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STAT 514
HW ②

1.

An article in the *Journal of Strain Analysis* (vol. 18, no.2 , 1983) compares several procedures for predicting the shear strength for steel plate girders. Data for nine girders in the form of the ratio of predicted to observed load for two of these procedures, the Karlsruhe and Lehigh Methods, are given as follows:

Girder	Karlsruhe Method	Lehigh Method
S1/1	1.186	1.061
S2/1	1.151	0.992
S3/1	1.322	1.063
S4/1	1.339	1.062
S5/1	1.200	1.065
S2/2	1.402	1.178
S2/3	1.365	1.037
S2/4	1.537	1.086
S2/5	1.559	1.052

a) Is there any evidence to support the claim that there is a difference in mean performance between the two methods? Use $\alpha = 5\%$.

b) What is the P-value for the test in part a?

c) Construct a 95 percent confidence interval for the difference in mean predicted to observed load ratio

d) Investigate the normality assumption for both samples.

e) Investigate the normality assumption for the difference in ratios of the two methods.

f) Which design principles have been used in the experiment. Comment on their advantages and disadvantages.

(Hint: Please read Section 2.5 of Montgomery and find formulas for this problem in the section)

a) $\bar{A}$ = the mean performance of Karlsruhe Method
$\bar{B}$ = the mean performance of Lehigh Method

$\bar{A} = 1.340 \qquad \bar{B} = 1.066$

$$S_d^2 = \frac{\Sigma d_j^2 - \frac{1}{n}(\Sigma d_j)^2}{n-1} = \frac{0.82 - 0.675}{8} \simeq 0.01825 \qquad \therefore S_d \simeq 0.1351$$

$$t_0 = \frac{1.340 - 1.066}{0.1351 / \sqrt{9}} = \frac{0.274}{0.045} \simeq 6.0844$$

$< t_{0.025, 8} = 2\ 306$

P-value for t_0 statistics is 0.000294

Since P-value < 0.05. Statistically, this evidence supports the claim, that is, there is a difference in mean performance.

b) P-value $\simeq$ 0.000294 //

Without being able to use t-table, can we say its P-value << 0.0005 ?

