

Application of Hermite Subdivision to SE(2)

document prepared by Jan Hakenberg, 2019-10-26

related documents: 20190621_applications_bi-invariant_mean

Background

Hermite subdivision on manifolds has been proposed by Moosmüller 2015. In her article *Hermite subdivision on manifolds via parallel transport*, 2016, the scheme H2[-1/5, 9/10] is applied to artificial data in SO(3). Until now, no publication has demonstrated how hermite subdivision performs on **real-world manifold-valued data**.

The position and velocity of the gokart is estimated at a **discrete rate**. The vehicle position is represented as an element (px, py, pa) in the 3-dimensional Lie-group SE(2). The velocity is encoded as an element (vx, vy, va) in the 3-dimensional Lie-algebra $\mathfrak{se}(2)$ of SE(2). Hermite subdivision of the discrete sequence results in a **continuous curve** in SE(2) that approximates or even interpolates the original data.

Purpose of Document

We **visualize** the application of the following hermite subdivision methods

H1[-1/8,-1/2], H2[-1/8,-1/2], H2[-1/5,9/10], H3[1/128,-1/16], H3.A1, H3.A2

to the **gokart position and velocity data**. (Among the schemes, only H1[-1/8,-1/2] produces limit curves that interpolate the input.)

The position data is recorded originally at 50[Hz]. We downsample the data from 50[Hz] to 2[Hz] by keeping every 25th sample. The sparse data is input to hermite subdivision. In the figures below, the sparse input is indicated by **blue dots**. The downsampling allows to compare the generated limit curve in **yellow** with the dense recorded data **green-dashed**.

Observation

The schemes H3.A1, and H3.A2 approximate the original data well in all coordinates. The scheme H2[-1/5, 9/10] approximates the original data well in all coordinates but results in high-frequency curvature.

For the other schemes, the coordinate $vy \in \mathfrak{se}(2)$ in the limit curves deviates from the input data and exhibits high frequencies. The coordinate functions for px, py, pa, vx, va approximate the original data well.

Conclusion

When applied to the gokart positional data, the schemes H3.A1, H3.A2, H2[-1/5, 9/10] give the most physically plausible limit curves among the investigated hermite subdivision schemes.

From the 2-parameter families $H1[\lambda, \mu]$, $H2[\lambda, \mu]$, $H3[\theta, \omega]$ we have investigated only selected schemes. Methods corresponding to different parameters are deferred to future investigation.

References

A reference for each hermite subdivision scheme is given in the respective section below.

Our open-source implementation of hermite subdivision for Lie-group-valued data is available at
<https://github.com/idsc-frazzoli/owl>

