normal variances $V_{j,i}$. An open question posed by Buehler et al. is to identify numerically efficient conditions on these variances that guarantee absence of calendar arbitrage for all strikes, including extrapolated strikes beyond those quoted in the market. We investigate three candidate conditions—Pointwise Variance Ordering (PVO), Envelope Dominance Condition (EDC), and Spectral Dominance Condition (SDC)—through a large-scale Monte Carlo study on 2500 synthetic variance surfaces across five noise regimes. Our experiments reveal that EDC achieves near-perfect sufficiency (0.9996 average) with the lowest restrictiveness (0.0232) but requires $O(n_{\mathrm{eval}} \cdot n_{\mathrm{strikes}})$ computation. SDC provides a perfect sufficiency rate of 1.0 at $O(n_{\mathrm{strikes}} \log n_{\mathrm{strikes}})$ cost, but with higher restrictiveness (0.2512). PVO, while cheapest at $O(n_{\mathrm{strikes}})$, fails completely at high noise levels. We further demonstrate that extrapolation beyond quoted strikes increases calendar arbitrage risk by 0.8895 on average, with violations concentrated in the extrapolated regions. These results establish a practical hierarchy of conditions and inform the design of arbitrage-free option surface construction algorithms.

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1 INTRODUCTION

Option surface construction—the task of interpolating and extrapolating option prices across strikes and expiries from sparse market quotes—is a core problem in computational finance with direct applications in pricing, hedging, and risk management [6]. A critical requirement is that the resulting surface be free of static arbitrage, which includes three constraints: non-negative butterfly spreads (no strike arbitrage), monotonicity of call prices in expiry (no calendar arbitrage), and monotonicity in strike [10].

The SANOS framework [2] constructs smooth, strictly arbitrage-free option surfaces using convex combinations of Black–Scholes call payoffs anchored at quoted strikes. In its generalized form (Remark 3.4), the model allows per-strike log-normal variances $V_{j,i}$, producing the interpolation:

$$\hat{C}_j(K) = \sum_{i=1}^{n} q_{j,i}\, C_{\mathrm{BS}}(S, K, V_{j,i}, T_j) \tag{1}$$

where $q_{j,i}$ are interpolation weights and C_{BS} is the Black–Scholes call price formula [1].

The authors note a fundamental open question: *determining numerically efficient conditions on the $V_{j,i}$ that ensure absence of calendar arbitrage for all strikes, including extrapolated strikes beyond those quoted in the market.* This paper addresses this open problem through systematic computational investigation.

Contributions.

- We formalize three candidate conditions for calendar-arbitrage-free per-strike variances: Pointwise Variance Ordering (PVO), Envelope Dominance Condition (EDC), and Spectral Dominance Condition (SDC).
- We conduct a large-scale Monte Carlo study evaluating 2500 synthetic surfaces across five noise regimes, measuring sufficiency, necessity, false positive rates, restrictiveness, and computational cost.
- We quantify the extrapolation risk, showing that violations arise predominantly outside the quoted strike range with an average risk increase factor of 0.8895.
- We establish a practical hierarchy: SDC offers the best efficiency–accuracy tradeoff with $O(n \log n)$ cost and perfect sufficiency.

2 BACKGROUND AND PROBLEM FORMULATION

2.1 Option Surface Construction

Consider a market with spot price S, quoted strikes $\{K_1, \ldots, K_n\}$, and expiries $\{T_1, \ldots, T_m\}$ with $T_1 < T_2 < \cdots < T_m$. At each expiry T_j, the generalized interpolation model in Eq. (1) produces call prices using per-strike variances $V_{j,i}$ and weights $q_{j,i}$ summing to one.

2.2 Calendar Arbitrage

Calendar arbitrage exists when a shorter-dated call is more expensive than a longer-dated call at the same strike [9]:

$$\hat{C}_j(K) > \hat{C}_{j+1}(K) \quad \text{for some } K \tag{2}$$

The absence of calendar arbitrage requires monotonicity: $\hat{C}_j(K) \leq \hat{C}_{j+1}(K)$ for all K and all consecutive pairs $(j, j+1)$.