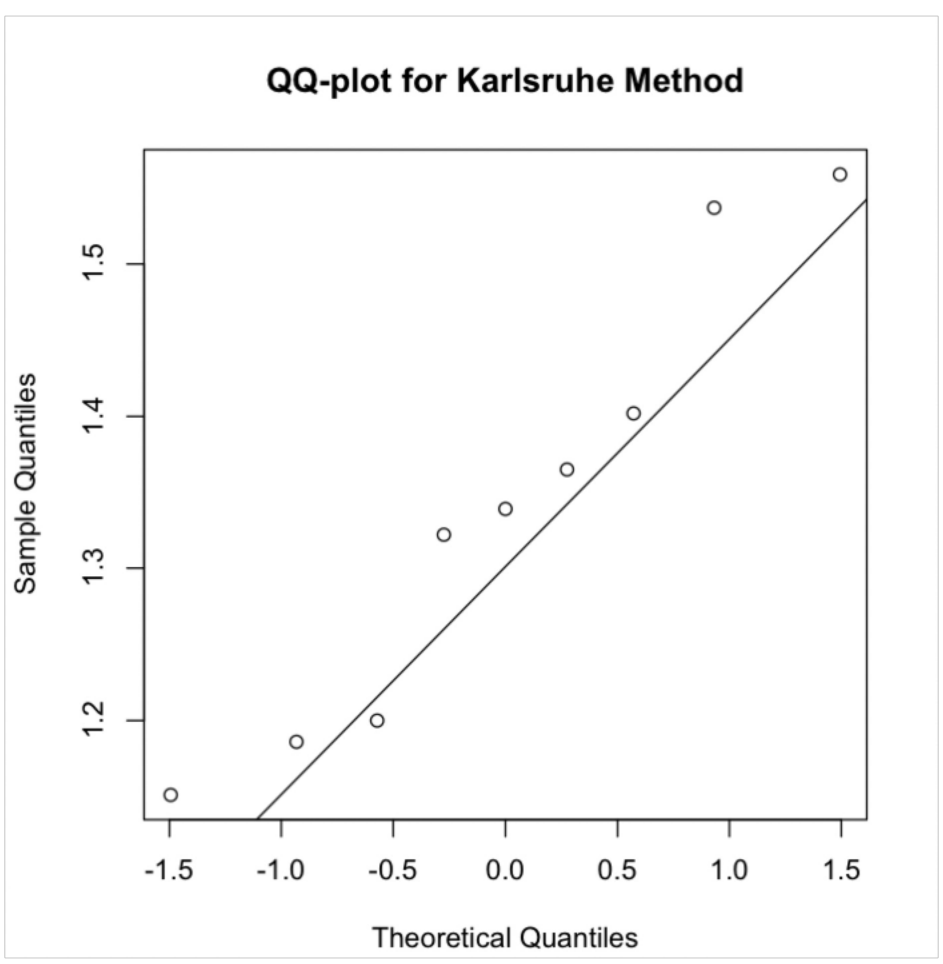
c) $\bar{A} - \bar{B} = 0.274$

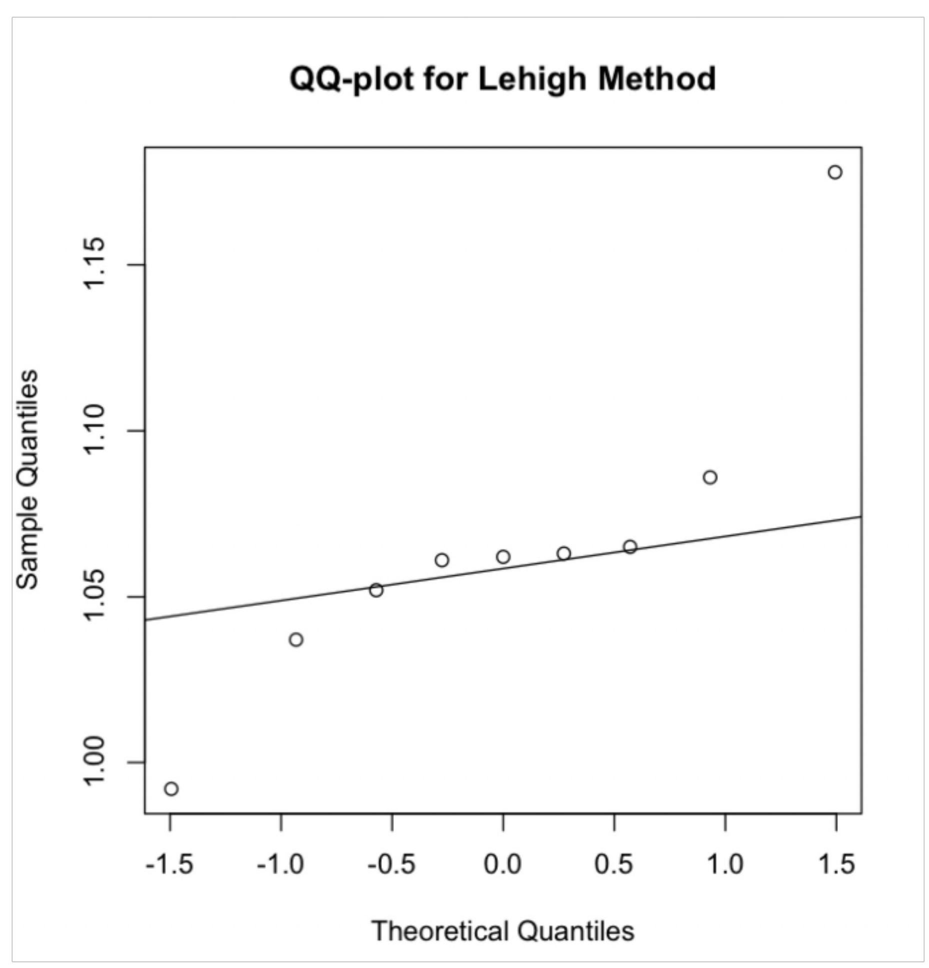
$$(L, U) = \left(\bar{Y} - t_{0.025}(\overset{8}{\cancel{16}}) \cdot S_d/\sqrt{n},\ \bar{Y} + t_{0.025}(\overset{8}{\cancel{16}}) \cdot S_d/\sqrt{n}\right)$$

