1994: The Communication Network Problem

By: Ashlyn Orr & Isaac Dugan

General Assumptions for all problems

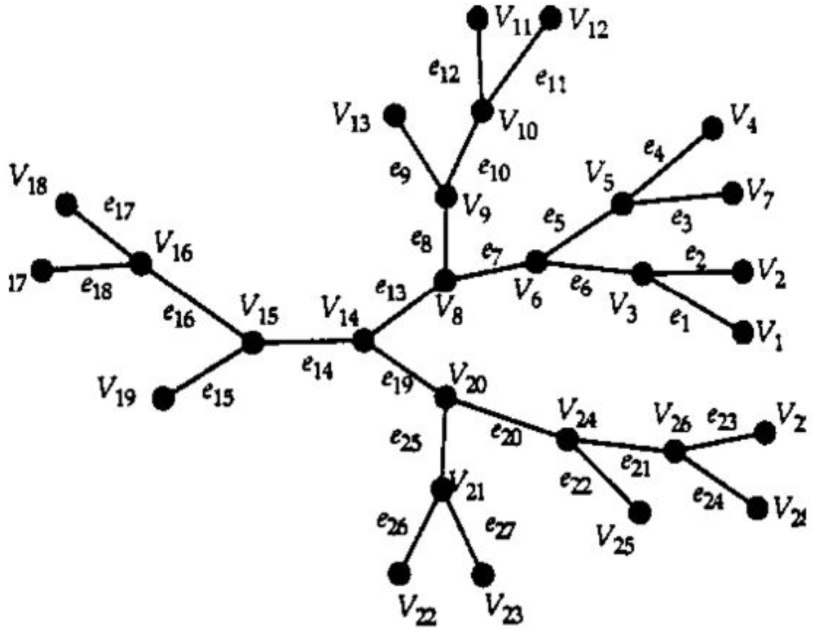
- Vertices/nodes represent departments
- Edges represent transfers between departments
- Transfers use both connecting departments for the transfer to occur
- Transfer times are in minutes
- You can not pause a transfer, it must be completed once it has started

Restatement of Problem (Situation A)

Assumptions:

- Each department can only handle one transfer at a time
- Transfer times between all departments are 1 minute

Problem: Find the fastest possible transfer time for this department.

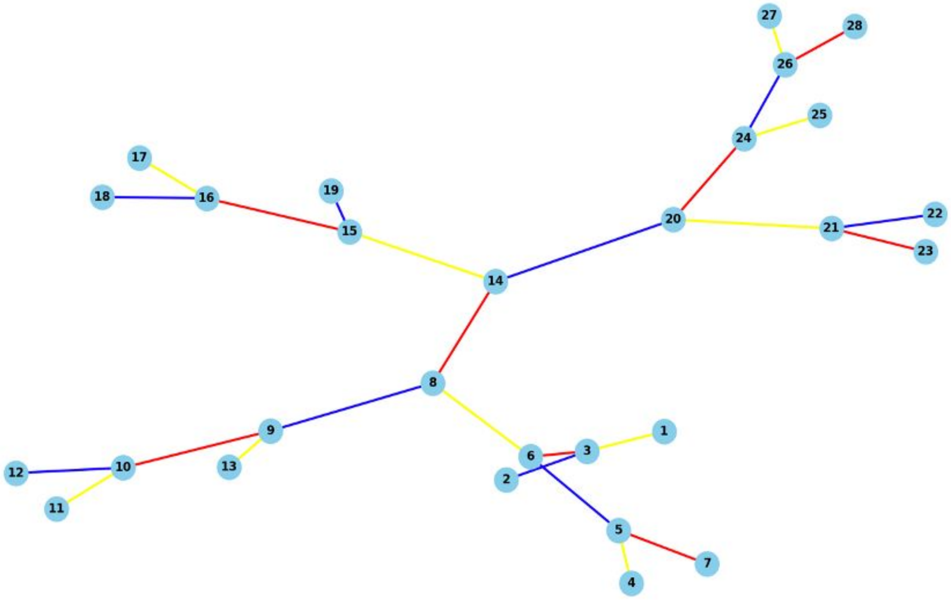


Summary (Situation A)

- We were able to devise an algorithm that goes through the departments and their transfers to create an optimal schedule.
- Because some of the departments are involved in three transfers and only one transfer can occur at a time, the fastest all transfers can be completed is in 3 minutes.
- You can see the schedule we created in on the next slide (each color corresponds to a different minute).
- We have written code to perform this algorithm, so it will be very easy to find a new optimal schedule if we change the number of departments or what transfers must occur.

