Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich

Institute for
Dynamic Systems and Control
IDSC
Institut für Dynamische Systeme
und Regelungstechnik

Thomas Andrews

Assistive Go-kart Racing

Assistive Torque Controllers using Model Predictive Control

Semester Project

Institute for Dynamic Systems and Control
Swiss Federal Institute of Technology (ETH) Zurich

Supervision

Mentor Enrico Mion
Scientific Supervisor Andrea Censi
Prof. Dr. Emilio Frazzoli

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Abstract

Assistive Controllers offers the opportunity for drivers to learn and improve as they race, with assistive torques correcting steering mistakes as they occur. This setup offers the driver rapid feedback and insight into the correct racing line for different types of corners. A model predictive controller is derived using a dynamic model of the go-kart and steering column that can assist the human in driving, the dynamic model letting the controller predict and manage tire slip, resulting in controlled drifts that increase performance when taking corners at speed. A set of fully autonomous controllers are also implemented, mimicking a range of driving abilities, with the fastest outperforming human drivers. These controllers allow the driver to experience high performance driving as a passenger and help with the identification of where improvement is needed in their driving.

Keywords: Autonomous Racing, Model Predictive Control, Collaborative Driving .

Contents

Chapter 1

Introduction

1.1 Autonomous Cars

The last decade has seen massive progress in the field of Autonomous vehicles, particularly driver-less cars. The number of publications on the topic of autonomous driving research has tripled since 2010 [1], on the industry side Waymo has been steadily expanding its driver-less taxi service since 2016 [2] and on the manufacturers side, functionally autonomous cars are currently available for purchase, with thousands of Telsa vehicles on the roads already equiped with Autopilot, and other autonomous functionality[3]. It is no longer a question of if there will be fully autonomous cars sharing the roads with us but when, and this change is likely to have a huge impact on peoples relationships with their vehicles.

1.2 Assistive driving

Even more prevelent than full Autonomy are Assistive driving technology's, systems such as adaptive cruise control have been around for decades[4]. Lane keeping assist has been steadily introduced by manufacturers since 2015, and it would be difficult to by a new car that didn't feature some for of anti-lock breaking and traction control. These systems don't drive for the driver, instead they help the driver avoid difficult or dangerous situations.

Figure 1.1: The Jaguar XK8, which featured radar based adaptive cruise control in 1998 [5]

With the advent of fully autonomous vehicles new forms of assistive driving will become possible, including systems that supervise and teach drivers, ready to correct small mistakes, or in the event of an emergency, seize control and avoid or reduce the impact of an accident. Many countries require learner drivers to have an experienced driver when they start learning to drive, and this can be made both easier and more effective with the introduction of supervisory assistive driving systems.

1.3 Go-kart Racing

Go-kart racing is a highly competitive sport that is seen as one of the main pathways into Formula 1 and other competitive motor-sport scenes. Go-karting is also a popular recreational activity around the world with both children and adults. The structure of go-kart racing , with its fixed track, fairly uniform driving surface, and limited external agents makes it a good test bed for Autonomous systems, without the full complexity that's introduced driving on real streets. Go-karting also provides the opportunity to gather data on maneuvers such as drifting and light collisions with minimal risks, something not possible with cars.

Figure 1.2: FIA E-Karting Challenge [6]

1.4 Overview

This project looks at the development of an assistive controller for go-kart racing as an extension of the existing autonomous go-kart that has been built by the Frazzoli group, part of the Institute for Dynamic Systems and Control at ETH Zürich. The current Autonomous controller uses model predictive control and is able to match or beat the lap times of most casual drivers [7]. The aim of the proposed assistive controller is to be able to assist a driver in driving high speed laps, providing corrective torque to the steering, and continue to drive the course safely Ad infinitum if the driver releases the steering wheel completely. This project also aims to create a set of driving profiles for the autonomous controller that can be used to demonstrate the different ways a course can be driven, from as aggressively as possible allowing for occasionally brushing obstacles, to a safe restrained approach that keeps to the center of the track. These profiles would allow drivers to gain experience firsthand and help them identify areas where they need work.

Chapter 2

Existing Setup

2.1 Go-kart Hardware

Figure 2.1: Autonomous go-kart setup at the Winterthur Lab

2.1.1 Actuation

The gokart shown in Figure 2.1 is currently setup with a ‘Fly-by-Wire’ throttle [8]. In manual driving the pedal position is passed to embedded controllers that control the motor torques. However this setup also allows the throttle commands to be interpreted or ignored by the controller when driving in an autonomous manner. The steering uses physical linkages so that the tire position is locked to the position of the steering wheel, with an actuator mounted on the steering column providing the torque from the controller. Braking controlled by a hydraulic system connected to the brake pedal, with autonomous braking achieved by a linear motor attached to the pedal that replicates the effect of human braking. This setup ensures that the driver can always break in an emergency situation by physically applying the brake.

2.1.2 Localization and Mapping

Localization of the go-kart is achieved using the Lidar mounted behind the driver and a set of IMUs recording the accelerations and rotations of the go-kart. This provides an accurate estimation of the go-kart pose and velocity to the controller at high frequency. Mapping of the track uses the Lidar to create an occupancy grid from which the track is calculated as a set of minimal b-splines to be passed to the controller[9].