$$= (0.274 - 2.306 \cdot 0.045,\ 0.274 + 2.306 \cdot 0.045)$$

$$= (0.17,\ 0.3778)$$ //

d)





As we can see from the plots above, both samples, even though both only have 20 sample data, have some outliers and the points do not necessarily fall around a straight line. Therefore, we can say both samples do not follow the normality assumption.

For example, I take a look at the points of S5/1

$(S5/1, \text{Karlsruhe}) = 1.200$,

$\rightarrow \alpha_3 = \dfrac{3 - 3/8}{9 + \frac{1}{4}} \approx 0.2838$

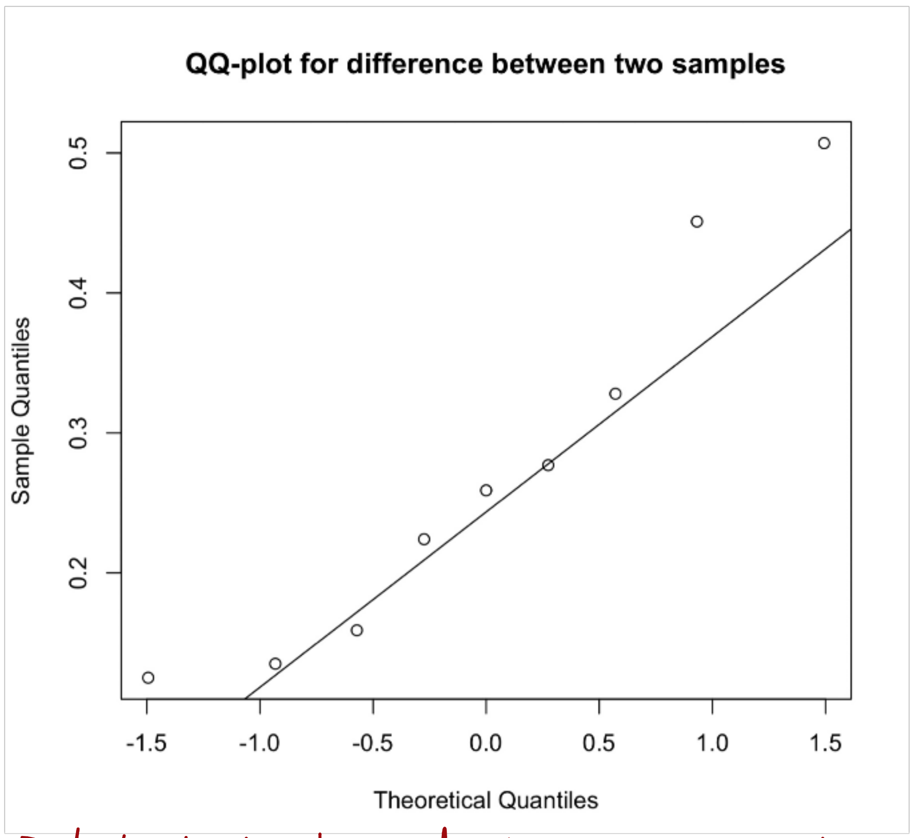
$r_{\alpha_3} = -0.572$

$(S5/1, \text{Lehigh}) = 1.065$

$\rightarrow \alpha_7 = \dfrac{7 - 3/8}{9 + 1/4} \approx 0.716$

$r_{\alpha_7} = 0.571$

e)



The difference between two values seems relatively follow the normality; yet, there are several outliers far away from a straight line.

