ENTRE [m] = $\hat{j}$

RE [m] = j^*

$$E = \sqrt{\frac{1}{n}\sum_{1}^{n}(\hat{a}-a^*)^2} \; MSE$$

$$= \frac{1}{n}\sum_{1}^{n}|\hat{a}-a^*|$$

feature map dim

$$= \sum_{i}^{n}\sum_{j}^{n} err(y^*-\hat{y})_{ij}$$

$err \in \{MAE, MSE, RMSE\}$

PREDICT

$$\bar{\mu}_t = A\mu_{t-1} + Bu_t$$

$$\bar{\Sigma}_t = A\Sigma_{t-1}A^T + R$$

UPDATE

$$K_t = \bar{\Sigma}_t \; C^T (C \; \bar{\Sigma}_t \; C^T + Q)^{-1}$$

$$2\times2 = 2\times2 \quad 2\times1 \quad 1\times2 \quad 2\times2 \quad 2\times1 \quad 1\times1$$

$$(1\times1)^{-1}$$

$$\mu = \begin{bmatrix} x \\ \dot{x} \end{bmatrix} = \begin{bmatrix} x \\ v \end{bmatrix} \qquad z = [x]$$

$$A = \begin{bmatrix} 1 & \Delta T \\ 0 & 1 \end{bmatrix} \qquad \begin{bmatrix} x \\ \dot{x} \end{bmatrix}$$

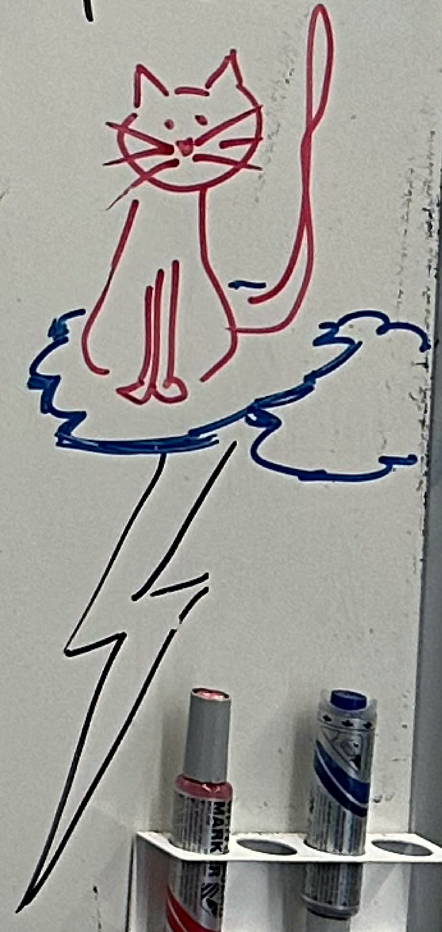
B: no control input

$$R = \begin{bmatrix} r_{xx} & r_{xv} \\ r_{xv} & r_{vv} \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & r_{vv} \end{bmatrix}$$

called q_{vv}

$$C = [1 \quad 0]$$

$$\bar{\Sigma}_t = \begin{bmatrix} \bar{\sigma}_{xx} & \bar{\sigma}_{xv} \\ \bar{\sigma}_{xv} & \bar{\sigma}_{vv} \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & \Delta T \\ 0 & 1 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} \sigma_{xx} & \sigma_{xv} \\ \sigma_{xv} & \sigma_{vv} \end{bmatrix}}_{\Sigma_t} \underbrace{\begin{bmatrix} 1 & 0 \\ \Delta T & 1 \end{bmatrix}}_{A^T} + R$$



$$\bar{\sigma}_{xx} = \sigma_{xx} + 2\Delta T\sigma_{xv} + \Delta T^2\sigma_{vv} + \cancel{r_{xx}}$$

$$\bar{\sigma}_{xv} = \sigma_{xv} + \Delta T\sigma_{vv} + \cancel{r_{xv}}$$

$$\bar{\sigma}_{vv} = \sigma_{vv} + r_{vv}$$

ENTRE $[m] = \hat{d}$

RE $[m] = d^*$

$E = \sqrt{\frac{1}{n}\sum_{1}^{n}(\hat{a}-a^*)^2}$ MSE

$= \frac{1}{n}\sum_{1}^{n}|\hat{a}-a^*|$

feature map dim

$\sum_{i}^{n}\sum_{j}^{n} err(y^*-\hat{y})_{ij}$

$r \in \{MAE, MSE, RMSE\}$

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$$\bar{\mu}_t = A\mu_{t-1} + Bu_t$$

