Approx. Clothoids with Ext. Winding#s

by Joel Gächter, and Jan Hakenberg, 2019-09-03

Background

The original clothoid appoximation scheme by [2019 Reif] performs all computations in the complex plane ℂ. The resulting clothoids are invariant under addition of multiples of 2π to the angles of the control points. For example, the clothoid between the two points ($x = 0$, $y = 0$, $\alpha = 0$) and (1, 0, 0) is identical to the clothoid generated between (0, 0, 0) and (1, 0, 2π).

Contribution

We map the complex-valued operations of the original implementation to the SE(2) covering group. The new scheme in $\overline{\mathrm{SE}(2)}$ preserves winding numbers beyond the range of $(-\pi, \pi)$.

Details

The original scheme involves approximating integrals of complex-valued functions. The main challenge in extending the scheme is to track the winding number in the numeric integration. Our current implementation [2019 IDSC/Frazzoli] handles one additional winding compared to the original scheme. The examples below show that the curvature profile of the clothoid approximation is non-linear, and even non-monotonous for some configurations. Investigations and possible enhancements are subject to future work.

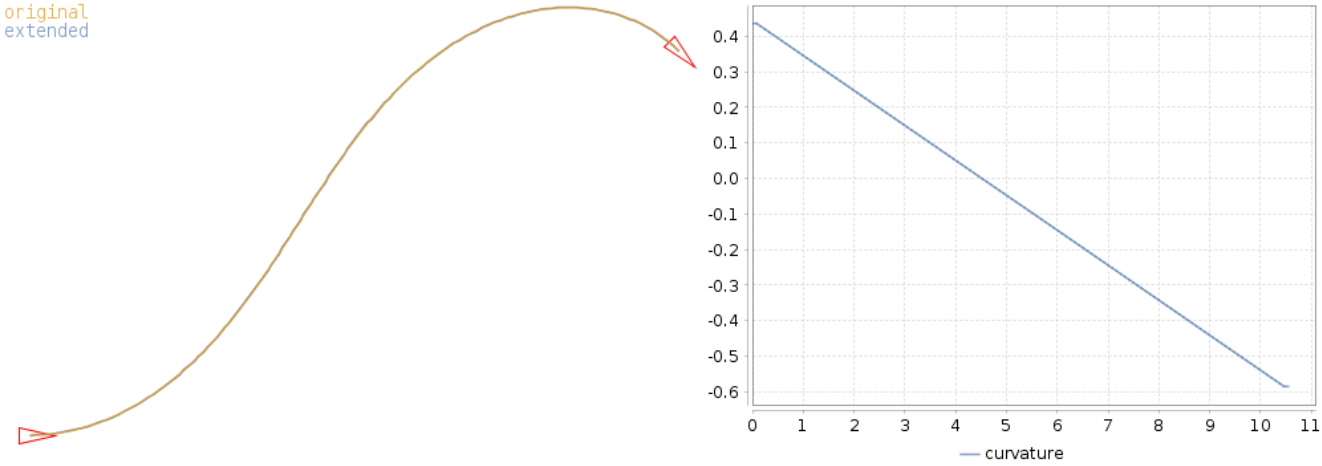
References

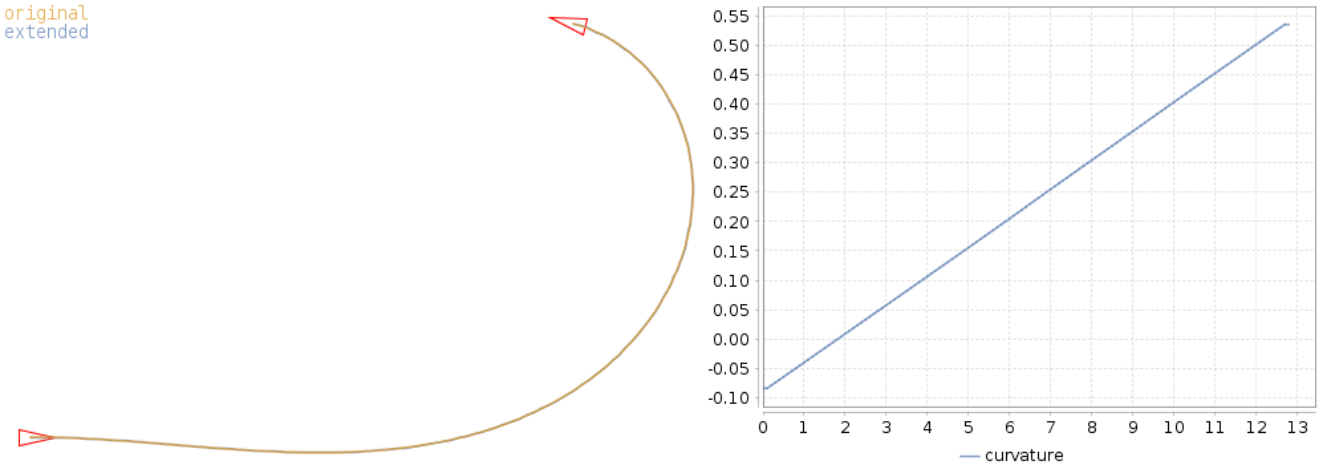
[2019 Reif] *Clothoids and Subdivision;* talk at ETH Zürich