Code

```

fmt[x_, y_] := Transpose[{x, y}]
shw[d_, k_, ly_] :=
  ListPlot[{fmt[ct, cpv[[All, d, k]], fmt[rt, rpv[[All, d, k]]], fmt[gt, gpv[[All, d, k]]]}, 
    PlotStyle -> {{Opacity[0.7], PointSize[Medium]}, {Opacity[0.8]}, {Opacity[0.8], Dashed}}, 
    Joined -> {False, True, True}, AspectRatio -> 1/6, ImageSize -> 700, AxesLabel -> {"[s]", ly}]
shc := ListPlot[fmt[rt, crv], Joined -> True, PlotRange -> All, AspectRatio -> 1/6, 
  ImageSize -> 700, AxesLabel -> {"[s]", "c[1/m]"}, PlotLabel -> "curvature (px,py)"]

dn = "/home/datahaki/Documents/20190701T163225_01";
gpv = Get[FileNameJoin[{dn, "gndrth.mathematica"}]];
gt = Get[FileNameJoin[{dn, "gndrth_domain.mathematica"}]];
cpv = Get[FileNameJoin[{dn, "control.mathematica"}]];
ct = Get[FileNameJoin[{dn, "control_domain.mathematica"}]];

get[mn_] := Module[{},
  rpv = Get[FileNameJoin[{dn, mn, "refined.mathematica"}]];