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UPDATE

$$K_t = \bar{\Sigma}_t C^T (C\bar{\Sigma}_t C^T + Q)^{-1}$$

$$\mu_t = \bar{\mu}_t + K_t(z_t + C\bar{\mu}_t)$$

$$\Sigma_t = (I - K_t C)\bar{\Sigma}_t$$

$$k_x = \sigma \quad (\sigma_{xx} + q_x)^{-1}$$

$$x_t = \bar{x}_t + k_x(x_{new} - \bar{x}_t)$$

$u = [\,]$ no control input

$$\mu = \begin{bmatrix} x \\ \dot{x} \end{bmatrix} = \begin{bmatrix} x \\ v \end{bmatrix} \qquad z = [x]$$

xNew

$$A = \begin{bmatrix} 1 & \Delta T \\ 0 & 1 \end{bmatrix} \quad \begin{bmatrix} x \\ \dot{x} \end{bmatrix}$$

$B = [\,]$

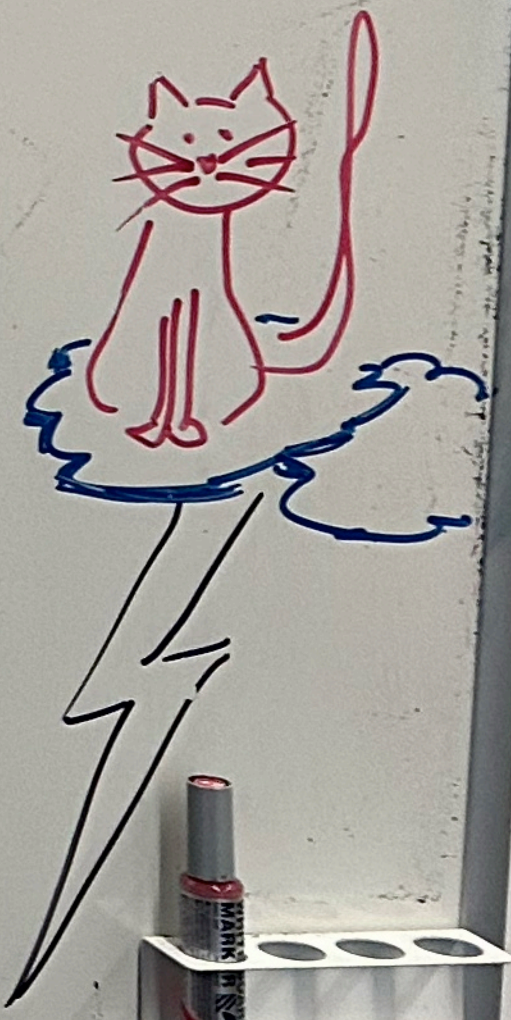
$$R = \begin{bmatrix} r_{xx} & r_{xv} \\ r_{xv} & r_{vv} \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & r_{vv} \end{bmatrix}$$

called q_{vv}

$$C = [1 \quad 0]$$

$$Q = [q_x]$$

called r_{xx}



$$\underbrace{[1 \quad 0]\begin{bmatrix} \sigma_{xx} & \sigma_{xv} \\ \sigma_{xv} & \sigma_{vv} \end{bmatrix}}_{2\times2}\underbrace{\begin{bmatrix} 1 \\ 0 \end{bmatrix}}_{2\times1} + q_x = [1 \quad 0]\begin{bmatrix} \sigma_{xx} \\ \sigma_{xv} \end{bmatrix} + q_x = \sigma_{xx} + q_x$$

