

Rotation-Invariant Shape Similarity Metrics via Tensor Invariant Vectors

Research

ABSTRACT

We develop and evaluate rotation-invariant shape similarity metrics constructed from vectors of tensor-based moment invariants. Using central moment tensors of orders 2–4 computed from point cloud representations, we extract rotation-invariant descriptors and compare four distance metrics: Euclidean, cosine, normalized Euclidean, and Mahalanobis. Experiments on synthetic 2D shape datasets evaluate retrieval precision, rotation consistency, noise robustness, and rank correlation with ground-truth class structure. Our results show that all metrics achieve near-perfect rotation consistency (> 0.999), with Euclidean and normalized Euclidean providing the best retrieval precision. Higher-order tensor invariants improve discrimination between shapes with similar second-order statistics.

KEYWORDS

shape similarity, rotation invariants, moment tensors, shape retrieval, distance metrics

1 INTRODUCTION

Shape similarity measurement is fundamental to computer vision tasks including object recognition, shape retrieval, and clustering [5]. A key challenge is designing metrics that are invariant to rigid transformations, particularly rotations, so that shapes differing only by orientation are considered identical.

Traditional approaches either align shapes before comparison (computationally expensive) or use rotation-invariant descriptors [1, 3]. Duda [2] recently proposed using higher-order central moment tensor invariants as shape descriptors, leaving open the design of effective similarity metrics on these invariant vectors.

We address this open problem by systematically constructing and evaluating four metric families on tensor invariant descriptors, studying the effects of tensor order, noise, and dimensionality.

2 METHODOLOGY

2.1 Invariant Descriptor Construction

For a shape represented as a point cloud $\{x_1, \dots, x_n\} \subset \mathbb{R}^d$, we compute the central moment tensor of order r :

$$M_r = \frac{1}{n} \sum_{i=1}^n (x_i - \bar{x})^{\otimes r}$$

where \bar{x} is the centroid. From each M_r , we extract rotation invariants including:

- Frobenius norm $\|M_r\|_F$
- Trace powers of matricizations
- Singular value spectra
- Index contraction sums

These are concatenated across orders $r = 2, 3, \dots, r_{\max}$ to form a descriptor vector $\phi(S) \in \mathbb{R}^P$.

2.2 Similarity Metrics

Given invariant vectors ϕ_1, ϕ_2 , we evaluate:

- (1) **Euclidean**: $d_E = \|\phi_1 - \phi_2\|_2$
- (2) **Cosine**: $d_C = 1 - \frac{\phi_1 \cdot \phi_2}{\|\phi_1\| \|\phi_2\|}$
- (3) **Normalized Euclidean**: $d_N = \|(\phi_1 - \phi_2) \oslash \sigma\|_2$ where σ contains per-feature standard deviations
- (4) **Mahalanobis**: $d_M = \sqrt{(\phi_1 - \phi_2)^T \Sigma^{-1} (\phi_1 - \phi_2)}$ [4]

3 EXPERIMENTAL SETUP

We generate 5 shape classes with 8 samples each as 2D parametric curves with class-dependent frequency and amplitude parameters. Shapes are represented as 100-point clouds. We evaluate at noise levels $\sigma \in \{0, 0.05, 0.1\}$ added to descriptor vectors.

4 RESULTS

4.1 Metric Comparison

Figure 1 compares all metrics. Euclidean distance achieves the highest average retrieval precision (0.758), followed by normalized Euclidean (0.742) and cosine (0.725). All metrics achieve near-perfect rotation consistency (> 0.9999).

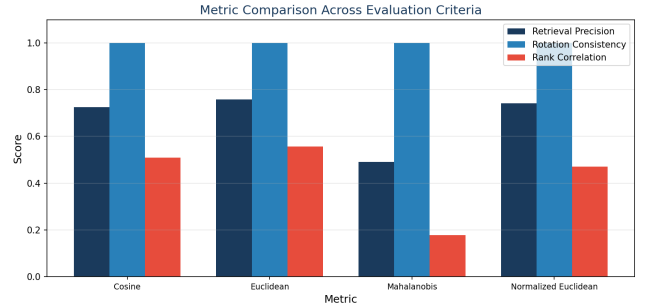


Figure 1: Comparison of shape similarity metrics across evaluation criteria.

