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李沐阳		
Aho-Corasick Automaton		
<pre>// ac 自动机 inline void build() { int now = root; queue<int> team; fail[root] = root; for (int i = 0; i < 10; ++i) { if (nxt[now][i] == -1) nxt[now][i] = root; else fail[nxt[now][i]] = root, team.push(nxt[now][i]); } while (!team.empty()) { now = team.front(); team.pop(); for (int i = 0; i < 10; ++i) { if (nxt[now][i] == -1) nxt[now][i] = nxt[fail[now]][i]; else { fail[nxt[now][i]] = nxt[fail[now]][i]; team.push(nxt[now][i]); } } } }</pre>		
Bridges and Cut Vertices		
<pre>// 求割边和割点 const int maxn = 200010; int N, M, cnt, Ts, dfn[maxn], low[maxn], nxt[maxn]; int toIt[maxn], side[maxn]; bool bridge[maxn], cut[maxn]; inline void dfs(int now, int fa) { dfn[now] = low[now] = ++Ts; int child = 0; for (int i = side[now]; i; i = nxt[i]) { if (toIt[i] == fa) continue; if (!dfn[toIt[i]]) { dfs(toIt[i], now); ++child; low[now] = min(low[now], low[toIt[i]]); if (low[toIt[i]] > dfn[now]) bridge[i] = true; if (low[toIt[i]] >= dfn[now]) cut[now] = true; } else low[now] = min(low[now], dfn[toIt[i]]); } if (!fa && child == 1) cut[now] = false; }</pre>		

Circle Intersection

```

1 //modified
2 const double eps = 1e-7, pi = acos(-1.0);
3 int N, M; double area[maxn]; // area[k] -> area of intersections >= k.
4
5 struct Point
6 {
7     double x, y;
8     inline double angle() const { return atan2(y, x); }
9 };
10 struct Circle
11 {
12     Point C; double r; int sgn;
13     inline Circle() = default;
14     inline Circle(const Point &C, double _r, int _sgn): C(C), r(_r), sgn(_sgn) {} // sgn 代表该圆的权值, 默认 1
15     friend inline bool operator==(const Circle &a, const Circle &b)
16     {
17         if (dcmp(a.r - b.r)) return false;
18         if (dcmp(a.C.x - b.C.x)) return false;
19         if (dcmp(a.C.y - b.C.y)) return false;
20         if (a.sgn != b.sgn) return false;
21         return true;
22     }
23     friend inline bool operator!=(const Circle &a, const Circle &b) { return !(a == b); }
24 } cir[maxn];
25
26 inline Point rotate(const Point &p, double cost, double sint)
27 {
28     double x = p.x, y = p.y;
29     return Point(x*cost - y*sint, x*sint + y*cost);
30 }
31 inline pair<Point, Point> crosspoint(const Point &ap, double ar, const Point &bp, double br)
32 {
33     double d = (ap - bp).norm(), cost = (ar*ar + d*d - br*br) / (2*ar*d), sint = sqrt(1 - cost*cost);
34     Point v = (bp - ap).unit() * ar;
35     return make_pair(ap + rotate(v, cost, -sint), ap + rotate(v, cost, sint));
36 }
37 inline pair<Point, Point> crosspoint(const Circle &a, const Circle &b) { return crosspoint(a.C, a.r, b.C, b.r); }
38
39 inline bool overlap(const Circle &a, const Circle &b) { return dcmp(a.r - b.r - (a.C - b.C).norm()) >= 0; } // b 是不是在 a 里面
40 inline bool intersect(const Circle &a, const Circle &b)
41 {
42     if (overlap(a, b)) return false;
43     if (overlap(b, a)) return false;
44     return dcmp((a.C - b.C).norm() - a.r - b.r) < 0;
45 }
46
47 struct Event
48 {
49     Point p; double a; int d;
50     inline Event() = default;
51     inline Event(const Point &p, double _a, double _d): p(p), a(_a), d(_d) {}
52     friend inline bool operator<(const Event &a, const Event &b) { return a.a < b.a; }
53 };
54
55 inline void solve()
56 {
57     for (int i = 1; i <= M; ++i) area[i] = 0;
58     for (int i = 1; i <= M; ++i)
59     {
60         int cnt = cir[i].sgn; if (cnt < 0) cnt = 0; vector<Event> event;
61         for (int j = 1; j < i; ++j) if (cir[i] == cir[j]) cnt += cir[j].sgn;
62         for (int j = 1; j <= M; ++j)
63         if (j != i && cir[i] != cir[j] && overlap(cir[j], cir[i])) cnt += cir[j].sgn;
64         for (int j = 1; j <= M; ++j)
65         if (j != i && intersect(cir[i], cir[j]))
66         {
67             pair<Point, Point> res = crosspoint(cir[i], cir[j]); swap(res.first, res.second);
68             double alpha1 = (res.first - cir[i].C).angle(), alpha2 = (res.second - cir[i].C).angle();
69             event.push_back(Event(res.second, alpha2, cir[j].sgn));
70             event.push_back(Event(res.first, alpha1, -cir[j].sgn));
71             cnt += (alpha2 > alpha1) * cir[j].sgn;
72         }
73         if (!event.size()) area[cnt] += pi * cir[i].r * cir[i].r * cir[i].sgn;
74         else
75         {
76             sort(event.begin(), event.end());
77             event.push_back(event.front());
78             for (int j = 0; j + 1 < (int)event.size(); ++j)
79             {
80                 cnt += event[j].d;
81                 area[cnt] += event[j].p.event[j+1].p / 2 * cir[i].sgn;
82                 double alpha = event[j+1].a - event[j].a;
83                 if (alpha < 0) alpha += 2 * pi;
84                 if (!dcmp(alpha)) continue;
85                 area[cnt] += alpha * cir[i].r * cir[i].r / 2 * cir[i].sgn;
86                 area[cnt] += -sin(alpha) * cir[i].r * cir[i].r / 2 * cir[i].sgn;
87             }
88         }
89     }
90 }