PREDICT

$$\overline{\mu_t} = A\mu_{t-1} + Bu_t$$

$$\overline{\Sigma_t} = A\Sigma_{t-1}A^T + R$$

UPDATE

$$K_t = \overline{\Sigma_t}\, C^T \left(C\,\overline{\Sigma_t}\, C^T + Q\right)^{-1}$$

$$\mu_t = \overline{\mu_t} + K_t\left(z_t + C\bar{\mu}_t\right)$$

$$\Sigma_t = \left(I - K_t C\right)\overline{\Sigma_t}$$

$$k_x = \sigma \quad \underbrace{\left(\sigma_{xx} + q_x\right)}_{\text{called } S}^{-1}$$

$$x_t = \overline{x_x} + k_x\left(x_{new} - \overline{x_t}\right)$$

$[m] = \partial$

$] = \partial^*$

$\frac{1}{n}\sum_1^n(\hat{a}-a^*)^2$ MSE

$\frac{1}{n}\sum_1^n|\hat{a}-a^*|$

ture map dim

$\sum err(y^* - \hat{y})_{ij}$

$\{$MAE, MSE, RMSE$\}$

$u = [\,]$ no control input

$$\mu = \begin{bmatrix} x \\ \dot{x} \end{bmatrix} = \begin{bmatrix} x \\ v \end{bmatrix} \quad z = [x]$$

xNew

$$A = \begin{bmatrix} 1 & \Delta T \\ 0 & 1 \end{bmatrix} \quad \begin{bmatrix} x \\ \dot{x} \end{bmatrix}$$

$$B = [\,]$$

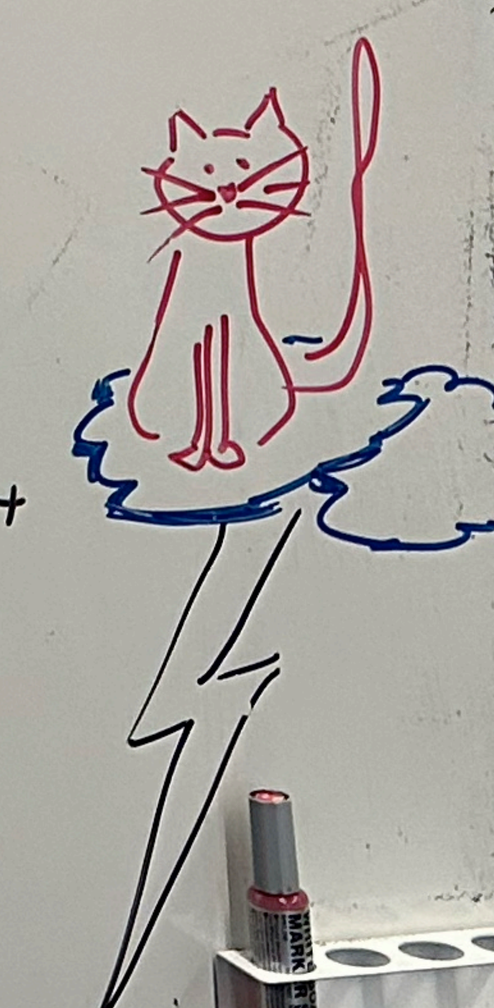
$$R = \begin{bmatrix} r_{xx} & r_{xv} \\ r_{xv} & r_{vv} \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & r_{vv} \end{bmatrix}$$

called q_{vr}

$$C = \begin{bmatrix} 1 & 0 \end{bmatrix}$$

$$Q = [q_x]$$

called r_{xx}



$$\begin{bmatrix} 1 & 0 \end{bmatrix}\begin{bmatrix} \sigma_{xx} & \sigma_{xv} \\ \sigma_{xv} & \sigma_{vv} \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} + q_x = \begin{bmatrix} 1 & 0 \end{bmatrix}\begin{bmatrix} \sigma_{xx} \\ \sigma_{xv} \end{bmatrix} + q_x = \sigma_{xx} + q_x$$

2x2   2x1

}

$-a^*)^2$ MSE

$-a^*|$

m

$*-\hat{y})_{ij}$

, MSE, RMSE}

PREDICT

$$\bar{\mu}_t = A\mu_{t-1} + Bu_t$$

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$$K_t = \bar{\Sigma}_t C^T (C\bar{\Sigma}_t C^T + Q)^{-1}$$

$$\mu_t = \bar{\mu}_t + K_t(z_t + C\bar{\mu}_t)$$

$$\Sigma_t = (I - K_t C)\bar{\Sigma}_t$$

$$K_x = \sigma_{xx} \underbrace{(\sigma_{xx} + q_x)}_{\text{called } S}^{-1}$$

$$x_t = \bar{x}_t + k_x(x_{new} - \bar{x}_t)$$

$$\begin{bmatrix}\sigma_{xx} & \sigma_{xv}\\ \sigma_{xv} & \sigma_{vv}\end{bmatrix}\begin{bmatrix}1\\ 0\end{bmatrix} = \begin{bmatrix}\sigma_{xx}\\ \sigma_{xv}\end{bmatrix} \frac{1}{[\sigma_{xx} + q_x]}$$