I take a look at the points (the difference of S5/1) = 0.135

$\rightarrow \alpha_2 = \dfrac{2 - 3/8}{9 + 1/4} \approx 0.1757$

$r_{\alpha_2} = -0.932$

The paired t-test only needs to assume that the differences form a sample drawn from a normal distribution. The QQ plot of the differences does not suggest any significant departure from the normality assumption.

(f) The design principle have used in this experiment is blocking and the paired comparison test which is a special case of the randomized block design.

The advantage of this principle is that it serves as a noise reduction design technique. By using this technique, experimenters can create a relatively homogeneous experimental unit, that is, Girders are the blocks in this experiment and can improve the precision with which the comparisons among the factors of interest are made.

The disadvantage of this principle is that people assume the block is a set of relatively homogeneous experimental conditions. However, there is Nine different conditions (= Girders) in each block, and they only have one value for each. Hence, The conditions among each block can vary than they expect, which can create the unnecessary bias.

Blocking was used. Blocking can effectively eliminate the variation due to girders and increase the power to detect the difference between the two measurement methods.

On the other hand, blocking will result in reduced degree of freedom, and reduce the power. The success of blocking depends on the trade-off between variation elimination and the loss of degree of freedom. ??

2.

A vendor submits lots of fabric to a textile manufacturer. The manufacturer wants to know if the lot average breaking strength exceeds 200 psi. if so, she wants to accept the lot. Past experience indicates that a reasonable value for the variance of breaking strength is 100 $(\text{psi})^2$. The hypotheses to be tested are

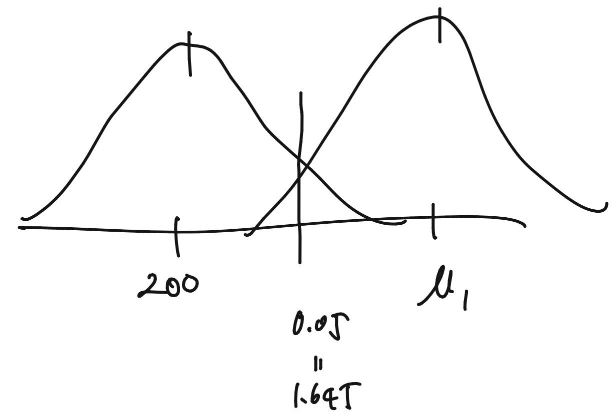
Variance is given so Z-test

$$H_0 : \mu = 200 \text{ vs. } H_1 : \mu > 200.$$

The manufacturer decides to randomly select a number of specimens, measure their breaking strengths and test the hypotheses with $\alpha = 5\%$.

(a) Draw (by hand) the operating characteristic curve when 9 specimens are selected for the test. (Hint: Z-test should be used).

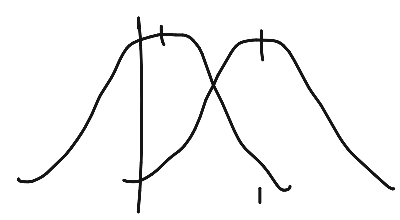
(a) $Z = \dfrac{\bar{x} - \mu}{\sigma/\sqrt{n}}$ where $\sigma = 10$, $\sqrt{n} = 3$



$\beta(d) = P(\text{accepting } H_0 \mid H_1 \text{ is true}) \leq 0.05$

$= P(\bar{x} \leq 200 \mid \overset{\mu}{\cancel{\bar{x}}} > 200)$

$= P\left(\dfrac{\bar{x} - 200}{10/3} \leq 1.645 \mid \overset{\mu}{\cancel{\bar{x}}} > 200\right)$



Let $d = \dfrac{\mu_1 - 200}{2 \times 10}$ where $\mu_1 = 202, 204, 206, 208, 210$