2.2 Dynamic tricycle model

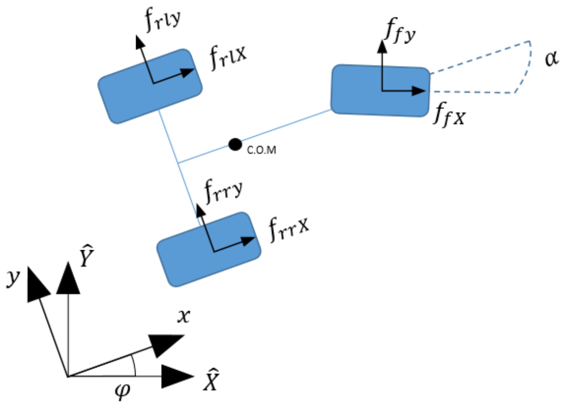

Figure 2.2: Illustration of the Dynamic Tricycle Model

To reduce the model complexity the go-kart is modeled as a single rigid body with a single front tire and a pair of rear tires. The motion of the go-kart is then calculated from a force balance of the inertial and grip forces, the tricycle model has an advantage over a bicycle model due to its inclusion the effects of Torque Vectoring for rear wheels to assist in turning. In this formulation the motion is controlled by three main inputs: the Steering angle (β), the Acceleration/Braking from the rear wheels (AB) and the difference between the rear tire accelerations (TV). This is equivalent to considering the control inputs $\left[\beta, \hat{f}_l, \hat{f}_r\right]$ with the desired left and right lateral accelerations equal to $AB + TV$ and $= AB - TV$ respectively.

2.3 Controller Architecture

The Autonomous controller shown in Figure 2.3 features a high level Model predictive controller that calculates the desired steering angle, braking actuation and motor accelerations. The high gain PID controller then determines the torque to be applied to the steering column. This setup allows the torque to be adjusted at the high frequency of the PID to ensue that the steering angle is accurately tracked[10].

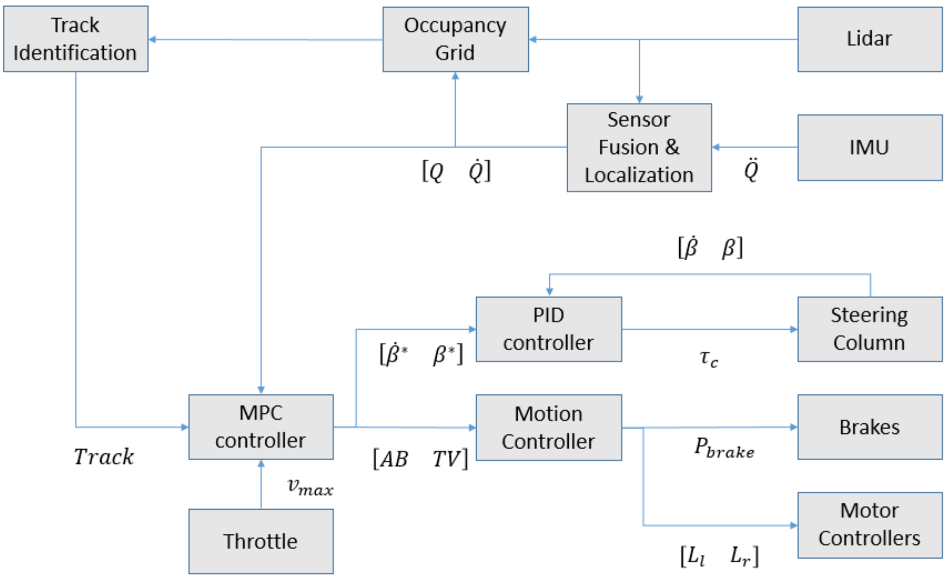

Figure 2.3: Autonomous go-kart control architecture

Chapter 3

Model Extension

To allow for a controller that allows collaborative control of the steering a number of extensions needed to be made to the existing model. This included creating an accurate representation of the steering column dynamics, identifying its parameter experimentally, and restructuring the controller architecture..

3.1 Steering Column Model

The steering column along with the mechanical linkage to the tires can modeled as a mass, spring, damper system [11]. The system is then described by the following differential equation 3.1 with β the steering column angle, $j, b, and\ k$ the inertia, damping and stiffness respectively.

$$J\ddot{\beta} = -b\dot{\beta} - k\beta + \tau_d + \tau_c \tag{3.1}$$

The torque applied by the driver τ_d is observable to the controller and can either be treated as a disturbance to be rejected, as happens in fully autonomous driving, or as a control input to be supplemented by the controller torque τ_c. This can be transformed into the standard state space shown in representation with $\boldsymbol{X}_\beta = \left[\begin{smallmatrix}\beta \\ \dot{\beta}\end{smallmatrix}\right]$

$$\dot{\boldsymbol{X}}_\beta = \begin{bmatrix} 0 & 1 \\ -\frac{k}{j} & -\frac{b}{j} \end{bmatrix} \boldsymbol{X}_\beta + \begin{bmatrix} 0 & 0 \\ \frac{1}{j} & \frac{1}{j} \end{bmatrix} \begin{bmatrix} \tau_d \\ \tau_c \end{bmatrix} = f_\beta(\boldsymbol{X}_\beta, \tau_d, \tau_c) \tag{3.2}$$

3.1.1 Extended state variables

The Extended MPC controller requires a change to the control and state vectors, replacing 3.3 with 3.5, and 3.4 with 3.6. This results in $\dot{\beta}$ shifting from control input to state vector and the addition of $\dot{\tau}_d$ and τ_d to the Control input and state vector respectively.