Model (Situation A)

- The graph to the right shows a schedule that will complete all transfers in 3 minutes
- We can use node 14 to prove that this is an optimal solution
- We implemented an algorithm in python that can solve this for any graph



Algorithm for Situation A

Do for each degree of the graph:

Sort nodes by the number of remaining edges to complete

Using the order above do for each node:

If the node is not currently “engaged”:

Find an “uncompleted” edge on this node that goes to another “unengaged” node with the highest remaining degree

If a eligible destination node is found:

Set both nodes as “engaged”

Set selected edge as “completed”

Set edge color to corresponding color of the “round”

Set all nodes back to “unengaged”

Sensitivity Analysis (Situation A)

	Add	Remove
Edge	This will never decrease the optimal time for all transfers but can increase it by 1 minute if this increases the degree of the entire graph	This will never increase the optimal time for all transfers, but can decrease it by 1 minute if this decreases the degree of the entire graph.
Node	This will never affect the optimal time for all transfer to occur because the newly added node will have no edges/transfers to perform	Removing a node necessarily removes its edges. If removing this node decreases the degree of the graph, it will decrease the optimal time by the same amount that the degree was decreased

Restatement of Problem (Situation B)

- Same network as Situation A
- Times are added to each of the transfers

File transfer time data for situation B.

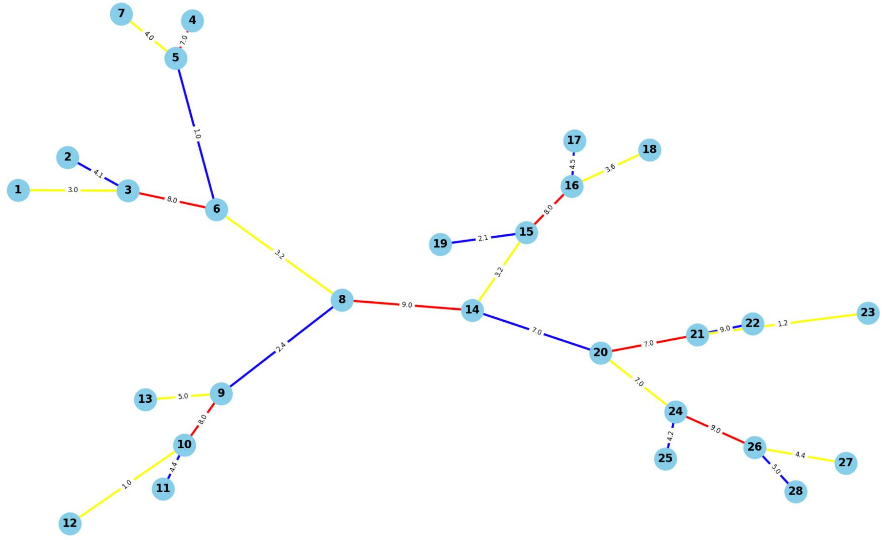
x	1	2	3	4	5	6	7	8	9	10	11	12	13	14
$T(e_x)$	3.0	4.1	4.0	7.0	1.0	8.0	3.2	2.4	5.0	8.0	1.0	4.4	9.0	3.2
x	15	16	17	18	19	20	21	22	23	24	25	26	27	
$T(e_x)$	2.1	8.0	3.6	4.5	7.0	7.0	9.0	4.2	4.4	5.0	7.0	9.0	1.2	

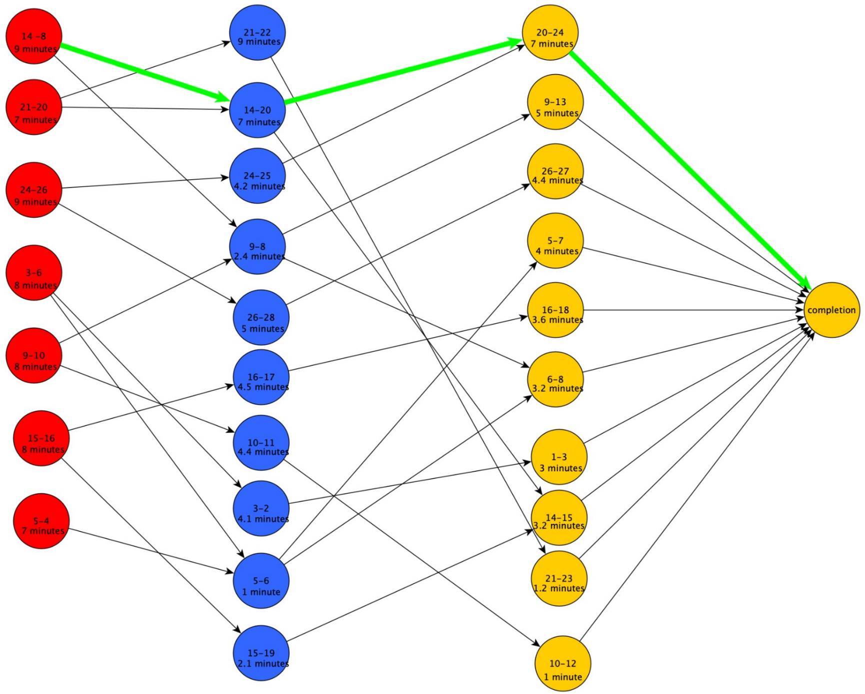
Problem: Find the fastest possible transfer time for this network involving transfer times.