  crv = Get[FileNameJoin[{dn, mn, "curvatu.mathematica"}]];
  rt = Get[FileNameJoin[{dn, mn, "refined_domain.mathematica"}]]]

trk :=
  ListPlot[{cpv[[All, 1, {1, 2}], rpv[[All, 1, {1, 2}], gpv[[All, 1, {1, 2}]]}, Joined -> {False, True, True}, 
    PlotStyle -> {{Opacity[0.5], PointSize[Medium]}, Opacity[0.5], Opacity[0.5]}, 
    AspectRatio -> Automatic, PlotLabel -> "track (px,py)", ImageSize -> 400]

spc := Spectrogram[rpv[[All, 2, 2]], AspectRatio -> 1/7, ImageSize -> 700, PlotLabel -> "spectrogram of vy"]

pr[img_] := Print[Rasterize[img, ImageResolution -> 144]]

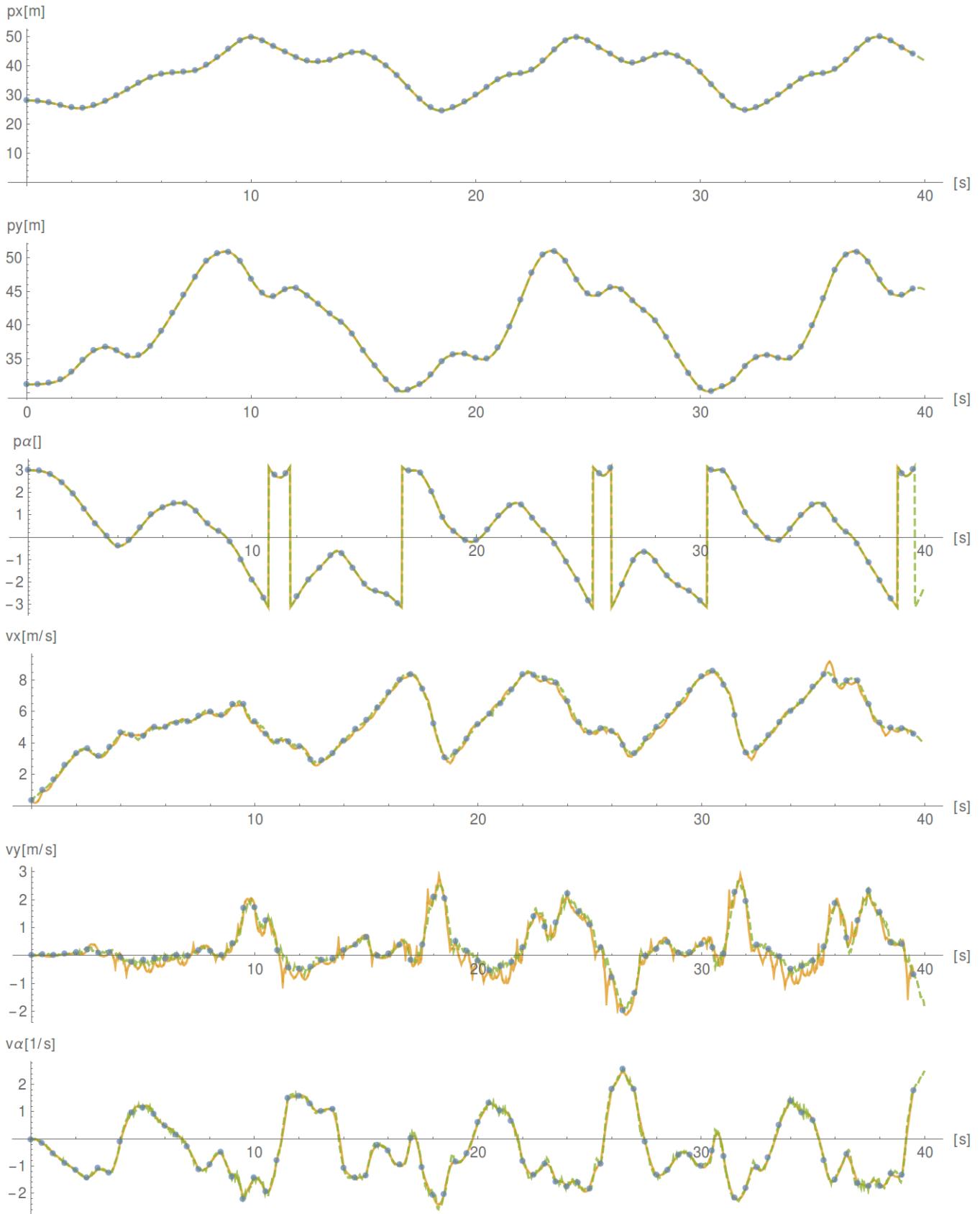
vis[mn_] := Module[{}, get[mn];
  pr[shw[1, 1, "px[m]"]]; pr[shw[1, 2, "py[m]"]]; pr[shw[1, 3, "p\alpha[]"]];
  pr[shw[2, 1, "vx[m/s]"]]; pr[shw[2, 2, "vy[m/s]"]]; pr[shw[2, 3, "v\alpha[1/s]"]];
  pr[trk];
  pr[shc];
  pr[spc]];

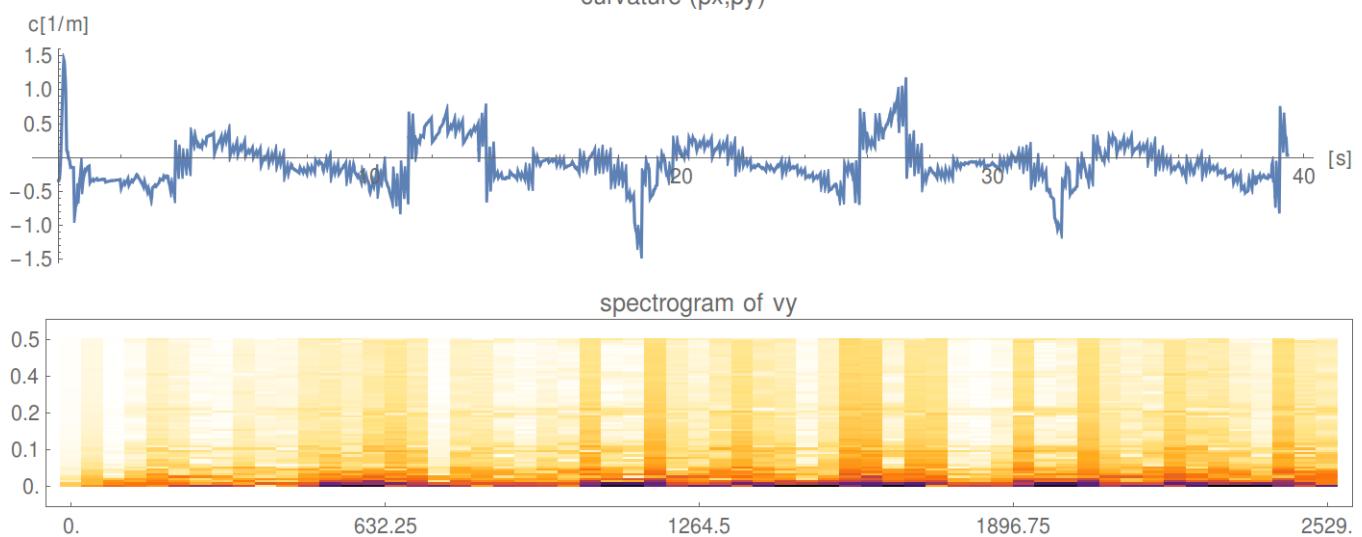
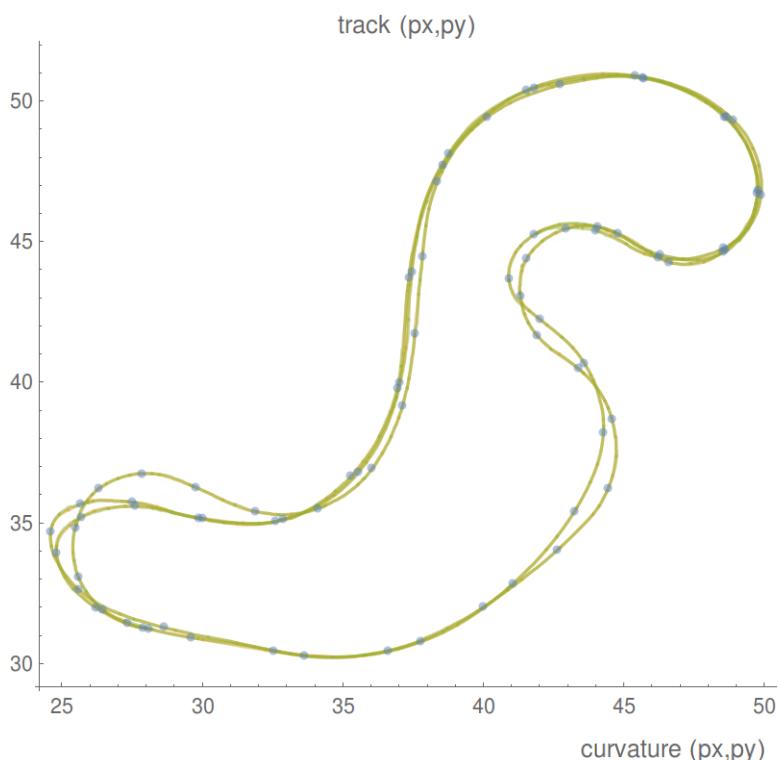
```