$$\boldsymbol{X}_{old} = \begin{bmatrix} \bar{x} \\ \bar{y} \\ \phi \\ v_x \\ v_y \\ \dot{\phi} \\ \beta \end{bmatrix} \tag{3.3}$$

$$\boldsymbol{U}_{old} = \begin{bmatrix} \dot{\beta} \\ AB \\ TV \end{bmatrix} \tag{3.4}$$

$$\boldsymbol{X}_{new} = \begin{bmatrix} \bar{x} \\ \bar{y} \\ \phi \\ v_x \\ v_y \\ \dot{\phi} \\ \beta \\ \dot{\beta} \\ \tau_d \end{bmatrix} \tag{3.5}$$

$$\boldsymbol{U}_{new} = \begin{bmatrix} \dot{\tau}_d \\ AB \\ TV \end{bmatrix} \tag{3.6}$$

3.1.2 Extended model dynamics

The additon of the steering column also requires an extension of the model dynamics. Previously the system dynamics were given by 3.7, with the full derivation of f_{dyn} coming from the dynamic tricycle model [9].

$$\dot{\boldsymbol{X}}_{old}(t) = \begin{bmatrix} f_{dyn}(X(t), U(t)) \\ \dot{\beta} \end{bmatrix} \tag{3.7}$$

The extended dynamics introduce f_β from 3.2 , while f_{dyn} remains unchanged giving:

$$\dot{\boldsymbol{X}}_{new}(t) = \begin{bmatrix} f_{dyn}(X(t), U(t)) \\ f_\beta(X(t), U(t)) \\ \dot{\tau}_d \end{bmatrix} = f_{ext}(X(t), U(t)) \tag{3.8}$$

3.2 System Identification

The extended system dynamics introduces three new go-kart dependant parameters j_{sc}, b_{sc}, and k_{sc} which represent the effective inertia, damping, and stiffness of the steering assembly. these could be derived from first principles however due to the complex nature of the assembly, System Identification techniques have been used to obtain estimate values. A range of experiments were conducted to gather data for the identification process, these included a setup with the front wheels raised above the ground, a setup with the go-kart stationary, and data from the go-kart driving autonomously. A preliminary analysis showed that between these experiments there was a significant difference in the relationship between Steering Angle and Applied Torque, due to the interactions between the tire, ground, and steering linkages. This lead to the decision to use the autonomous driving data for system identification as this is closest to the situation in which the model will be applied. A section of the data used is shown in figure 3.1. The units that have been used are the go-kart specific units, Steering Column Torque (SCT) and Steering Column Encoder (SCE), which are the units returned by the equipment installed on the go-kart.

Parameter estimation was performed by performing Subspace Identification to fit a linear 2nd order model to the data[12], which was then used as the base for a Hammerstein-Wiener model with output saturation. The Hammerstein-Wiener model was restricted to the general form given in equation 3.1, so that the parameters can be read directly from the resulting model. The parameter estimates are shown in table 3.1.

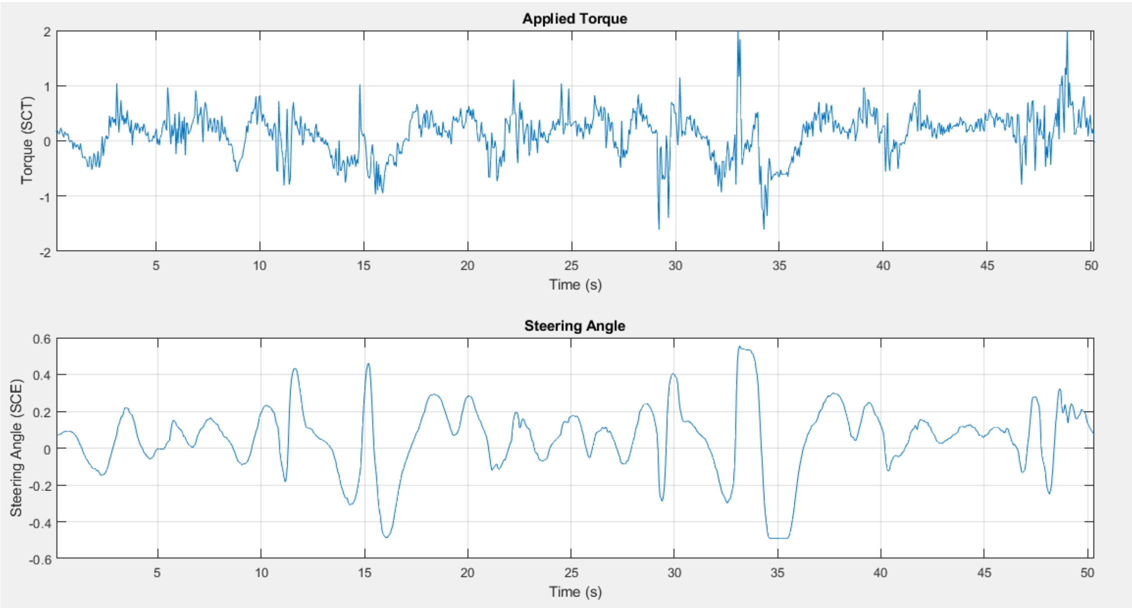

Figure 3.1: Torque v.s. Steering angle for an Autonomous lap

Table 3.1: Estimated Parameters.

parameter	**value**
Inertia	2.0000
Damping	-0.8906
Stiffness	0.3997

3.3 Updated Controller Architecture

The Assistive controller removes the PID controller that was previously used to calculate the required torque, instead extending the MPC controller to output the torque to be applied directly, this allows the MPC to have more direct control over the torque. This comes at the cost of losing the high speed tracking of desired steering, which was possible through the large torques the PID commanded, which could injure a human driver in an assistive driving situation.