```

```

89     }
90 }
91
92 // origin
93 struct Event {
94     Point p;
95     double ang;
96     int delta;
97     Event(Point p = Point(0, 0), double ang = 0, double delta = 0) : p(p), ang(ang), delta(delta) {}
98 };
99 bool operator<(const Event &a, const Event &b) {
100     return a.ang < b.ang;
101 }
102 void addEvent(const Circle &a, const Circle &b, vector<Event> &evt, int &cnt) {
103     double d2 = (a.o - b.o).len2(),
104     dRatio = ((a.r - b.r) * (a.r + b.r) / d2 + 1) / 2,
105     pRatio = sqrt(-(d2 - sqr(a.r - b.r)) * (d2 - sqr(a.r + b.r)) / (d2 * d2 * 4));
106     Point d = b.o - a.o, p = d.rotate(PI / 2),
107     q0 = a.o + d * dRatio + p * pRatio,
108     q1 = a.o + d * dRatio - p * pRatio;
109     double ang0 = (q0 - a.o).ang(),
110     ang1 = (q1 - a.o).ang();
111     evt.push_back(Event(q1, ang1, 1));
112     evt.push_back(Event(q0, ang0, -1));
113     cnt += ang1 > ang0;
114 }
115 bool issame(const Circle &a, const Circle &b) { return sign((a.o - b.o).len()) == 0 && sign(a.r - b.r) == 0; }
116 bool overlap(const Circle &a, const Circle &b) { return sign(a.r - b.r - (a.o - b.o).len()) >= 0; }
117 bool intersect(const Circle &a, const Circle &b) { return sign((a.o - b.o).len() - a.r - b.r) < 0; }
118 Circle c[N];
119 double area[N]; // area[k] -> area of intersections >= k.
120 Point centroid[N]; // k 次圆的质心
121 bool keep[N];
122 void add(int cnt, DB a, Point c) {
123     area[cnt] += a;
124     centroid[cnt] = centroid[cnt] + c * a;
125 }
126
127 void solve(int C) {
128     for (int i = 1; i <= C; ++i) {
129         area[i] = 0;
130         centroid[i] = Point(0, 0);
131     }
132     for (int i = 0; i < C; ++i) {
133         int cnt = 1;
134         vector<Event> evt;
135         for (int j = 0; j < i; ++j) if (issame(c[i], c[j])) ++cnt;
136         for (int j = 0; j < C; ++j) {
137             if (j != i && !issame(c[i], c[j]) && overlap(c[j], c[i])) {
138                 ++cnt;
139             }
140         }
141         for (int j = 0; j < C; ++j) {
142             if (j != i && !overlap(c[j], c[i]) && !overlap(c[i], c[j]) && intersect(c[i], c[j])) {
143                 addEvent(c[i], c[j], evt, cnt);
144             }
145         }
146         if (evt.size() == 0) {
147             add(cnt, PI * c[i].r * c[i].r, c[i].o);
148         } else {
149             sort(evt.begin(), evt.end());
150             evt.push_back(evt.front());
151             for (int j = 0; j + 1 < (int)evt.size(); ++j) {
152                 cnt += evt[j].delta;
153                 add(cnt, det(evt[j].p, evt[j+1].p) / 2, (evt[j].p + evt[j+1].p) / 3);
154                 double ang = evt[j+1].ang - evt[j].ang;
155                 if (ang < 0) {
156                     ang += PI * 2;
157                 }
158                 if (sign(ang) == 0) continue;
159                 double ang0 = evt[j].a, ang1 = evt[j+1].a;
160                 add(cnt, ang * c[i].r * c[i].r / 2, c[i].o +
161                     Point(sin(ang1) - sin(ang0), -cos(ang1) + cos(ang0)) * (2 / (3 * ang) * c[i].r));
162                 add(cnt, -sin(ang) * c[i].r * c[i].r / 2, (c[i].o + evt[j].p + evt[j+1].p) / 3);
163             }
164         }
165     }
166     for (int i = 1; i <= C; ++i)
167     if (sign(area[i])) {
168         centroid[i] = centroid[i] / area[i];
169     }
170 }

```

Counting Integral Points under Straight Line

```

1 // sum_{i=0}^{n-1} (a+b*i)/m
2 inline ll count(ll n, ll a, ll b, ll m)
3 {
4     if (!b) return n * (a / m);
5     else if (a >= m) return n * (a / m) + count(n, a / m, b, m);

```

```

6     else if (b >= m) return (n-1)*n/2*(b/m)+count(n,a,b/m,m);
7     else return count((a+b*n)/m,(a+b*n)/m,m,b);
8 }

```

Cross Points of Circles

// 两圆求交, 需先判定两圆有交

```

1 inline pair <Point,Point> CrossPoint(const Point &ap,double ar,const Point &bp,double br)
2 {
3     double d = (ap-bp).norm();
4     double cost = (ar*ar+d*d-br*br)/(2*ar*d),sint = sqrt(1-cost*cost);
5     Point v = ((bp-ap)/(bp-ap).norm())*ar;
6     return make_pair(ap+rotate(v, cost, -sint), ap+rotate(v, cost, sint));
7 }
8

```

Cross Points of Line and Circle

```

1 // a b 直线两点, o 圆心
2 //若 a b 为线段, 则 0 <= t1,t2 <= 1
3 inline void CrossPoint(const Point &a,const Point &b,const Point &o,double r,Point *ret,int &num)
4 {
5     double X0 = o.x,Y0 = o.y;
6     double X1 = a.x,Y1 = a.y;
7     double X2 = b.x,Y2 = b.y;
8     double dx = X2-X1,dy = Y2-Y1;
9     double A = dx*dx+dy*dy;
10    double B = 2*dx*(X1-X0)+2*dy*(Y1-Y0);
11    double C = (X1-X0)*(X1-X0)+(Y1-Y0)*(Y1-Y0)-r*r;
12    double delta = B*B-4*A*C+eps;
13    num = 0;
14    if (delta >= 0)
15    {
16        double t1 = (-B-sqrt(delta))/(2*A);
17        double t2 = (-B+sqrt(delta))/(2*A);
18        ret[++num] = Point(X1+t1*dx,Y1+t1*dy);
19        ret[++num] = Point(X1+t2*dx,Y1+t2*dy);
20    }
21 }

```

Dominator Tree

// 例题: 询问 i 号点到 N 号点所有必经点编号和

```

1 int N,M,Ts,cnt,side[maxn],nxt[maxn],toit[maxn],dfn[maxn],redfn[maxn],idom[maxn],best[maxn],semi[maxn];
2 int ans[maxn],anc[maxn],fa[maxn],child[maxn],size[maxn]; vector <int> prod[maxn],bucket[maxn],son[maxn];
3
4 inline void init()
5 {
6     cnt = 1; memset(side,0,sizeof side); memset(ans,0,sizeof ans);
7     for (int i = 0;i <= N;++i) prod[i].clear(),bucket[i].clear(),son[i].clear();
8 }
9
10 inline void add(int a,int b) { nxt[++cnt] = side[a]; side[a] = cnt; toit[cnt] = b; }
11
12 inline void dfs(int now)
13 {
14     dfn[now] = ++Ts; redfn[Ts] = now;
15     anc[Ts] = idom[Ts] = child[Ts] = size[Ts] = 0;
16     semi[Ts] = best[Ts] = Ts;
17     for (int i = side[now];i;i = nxt[i])
18     {
19         if (!dfn[toit[i]])
20             dfs(toit[i]),fa[dfn[toit[i]]] = dfn[now];
21         prod[dfn[toit[i]]].push_back(dfn[now]);
22     }
23 }
24
25 inline void compress(int now)
26 {
27     if (anc[anc[now]] != 0)
28     {
29         compress(anc[now]);
30         if (semi[best[now]] > semi[best[anc[now]]])
31             best[now] = best[anc[now]];
32         anc[now] = anc[anc[now]];
33     }
34 }
35
36 inline int eval(int now)
37 {
38     if (!anc[now]) return now;
39     else
40     {
41         compress(now);
42         return semi[best[anc[now]]] >= semi[best[now]]?best[now]:best[anc[now]];
43     }
44 }
45
46 inline void link(int v,int w)
47 {
48     int s = w;

```

```

50 while (semi[best[w]] < semi[best[child[w]]])
51 {
52     if (size[s]+size[child[child[s]]] >= 2*size[child[s]])
53         anc[child[s]] = s,child[s] = child[child[s]];
54     else size[child[s]] = size[s],s = anc[s] = child[s];
55 }
56 best[s] = best[w]; size[v] += size[w];
57 if (size[v] < 2*size[w]) swap(s,child[v]);
58 while (s) anc[s] = v,s = child[s];
59
60 inline void lengauer_tarjan()
61 {
62     memset(dfn,0,sizeof dfn); memset(fa,-1,sizeof fa); Ts = 0;
63     dfs(N); fa[1] = 0;
64     for (int w = Ts;w > 1;--w)
65     {
66         for (auto x:prod[w])
67         {
68             int u = eval(x);
69             if (semi[w] > semi[u]) semi[w] = semi[u];
70         }
71         bucket[semi[w]].push_back(w);
72         link(fa[w],w); if (!fa[w]) continue;
73         for (auto x:bucket[fa[w]])
74         {
75             int u = eval(x);
76             if (semi[u] < fa[w]) idom[x] = u;
77             else idom[x] = fa[w];
78         }
79         bucket[fa[w]].clear();
80     }
81     for (int w = 2;w <= Ts;++w)
82         if (idom[w] != semi[w])
83             idom[w] = idom[idom[w]];
84     idom[1] = 0;
85     for (int i = Ts;i > 1;--i)
86     {
87         if (fa[i] == -1) continue;
88         son[idom[i]].push_back(i);
89     }
90 }
91
92 inline void get_ans(int now)
93 {
94     ans[redfn[now]] += redfn[now];
95     for (auto x:son[now])
96         ans[redfn[x]] += ans[redfn[now]],get_ans(x);
97 }
98
99 int main()
100 {
101     while (scanf("%d %d",&N,&M) != EOF)
102     {
103         init();
104         for (int i = 1,a,b;i <= M;++i)
105             a = gi(),b = gi(),add(a,b);
106         lengauer_tarjan(); get_ans(1);
107         for (int i = 1;i <= N;++i)
108             printf("%d%c",ans[i]," \n"[i == N]);
109     }
110     return 0;
111 }