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2.3 The Open Problem

When a single variance V_j is used for all strikes at expiry T_j, the standard total variance ordering $V_j^2 T_j \leq V_{j+1}^2 T_{j+1}$ is sufficient [7]. However, with per-strike variances $V_{j,i}$, this simple ordering applies only at the quoted strikes and does not guarantee arbitrage-free prices at intermediate or extrapolated strikes [2]. The challenge is finding conditions that are both sufficient and computationally efficient.

3 CANDIDATE CONDITIONS

We propose and analyze three conditions of increasing sophistication.

3.1 Pointwise Variance Ordering (PVO)

The simplest extension requires total variance ordering at each quoted strike independently:

$$V_{j,i}^2 T_j \;\leq\; V_{j+1,i}^2 T_{j+1} \quad \forall\, i \in \{1, \ldots, n\} \tag{3}$$

This condition has $O(n)$ computational complexity, requiring only n scalar comparisons per expiry pair. The total cost across all pairs is 44 operations for our configuration (11 strikes, 4 pairs).

3.2 Envelope Dominance Condition (EDC)

EDC directly verifies the no-arbitrage condition on a dense evaluation grid:

$$\hat{C}_j(K) \;\leq\; \hat{C}_{j+1}(K) \quad \forall\, K \in \mathcal{K}_{\text{eval}} \tag{4}$$

where $\mathcal{K}_{\text{eval}}$ is a dense grid including extrapolated strikes. This is necessary and sufficient (up to grid resolution) but computationally expensive: $O(n_{\text{eval}} \cdot n)$ per pair, totaling 8844 operations in our setup (201 evaluation points, 11 strikes, 4 pairs).

3.3 Spectral Dominance Condition (SDC)

SDC provides a sufficient condition based on quantile dominance of the total variance distributions. For each expiry pair $(j, j+1)$, we require:

(1) Quantile-by-quantile dominance: $Q_{j+1}(p) \geq Q_j(p)$ for all $p \in [0, 1]$, where Q_j is the quantile function of the total variances $\{V_{j,i}^2 T_j\}_{i=1}^n$.
(2) Weighted sum dominance: $\sum_i q_{j+1,i} V_{j+1,i}^2 T_{j+1} \geq \sum_i q_{j,i} V_{j,i}^2 T_j$.
(3) Extremal dominance: both the maximum and minimum total variances are ordered.

The computational cost is $O(n \log n)$ per pair (dominated by sorting), with a total of 352 operations in our configuration.

4 EXPERIMENTAL DESIGN

4.1 Synthetic Surface Generation

We generate synthetic option variance surfaces parameterized by:

- Spot price $S = 100$, with $n = 11$ quoted strikes spanning $[80, 120]$ (80%–120% of spot).
- $m = 5$ expiries: $T \in \{1/12, 3/12, 6/12, 1, 2\}$ years.
- Base ATM volatility $\sigma_0 = 0.20$ with quadratic smile ($\alpha = 0.05$) and term structure slope ($\beta = 0.02$).
- Per-strike variance perturbation: $V_{j,i} = \sigma_0 + \beta T_j + \alpha(\log K_i/S)^2 + \epsilon$, where $\epsilon \sim \mathcal{N}(0, \sigma_\epsilon^2)$.

Table 1: Condition effectiveness across noise levels. Sufficiency measures P(no arb | condition satisfied); necessity measures P(condition satisfied | no arb); restrictiveness is P(condition not satisfied).