[2019 IDSC/Frazzoli] https://github.com/idsc-frazzoli/owl

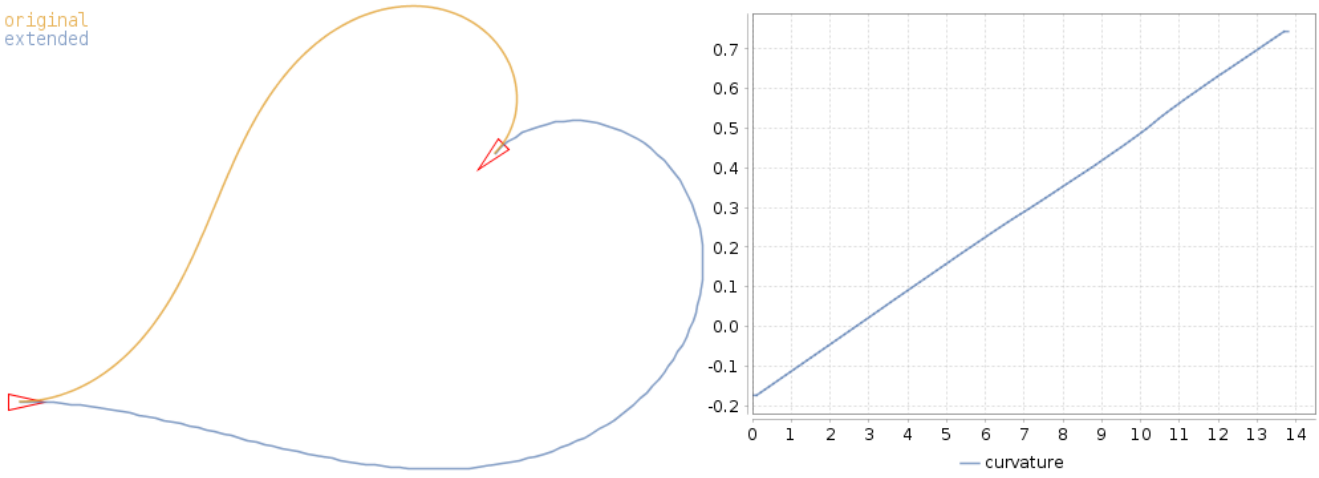
Examples

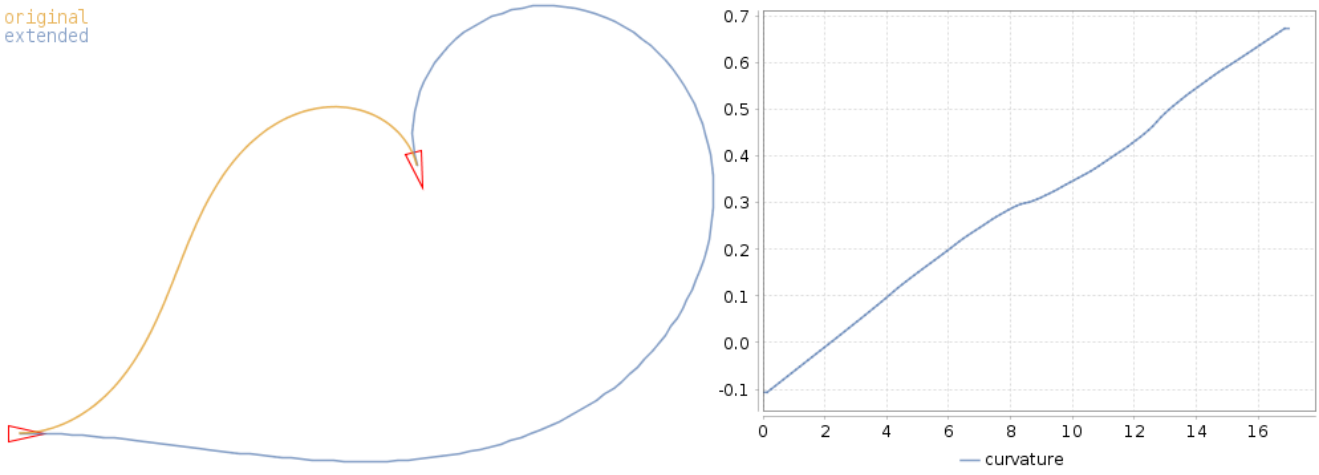
There is no difference between the original and extended schemes for angle differences in the range $(-\pi, \pi)$.