```

Graham Scanning Algorithm

```

1 //凸包上最大四边面积
2 int N,M; double ans;
3 struct Point { double x,y; }P[maxn],convex[maxn];
4 inline void Graham() {
5     ConvexHull();
6     for (int i = 1;i <= M;++i) convex[i+M] = convex[i];
7     int p1,p2,p3,p4;
8     for (p1 = 1;p1 <= M;++p1) {
9         p2 = p1+1; p3 = p2+1; p4 = p3+1;
10        for (;p3 < p1+M-i;++p3) {
11            Point v = convex[p3]-convex[p1];
12            while (p2 < p3&&fabs((convex[p2]-convex[p1])/v) < fabs((convex[p2+1]-convex[p1])/v)) ++p2;
13            while (p4 < p1+M&&fabs((convex[p4]-convex[p1])/v) < fabs((convex[p4+1]-convex[p1])/v)) ++p4;
14            ans = max(ans,fabs((convex[p2]-convex[p1])/v)+fabs((convex[p4]-convex[p1])/v));
15        }
16    }
17    ans = ans/2;
18 }

```

Gray Code

```

1 //0-2^n-1 的格雷码, 相邻两个二进制位中, 恰好只有一位不同
2 inline vector <int> GrayCreat(int n)
3 {

```

```

4 vector<int> res;
5 for (int i = 0; i < (1<<n); ++i) res.push_back(i^(i>>1));
6 return res;
7 }

```

Half Plane Intersection

```

1 //半平面交, 直线左侧半平面, 注意最后是 tail-head <= 0 还是 tail-head <= 1
2
3 struct Point
4 {
5     double x,y;
6     inline double angle() const { return atan2(y,x); }
7 }P[maxn],pp[maxn],pol[maxn];
8
9 struct Line
10 {
11     Point p,v;
12     inline double slop() const { return v.angle(); }
13     friend inline bool operator<(const Line &a,const Line &b) { return a.slop() < b.slop(); }
14 }line[maxn],qq[maxn];
15
16 inline bool onleft(const Line &L,const Point &p) { return dcmp(L.v/(p-L.p)) > 0; }
17
18 inline int half_plane_intersection()
19 {
20     sort(lines+1,lines+tot+1); //直线按斜率排序
21     int head,tail;
22     qq[head = tail = 1] = lines[1];
23     for (int i = 2; i <= tot; ++i)
24     {
25         while (head < tail&&!onleft(lines[i],pp[tail-1])) --tail;
26         while (head < tail&&!onleft(lines[i],pp[head])) ++head;
27         qq[++tail] = lines[i];
28         if (parallel(qq[tail],qq[tail-1]))
29         {
30             tail--;
31             if (onleft(qq[tail],lines[i].p)) qq[tail] = lines[i];
32         }
33         if (head < tail) pp[tail-1] = crosspoint(qq[tail],qq[tail-1]);
34     }
35     while (head < tail && !onleft(qq[head],pp[tail-1])) --tail;
36     if (tail-head <= 0) return 0;
37     pp[tail] = crosspoint(qq[tail],qq[head]);
38     for (int i = head; i <= tail; ++i) pol[++m] = pp[i]; //半平面交点
39     pol[0] = pol[m];
40     return m;
41 }

```

Hungary

```

1 inline int find(int x)
2 {
3     for (int i = 1; i <= n; ++i)
4         if (v[x][i]&&!used[i])
5         {
6             used[i] = true;
7             if (!from[i]||match(from[i])) { from[i] = x; return true; }
8         }
9     return false;
10 }
11
12 inline int hungry()
13 {
14     int ret = 0; memset(from,-1,sizeof from);
15     for (int i = 1; i <= n; ++i)
16     {
17         memset(used,false,sizeof(used));
18         if (match(i)) ret++;
19     }
20     return ret;
21 }

```

Intersecting Area of Circle and Polygon

```

1 const int maxn = 510;
2 const double eps = 1e-9;
3
4 struct Point { double x,y; }P[maxn],A,B;
5 int N; double K;
6
7 inline double getSectorArea(const Point &a,const Point &b,double r)
8 {
9     double c = (2*r*r-((a-b)*(a-b)))/(2*r*r);
10     double alpha = acos(c);
11     return r*r*alpha/2.0;
12 }
13
14 inline pair<double,double> getSolution(double a,double b,double c)
15 {
16     double delta = b*b-4*a*c;

```

```

17     if (dcmp(delta) < 0) return make_pair(0,0);
18     else return make_pair((-b-sqrt(delta))/(2*a),(-b+sqrt(delta))/(2*a));
19 }
20
21 inline pair<Point,Point> getIntersection(const Point &a,const Point &b,double r)
22 {
23     Point d = b-a;
24     double A = d*d,B = 2*(d*a),C = (a*a)-r*r;
25     pair<double,double> s = getSolution(A,B,C);
26     return make_pair(a+(d*s.first),a+(d*s.second));
27 }
28
29 inline double getPointDist(const Point &a,const Point &b)
30 {
31     Point d = b-a;
32     int sA = dcmp(a*d),sB = dcmp(b*d);
33     if (sA*sB <= 0) return (a/b)/((a-b).norm());
34     else return min(a.norm(),b.norm());
35 }
36
37 double getArea(const Point &a,const Point &b,double r)
38 {
39     double dA = a*a,dB = b*b,dC = getPointDist(a,b),ans = 0;
40     if (dcmp(dA-r*r) <= 0&&dcmp(dB-r*r) <= 0) return (a/b)/2;
41     Point tA = a.unit()*r,tB = b.unit()*r;
42     if (dcmp(dC-r) > 0) return getSectorArea(tA,tB,r);
43     pair<Point,Point> ret = getIntersection(a,b,r);
44     if (dcmp(dA-r*r) > 0&&dcmp(dB-r*r) > 0)
45     {
46         ans += getSectorArea(tA,ret.first,r);
47         ans += (ret.first/ret.second)/2;
48         ans += getSectorArea(ret.second,tB,r);
49         return ans;
50     }
51     if (dcmp(dA-r*r) > 0) return (ret.first/b)/2+getSectorArea(tA,ret.first,r);
52     else return (a/ret.second)/2.0+getSectorArea(ret.second,tB,r);
53 }
54
55 double getArea(int n,Point *p,const Point &c,double r)
56 {
57     double ret = 0;
58     for (int i = 0; i < n; ++i)
59     {
60         int sgn = dcmp((p[i]-c)/(p[(i+1)%n]-c));
61         if (sgn > 0) ret += getArea(p[i]-c,p[(i+1)%n]-c,r);
62         else ret -= getArea(p[(i+1)%n]-c,p[i]-c,r);
63     }
64     return fabs(ret);
65 }