H1[-1/8, -1/2]

[2007 Dubuc, Merrien] de Rham Transform of a Hermite Subdivision Scheme, p. 9

```
vis["h1standard"]
```

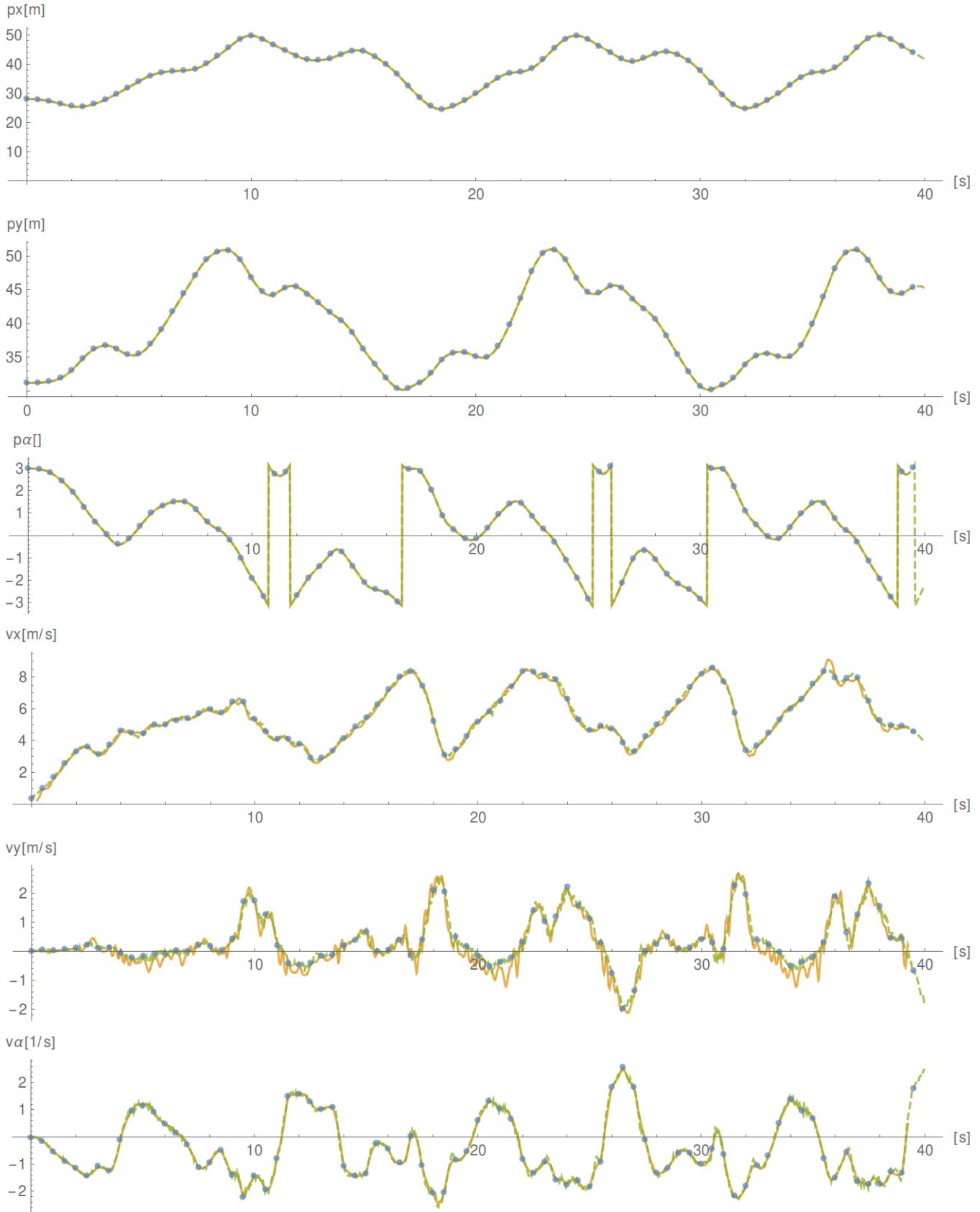




H2[-1/8, -1/2]

[2018 Conti, Huening] *An algebraic approach to polynomial reproduction of hermite subdivision schemes*, p. 14

```
vis["h2standard"]
```