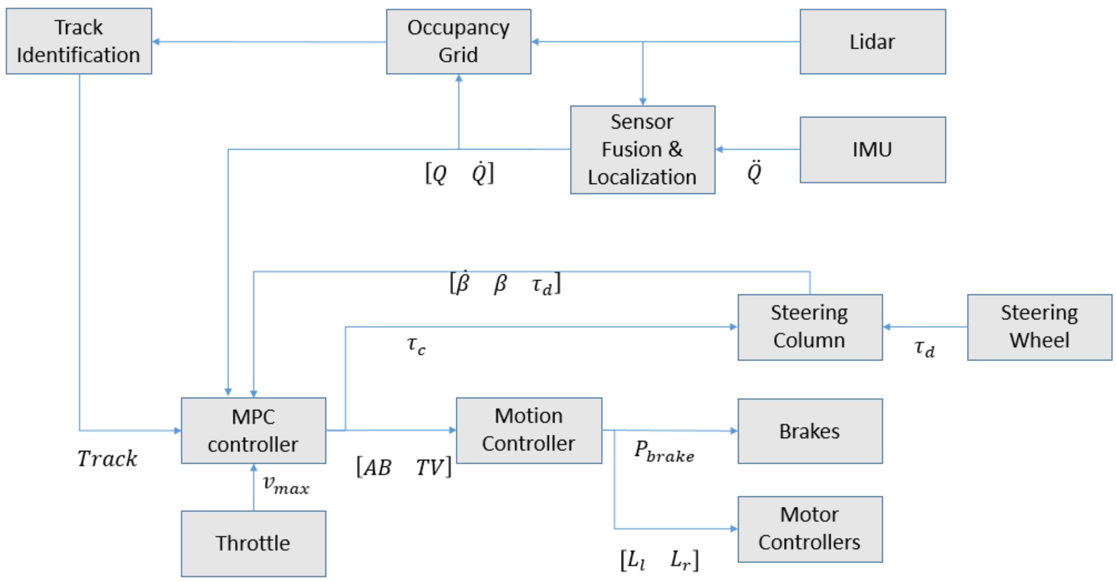

Figure 3.2: Assistive go-kart control architecture

Chapter 4

Implementation

4.1 Torque Mode

The new extended dynamics can be brought into the form of an Model Predict Control Optimisation Problem by introducing the cost function $J(X, U, \lambda)$, which contains quadratic costs on acceleration, breaking and other performance metrics as shown in Equation 4.1 along with a reward on progress which is given by $s_f(X)$ which is a projection of the position onto the center line of the track. λ_v and λ_s are slack variables that are zero except when the to go-kart is outside the bounds of the track or over its speed limit.

$$J(X,U,\lambda) = r_p S_f(X) - \begin{bmatrix} \dot{\beta} \\ \dot{\tau} \\ AB \\ TV \\ e_{lat} \\ \lambda_v \\ \lambda_s \end{bmatrix}^T \begin{bmatrix} r_\beta & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & r_\tau & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & r_{AB} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & r_{TV} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & r_{lat} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & r_v & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & r_s \end{bmatrix} \begin{bmatrix} \dot{\beta} \\ \dot{\tau} \\ AB \\ TV \\ e_{lat} \\ \lambda_v \\ \lambda_s \end{bmatrix} \tag{4.1}$$

The Model Predict Control optimisation problem also requires the introduction of a set of constraints to reflect both the physical constrains of the system and the requirements introduced by the race setup. Equation 4.2 represents the system dynamics while equations 4.3 and 4.4 deal with the traction limits of the tires. Equation 4.5 ensures that the go-kart remains on the track and Equation 4.6 enforces the maximum speed and no reversing requirements. The cost function combined with these constraints gives the problem formulation for the optimal control sequence U as shown in equation 4.7. $\mathcal{U}$ is the set feasible control sequences, which ensures the control actuation remains within the safety limits. Note that for clarity inequalities that must be enforced at all time steps have had the time indices dropped (e.g. $v_x \geqslant 0$ instead of $v_x(t) \geqslant 0,\ \forall\ t\epsilon[1,..,N]$).

$$\begin{aligned} X(0) &= X_0 \\ \dot{X}(t) &= f_{ext}(X(t), U(t)) \end{aligned} \tag{4.2}$$

$$\begin{aligned} AB + TV &\leqslant f_{max}(v_x) \\ AB - TV &\leqslant f_{max}(v_x) \end{aligned} \tag{4.3}$$

$$\begin{aligned} (v_x^2 + v_y^2) \cdot (AB + TV) - v_x^2 k_p &\leqslant 1 \\ (v_x^2 + v_y^2) \cdot (AB - TV) - v_x^2 k_p &\leqslant 1 \end{aligned} \tag{4.4}$$

$$\begin{aligned} e_{lat} - \lambda_s &\leqslant W_{track}(x,y) \\ e_{lat} + \lambda_s &\geqslant -W_{track}(x,y) \end{aligned} \tag{4.5}$$

$$\begin{aligned} v_x - \lambda_v &\leqslant v_{max} \\ v_x &\geqslant 0 \end{aligned} \tag{4.6}$$

$$\begin{aligned} U = &\operatorname*{argmax}_{u \epsilon \mathcal{U}} \sum_{t=1}^{N} J(X(t), u(t), \lambda(t)) \\ s.t.& \\ & X(0) = X_0 \\ & \dot{X}(t) = f_{ext}(X(t), U(t)) \\ & AB + TV \leqslant f_{max}(v_x) \\ & AB - TV \leqslant f_{max}(v_x) \\ & (v_x^2 + v_y^2) \cdot (AB + TV) - v_x^2 k_p \leqslant 1 \\ & (v_x^2 + v_y^2) \cdot (AB - TV) - v_x^2 k_p \leqslant 1 \\ & e_{lat} - \lambda_s \leqslant W_{track} \\ & e_{lat} + \lambda_s \geqslant -W_{track} \\ & v_x - \lambda_v \leqslant v_{max} \\ & v_x \geqslant 0 \end{aligned} \tag{4.7}$$