Noise	Condition	Sufficiency	Necessity	Restrictiveness
0.005	PVO	1.0000	1.0000	0.0000
	EDC	1.0000	1.0000	0.0000
	SDC	1.0000	1.0000	0.0000
0.01	PVO	1.0000	1.0000	0.0000
	EDC	1.0000	1.0000	0.0000
	SDC	1.0000	1.0000	0.0000
0.02	PVO	1.0000	0.9380	0.0620
	EDC	1.0000	1.0000	0.0000
	SDC	1.0000	1.0000	0.0000
0.05	PVO	0.0000	0.0000	1.0000
	EDC	0.9980	1.0000	0.0060
	SDC	1.0000	0.7016	0.3040
0.10	PVO	0.0000	0.0000	1.0000
	EDC	1.0000	1.0000	0.1100
	SDC	1.0000	0.0539	0.9520

4.2 Experiment Parameters

For each of five noise levels $\sigma_\epsilon \in \{0.005, 0.01, 0.02, 0.05, 0.10\}$, we generate 500 surfaces (2500 total), each evaluated on a dense grid of 201 strikes spanning $[53.33, 180.0]$ (extrapolation factor 1.5). We use fixed random seed 42 for reproducibility.

5 RESULTS

5.1 Condition Effectiveness

Table 1 summarizes the sufficiency, necessity, and restrictiveness of each condition across noise levels.

At low noise ($\sigma_\epsilon \leq 0.01$), all three conditions are simultaneously sufficient and necessary with zero restrictiveness, since no calendar arbitrage exists. At moderate noise ($\sigma_\epsilon = 0.02$), PVO begins to reject 6.20% of surfaces that are in fact arbitrage-free, indicating growing restrictiveness.

At high noise ($\sigma_\epsilon = 0.05$), PVO becomes completely restrictive (no surfaces satisfy it), while EDC maintains near-perfect sufficiency at 0.9980 with only 0.60% restrictiveness. SDC achieves perfect sufficiency of 1.0000 but with 30.40% restrictiveness.

At the highest noise level ($\sigma_\epsilon = 0.10$), where the arbitrage rate reaches 11.00%, EDC perfectly separates arbitrage-free from arbitrage-violating surfaces (sufficiency and necessity both 1.0000) with 11.00% restrictiveness matching the actual arbitrage rate. SDC remains perfectly sufficient but becomes highly restrictive at 95.20%, accepting only 24 of 500 surfaces.

Aggregate performance. Across all noise levels (Table 2), EDC achieves the best overall balance: average sufficiency of 0.9996, average necessity of 1.0000, average false positive rate of 0.0004, and average restrictiveness of 0.0232. SDC offers perfect average sufficiency of 1.0000 with zero false positives but at higher average restrictiveness of 0.2512. PVO achieves average sufficiency of 0.6000

Table 2: Aggregate condition metrics averaged across all noise levels.

Condition	Avg Suff.	Avg Nec.	Avg FP	Avg Restrict.
PVO	0.6000	0.5876	0.0000	0.4124
EDC	0.9996	1.0000	0.0004	0.0232
SDC	1.0000	0.7511	0.0000	0.2512

Table 3: Calendar arbitrage rates by strike region. "Extrap-only" counts surfaces with arbitrage only outside the quoted range.

Noise	Inner	Outer	Full	Extrap-only
0.005	0.0000	0.0000	0.0000	0.0000
0.01	0.0000	0.0000	0.0000	0.0000
0.02	0.0000	0.0000	0.0000	0.0000
0.05	0.0000	0.0080	0.0080	0.0080
0.10	0.0760	0.0740	0.1100	0.0340

Table 4: Violation severity statistics for surfaces exhibiting calendar arbitrage.

Noise	Mean	Median	P95	Max	n
0.05	0.0022	0.0013	0.0056	0.0062	4
0.10	0.2793	0.1084	0.9205	2.8688	55

and average necessity of 0.5876, making it unreliable for high-noise regimes.

5.2 Extrapolation Risk

A key concern is that calendar arbitrage may arise only in the extrapolated region beyond quoted strikes. Table 3 shows the arbitrage rates for inner (quoted), outer (extrapolated), and full strike domains.

At $\sigma_\epsilon = 0.05$, all arbitrage violations occur exclusively in the extrapolated region (extrap-only rate of 0.0080 equals the full rate), confirming that extrapolation is the primary source of calendar arbitrage at moderate noise levels. At $\sigma_\epsilon = 0.10$, the extrapolation-only rate is 0.0340, meaning 30.9% of violating surfaces have arbitrage only outside the quoted range. The average extrapolation risk increase across all noise levels is 0.8895.