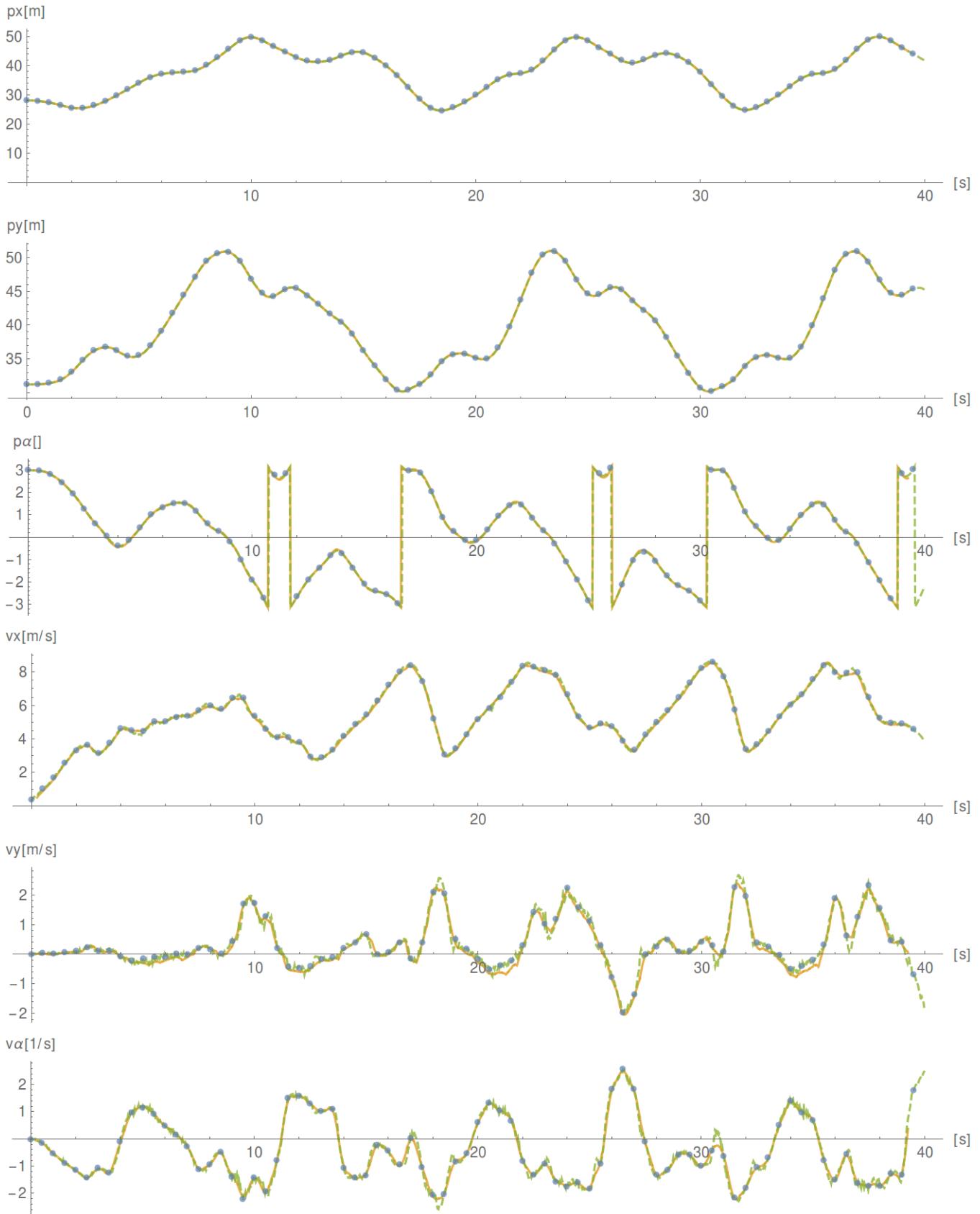


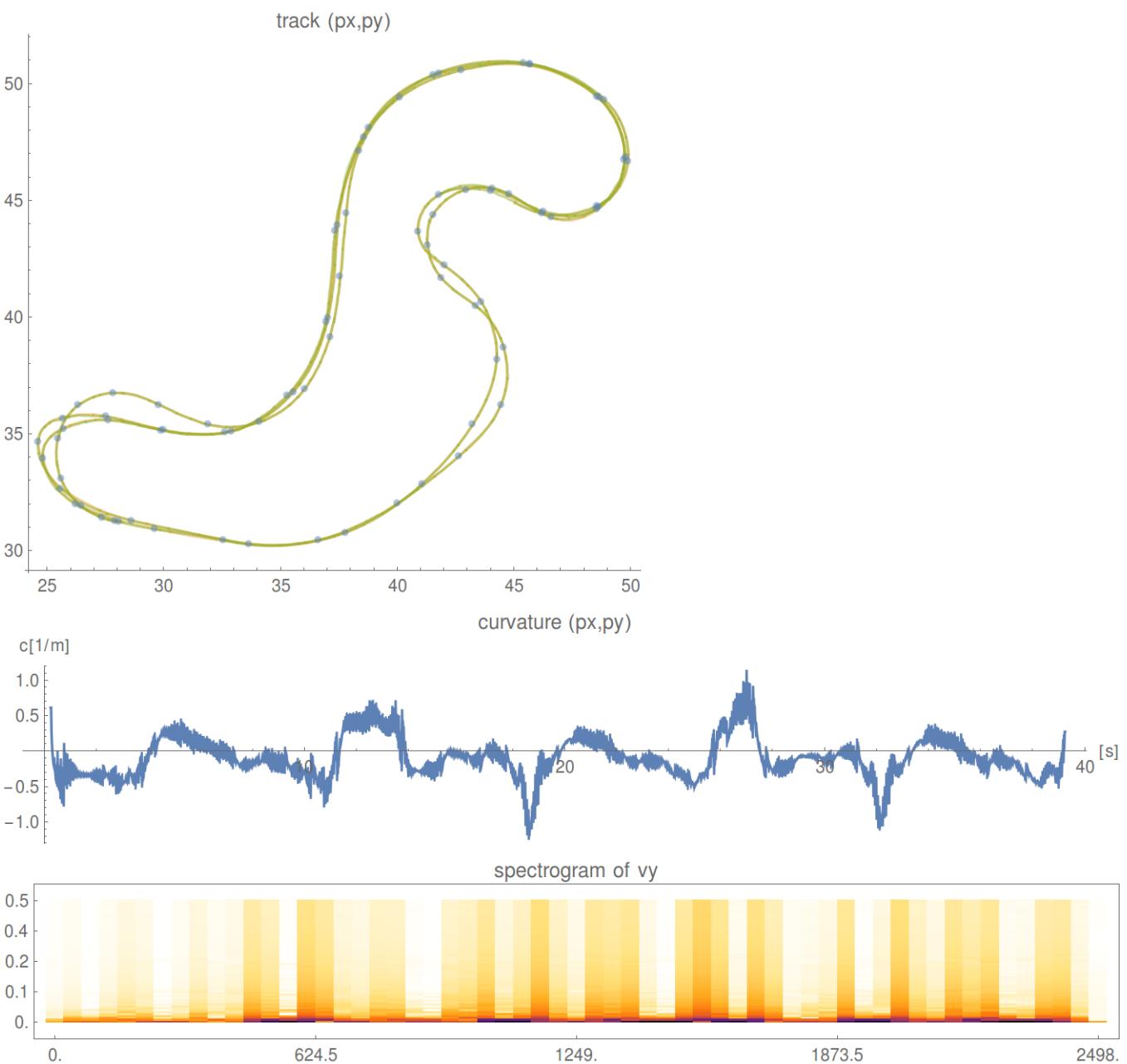


H2[-1/5, 9/10]

[2017 Moosmüller] Hermite subdivision on manifolds via parallel transport, p. 1063

```
vis["h2manifold"]
```