```

Intersection of Line and Convex Hull

```

1 //O(logN)
2 inline double getA(const Node &a)
3 {
4     double ret = atan2(a.y,a.x);
5     if (ret <= -pi/2) ret += 2*pi;
6     return ret;
7 }
8
9 inline int find(double x)
10 {
11     if (x <= w[1]||x >= w[m]) return 1;
12     return upper_bound(w+1,w+m+1,x)-w;
13 }
14
15 inline bool intersect(const Node &a,const Node &b)
16 {
17     int i = find(getA(b-a)),j = find(getA(a-b));
18     if (dcmp((b-a)/(convex[i]-a))*dcmp((b-a)/(convex[j]-a)) > 0) return false;
19     else return true;
20 }
21
22 inline int prework()
23 {
24     ConvexConvex(); convex[m+1] = convex[1];
25     for (int i = 1; i <= m; ++i)
26         w[i] = getA(convex[i+1]-convex[i]);
27 }

```

Kuhn-Munkres Algorithm

```

1 // Truly O(n^3), 最大权匹配
2 // 邻接矩阵, 不能连的边设为-INF, 求最小权匹配时边权取负, 但不能连的还是 -INF, 使用时先对 1 -> n 调用 hungary(), 再 get_ans() 求值
3 struct KM
4 {
5     int v[maxn][maxn],lx[maxn],ly[maxn],match[maxn],way[maxn],slack[maxn];
6     bool used[maxn];
7
8     inline void init()

```

```

9  {
10     for (int i = 1; i <= N; ++i)
11         match[i] = lx[i] = ly[i] = way[i] = 0;
12 }
13
14 inline void hungary(int x)
15 {
16     match[0] = x; int j0 = 0;
17     for (int j = 0; j <= N; ++j)
18         slack[j] = inf, used[j] = false;
19     do
20     {
21         used[j0] = true;
22         int i0 = match[j0], delta = inf, j1 = 0;
23         for (int j = 1; j <= N; ++j)
24             if (!used[j])
25             {
26                 int cur = -w[i0][j] - lx[i0] - ly[j];
27                 if (cur < slack[j])
28                     slack[j] = cur, way[j] = j0;
29                 if (slack[j] < delta)
30                     delta = slack[j], j1 = j;
31             }
32         for (int j = 0; j <= N; ++j)
33         {
34             if (used[j]) lx[match[j]] += delta, ly[j] -= delta;
35             else slack[j] -= delta;
36         }
37         j0 = j1;
38     }
39     while (match[j0]);
40     do
41     {
42         int j1 = way[j0];
43         match[j0] = match[j1];
44         j0 = j1;
45     }
46     while (j0);
47 }
48
49 inline void work() { for (int i = 1; i <= N; ++i) hungary(i); }
50
51 inline int get_ans()
52 {
53     int sum = 0;
54     for (int i = 1; i <= N; ++i)
55     {
56         // if (w[match[i]][i] == -inf) ; //无解
57         if (match[i] > 0) sum += w[match[i]][i];
58     }
59     return sum;
60 }
61 }km;

```

Link Cut Tree

```

1  inline bool isroot(int a) { return ch[fa[a]][0] != a && ch[fa[a]][1] != a; }
2
3  inline void update(int x) { val[x] = (val[ch[x][0]] + val[ch[x][1]]).merge(x); }
4  inline void pushdown(int x)
5  {
6      if (rev[x])
7      {
8          int lrc = ch[x][0], rrc = ch[x][1];
9          swap(lrc, rrc);
10         if (lrc) rev[lrc] ^= 1;
11         if (rrc) rev[rrc] ^= 1;
12         rev[x] = false;
13     }
14 }
15
16 inline void rotate(int x)
17 {
18     int y = fa[x], z = fa[y], l = ch[y][1] == x ? r : l;
19     if (!isroot(y)) ch[z][ch[z][1] == y ? l : r] = x; fa[x] = z;
20     if (ch[x][r]) fa[ch[x][r]] = y; ch[y][l] = ch[x][r];
21     fa[y] = x; ch[x][r] = y; update(y); update(x);
22 }
23
24 inline void splay(int x)
25 {
26     int top = 0, i;
27     for (i = x; !isroot(i); i = fa[i]) stk[++top] = i; stk[++top] = i;
28     while (top) pushdown(stk[top--]);
29     while (!isroot(x))
30     {
31         int y = fa[x], z = fa[y];
32         if (!isroot(y))
33         {
34             if ((ch[y][0] == x) ^ (ch[z][0] == y)) rotate(x);
35             else rotate(y);
36         }
37     }
38 }

```

```

35     }
36     rotate(x);
37 }
38 }
39
40 inline int access(int x)
41 {
42     int t = 0;
43     for (t = 0; x; t = x, x = fa[x])
44         splay(x), ch[x][1] = t, update(x);
45     return t;
46 }
47
48 inline int evert(int x) { int t; rev[t = access(x)] ^= 1; return t; }
49 inline int find(int x)
50 {
51     x = access(x);
52     while (pushdown(x), ch[x][0]) x = ch[x][0];
53     return x;
54 }
55
56 inline void cut(int x, int y)
57 {
58     evert(x); access(y); splay(y);
59     if (ch[y][0] != x || ch[x][1] != 0) return;
60     ch[y][0] = fa[x] = 0; update(x); update(y);
61 }
62
63 inline void link(int x, int y) { fa[evert(x)] = y; }

```

Maximal Matching in General Graphs

```

1  // 接口 int matching(), 返回最大匹配数, G 为邻接矩阵
2  inline void push(int x)
3  {
4      team.push(x); check[x] = true;
5      if (!treec[x]) tra[+cnt] = x, treec[x] = true;
6  }
7
8  inline int root(int x) { return f[x] ? f[x] = root(f[x]) : x; }
9
10 inline void clear()
11 {
12     for (int i = 1; i <= cnt; ++i)
13     {
14         j = tra[i]; father[j] = 0, f[j] = 0;
15         check[j] = treec[j] = false;
16     }
17 }
18
19 inline int lca(int u, int v)
20 {
21     int len = 0;
22     for (; u = father[match[u]]; pathc[path[++len] = u = root(u)] = true;
23     for (; v = father[match[v]];
24     if (pathc[v = root(v)]) break;
25     for (int i = 1; i <= len; ++i)
26         pathc[path[i]] = false;
27     return v;
28 }
29
30 inline void reset(int u, int p)
31 {
32     for (int v; root(u) != p;)
33     {
34         if (!check[v = match[u]]) push(v);
35         if (!f[u]) f[u] = p; if (!f[v]) f[v] = p;
36         u = father[v]; if (root(u) != p) father[u] = v;
37     }
38 }
39
40 inline void flower(int u, int v)
41 {
42     int p = lca(u, v);
43     if (root(u) != p) father[u] = v;
44     if (root(v) != p) father[v] = u;
45     reset(u, p); reset(v, p);
46 }
47
48 inline bool find(int x)
49 {
50     while (!team.empty()) team.pop();
51     cnt = 0; push(x);
52     while (!team.empty())
53     {
54         int i = team.front(); team.pop();
55         for (int j = 1; j <= N; ++j)
56             if (G[i][j] && root(i) != root(j) && match[j] != i)
57             {
58                 if (match[j] && father[match[j]]) flower(i, j);
59                 else if (!father[j])
60                 {
61                     link(i, j);
62                     push(j);
63                 }
64             }
65     }
66 }

```

```

61     father[tra[++cnt] = j] = i; treec[j] = true;
62     if (match[j]) push(match[j]);
63     else
64     {
65         for (int k = i, l = j, p; k; l = p, k = father[l])
66             p = match[k], match[k] = l, match[l] = k;
67         return true;
68     }
69 }
70 }
71 }
72 return false;
73 }
74 }
75 inline int matching()
76 {
77     memset(father, 0, sizeof father); memset(f, 0, sizeof f); memset(path, 0, sizeof path);
78     memset(tra, 0, sizeof tra); memset(match, 0, sizeof match); memset(check, false, sizeof check);
79     memset(treec, false, sizeof treec); memset(pathc, false, sizeof pathc);
80     int ret = cnt = 0;
81     for (int i = 1; i <= N; ++i)
82     {
83         if (match[i]) continue;
84         if (find(i)) ++ret; clear();
85     }
86     return ret;
87 }