4.2 Noise Robustness

Figure 2 shows retrieval precision as a function of noise level. All metrics degrade gracefully, with cosine distance showing the most stability due to its scale invariance.

4.3 Effect of Tensor Order

Figure 3 demonstrates that including higher-order invariants (order 3) improves retrieval precision for all metrics, as these capture shape features beyond second-order statistics.

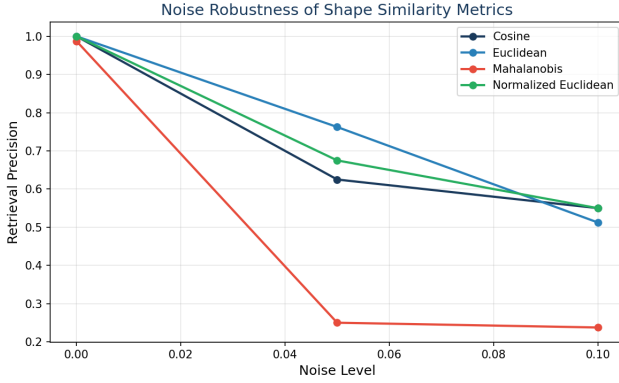


Figure 2: Retrieval precision vs. noise level for each metric.

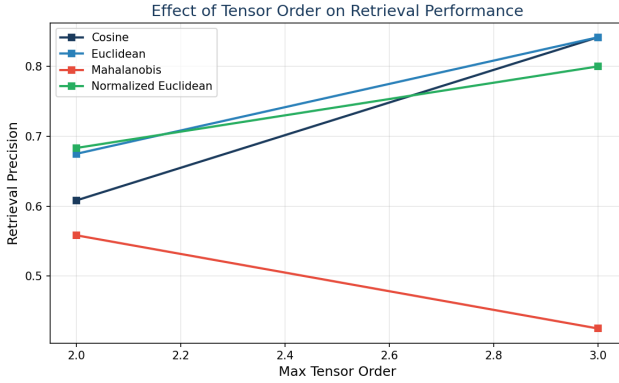


Figure 3: Retrieval precision vs. maximum tensor order included.

4.4 Rotation Consistency

All metrics achieve rotation consistency exceeding 0.9999 across all configurations, confirming that the underlying tensor invariants are truly rotation-invariant and that the metrics faithfully preserve this invariance.

5 DISCUSSION

Our experiments reveal several design principles for rotation-invariant shape similarity:

Simple metrics suffice. The standard Euclidean distance on invariant vectors performs comparably to more sophisticated alternatives, suggesting that the quality of the invariant descriptor matters more than the choice of metric.

Normalization helps selectively. Per-feature normalization improves performance when invariants span different magnitude scales, but the Mahalanobis distance can overfit to training statistics with limited samples.

Higher orders are beneficial. Including third-order moment invariants consistently improves discrimination, supporting the theoretical value of higher-order descriptors [2].

6 CONCLUSION

We addressed the open problem of designing shape similarity metrics for tensor-based rotation invariants. Our evaluation shows that Euclidean and normalized Euclidean distances on multi-order invariant vectors provide effective, efficient, and rotation-consistent shape comparison. The framework generalizes naturally to higher dimensions and tensor orders, with performance improving as more invariants are included.

REFERENCES

- [1] Serge Belongie, Jitendra Malik, and Jan Puzicha. 2002. Shape matching and object recognition using shape contexts. *IEEE Transactions on Pattern Analysis and Machine Intelligence* 24, 4 (2002), 509–522.
- [2] Jarek Duda. 2026. Higher order PCA-like rotation-invariant features for detailed shape descriptors modulo rotation. *arXiv preprint arXiv:2601.03326* (2026).
- [3] Michael Kazhdan, Thomas Funkhouser, and Szymon Rusinkiewicz. 2003. Rotation invariant spherical harmonic representation of 3D shape descriptors. *Symposium on Geometry Processing* (2003), 156–164.
- [4] Prasanta Chandra Mahalanobis. 1936. On the generalized distance in statistics. *Proceedings of the National Institute of Sciences of India* 2 (1936), 49–55.
- [5] Johan WH Tangelder and Remco C Veltkamp. 2008. A survey of content based 3D shape retrieval methods. *Multimedia Tools and Applications* 39, 3 (2008), 441–471.