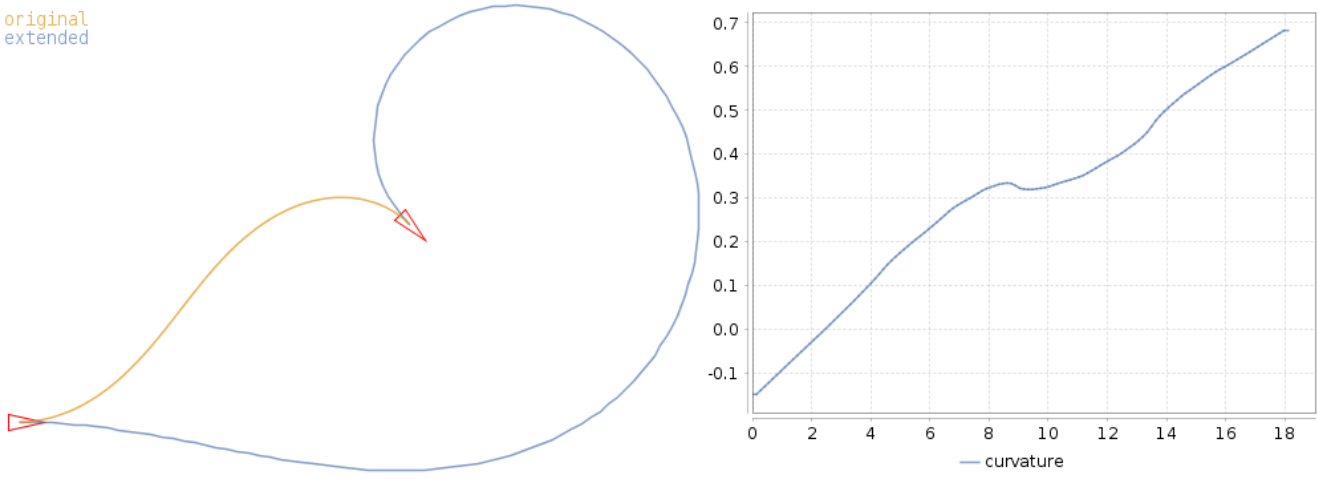


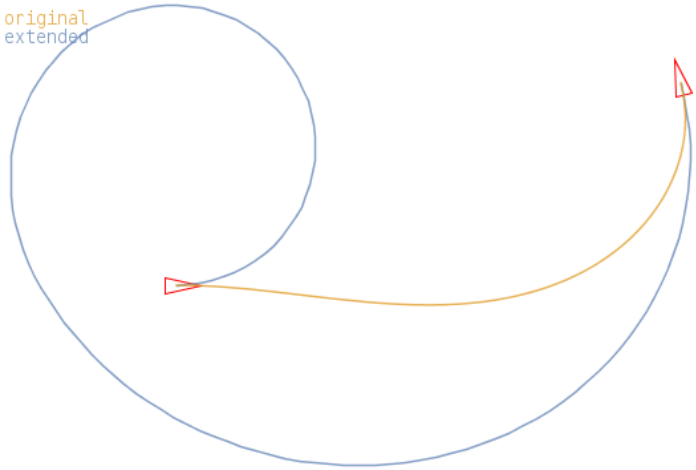


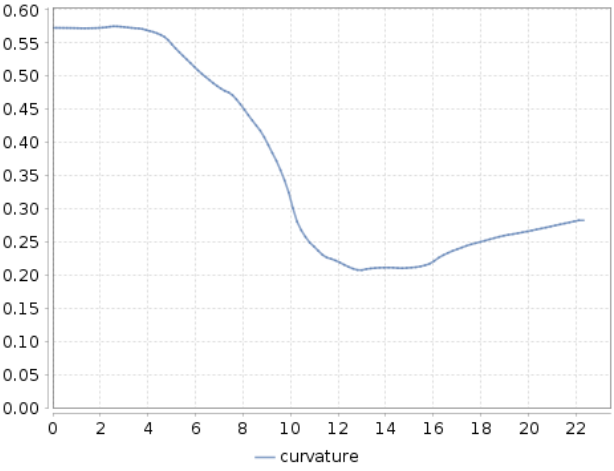
The examples below feature angle differences larger than π.