2x1 1x1

$$K_t = \begin{bmatrix}k_x\\ k_v\end{bmatrix} = \begin{bmatrix}\frac{\sigma_{xx}}{\sigma_{xx}+q_x}\\ \frac{\sigma_{xv}}{\sigma_{xx}+q_x}\end{bmatrix}$$

$u = [\,]$ no control input

$$\mu = \begin{bmatrix}x\\ \dot{x}\end{bmatrix} = \begin{bmatrix}x\\ v\end{bmatrix} \quad z = \underbrace{[x]}_{\text{xNew}}$$

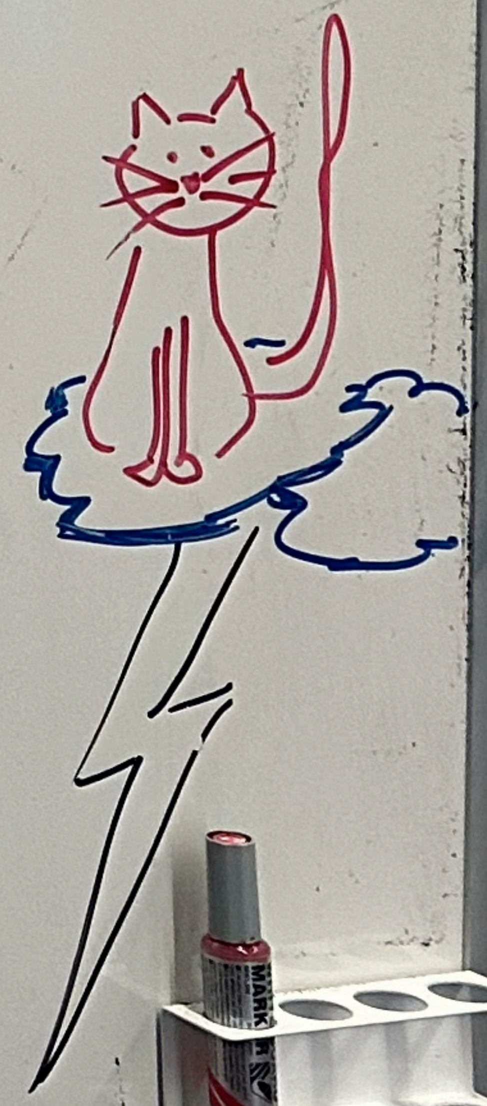
$$A = \begin{bmatrix}1 & \Delta T\\ 0 & 1\end{bmatrix} \quad \begin{bmatrix}x\\ \dot{x}\end{bmatrix}$$

$B = [\,]$

$$R = \begin{bmatrix}r_{xx} & r_{xv}\\ r_{xv} & r_{vv}\end{bmatrix} = \begin{bmatrix}0 & 0\\ 0 & \underbrace{r_w}_{\text{called } q_{vv}}\end{bmatrix}$$

$$C = [1 \quad 0]$$

$$Q = [\underbrace{q_x}_{\text{called } r_{xx}}]$$



TRE $[m] = \hat{d}$

$[m] = d^*$

$= \sqrt{\frac{1}{n}\sum_1^n (\hat{a}-a^*)^2}$ MSE

$= \frac{1}{n}\sum_1^n |\hat{a}-a^*|$

feature map dim

$\sum_i^n \sum_j^n err(y^* - \hat{y})_{ij}$

$r \in \{MAE, MSE, RMSE\}$

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$$x_t = \bar{x}_t + k_x(x_{new} - \bar{x}_t)$$

$$v_t = \bar{v}_t + k_v(x_{new} - \bar{x}_t)$$

$$\begin{bmatrix} \sigma_{xx} & \sigma_{xv} \\ \sigma_{xv} & \sigma_{vv} \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} \sigma_{xx} \\ \sigma_{xv} \end{bmatrix} \frac{1}{[\sigma_{xx} + q_x]}$$

$2\times 1 \qquad 1\times 1$

$$K_t = \begin{bmatrix} k_x \\ k_v \end{bmatrix} = \begin{bmatrix} \frac{\sigma_{xx}}{\sigma_{xx} + q_x} \\ \frac{\sigma_{xv}}{\sigma_{xx} + q_x} \end{bmatrix}$$