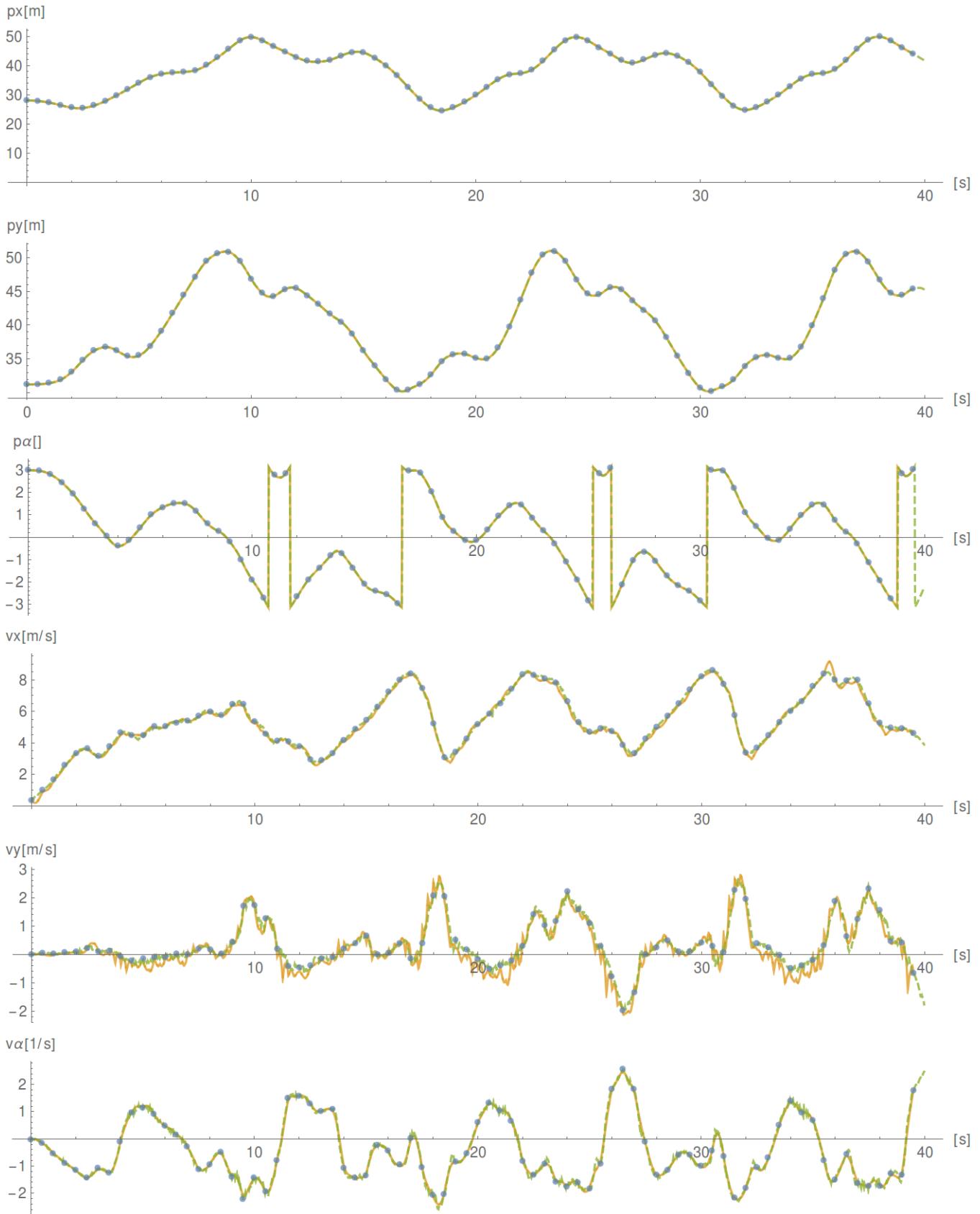


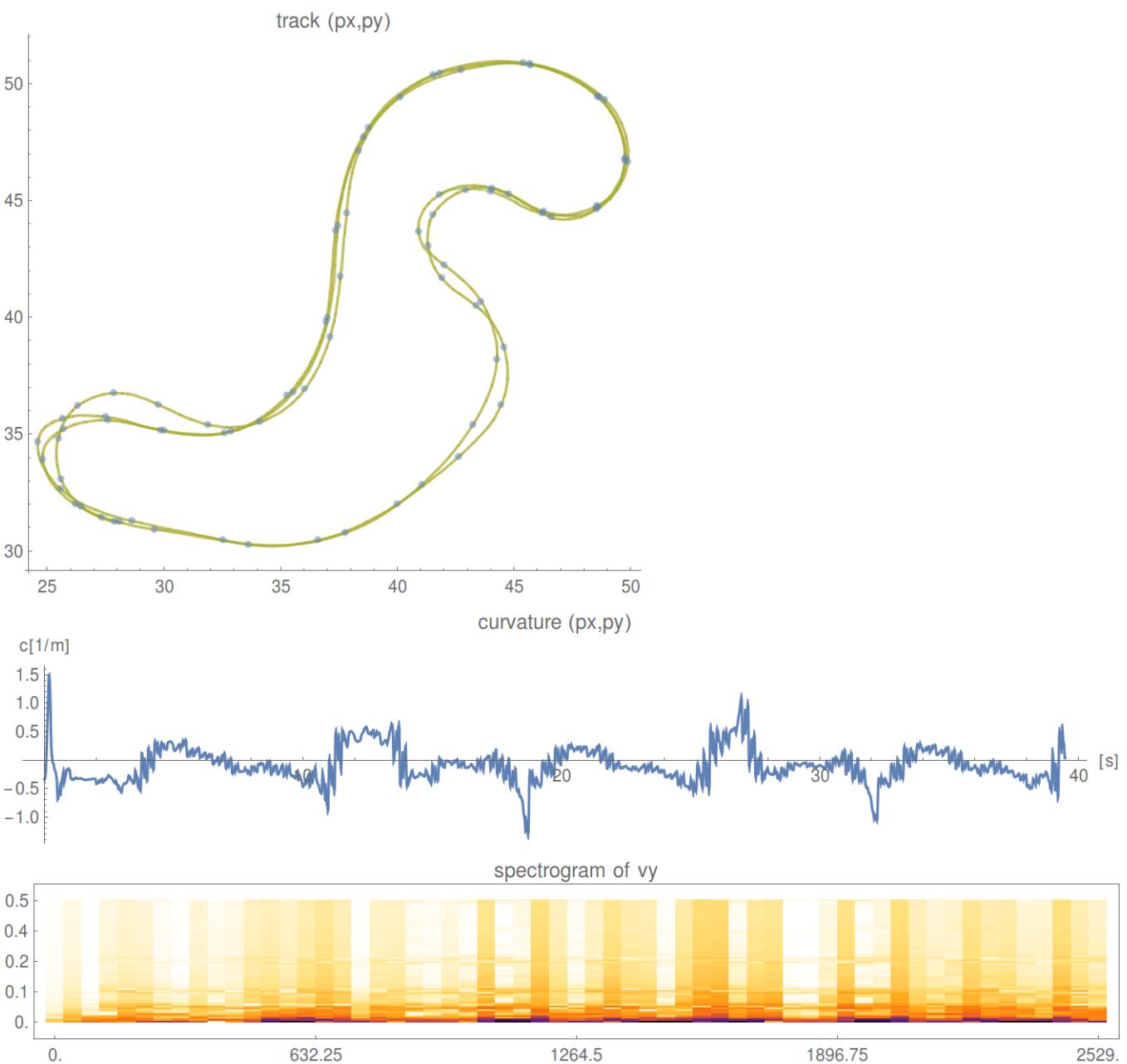


H3[1/128, -1/16]

[2017 Jeong, Yoon] *Construction of Hermite subdivision schemes reproducing polynomials*, p. 572

```
vis["h3standard"]
```