4.2 Driver Profiles

To create a system capable of teaching and demonstrating driving with various skill levels, the controller has been adapted to have distinct driver profiles; beginner, moderate, and advanced. The difference in the profile is achieved by varying the weights in the cost functions. For the beginner profile the maximum speed is reduced and acceleration and deviation from the center-line carry higher costs, this is to create a more relaxed and safe experience, ideal for a driver's first introduction to a go-kart. The advanced controller by contrast is much more aggressive with weighting placed toward making the most progress possible at the expense of a smoother ride. Table 4.1 shows the full list of weights for the 3 profiles in autonomous along with assistive mode.

Table 4.1: Profile weights.

Parameter	**Beginner**	**Moderate**	**Advanced**	**Assistive**
Maximum Speed	5 $\frac{m}{s}$	7 $\frac{m}{s}$	9 $\frac{m}{s}$	8 $\frac{m}{s}$
Speeding cost	0.04	0.04	0.005	0.04
Lateral error cost	0.03	0.01	0.015	0.03
Progress reward	0.15	0.15	0.2	0.15
Acceleration cost	0.001	6e-4	4e-4	6e-4
Torque Vectoring cost	0.01	0.005	0.0075	0.005
Collision cost	10	8	7	10
Steering angle cost	0.2	0.2	0.2	0.1
Steering torque cost	~	~	~	0.01

4.3 Testing Setup

The track has been set up to be challenging to navigate at high speed and contains a variety of different corner types, Figure 4.2 shows the track as recorded by the go-karts on-board LIDAR,

(a) High Speed Corner (b) Triple Hairpin Section

Figure 4.1: Images of the test track showing different sections

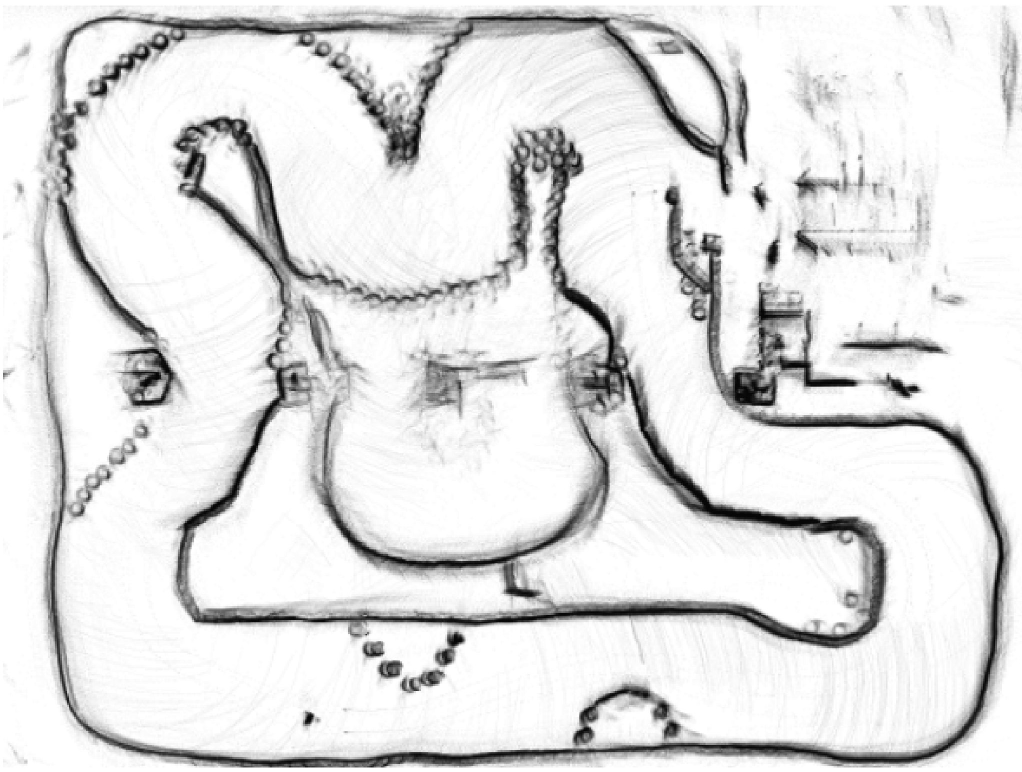

Figure 4.2: Lidar Visualisation of the track

while Figure 4.1 shows two of the main features: The high speed corner at the end of the back straight which is designed to test breaking point selection, and the triple hairpin, which requires linking drifts between the corners to navigate quickly. Both of these features caused frequent loss of control event when driven manually and during the initial testing of the layout by a member of the lab group. No-one was able to avoid sliding into the walls while completing practice laps.

Chapter 5

Results

5.1 Driving Modes

The different driving modes produces noticeably different driving styles. The beginner mode never exceeds $5Ms^-1$ and takes corners with plenty of space on the inside. The moderate controller follows nearly the same line as the advanced controller, however at reduced speed. The paths of the three modes can be seen in Figure 5.1. The lap times show steady improvement moving from beginner mode to advanced, with the best manually driven lap times of the lab falling between the moderate and advanced times as shown in Table 5.1. The manual laps were completed by the best driver of the group, after having already had a set of practice laps to familiarise themselves with the track.