original
extended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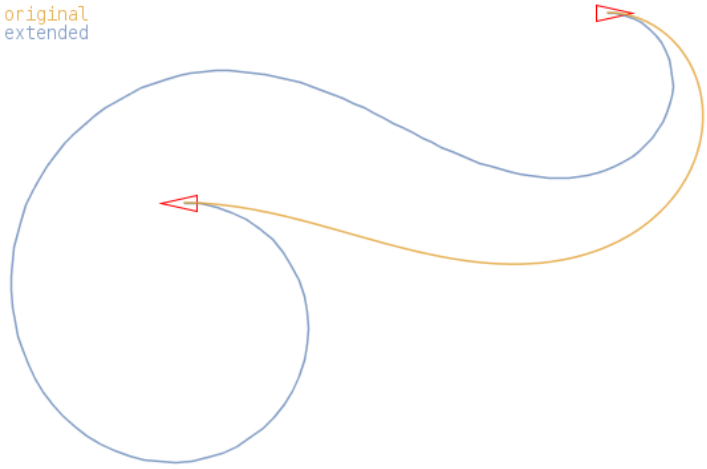
curvature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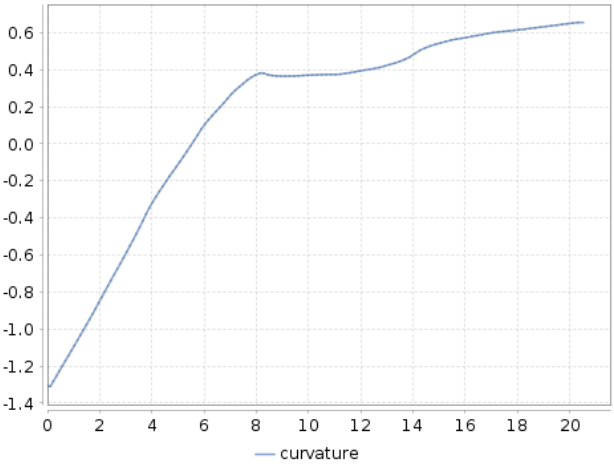
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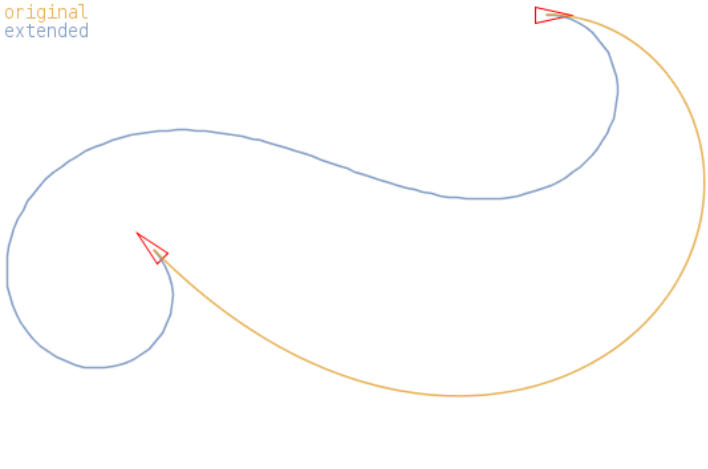
curvature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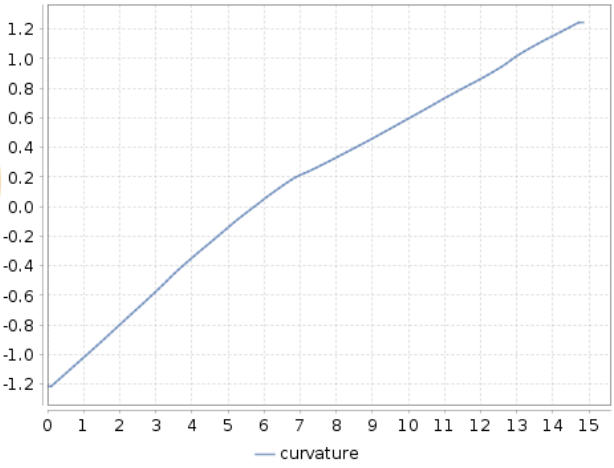