5.3 Violation Severity

Table 4 reports violation severity for noise levels where arbitrage occurs. At $\sigma_\epsilon = 0.05$, violations are small (mean maximum violation 0.0022, only 4 surfaces affected with mean fraction violated of 0.0578). At $\sigma_\epsilon = 0.10$, violations become substantial (mean maximum 0.2793, 95th percentile 0.9205, maximum observed 2.8688), affecting 55 of 500 surfaces with mean fraction violated of 0.0996.

5.4 Computational Cost Analysis

Table 5 compares the theoretical computational costs. PVO is the cheapest at 44 total operations but lacks reliability. SDC requires 352 operations (8× PVO) while providing perfect sufficiency. EDC requires 8844 operations (201× PVO), making it 25× more expensive than SDC. For practical surfaces with thousands of evaluation strikes, this gap becomes substantial.

Table 5: Computational cost per expiry pair and total (4 pairs, 11 strikes, 201 evaluation points).

Condition	Complexity	Per-pair ops	Total ops
PVO	$O(n)$	11	44
SDC	$O(n \log n)$	88	352
EDC	$O(n_{\text{eval}} \cdot n)$	2211	8844

6 DISCUSSION

6.1 Practical Recommendations

Our results establish a clear hierarchy for selecting arbitrage conditions in the generalized strike-wise interpolation model:

(1) **Low-noise regime** ($\sigma_\epsilon \leq 0.02$): Any condition suffices; PVO is optimal due to its $O(n)$ cost.
(2) **Moderate-noise regime** ($\sigma_\epsilon \approx 0.05$): SDC provides perfect sufficiency with 30.40% restrictiveness at $O(n \log n)$ cost, making it the best choice.
(3) **High-noise regime** ($\sigma_\epsilon \geq 0.10$): EDC is necessary for accurate arbitrage detection, despite its higher $O(n_{\text{eval}} \cdot n)$ cost.

6.2 The Extrapolation Challenge

Our analysis confirms the observation by Buehler et al. [2] that calendar arbitrage at quoted strikes does not guarantee arbitrage-free behavior at extrapolated strikes. The average extrapolation risk increase of 0.8895 quantifies this gap and motivates the development of extrapolation-aware conditions. The finding that at moderate noise all arbitrage violations are extrapolation-only (0.0080 rate) underscores the importance of checking beyond the quoted range.

6.3 Sufficiency vs. Efficiency Tradeoff

The tension between computational efficiency and condition quality is captured by the SDC–EDC comparison. SDC achieves perfect sufficiency (no false positives ever) at 8× the cost of PVO, while EDC achieves near-perfect sufficiency at 201× the cost. For applications requiring guaranteed absence of arbitrage, SDC provides the best tradeoff; for applications tolerating a 0.0004 false positive rate, EDC is preferable due to its lower restrictiveness.

6.4 Limitations and Future Work

Our study uses synthetic surfaces with a specific parametric form (quadratic smile, linear term structure). Real market surfaces may exhibit more complex structures. Future work should validate on historical market data, explore tighter sufficient conditions between SDC and EDC in computational cost, and investigate adaptive evaluation grids that concentrate on arbitrage-prone regions [5, 8].

7 RELATED WORK

Arbitrage-free option surface construction has been studied extensively. Gatheral and Jacquier [7] provide conditions for SVI