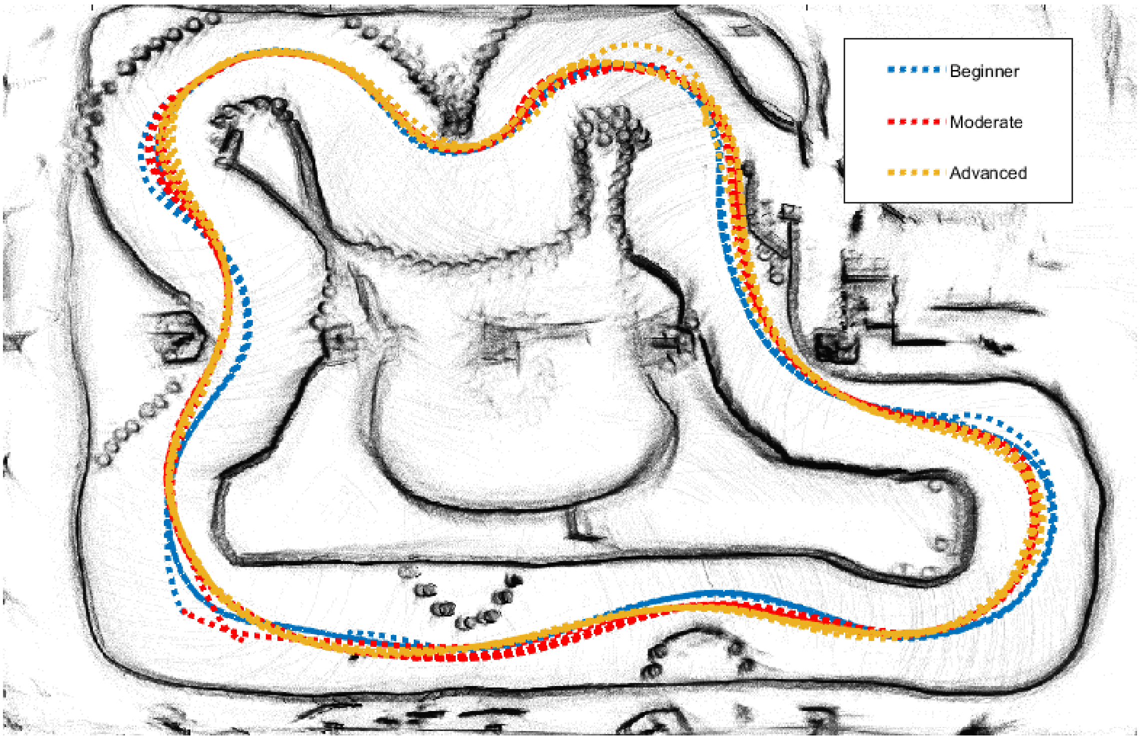

Figure 5.1: Paths of the different Driver Modes

Table 5.1: Lap Times.

Mode	**min**	**mean**	**max**
Beginner	21.2	22.4	23.6
Moderate	17.6	18.1	18.8
Advanced	16.5	16.8	17.1
Manual	17.0	17.5	18.2

Speeds of manually driven laps are compared to laps driven autonomously in advanced mode are shown in figure 5.2.This figure shows that the human driver had higher top speeds, however was also less consistent, especially when approaching the high speed corner (see Figure 4.1 a). The human driver regularly dropped from over 10 m/s to around 4 or 5 m/s, while the autonomous controller carried slightly less speed into the corner but would exit at speeds around 6 m/s. This loss of speed is for the human driver is caused in part by taking the corner too fast and too wide, as can be seen in Figure 5.3. This figure displays the paths of the manual and advanced autonomous laps.

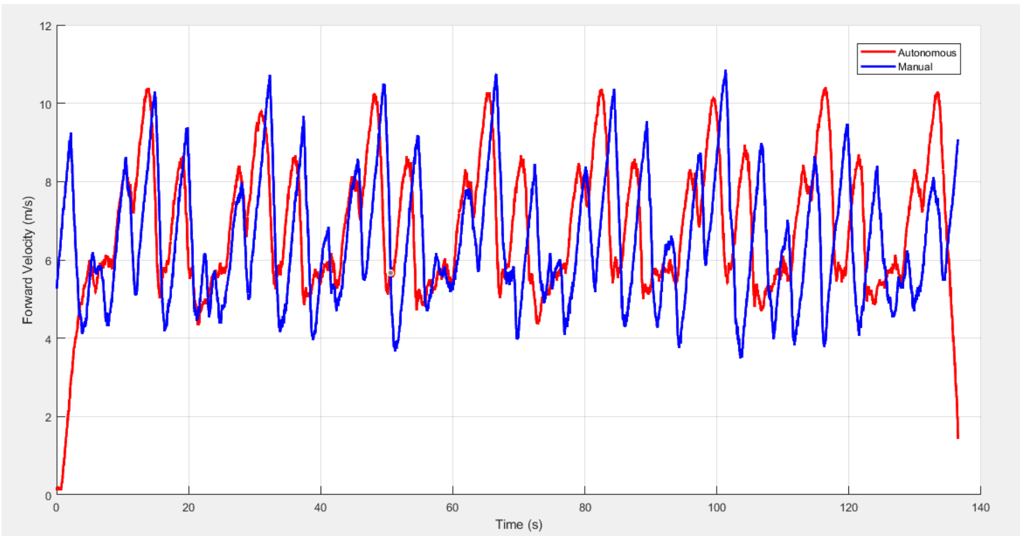

Figure 5.2: Forward velocity of manual and autonomous laps

5.2 Collaborative Driving

5.2.1 Driving lines

The advantage of assistive driving is that it can help a driver maintain a suitable driving line. Figure 5.4 shows the 3 sets of paths. For the first set driver was told to drive around the track as quickly as they could. The resulting manually driven laps have far more variance, in particular the high speed corner often has the driver going wide. In comparison, both the Autonomous and Assistive modes follow a much tighter more regular driving line.