$u = [\,]$

$$\mu = \begin{bmatrix} x \\ \dot{x} \end{bmatrix} = \begin{bmatrix} x \\ v \end{bmatrix}$$

$z = [x]$

xNe

$$A = \begin{bmatrix} 1 & \Delta T \\ 0 & 1 \end{bmatrix} \quad \begin{bmatrix} x \\ \dot{x} \end{bmatrix}$$

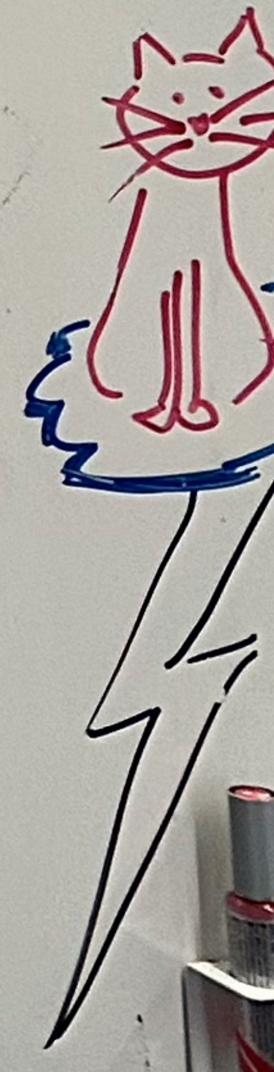
$B = [\,]$

$$R = \begin{bmatrix} r_{xx} & r_{xv} \\ r_{xv} & r_{vv} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$C = [1 \quad 0]$

calle

$$Q = [\underbrace{q_x}_{\text{called } r_{xx}}]$$



$n] = \hat{j}$

j^*

$\sum_1^n (\hat{a} - a^*)^2$ MSE

$\sum |\hat{a} - a^*|$

map dim

$rr(y^* - \hat{y})_{ij}$

MAE, MSE, RMSE }

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$$K_x = \sigma_{xx} (\sigma_{xx} + q_x)^{-1}$$

called S

$$x_t = \bar{x}_t + k_x(x_{new} - \bar{x}_t)$$

$$v_t = \bar{v}_t + K_v(x_{new} - \bar{x}_t)$$

$$\begin{bmatrix} \sigma_{xx} & \sigma_{xv} \\ \sigma_{xv} & \sigma_{vv} \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} \sigma_{xx} \\ \sigma_{xv} \end{bmatrix} \frac{1}{[\sigma_{xx} + q_x]^{-1}}$$

2x1 1x1

$$K_t = \begin{bmatrix} k_x \\ k_v \end{bmatrix} = \begin{bmatrix} \dfrac{\sigma_{xx}}{\sigma_{xx} + q_x} \\ \dfrac{\sigma_{xv}}{\sigma_{xx} + q_x} \end{bmatrix}$$

$u = [\,]$ no control input

$$\mu = \begin{bmatrix} x \\ \dot{x} \end{bmatrix} = \begin{bmatrix} x \\ v \end{bmatrix} \qquad z = [x]$$

xNew

$$A = \begin{bmatrix} 1 & \Delta T \\ 0 & 1 \end{bmatrix} \qquad \begin{bmatrix} x \\ \dot{x} \end{bmatrix}$$

$$B = [\,]$$

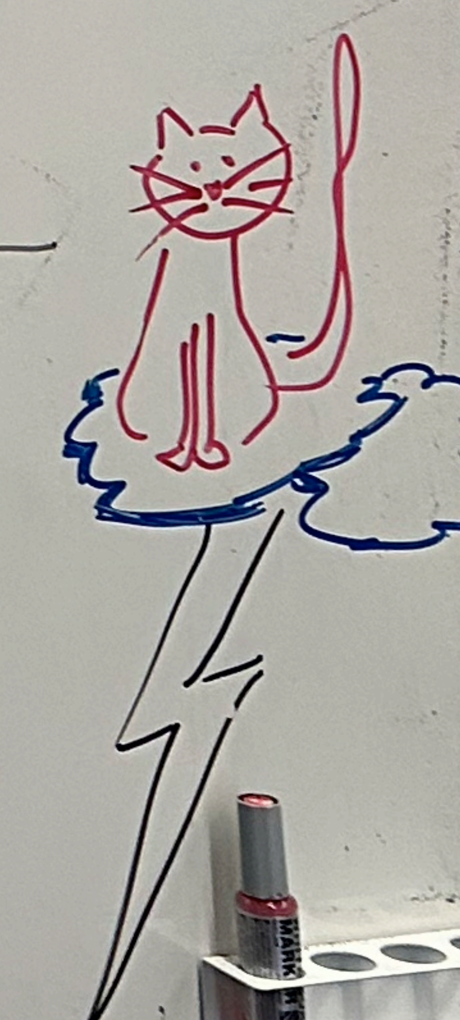
$$R = \begin{bmatrix} r_{xx} & r_{xv} \\ r_{xv} & r_{vv} \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & r_w \end{bmatrix}$$