Summary (Situation B)

- In this situation, we have computers and transfers that must occur (building off of situation A), but the transfers now take different amounts of time.
- By modifying our algorithm from situation A, and using a technique called a dependency graph, we were able to find a schedule that is optimal and takes 23 minutes to complete all transfers between departments (you can see the order in which the transfers should take place on the next slide).
- We will have to do some further work to confirm that our process will find an optimal schedule for any combination of transfers and departments.
- Transfers 14 to 8, 14 to 20, and 20 to 24 are bottlenecks, so we should look for ways to decrease the time those specific transfers take.

Model (Situation B)





Algorithm (Situation B)

Do for each degree of the graph:

Sort nodes by the number of remaining edges to complete **and then by the weight of the largest uncompleted edge from that node**

Using the order above do for each node:

If the node is not currently “engaged”:

Find an “uncompleted” edge on this node that goes to another “unengaged” node with the highest remaining degree. **If there is a tie, chose the edge with the highest weight**

If a eligible destination node is found:

Set both nodes as “engaged”

Set selected edge as “completed”

Set edge color to corresponding color of the “round”

Set all nodes back to “unengaged”

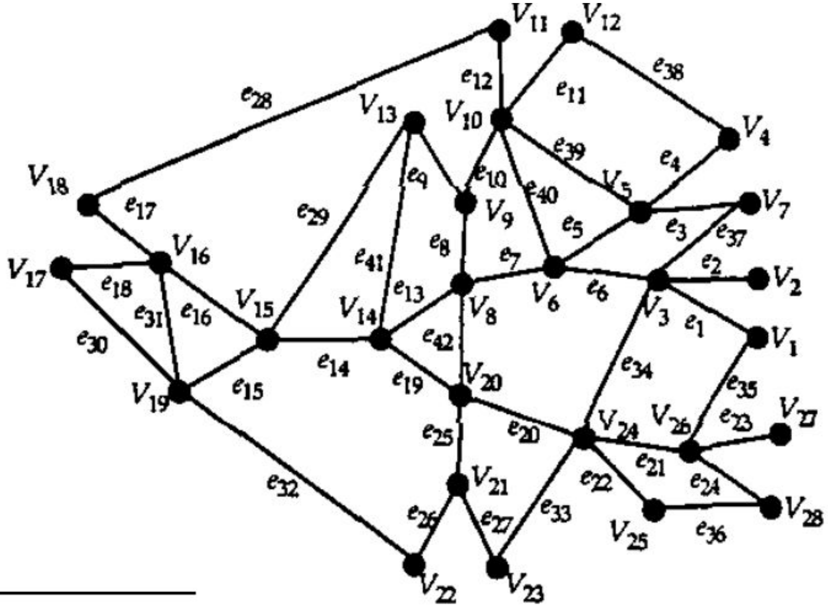
Sensitivity Analysis (Situation B)

Edge	Current Weight	Effect of Increase	Effect of Decrease
Node 14 - Node 8	9 minutes	Any increase of the time it takes to do this transfer will increase the optimal time by the same amount.	A decrease in this time will cause a decrease in the optimal time until this edge is below 7 minutes and is no longer a part of the critical path.
Node 14 - Node 20	7 minutes	Any increase of the time it takes to do this transfer will increase the optimal time by the same amount.	A decrease in this time will cause a decrease in the optimal time until this edge is below 3.2 minutes and is no longer on the critical path
Node 20 - Node 24	7 minutes	Any increase of the time it takes to do this transfer will increase the optimal time by the same amount.	A decrease in this time will cause a decrease in the optimal time until this edge is below 3.2 minutes and is no longer a part of the critical path

Sensitivity analysis done on our binding constraints

Restatement of Problem (Situation C)

- New edges are added, so more transfers must occur (can be seen to the right)
- Some departments have added capacity (can handle more than one transfer at a time) (Table seen below)



Problem: Find shortest possible transfer times.