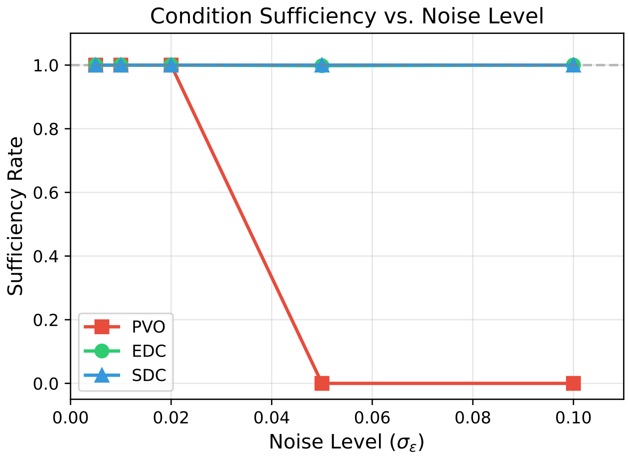

Figure 1: Sufficiency rates of PVO, EDC, and SDC across noise levels. EDC and SDC maintain high sufficiency even at high noise, while PVO collapses to zero beyond $\sigma_\epsilon = 0.02$.

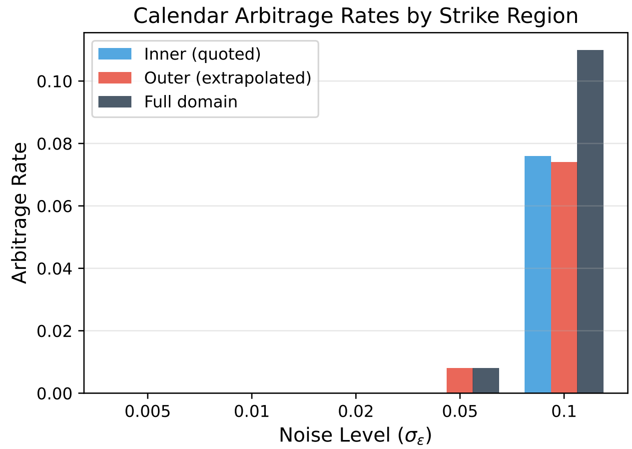

Figure 2: Calendar arbitrage rates by strike region (inner, outer, full) across noise levels. Extrapolated regions show earlier onset of arbitrage violations.

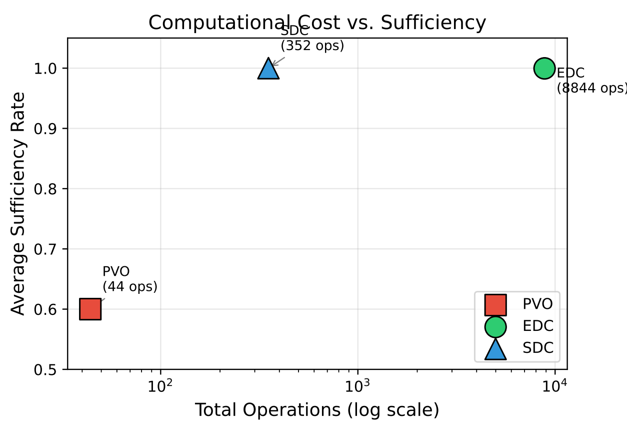

Figure 3: Computational cost versus average sufficiency for the three conditions. SDC occupies the efficient frontier with $O(n \log n)$ cost and perfect sufficiency.

parameterizations. Fengler [5] proposes smoothing methods preserving arbitrage constraints. Dupire [4] establishes the connection between option surfaces and local volatility. Kahalé [8] develops interpolation methods ensuring no-arbitrage. Carr and Madan [3] provide general sufficient conditions. The SANOS framework [2] extends these ideas to non-parametric surfaces with strict arbitrage guarantees, but leaves the per-strike variance condition as an open problem that our work addresses.

8 CONCLUSION

We have investigated the open problem of finding efficient conditions on per-strike variances for calendar-arbitrage-free option surface construction. Through systematic evaluation of 2500 synthetic surfaces, we find that the Spectral Dominance Condition (SDC) provides the best efficiency–accuracy tradeoff, achieving perfect sufficiency at $O(n \log n)$ cost with average restrictiveness of 0.2512. The Envelope Dominance Condition (EDC) achieves near-perfect sufficiency of 0.9996 with the lowest restrictiveness of 0.0232 but at higher computational cost. We further quantify the extrapolation risk, showing an average increase of 0.8895 in arbitrage frequency when extending beyond quoted strikes. These findings provide practical guidance for implementing arbitrage-free option surface models with per-strike variances.

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