5.2.2 Lap Times

The Assistive mode produced consistent lap times between 19.7 and 20.2 seconds. Figure 5.5 shows how lap times compare to the lap times of the manual and autonomous modes. The Assistive driving mode is slower than both the manual laps and two of the three autonomous modes. This matches the intuition of driving the go-kart in the assistive mode, where the driver doesn't directly control

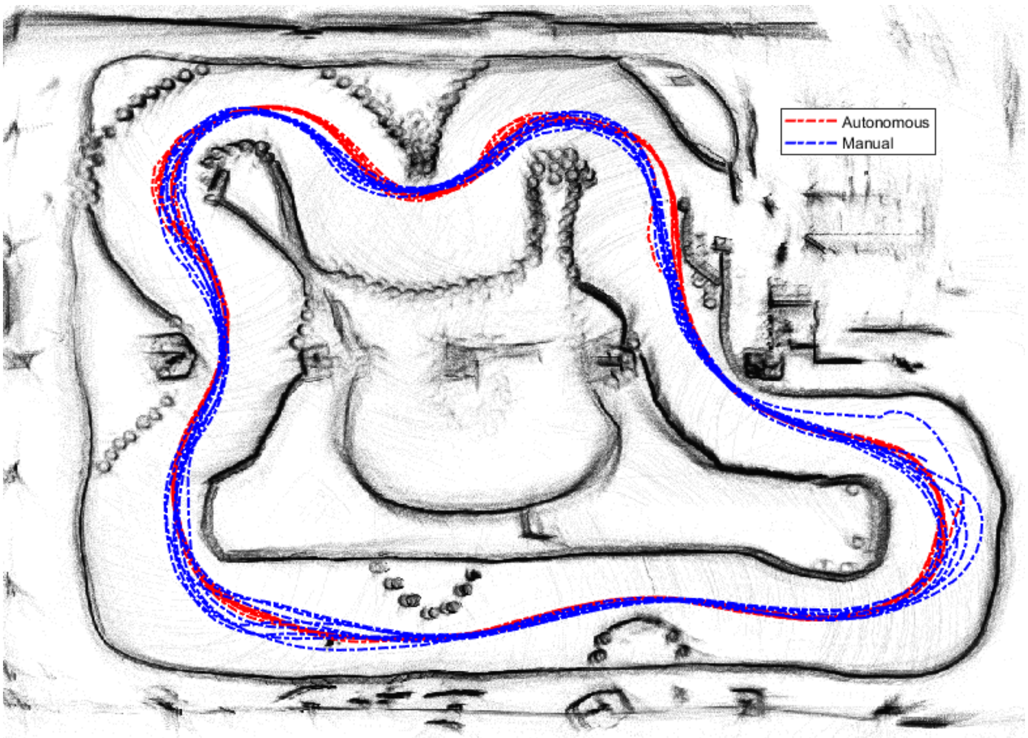

Figure 5.3: Paths of Manual and Advanced Autonomous Laps

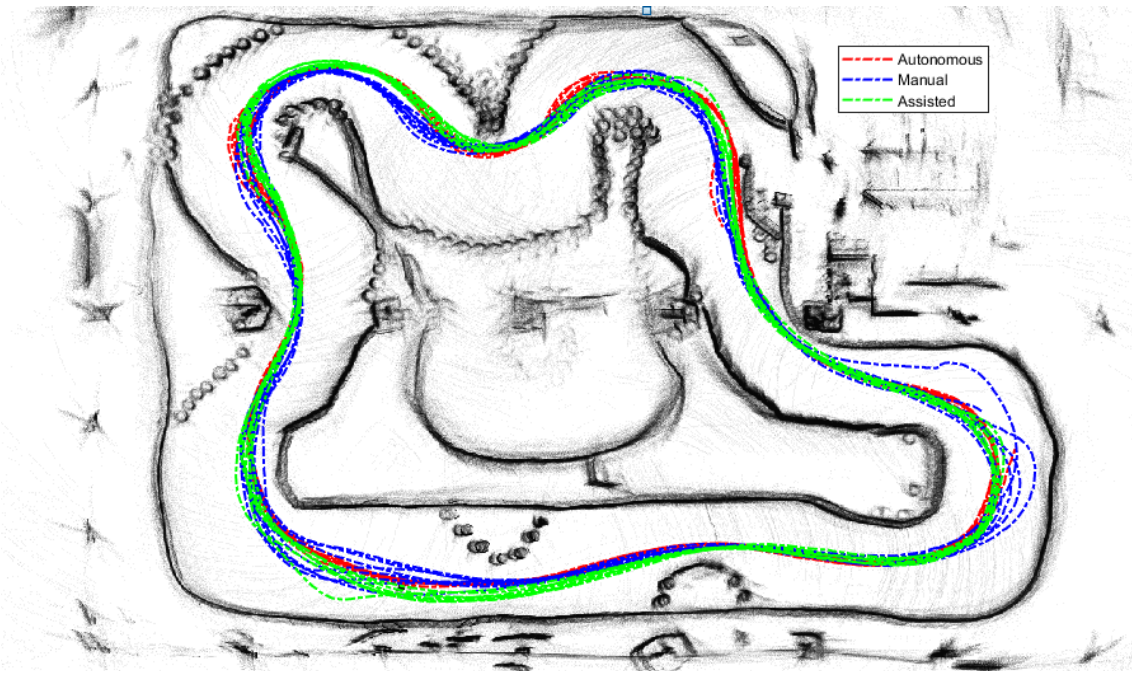

Figure 5.4: Manual v.s. Assisted Paths

the throttle or braking and the go-kart maintains a suitable speed based on the current driving line. Splitting the responsibility of driving between the two agents: driver and controller reduces the certainty of both about future control actions, which results in a more conservative speed.