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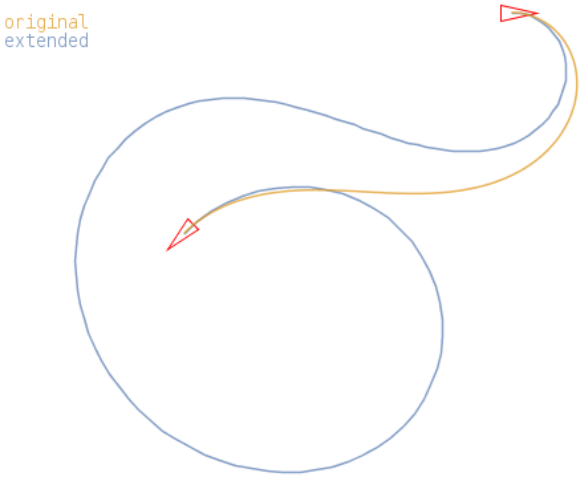
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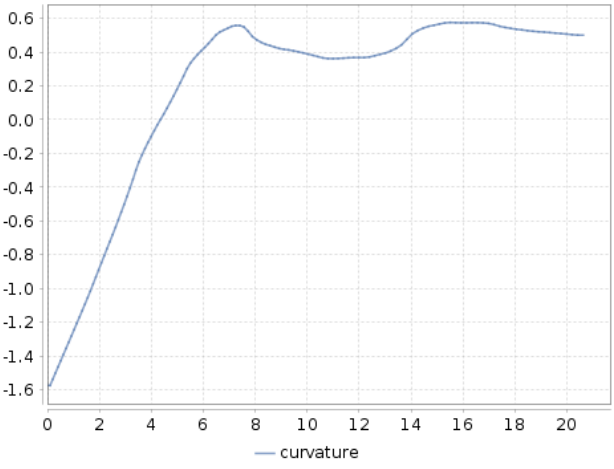
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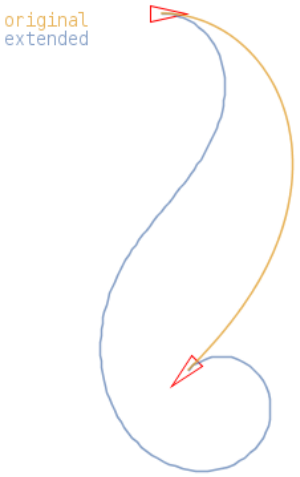
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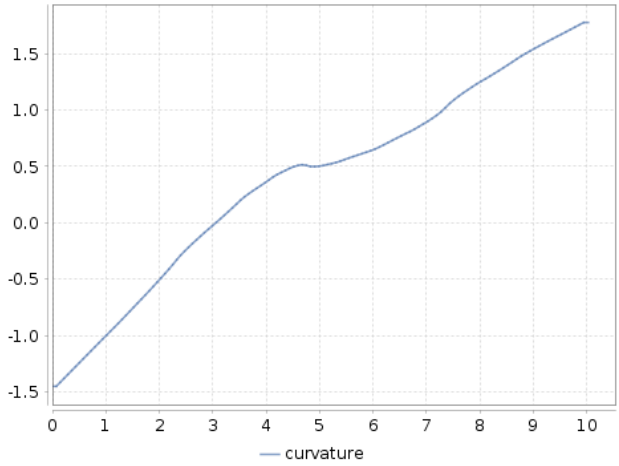