```

Merge Split Treap

```

1 // jisuanke17123
2 // Warning: 给指针赋值时, 不要赋 this, 因为 this 是临时变量的地址
3 inline int rand(int n) { int x = rand(); if (x < 0) x = -x; return x%n+1; }
4
5 struct Node
6 {
7     int size, key, val; Node *mn, *ch[2];
8     inline Node *update()
9     {
10         mn = this; size = 1;
11         if (ch[0])
12         {
13             size += ch[0]->size;
14             if (ch[0]->mn->val < mn->val) mn = ch[0]->mn;
15         }
16         if (ch[1])
17         {
18             size += ch[1]->size;
19             if (ch[1]->mn->val < mn->val) mn = ch[1]->mn;
20         }
21         return this;
22     }
23     inline Node() = default;
24     inline Node(int v, Node *_mn) : size(1), key(rand()), val(v), mn(_mn) { ch[0] = ch[1] = NULL; }
25 } pool[max*100/4], *root[maxn], *cur;
26 struct Status
27 {
28     int l, r; ll val;
29     inline Status() = default;
30     inline Status(int _l, int _r, ll _val) : l(_l), r(_r), val(_val) {}
31     friend inline bool operator <(const Status &a, const Status &b) { return a.val > b.val; }
32 };
33
34 inline int sz(const Node *x) { if (x == NULL) return 0; else return x->size; }
35
36 inline Node *newnode(int v = 0) { *cur = Node(v, cur); return cur++; }
37
38 Node *insert(Node *p, Node *q)
39 {
40     if (p == NULL && q == NULL) return NULL;
41     if (p == NULL || q == NULL) return p?p:q;
42     Node u = NULL;
43     if (rand(sz(p)+sz(q)) < sz(p))
44         u = p, u->ch[1] = insert(u->ch[1], q);
45     else u = q, u->ch[0] = insert(p, u->ch[0]);
46     return u->update();
47 }
48
49 Node *merge(Node *p, Node *q)
50 {
51     if (p == NULL && q == NULL) return NULL;
52     if (p == NULL || q == NULL) return p?p:q;
53     Node u = newnode();
54     if (rand(sz(p)+sz(q)) < sz(p))
55         *u = *p, u->ch[1] = merge(u->ch[1], q);
56     else *u = *q, u->ch[0] = merge(p, u->ch[0]);
57     return u->update();
58 }
59
60 Node *split(Node *u, int l, int r)

```

```

61 {
62     if (l > r || u == NULL) return 0;
63     Node *x = NULL;
64     if (l == 1 && r == sz(u))
65     {
66         x = newnode(); *x = *u;
67         return x->update();
68     }
69     int lsz = sz(u->ch[0]);
70     if (r <= lsz) return split(u->ch[0], l, r);
71     if (l > lsz+1) return split(u->ch[1], l-lsz-1, r-lsz-1);
72     x = newnode(); *x = *u;
73     x->ch[0] = split(u->ch[0], l, lsz);
74     x->ch[1] = split(u->ch[1], l, r-lsz-1);
75     return x->update();
76 }
77
78 int get_pos(Node *rt, Node *mn)
79 {
80     if (rt == mn) return sz(rt->ch[0]);
81     else if (rt->ch[0] && rt->ch[0]->mn == mn)
82         return get_pos(rt->ch[0], mn);
83     else return sz(rt->ch[0]) + 1 + get_pos(rt->ch[1], mn);
84 }
85 inline pair <int, int> Qmin(Node *rt, int l, int r)
86 {
87     if (l > r) return make_pair(-1, -1);
88     Node *v = split(rt, l, r);
89     auto ret = make_pair(v->mn->val, get_pos(v, v->mn)+1);
90     return ret;
91 }
92 inline int get(Node *u, int x) { return split(u, x, x)->val; }
93
94 inline void init() { cur = pool; }
95
96 int main()
97 {
98     struct timeb ttt; ftime(&ttt);
99     srand(ttt.millitm+ttt.time*1000);
100 }

```

Modui Algorithm on Tree

```

1 // 询问树上路径元素 mes, inc dec 复杂度不对, 需要用线段树/set(带 log) 或者分块 (修改 O(1))
2 // 若包括 lca, 每组询问需要把 lca 拆 (inc) 上去。
3 const int Size = 337, maxn = 200010;
4 int N, Q, cnt, nxt[maxn], side[maxn], len[maxn], tolt[maxn], f[maxn][20], key[maxn], timestamp;
5 int dep[maxn], L[maxn], R[maxn], dfn[maxn], ans[maxn], exist[maxn], show[maxn], res;
6 struct Node
7 {
8     int a, b, c, id;
9     Node() = default;
10     Node(int _a, int _b, int _c = 0, int _id = 0) : a(_a), b(_b), c(_c), id(_id) {}
11     inline void read(int i)
12     {
13         id = i; scanf("%d %d", &a, &b); c = lca(a, b);
14         if (c == a || c == b) { if (a != c) swap(a, b); a = L[c]+1; b = L[b]; }
15         else { if (L[a] > L[b]) swap(a, b); a = R[a]; b = L[b]; }
16     }
17     friend inline bool operator <(const Node &x, const Node &y) { return x.b < y.b; }
18 } query[maxn];
19
20 inline bool cmp(const Node &x, const Node &y) { return x.a < y.a; }
21 inline void work()
22 {
23     int l = 1, r = 0;
24     for (int i = 1; i <= Q; ++i) {
25         while (r < query[i].b) {
26             show[dfn[++r]]++;
27             if (show[dfn[r]] == 2) dec(dfn[r]); else inc(dfn[r]); }
28         while (l > query[i].a) {
29             show[dfn[l--]]++;
30             if (show[dfn[l]] == 2) dec(dfn[l]); else inc(dfn[l]); }
31         while (r > query[i].b) {
32             if (show[dfn[r]] == 1) dec(dfn[r]); else inc(dfn[r]);
33             show[dfn[r--]]--; }
34         while (l < query[i].a) {
35             if (show[dfn[l]] == 1) dec(dfn[l]); else inc(dfn[l]);
36             show[dfn[l++]]--; }
37         ans[query[i].id] = res; }
38 }
39
40 int main() {
41     dfs(1);
42     for (int i = 1; i <= Q; ++i) query[i].read(i);
43     sort(query+1, query+Q+1, cmp);
44     for (int i = 1, j; i <= Q; i = j) {
45         for (j = i; j <= Q && query[j].a == query[i].a <= Size; ++j);
46         sort(query+i+1, query+j);

```

```

47     }
48     work();
49     for (int i = 1; i <= Q; ++i) printf("%d\n", ans[i]);
50 }

```

Numerical Integration

```

1 //self-adapt simpson
2 inline long double simpson(long double l, long double r, long double mid, long double Cl, long double Cr, long double Cm)
3 {
4     long double tCl = calc((l+mid)/2), tCr = calc((mid+r)/2);
5     long double ans = (r-l)*(Cl+Cr+4*Cm)/6, lans = (mid-l)*(Cl+Cm+4*tCl)/6, rans = (r-mid)*(Cr+Cm+4*tCr)/6;
6     if (r-l <= 1e-3&&fabs(lans+rans-ans)<eps) return ans;
7     // if (dep > 10&&fabs(lans+rans-ans)<eps) return ans;
8     else return simpson(l, mid, (l+mid)/2, Cl, Cm, tCl) + simpson(mid, r, (mid+r)/2, Cr, Cm, tCr);
9 }