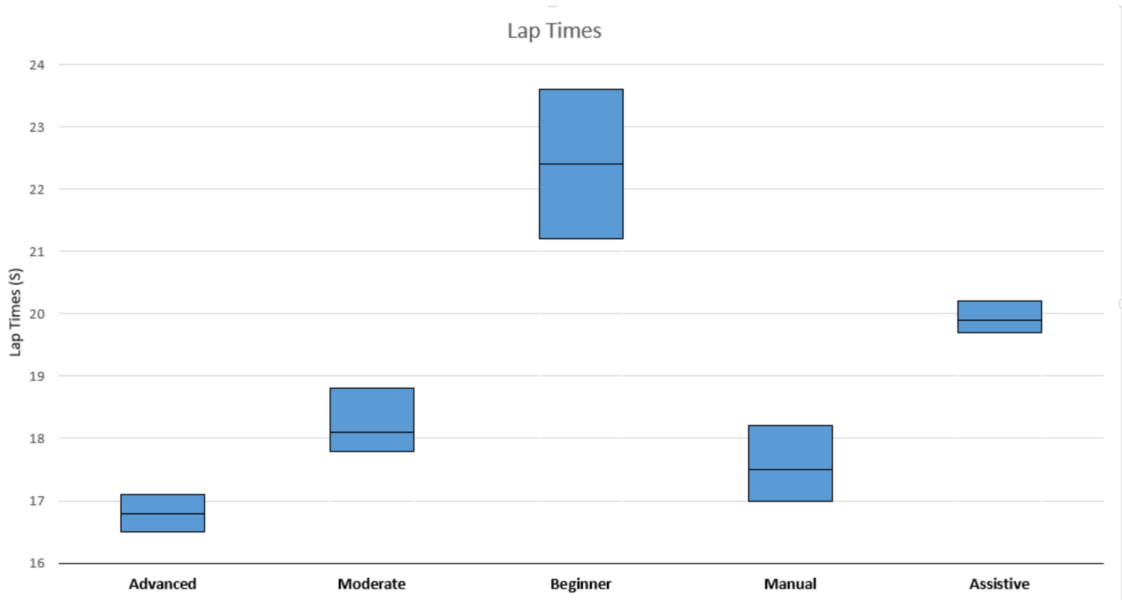

Figure 5.5: Plot of lap times between controllers

5.3 Assistive Controller Limitations

The loss of time for the Assistive mode comes from the fact that in its current iteration the model formulation requires the controller to be able to maintain control of the go-kart without assuming any additional assistance from the driver. This is coupled with the limits placed on torque in order to prevent discomfort to the driver. These requirements constrain the set of feasible trajectories compared with the fully autonomous MPC problem, where the controller could simply demand a steering angle which would then be achieved by the PID controller, which ultimately could apply up to 30N/m of torque to the steering column while in autonomous mode. This amount of torque makes difficult and unpleasant to hold onto the wheel.

The problem of reduced speed could be addressed by introducing a model of the driver into the MPC problem, which would assume a level of competency from the driver in tackling the required steering angle. Adding a driver model allows the controller to drive at speeds higher than it could achieve with the current setup.

Chapter 6

Conclusion

This project has implemented an assistive torque controller. The controller uses Model Predictive Control to find the appropriate torque and motor accelerations to complete a course with or without the assistance of a human driver. Additionally, this project has led to the creation of a set of Driver Profiles that allow real time modification of the go-kart's behavior through an on-board GUI. The most aggressive of these profiles is capable of out-driving members of the lab group consistently on a track that requires controlled loss of traction through several corners and caused frequent crashes and spins for human drivers.

From a teaching perspective, the introduced driver profiles provide a powerful learning tool. Drivers wishing to improve their lap times can experience first hand how a difficult corner should be taken, and can get the most accurate experience of it possible; how close the walls should be, when they should start breaking, how sharp it should feel, and how the go-kart's behaviour changes as it goes into a controlled slide. All of this can occur without even requiring that the driver knows how to drive. The Assistive controller can then provide a safe environment to test their own skills. The assistive torque will guide them back to the racing line, or even take over if the wheel is released in a moment of excitement.

Finally, the Assistive controller could further be improved by the addition of a driver model, which would allow it to predict or anticipate the driver actions. In turn this would lead to higher speeds and faster lap times, further improving the controller as a teaching tool. Additional Driver profiles could also be developed to provide further exams of what to-do and not to-do while driving. Examples of this could include a profile with a tendency to overuse the breaks leading to unnecessary loss of traction in corners, or a short-sighted mode, that ignores corners until the last second, leading to sharp entries and missing the apex of the turn.

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Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich

Institute for Dynamic Systems and Control

Prof. Dr. R. D'Andrea, Prof. Dr. E. Frazzoli, Prof. Dr. C. Onder, Prof. Dr. M. Zeilinger

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Supervision:

Mentor Enrico Mion
Scientific Supervisor Andrea Censi
Prof. Dr. Emilio Frazzoli

Student:

Name:	Thomas Andrews
E-mail:	muster@student.ethz.ch
Legi-Nr.:	18-940-866
Semester:	fall

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Zurich, 31.1.2020:

Thomas Andrews