Using the trial wavefunction

$$\Psi_T(\boldsymbol{r}_1, \boldsymbol{r}_2) = e^{-\alpha\omega(r_1^2+r_2^2)/2} e^{ar_{12}/(1+\beta r_{12})}$$

where $r_i = \sqrt{r_{ix}^2 + r_{iy}^2}$, $r_{ij} = |\boldsymbol{r}_i - \boldsymbol{r}_j|$, redefining the exponentials as

$$e^{-\alpha\omega(r_1^2+r_2^2)/2} = e^{O_\alpha}, \quad e^{ar_{12}/(1+\beta r_{12})} = e^{O_\beta}$$

$$\nabla_{i_x}\Psi_T = e^{O_\alpha}e^{O_\beta}\left(-\alpha\omega x_i + (-1)^{i+1}(x_1 - x_2)\frac{a}{r_{12}(1+\beta r_{12})^2}\right)$$

where i and j are particle number. This will look the same for the y-coordinate and one sign change for the second term depending on the particle number. Defining the parenthesis as $[\cdot\cdot]$, taking the Laplacian and dividing with Ψ_T to get the local energy E_L.

$$\frac{\nabla_{i_x}^2\Psi_T}{\Psi_T} = [\cdot\cdot]^2 - \alpha\omega + \frac{a}{r_{12}^2(1+\beta r_{12})^2}\left(r_{12} - \frac{(x_1-x_2)(1-\beta r_{12})}{r_{12}(1+\beta r_{12})}\right)$$

.1 Calculations

.1.1 Local Energy E_L

Derivative of first exponential

$$\frac{\partial}{\partial x_1}e^{-\alpha\omega(r_1^2+r_2^2)/2} = -\alpha\omega x_1 e^{-\alpha\omega(r_1^2+r_2^2)/2}$$
$$\frac{\partial}{\partial x_2}e^{-\alpha\omega(r_1^2+r_2^2)/2} = -\alpha\omega x_2 e^{-\alpha\omega(r_1^2+r_2^2)/2}$$

Derivative of second exponential

$$\frac{\partial}{\partial x_1}e^{ar_{12}/(1+\beta r_{12})} \Rightarrow$$

with change of variables

$$\frac{\partial r_{12}}{\partial x_1} = \frac{x_1 - x_2}{r_{12}}, \quad \frac{\partial r_{12}}{\partial x_2} = -\frac{x_1 - x_2}{r_{12}}$$
$$\frac{\partial r_{12}}{\partial x_1}\frac{\partial}{\partial r_{12}}e^{ar_{12}/(1+\beta r_{12})} = e^{ar_{12}/(1+\beta r_{12})}\frac{(x_1-x_2)a}{r_{12}(1+\beta r_{12})^2}$$
$$\frac{\partial r_{12}}{\partial x_2}e^{ar_{12}/(1+\beta r_{12})} = -e^{ar_{12}/(1+\beta r_{12})}\frac{(x_1-x_2)a}{r_{12}(1+\beta r_{12})^2}$$

The result will be the same for other dimensions too. Collected the term is

$$e^{-\alpha\omega(r_1^2+r_2^2)/2}e^{ar_{12}/(1+\beta r_{12})}\left(-\alpha\omega x_1 + (-1)^{i+1}\frac{a(x_1-x_2)}{r_{12}(1+\beta r_{12})^2}\right)$$

Defining the parenthesis as $[\cdot\cdot]$

$$\frac{\partial}{\partial x_1}[\cdot\cdot] = a\left(\frac{1}{r_{12}(1+\beta r_{12})^2} - \frac{(x_1-x_2)^2}{r_{12}^3(1+\beta r_{12})^2} - \frac{2(x_1-x_2)^2}{r_{12}^2(1+\beta r_{12})^3}\right)$$
$$= \frac{a}{r_{12}^2(1+\beta r_{12})^2}\left(r_{12} - (x_1-x_2)^2\left(\frac{1}{r_{12}} - \frac{2\beta}{r_{12}(1+\beta r_{12})}\right)\right)$$
$$= \frac{a}{r_{12}^2(1+\beta r_{12})^2}\left(r_{12} - (x_1-x_2)^2\left(\frac{1-\beta r_{12}}{r_{12}(1+\beta r_{12})}\right)\right)$$

Comibining these calculations gives the gradient and local energy E_L

$$\frac{\nabla_{i_x}\Psi_T}{\Psi_T} = -\alpha\omega x_1 + (-1)^{i+1}\frac{a(x_1-x_2)}{r_{12}(1+\beta r_{12})^2} \tag{1}$$

$$\frac{\nabla_{i_x}^2\Psi_T}{\Psi_T} = [\cdot\cdot]^2 - \alpha\omega + \frac{a}{r_{12}^2(1+\beta r_{12})^2}\left(r_{12} - (x_1-x_2)^2\left(\frac{1-\beta r_{12}}{r_{12}(1+\beta r_{12})}\right)\right) \tag{2}$$