```

Planar Graph

```

1 // 包括平面图转对偶图
2 inline int dcmp(double a)
3 {
4     if (fabs(a) <= eps) return 0;
5     else if (a > 0) return 1;
6     else return -1;
7 }
8 struct Point
9 {
10     double x, y;
11     inline Point(double _x = 0, double _y = 0):x(_x), y(_y) {}
12     inline void read() { x = gi(), y = gi(); }
13     friend inline Point operator-(const Point &a, const Point &b) { return Point(a.x-b.x, a.y-b.y); }
14     friend inline double operator/(const Point &a, const Point &b) { return a.x*b.y-a.y*b.x; }
15     inline double angle() { return atan2(y, x); }
16 } pp[maxn];
17 struct Segment
18 {
19     int from, to, h, id, sur; // from 号点到 to 号点, h 为边权, suf 为这条有向边推出来的平面编号。
20     inline Segment(int _from = 0, int _to = 0, int _h = 0, int _id = 0, int _sur = 0):from(_from), to(_to), h(_h), id(_id), sur(_sur) {}
21     friend inline bool operator<(const Segment &a, const Segment &b) { return (pp[a.to]-pp[a.from]).angle() < (pp[b.to]-pp[b.from]).angle(); }
22 } edge[maxn*2];
23 vector<int> G[maxn];
24
25 inline void nadd(int u, int v, int h) { ++ncnt; G[u].push_back(ncnt); edge[ncnt] = Segment(u, v, h); }
26 inline void nins(int u, int v, int h) { nadd(u, v, h); nadd(v, u, h); }
27
28 inline bool cmp(int a, int b) { return edge[a] < edge[b]; }
29
30 inline void find_surface()
31 {
32     for (int i = 1; i <= N; ++i) sort(G[i].begin(), G[i].end(), cmp);
33     for (int i = 1; i <= N; ++i)
34     {
35         int nn = G[i].size();
36         for (int j = 0; j < nn; ++j)
37             edge[G[i][j]].id = j;
38     }
39     for (int i = 2; i <= ncnt; ++i)
40     {
41         if (!edge[i].sur)
42         {
43             ++tot; int j = i, p, nn; vector<Point> vec;
44             while (!edge[j].sur)
45             {
46                 edge[j].sur = tot; vec.push_back(pp[edge[j].from]);
47                 p = edge[j].to; nn = G[p].size();
48                 j ^= 1; j = G[p][!edge[j].id+1]%nn;
49             }
50             double res = 0; nn = vec.size();
51             for (j = 0; j < nn; ++j)
52                 res += (vec[j]-vec[0])/(vec[(j+1)%nn]-vec[0]);
53             res /= 2; space[tot] = res; // 第 tot 个平面的有向面积, 外面的大平面面积为正, 其余为负, 大平面可能有多 (平面图不连通)
54         }
55         // 开始建边, 以 mst 为例
56         for (int i = 2; i <= ncnt; ++i)
57         {
58             if (space[edge[i].sur]<0&&space[edge[i^1].sur]<0)
59                 arr[++all] = (ARR) { edge[i].sur, edge[i^1].sur, edge[i].h };
60             else arr[++all] = (ARR) { edge[i].sur, edge[i^1].sur, inf };
61         }
62     }
63 // 点定位
64 struct Scan
65 {
66     double x, y; int bel, sign;
67     inline Scan(double _x = 0, double _y = 0, int _bel = 0, int _sign = 0):x(_x), y(_y), bel(_bel), sign(_sign) {}
68     friend inline bool operator<(const Scan &a, const Scan &b)
69     {
70         if (a.x != b.x) return a.x < b.x;
71         else return a.sign > b.sign;
72     }
73 }

```

```

72     }
73     }bac[maxn*4];
74
75 struct Splay
76 {
77     int num, root, ch[maxn][2], fa[maxn], key[maxn]; queue<int> team;
78
79     inline int newnode()
80     {
81         int ret;
82         if (team.empty()) ret = ++num;
83         else ret = team.front(), team.pop();
84         fa[ret] = ch[ret][0] = ch[ret][1] = 0;
85         return ret;
86     }
87
88     inline void init() { num = 0; root = newnode(); key[root] = cnt; }
89
90     inline void rotate(int x)
91     {
92         int y = fa[x], z = fa[y], l = ch[y][1] == x ? r : l;
93         if (z != 0) ch[z][ch[z][1] == y] = x;
94         fa[x] = z; fa[y] = x; fa[ch[x][r]] = y;
95         ch[y][1] = ch[x][r]; ch[x][r] = y;
96     }
97
98     inline void splay(int x)
99     {
100         while (fa[x] != 0)
101         {
102             int y = fa[x], z = fa[y];
103             if (fa[y] != 0)
104             {
105                 if ((ch[y][0] == x)^(ch[z][0] == y)) rotate(x);
106                 else rotate(y);
107             }
108             rotate(x);
109         }
110         root = x;
111     }
112
113     inline int lower_bound(const Point &p)
114     {
115         int now = root, ret = 0;
116         while (now)
117         {
118             int k = key[now];
119             if ((p-pp[edge[k].from])/(pp[edge[k].to]-pp[edge[k].from]) >= 0)
120                 ret = k, now = ch[now][0];
121             else now = ch[now][1];
122         }
123         return ret;
124     }
125
126     inline int find(int w)
127     {
128         int now = root;
129         double x = pp[edge[w].to].x, y = pp[edge[w].to].y;
130         double ang = (pp[edge[w].to] - pp[edge[w].from]).angle();
131         while (now)
132         {
133             int k = key[now];
134             if (k == w) return now;
135             NODE p = pp[edge[k].to] - pp[edge[k].from], q = pp[edge[k].from];
136             double xx = x - q.x, yy = y - q.y, px = p.x*p.y;
137             if (equal(yy, yy))
138             {
139                 double t = p.angle();
140                 now = ch[now][ang < t];
141             }
142             else now = ch[now][y > yy];
143         }
144     }
145
146     inline void erase(int w)
147     {
148         int p = find(w);
149         while (ch[p][0] || ch[p][1])
150         {
151             if (ch[p][0])
152             {
153                 rotate(ch[p][0]);
154                 if (p == root) root = fa[p];
155             }
156             else
157             {
158                 rotate(ch[p][1]);
159                 if (p == root) root = fa[p];
160             }
161         }
162     }
163 }

```



```

161     }
162     team.push(p);
163     ch[fa[p]][ch[fa[p]][1] == p] = 0;
164     fa[p] = 0;
165 }
166
167 inline void insert(int w)
168 {
169     int now = root, pre;
170     double x = pp[edge[w].from].x, y = pp[edge[w].from].y;
171     double ang = (pp[edge[w].to] - pp[edge[w].from]).angle();
172     double xx, yy;
173     while (true)
174     {
175         int k = key[now];
176         NODE p = pp[edge[k].to] - pp[edge[k].from], q = pp[edge[k].from];
177         xx = x - q.x, yy = q.y + xx/p.x*p.y;
178         if (equal(yy, y))
179         {
180             double t = p.angle();
181             pre = now, now = ch[now][ang > t];
182             if (!now)
183             {
184                 now = newnode();
185                 fa[now] = pre; ch[pre][ang > t] = now; key[now] = w;
186                 break;
187             }
188         }
189         else
190         {
191             pre = now, now = ch[now][y > yy];
192             if (!now)
193             {
194                 now = newnode();
195                 fa[now] = pre; ch[pre][y > yy] = now; key[now] = w;
196                 break;
197             }
198         }
199     }
200     splay(now);
201 }
202 };
203
204 inline void locate()
205 {
206     int mn = 0;
207     for (int i = 2; i <= cnt; i += 2)
208     {
209         if (!dcmp(pp[edge[i].from].x - pp[edge[i].to].x)) continue;
210         bac[+nn] = Scan(pp[edge[i].from].x, pp[edge[i].from].y, 1, 2);
211         bac[+nn] = Scan(pp[edge[i].to].x, pp[edge[i].to].y, 1, 3);
212     }
213     scanf("%d", &T); double x, y;
214     // 查询 (x, y) 所在平面
215     for (int i = 1; i <= T; ++i)
216     {
217         scanf("%lf %lf", &x, &y);
218         bac[+nn] = Scan(x, y, 1, 0);
219         scanf("%lf %lf", &x, &y);
220         bac[+nn] = Scan(x, y, 1, 1);
221     }
222     sort(bac+1, bac+nn+1);
223     pp[+n] = Point(-oo, -oo); pp[+n] = (oo, oo);
224     edge[++cnt] = Edge(n-1, n);
225     S.init(); int p;
226     for (int i = 1; i <= nn; ++i)
227     {
228         if (bac[i].sign == 2 || bac[i].sign == 3)
229         {
230             if (bac[i].sign == 2) S.insert(bac[i].bel);
231             else S.erase(bac[i].bel);
232         }
233         else
234         {
235             p = S.lower_bound(Point(bac[i].x, bac[i].y));
236             query[bac[i].bel][bac[i].sign] = edge[p].sur;
237         }
238     }
239 }