called q_{vv}

$$C = [1 \quad 0]$$

$$Q = [q_x]$$

called r_{xx}



$\cdot] = \partial$

$*$

$(\hat{a} - a^*)^2$ MSE

$\hat{} - a^* |$

m

$- \hat{y})_{ij}$

SE, RMSE]

PREDICT

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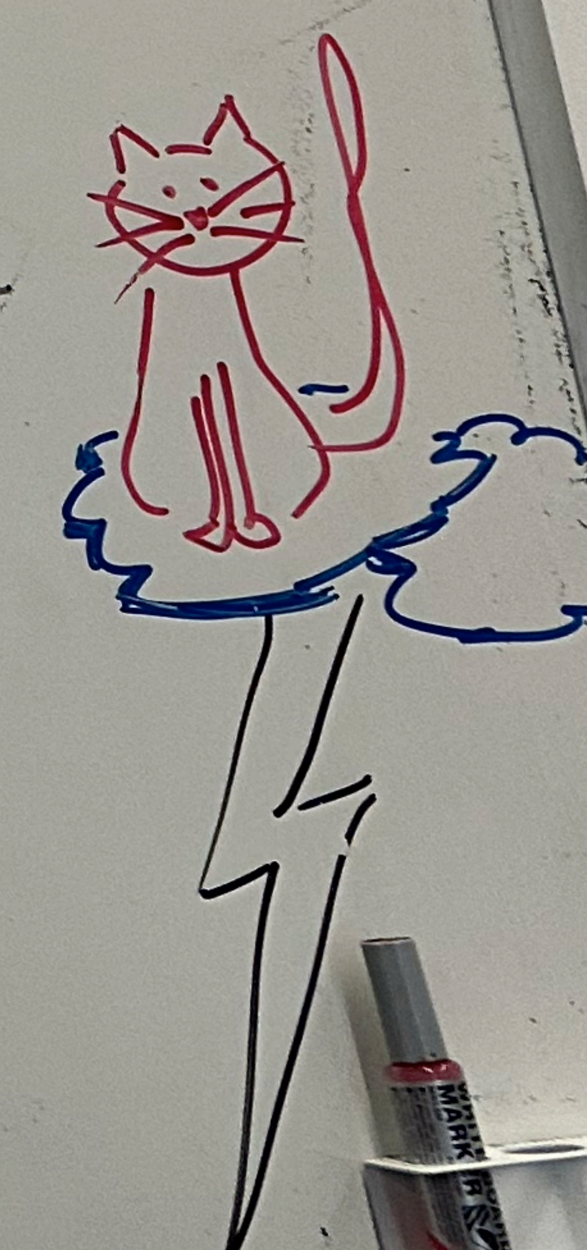
$$R = \begin{bmatrix} r_{xx} & r_{xv} \\ r_{xv} & r_{vv} \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & r_w \end{bmatrix}$$

called qvr

$$C = [1 \quad 0]$$

$$Q = [q_x]$$

called r_{xx}



$$\begin{bmatrix} \frac{\sigma_{xx}}{\sigma_{xx}+q_x} \\ \frac{\sigma_{xv}}{\sigma_{xx}+q_x} \end{bmatrix} [1 \quad 0]$$

2×1 $\quad 1 \times 2$

$$\begin{bmatrix} 1 - \frac{\sigma_{xr}}{\sigma_{xx}+q_x} & 0 \\ -\frac{q_{xv}}{\sigma_{xx}+q_x} & 1 \end{bmatrix} \bar{\Sigma}_t$$