Computer capacity data for situation C.

y	1	2	3	4	5	6	7	8	9	10	11	12	13	14
$C(V_y)$	2	2	1	1	1	1	1	1	2	3	1	1	1	2

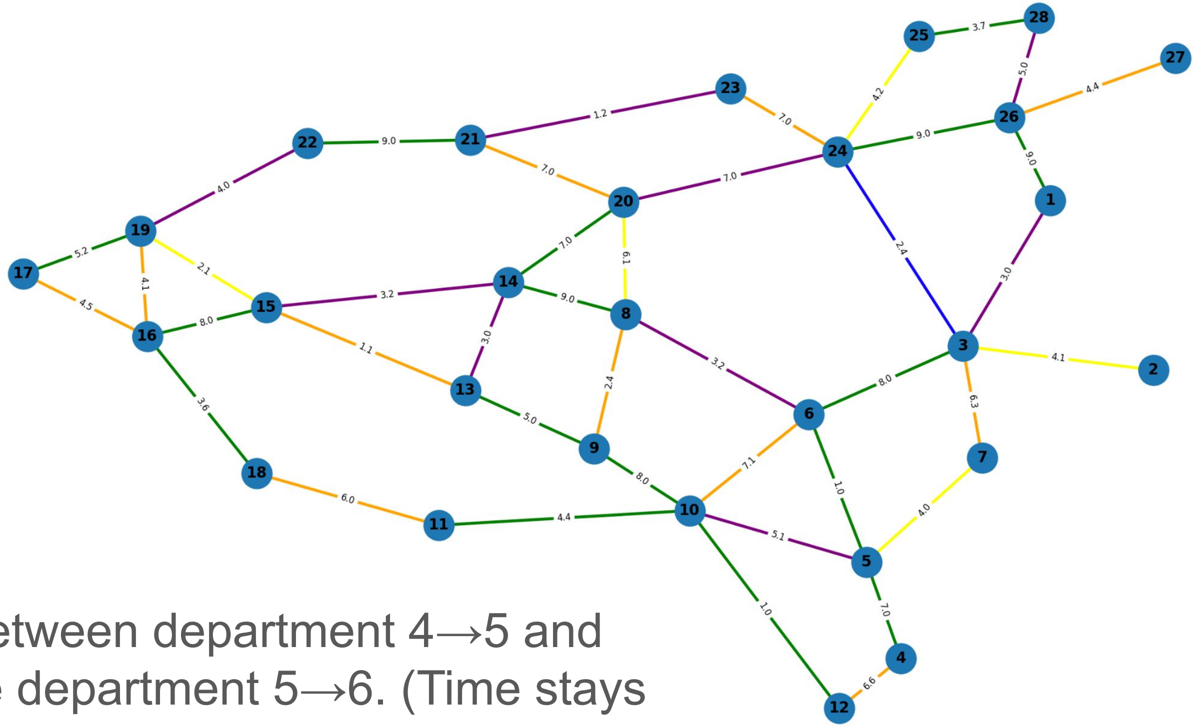
y	15	16	17	18	19	20	21	22	23	24	25	26	27	28
$C(V_y)$	1	2	1	1	1	1	1	2	1	1	1	2	1	1

Summary (Situation C)

- Department 24 must complete 5 transfers and its computer can only handle one at a time. Therefore our optimal time is those edges added together. A computer upgrade for this department would speed up the entire transfer process.
- We were able to find an optimal solution that will complete all transfers in 29.6 minutes.
- Unfortunately, we were unable to create a scheduling framework that would work for any combination of departments, transfers, and computer capacities. If we were to change any of these parameters, we would have to go back to the drawing board and find a new optimal transfer schedule.

Model (Situation C)

- This model is the version that allowed us to get that optimal transfer time (There may be other ways to also get this time).



Important Factors:

- (Green round) Transfers between department 4→5 and department 3→6 go before department 5→6. (Time stays within 9 minutes)
- (Purple round) Transfers between 6→10 start during this round to shave off time
- (Yellow round) Transfers between 8→20 will finish during the blue round so this round is 4.2 minutes long

Sensitivity Analysis (Situation C)

- Since department 24 is our binding constraint we will consider this department.

	Add	Remove
Edge	If we add an edge to department 24 and it still only has the capacity to do one transfer at a time, then the length of that edge will need to be added to our solution time. For instance, if it needs to do another transfer that takes 5 minutes long, then the optimal solution will change from 29.6 to 34.6.	If we remove an edge our new binding constraint would be department 20 because it has four transfers and the capacity of one. Our new optimal time would become 27.1.

Conclusion

- Optimal transfer time:
  - Situation A: 3 minutes
  - Situation B: 23 minutes
  - Situation C: 29.6 minutes
- Future work:
  - Situation A: Alter model to consider transfers of priority (transfers that may need to occur earlier in the day)
  - Situation B: Create a formal proof as to why we can not shorten our transfer time to the optimal solution of 21 minutes
  - Situation C: Search for other solutions that match our optimal solution time to find flexibility within our network