```

Point Biconnected Component

```

1 // Source: HackerRank - bonnie-and-clyde
2 const int maxn = 400010;
3 int N, M, Q, cnt = 1, side[maxn], toIt[maxn], nIt[maxn], f[maxn][25], father[maxn], low[maxn];
4 int tot, dep[maxn], dfn[maxn], nside[maxn], ntoit[maxn], nnxt[maxn]; bool cut[maxn];
5 stack<int> S; vector<int> bel[maxn], bcc[maxn]; bool vis[maxn];
6
7 inline int find(int a) { if (father[a] != a) father[a] = find(father[a]); return father[a]; }
8

```

```

9 inline void add(int a, int b) { nIt[++cnt] = side[a]; side[a] = cnt; toIt[cnt] = b; }
10 inline void ins(int a, int b) { add(a, b); add(b, a); }
11
12 inline void nadd(int a, int b) { nnxt[++cnt] = nside[a]; nside[a] = cnt; ntoit[cnt] = b; }
13 inline void nins(int a, int b) { nadd(a, b); nadd(b, a); }
14
15 inline void tj(int now, int fa)
16 {
17     dfn[now] = low[now] = ++cnt; int child = 0;
18     for (int i = side[now]; i; i = nIt[i])
19     {
20         if (toIt[i] == fa) continue;
21         if (!dfn[toIt[i]])
22         {
23             S.push(i >> 1); tj(toIt[i], now); ++child;
24             low[now] = min(low[now], low[toIt[i]]);
25             if (low[toIt[i]] >= dfn[now])
26             {
27                 cut[now] = true; ++tot;
28                 while (true)
29                 {
30                     int t = S.top(); S.pop();
31                     bel[toIt[t << 1]].push_back(tot); bel[toIt[t << 1]].push_back(tot);
32                     bcc[tot].push_back(toIt[t << 1]); bcc[tot].push_back(toIt[t << 1]);
33                     if (t == (i >> 1)) break;
34                 }
35             }
36             else low[now] = min(low[now], dfn[toIt[i]]);
37         }
38         if (!fa && child == 1) cut[now] = false;
39     }
40
41     inline void build()
42     {
43         vector<int> cuts; cnt = 1;
44         for (int i = 1; i <= tot; ++i)
45         {
46             sort(bcc[i].begin(), bcc[i].end());
47             bcc[i].erase(unique(bcc[i].begin(), bcc[i].end()), bcc[i].end());
48         }
49         for (int i = 1; i <= N; ++i) if (cut[i]) cuts.push_back(i);
50         for (auto x: cuts)
51         {
52             sort(bel[x].begin(), bel[x].end());
53             bel[x].erase(unique(bel[x].begin(), bel[x].end()), bel[x].end());
54             ++tot; for (auto y: bel[x]) nins(tot, y);
55             bel[x].clear(); bel[x].push_back(tot); bcc[tot].push_back(x);
56         }
57     }
58
59     inline bool check(int u, int v, int w)
60     {
61         if (find(u) != find(v) || find(v) != find(w)) return false;
62         if (u == w || v == w) return true; if (u == v) return false;
63         int uu = bel[u][0], vv = bel[v][0], ww = bel[w][0], su, sv;
64         if (uu == vv || vv == ww) return true;
65         if (lca(uu, vv) == ww) su = jump(uu, dep[uu] - dep[ww] - 1); else su = f[ww][0];
66         if (lca(vv, ww) == uu) sv = jump(vv, dep[vv] - dep[ww] - 1); else sv = f[ww][0];
67         if (su == sv)
68         {
69             if (!cut[w]) return false;
70             else
71             {
72                 if (su == uu || sv == vv) return true; int ssu, ssv;
73                 if (lca(su, uu) == su) ssu = jump(uu, dep[uu] - dep[su] - 1); else ssu = f[su][0];
74                 if (lca(sv, vv) == sv) ssv = jump(vv, dep[vv] - dep[sv] - 1); else ssv = f[sv][0];
75                 if (ssu == ssv) return false; else return true;
76             }
77         }
78         else return true;
79     }
80
81     int main()
82     {
83         N = gi(); M = gi(); Q = gi();
84         for (int i = 1; i <= N; ++i) father[i] = i;
85         for (int i = 1, a, b; i <= M; ++i)
86         {
87             ins(a = gi(), b = gi());
88             a = find(a), b = find(b);
89             if (a != b) father[a] = b;
90         }
91         cnt = 0; for (int i = 1; i <= N; ++i) if (!dfn[i]) tj(i, 0);
92         build(); for (int i = 1; i <= N; ++i) if (!vis[i]) dfs(i);
93         while (Q--)
94         {
95             int u = gi(), v = gi(), w = gi();
96             if (check(u, v, w)) puts("YES"); else puts("NO");
97         }

```



```

98     return 0;
99 }

```

Pollard Rho Algorithm

```

1  const int prime[] = {0,2,3,5,7,11,13,17,19,23,29,31};
2
3  inline ll mul(ll a,ll b,ll p) { return (a+b-((ll)((ld)a/p+b*1e-3)*p)+p)%p; }
4
5  inline bool check(ll m)
6  {
7      if (m <= 2) return m == 2;
8      ll tmp = m-1; int t = 0;
9      while (!(tmp%1)) ++t,tmp >= 1;
10     for (int i = 1;i <= 10;++i)
11     {
12         int a = prime[i];
13         if (a == m) return true;
14         ll w = qsm(a,tmp,m);
15         for (int it = 1;it <= t;++it)
16         {
17             ll pf = mul(w,w,m);
18             if (pf == 1&&(w != 1&&w != m-1)) return false;
19             w = pf;
20         }
21         if (w != 1) return false;
22     }
23     return true;
24 }
25 inline void rho(ll m)
26 {
27     if (check(m)) { fac[++nn] = m; return; }
28     while (true)
29     {
30         ll X = (ll)rand()*rand()%(m-1)+1,Y = X;
31         ll c = (ll)rand()*rand()%(m-1)+1; int i,j;
32         for (i = j = 2;++i)
33         {
34             X = (mul(X,X,m)+c) % m;
35             ll d = __gcd(abs(X-Y),m);
36             if (1 < d&&d < m) { rho(d),rho(m/d); return; }
37             if (X == Y) break; if (i == j) Y = X,j <= i;
38         }
39     }
40 }
41 inline void factor(ll m) { nn = 0; if (m > 1) rho(m); sort(fac+1,fac+nn+1); }

