Topology Optimization for Maximizing Feasible Wealth Volume in Payment Channel Networks

Research Investigation
Open Problems in Network Architecture

ABSTRACT

We investigate the problem of finding payment channel network topologies that maximize $r(G) = |W_G|/|\mathcal{W}(C,n)|$, the ratio of feasible off-chain wealth distributions to total on-chain distributions. Through exhaustive enumeration over all connected graphs for $n \leq 5$ and evolutionary search for $n = 6$, we find that the complete graph (K_n) maximizes $r(G)$ for $n \leq 4$ with $r(K_4) = 0.441$, while for $n \geq 5$, sparser topologies with moderate connectivity dominate. Across all tested configurations, cycle graphs consistently achieve the highest $r(G)$ among standard graph families for $n \geq 5$, with $r(C_6) = 0.100$ compared to $r(K_6) = 0.001$. The optimal edge count for $n = 4$ is consistently 6 (complete graph) regardless of capacity, while the optimal degree sequence transitions from regular to near-regular as n grows. These findings provide practical guidance for designing payment channel networks that maximize off-chain payment feasibility.

1 INTRODUCTION

Payment channel networks (PCNs) such as the Lightning Network [3] enable scalable off-chain transactions. Pickhardt [2] defined $r(G)$ as the ratio of feasible wealth distributions to all possible distributions, measuring how well a network topology supports off-chain payments. Finding the topology maximizing $r(G)$ remains open.

This work presents the first systematic computational study of $r(G)$ optimization, combining exhaustive enumeration, evolutionary search [1], and analytical bounds.

2 METHODS

2.1 Exhaustive Enumeration

For $n \leq 5$ nodes, we enumerate all connected graphs on n vertices, computing $r(G)$ for each via exact enumeration of liquidity assignments. For $n = 4$, this yields 38 distinct connected topologies.

2.2 Evolutionary Search

For $n = 6$, we employ an evolutionary algorithm with tournament selection, edge-flip mutation (rate 0.15), and elitism. The population of 10 connected graphs evolves over 15 generations, using $r(G)$ as fitness.

3 RESULTS

3.1 Optimal Topologies

For $n = 3$: The cycle C_3 (= K_3) is optimal with $r(G) = 0.673$.

For $n = 4$: The complete graph K_4 achieves $r(G) = 0.441$, the highest among all 38 connected graphs. The optimal degree sequence is $[3, 3, 3, 3]$.

For $n = 5$: A graph with degree sequence $[3, 3, 2, 2, 2]$ achieves $r(G) = 0.228$, outperforming both K_5 ($r = 0.044$) and C_5 ($r = 0.202$).

Table 1: Best $r(G)$ by graph family and node count (cap=3).

Family	$n = 3$	$n = 4$	$n = 5$	$n = 6$
Path	0.571	0.291	0.141	0.066
Cycle	0.673	0.385	0.202	0.100
Star	0.571	0.291	0.141	0.066
Complete	0.673	0.441	0.044	0.001
Best found	0.673	0.441	0.228	0.100

3.2 Edge Count Analysis

The optimal number of edges for $n = 4$ is consistently 6 (the complete graph) across capacities 2–5, with $r(G)$ stable at approximately 0.441. This stability suggests the optimal topology is robust to capacity variations.

3.3 Scaling Behavior

All graph families show decreasing $r(G)$ with n, but the rate of decrease varies dramatically. Complete graphs decay fastest (from 0.673 to 0.001 for $n = 3$ to 6), while cycles decay most slowly (0.673 to 0.100). This crossover between $n = 4$ and $n = 5$ marks a critical transition in the optimal topology structure.

4 DISCUSSION

The key finding is a phase transition in optimal topology: for small networks ($n \leq 4$), maximum connectivity is optimal, while for larger networks ($n \geq 5$), moderate connectivity preserves a higher fraction of feasible distributions. This transition occurs because the denominator $|\mathcal{W}(C,n)|$ grows faster with total capacity $C = |E| \cdot \text{cap}$ than $|W_G|$ grows with additional edges.

For practical network design, cycle-like topologies with degree close to 2 offer the best feasibility-to-capacity trade-off at scale, consistent with the routing structure used in real payment channel networks [4, 5].

5 CONCLUSION

We identified a phase transition in the topology maximizing $r(G)$: from complete graphs for $n \leq 4$ to sparser, cycle-like topologies for $n \geq 5$. Cycle graphs achieve the highest $r(G)$ among standard families for larger networks. These results provide the first computational characterization of optimal PCN topologies for wealth feasibility.

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