Assume $\mu_1 = 202$, $\qquad \mu \sim N(202, 100)$

$\rightarrow d = \cancel{0.2}\ 0.1 \qquad \bar{x} \sim N\left(202, \frac{100}{9}\right)$

$\rightarrow \beta(0.2) = P\left(\dfrac{\bar{x} - 200}{10/3} \leq 1.645 \mid \mu = 202\right)$

$= P\left(\bar{x} \leq 200 + 1.645 \cdot \frac{10}{3} \mid \mu = 202\right)$

$= P\left(\dfrac{\bar{x} - 202}{10/3} \leq 1.045\right)$

$= P(z^* \leq 1.045)$

$\simeq 0.85$

For $\mu_1 = 204$,

$d = \cancel{0.4}\ 0.2$

$\beta(0.4) = P\left(\frac{\bar{x}-204}{10/3} \le 0.445\right) \cong 0.67$

For $\mu_1 = 206$,

$d = \cancel{0.6}\ 0.3$

$\beta(0.6) = P\left(\frac{\bar{x}-206}{10/3} \le -0.155\right) \cong 0.438$

For $\mu_1 = 208$

$d = \cancel{0.8}\ 0.4$

$\beta(0.8) = P\left(\frac{\bar{x}-208}{10/3} \le -0.755\right) \cong 0.225$

For $\mu_1 = 210$

$d = \cancel{1}\ 0.5$

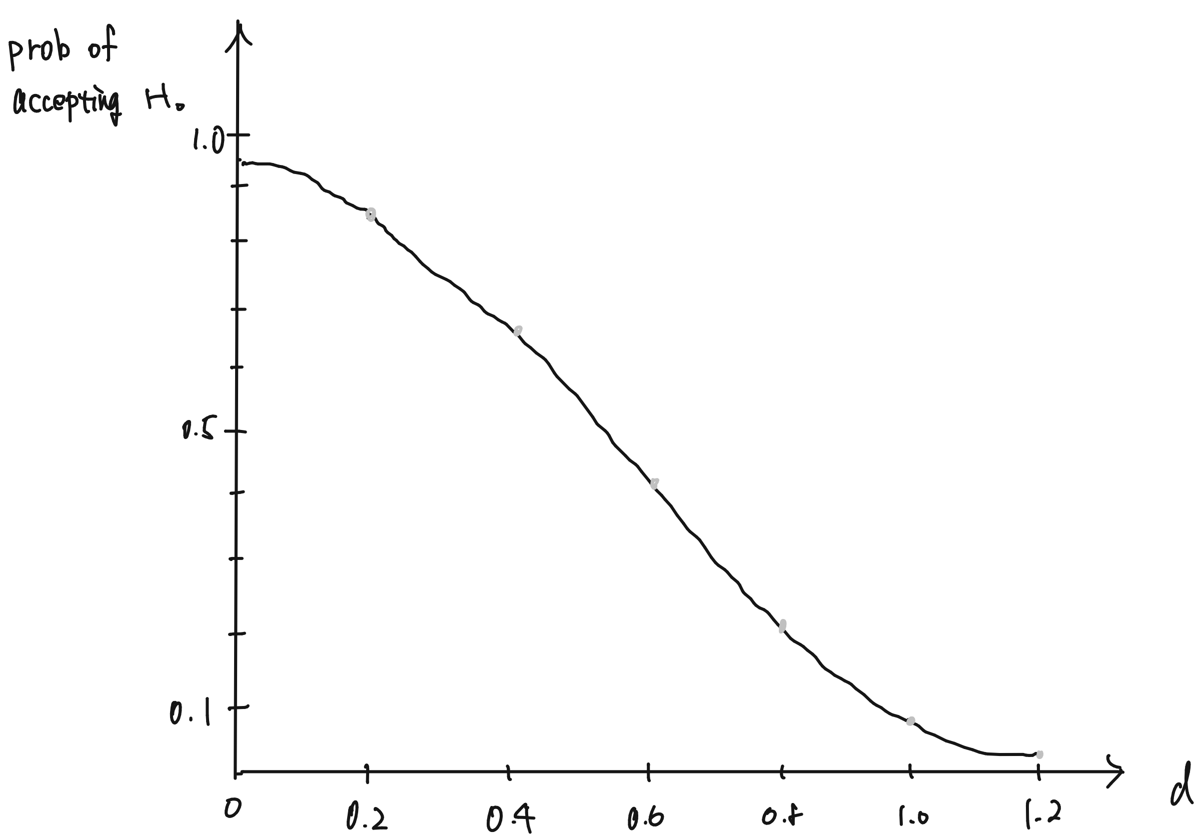
$\beta(1) = P\left(\frac{\bar{x}-210}{10/3} \le -1.355\right) \cong 0.088$