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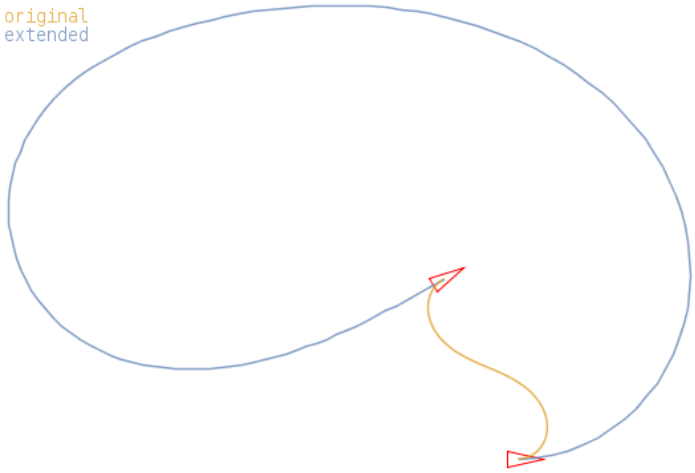
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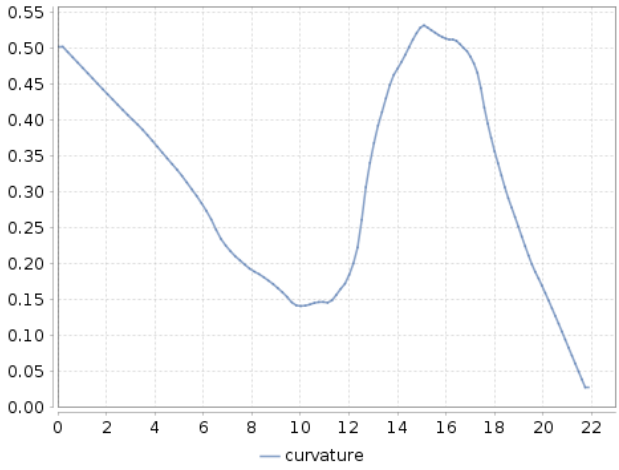
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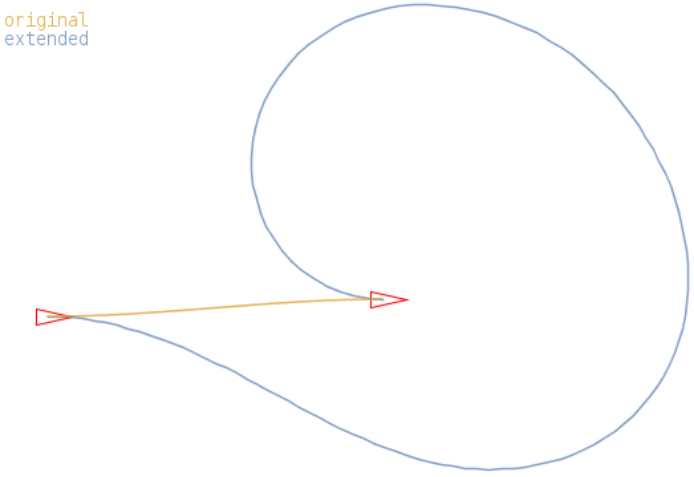
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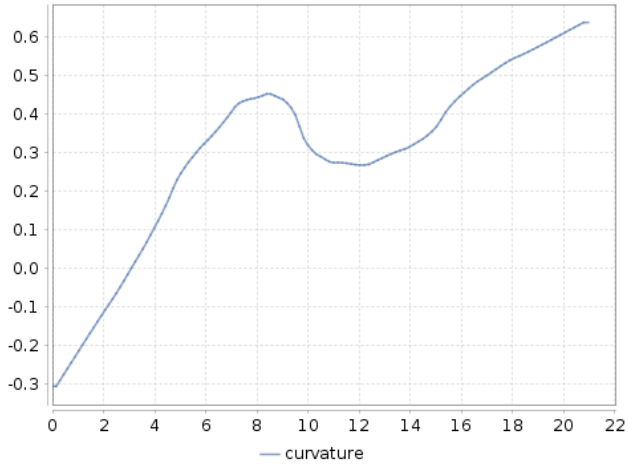
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curvature