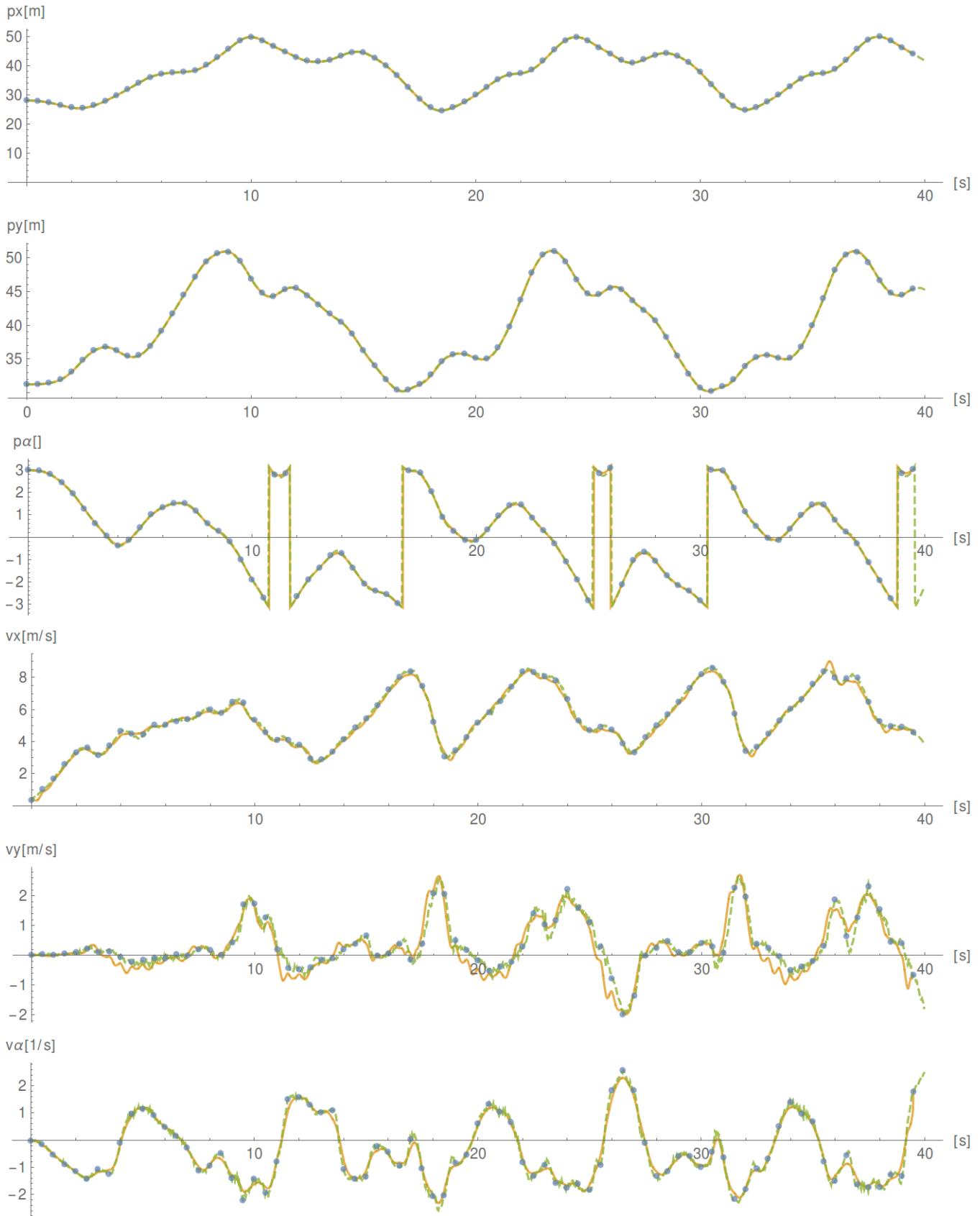


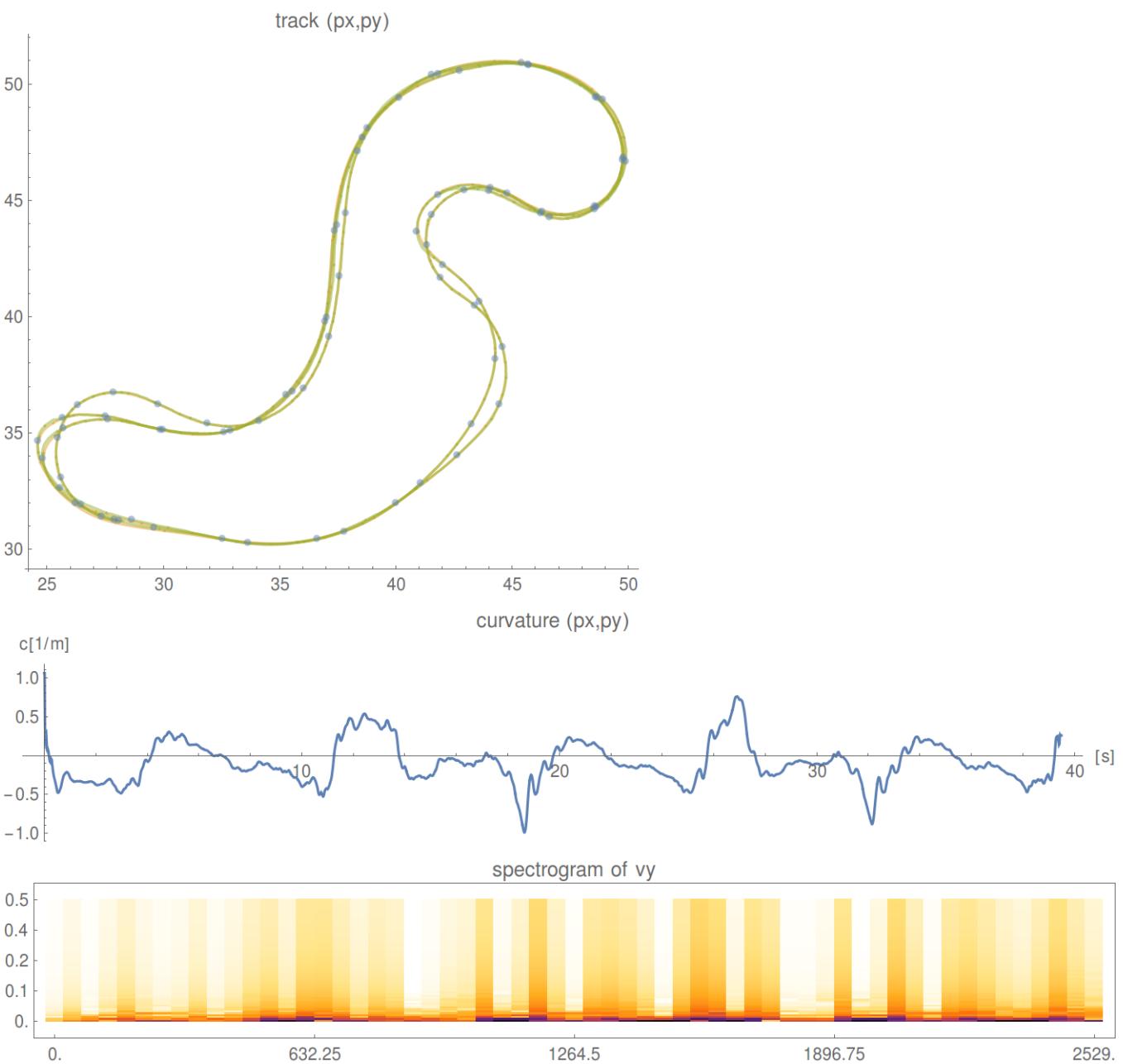


H3.A1

[2004 Han, Yu, Xue] Noninterpolatory Hermite subdivision schemes, p. 1358

```
vis["h3a1"]
```





H3.A2

[2004 Han, Yu, Xue] Noninterpolatory Hermite subdivision schemes, p. 1358

```
vis["h3a2"]
```