For $\mu_1 = 212$

$d = \cancel{1.2}\ 0.6$

$\beta(1.2) = P\left(\frac{\bar{x}-212}{10/3} \le -1.955\right) \cong 0.025$

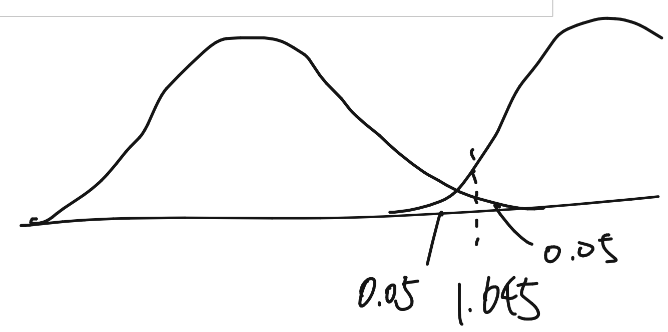
$\therefore$ OC curve look like below :



$\beta \le 0.05$

(b) Suppose the manufacturer wants to guarantee that average breaking strength 210 or higher should be detected with probability at least 95%. What is the minimum number of specimens he/she should check? (Hint: Because the distribution of the sample mean under $\mu = 210$ is normal, you can determine the necessary sample size without using the O.C. curves.)

$$Z = \frac{\bar{x} - 200}{\sigma/\sqrt{n}} \quad \text{where } \sigma = 10$$



Rejecting H_0 when $Z > 1.645$

$$\beta = P(\text{accepting } H_0 \mid H_1 \text{ is True})$$

$$= P\left(\bar{x} \le 200 + Z \cdot \frac{\sigma}{\sqrt{n}} \;\middle|\; \mu = 210\right) \le 0.05$$

$$= P\left(\frac{\bar{x} - 210}{10/\sqrt{n}} \le \left(-10 + Z \cdot \frac{10}{\sqrt{n}}\right) / \left(10/\sqrt{n}\right)\right) \le 0.05$$

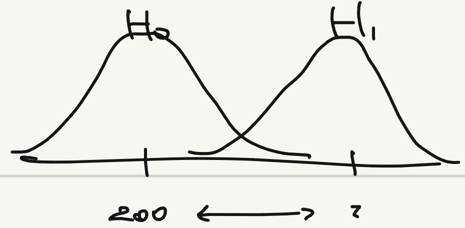
$$= P\left(Z^* \le -\sqrt{n} + Z_{0.05}\right) \le 0.05$$

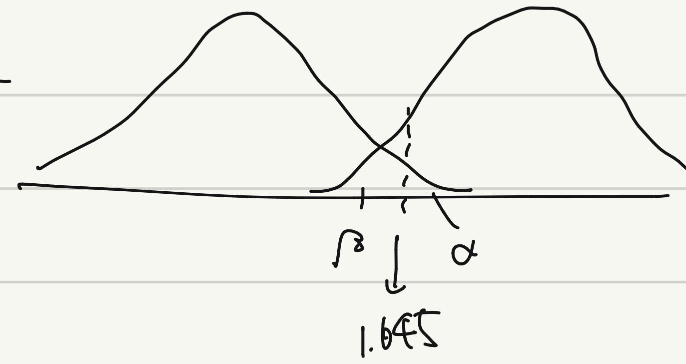
$$\therefore \; -\sqrt{n} + 1.645 \le -1.645$$

$$n \ge 10.8$$

$\therefore$ At least, we need 11 specimens. //

$$Z = \frac{(\bar{X} - \mu_0)}{s} \;??$$





(a) $\beta(d, q)$

$d = 0 \; : \; \beta(0, q)$

$d = 1 \; : \; \beta(1, q) = P(Z \leq 0.3) =$

$d = 2 \; : \; \beta(2, q) = P(Z \leq 0.6) =$

$$\frac{\mu_2 - 200}{2 \cdot 10} = 1$$

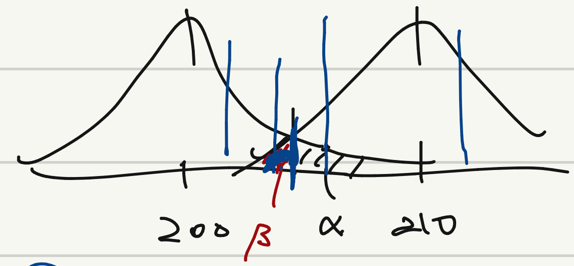
(b) $d = \dfrac{|\mu_1 - \mu_0|}{2\sigma} = \dfrac{10}{2 \cdot 10} = 0.5$

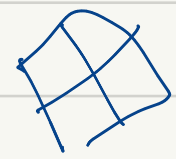
$P(\text{Type II error}) = \beta(0.5, n) < 1 - 0.95 = 0.05$

$\beta = P\left(Z_{0.05} \leq \dfrac{0.5}{10/\sqrt{n}}\right) = 0.05$

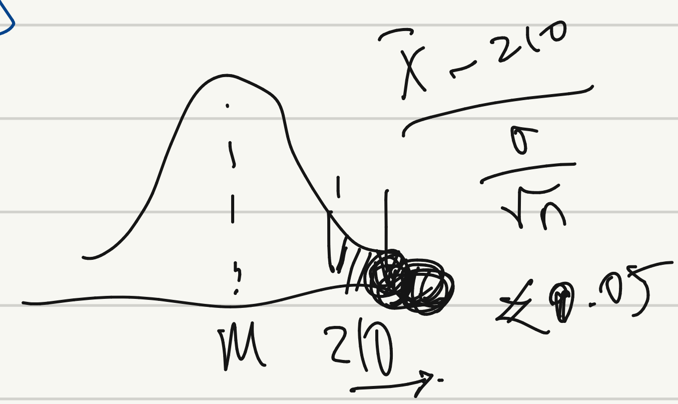
$Z_{0.05} = -2.375 \longrightarrow$ n became negative !! ??

$\sigma^2(x) = 100 \quad \therefore \quad \sigma(x) = 10$





(4) $d = \frac{|\mu_2 - \mu_1|}{2\sigma} = \frac{210 - 200}{20} = 0.5$



$\beta(0.5, n) < 0.05$

$\beta = P\left(Z_{0.05} \le \frac{0.5}{10/n}\right) \le 0.05$

$Z_{0.05} = -2.575$

$-2.575 \cdot \frac{10}{n} = 0.5n$

$-25.75 = 0.5n$

$n =$

$\beta = P($

$d = \frac{210 - 200}{20}$ $\quad n = 9 \quad$ $Z_{0.05} \le \frac{\quad}{\sqrt{\frac{100}{9} + \frac{100}{9}}}$

$X \sim N(210, \sigma = 10^2)$ $\quad H_0 : 200 \quad H_1 : \mu_1 > 200$

$z = (200 - 210) / 10$