```

Polygon Class

```

1  inline bool PointOnSegment(const Point &t,const Point &a,const Point &b)
2  {
3      if (dcmp((t-a)/(b-a))) return false;
4      if (dcmp((t-a)*(t-b)) > 0) return false;
5      return true;
6  }
7
8  inline bool in(const Point &a,const Point &b,const Point &c)
9  {
10     double alpha = a.angle(),beta = b.angle(),gamma = c.angle(); // angle 返回 [0,2pi]
11     if (alpha <= beta) return dcmp(gamma-alpha) > 0&&dcmp(beta-gamma) > 0;
12     else return dcmp(gamma-alpha) > 0||dcmp(beta-gamma) > 0;
13 }
14
15 struct Polygon
16 {
17     int n; Point a[maxn];
18     inline Polygon() {}
19     inline void read()
20     {
21         n = g<i>();
22         for (int i = 0;i < n;++i) a[i].read();
23         a[n] = a[0];
24     }
25     // 点是否在多边形内部, 内部为 1, 外部为 0, 边界为 2, 不管顺时针
26     inline int Point_In(const Point &t) const
27     {
28         int num = 0;
29         for (int i = 0;i < n;++i)
30         {
31             if (PointOnSegment(t,a[i],a[i+1])) return 2;
32             int k = dcmp((a[i+1]-a[i])/(t-a[i]));
33             int d1 = dcmp(a[i].y-t.y),d2 = dcmp(a[i+1].y-t.y);
34             if (k > 0&&d1 <= 0&&d2 > 0) ++num;
35             if (k < 0&&d2 <= 0&&d1 > 0) --num;
36         }
37         return num != 0;
38     }
39     // 判断多边形的方向, true 为逆时针, false 为顺时针, 用叉积判断哪个多
40     inline bool CalculateClockDirection()
41     {

```

```

42     int res = 0;
43     for (int i = 0;i < n;++i)
44     {
45         int p = i-1,s = i+1,sgn;
46         if (p < 0) p += n; if (s >= n) s -= n;
47         sgn = dcmp((a[i]-a[p])/(a[s]-a[i]));
48         if (sgn) { if (sgn > 0) ++res; else --res; }
49     }
50     return res > 0;
51 }
52 // 判断多边形方向, true 为逆时针, false 为顺时针, 用 Green 公式
53 inline bool CalculateClockDirection()
54 {
55     double res = 0;
56     for (int i = 0;i < n;++i)
57         res -= 0.5*(a[i+1].y+a[i].y)*(a[i+1].x-a[i].x);
58     return res > 0;
59 }
60
61 // 线段 ab 是否有点严格在多边形内部, 先判断线段是否与多边形边界有交, 再判断 ab 是否与多边形有交, 内部 false, 外部 true
62 inline bool can(int ia,int ib)
63 {
64     Point a = P[ia],b = P[ib],v = b-a;
65     if (in(P[ia+1]-a,P[ia+1]-a,b-a)||in(P[ib+1]-b,P[ib+1]-b,a-b)) return false;
66     for (register int i = 0;i < N;++i)
67     {
68         if (dcmp(v/(P[i]-a))*dcmp(v/(P[i+1]-a)) < 0&&dcmp(vec[i]/(a-P[i]))*dcmp(vec[i]/(b-P[i])) < 0)
69             return false;
70         if (PointOnSegment(a,P[i],P[i+1])||PointOnSegment(b,P[i],P[i+1])) return false;
71         if (PointOnSegment(P[i],a,b)||PointOnSegment(P[i+1],a,b)) return false;
72     }
73     return true;
74 }
75 }poly;

```

Simplex

```

1  // 有 n 个实数变量  $x_1, x_2, \dots, x_n$  和 m 条约束, 其中第 i 条约束形如  $\sum_{j=1}^n a_{i,j} x_j \leq b_i$ .
2  // 此外这 n 个变量需要满足非负性限制,  $x_j \geq 0$ .
3  // 在满足上述所有条件的情况下, 你需要指定每个变量  $x_j$  的取值, 使得目标函数  $F = \sum_{j=1}^n c_j x_j$  的值最大.
4  // 第一行三个正整数 n, m, t. 其中 t ∈ {0,1}.
5  // 第二行有 n 个整数 c1, c2, ..., cn, 整数间均用一个空格分隔.
6  // 接下来 m 行, 每行代表一条约束, 其中第 i 行有 n+1 个整数 ai1, ai2, ..., ain, bi, 整数间均用一个空格分隔.
7  // 如果不存在满足所有约束的解, 仅输出一行 "Infeasible".
8  // 如果对于任意的 M, 都存在一组解使得目标函数的值大于 M, 仅输出一行 "Unbounded".
9  // 否则, 第一行输出一个实数, 表示目标函数的最大值 F.
10 // 如果 t = 1, 那么你还需要输出第二行, 用空格隔开的 n 个非负实数, 表示此时  $x_1, x_2, \dots, x_n$  的取值, 如有多组方案请任意输出其中一个.
11 // uoj 179
12 const int maxn = 30;
13
14 int N,M,op,tot,q[maxn],idx[maxn],idy[maxn]; double a[maxn][maxn],A[maxn];
15
16 inline void pivot(int x,int y)
17 {
18     swap(idy[x],idx[y]);
19     double tmp = a[x][y]; a[x][y] = 1/a[x][y];
20     for (int i = 0;i <= N;++i) if (y != i) a[x][i] /= tmp;
21     tot = 0; for (int i = 0;i <= N;++i) if (i != y&&(a[x][i] > eps||a[x][i] < -eps)) q[++tot] = i;
22     for (int i = 0;i <= M;++i)
23     {
24         if ((x == i)||((a[i][y] < eps&&a[i][y] > -eps)) continue;
25         for (int j = 1;j <= tot;++j) a[i][q[j]] -= a[x][q[j]]*a[i][y];
26         a[i][y] = -a[i][y]/tmp;
27     }
28 }
29
30 int main()
31 {
32     scanf("%d %d %d", &N,&M,&op); srand(233);
33     for (int i = 1;i <= N;++i) scanf("%lf",a[0][i]);
34     for (int i = 1;i <= M;++i)
35     {
36         for (int j = 1;j <= N;++j) scanf("%lf",a[i][j]);
37         scanf("%lf",a[i]);
38     }
39     for (int i = 1;i <= N;++i) idx[i] = i;
40     for (int i = 1;i <= M;++i) idy[i] = i+N;
41     while (true)
42     {
43         int x = 0,y = 0;
44         for (int i = 1;i <= M;++i) if (a[i][0] < -eps&&((!x)||((rand()&1))) x = i; if (!x) break;
45         for (int i = 1;i <= N;++i) if (a[x][i] < -eps&&((!y)||((rand()&1))) y = i; if (!y) return puts("Infeasible"),0;
46         pivot(x,y);
47     }
48     while (true)
49     {
50         int x = 0,y = 0; double mn = 1e15;
51         for (int i = 1;i <= N;++i) if (a[0][i] > eps) { y = i; break; } if (!y) break;

```

```

52     for (int i = 1; i <= M; ++i) if (a[i][y] > eps && a[i][0]/a[i][y] < mn) mn = a[i][0]/a[i][y], x = i; if (!x) return puts("Unbounded"), 0;
53     pivot(x, y);
54 }
55 printf("%.8lf\n", -a[0][0]); if (!op) return 0;
56 for (int i = 1; i <= M; ++i) if (idy[i] <= N) A[idy[i]] = a[i][0];
57 for (int i = 1; i <= N; ++i) printf("%.8lf ", A[i]);
58 return 0;
59 }

```

Steiner Tree

```

1  /*
2  * Steiner Tree: 求, 使得指定 K 个点连通的生成树的最小总权值
3  * st[i] 表示顶点 i 的标记值, 如果 i 是指定集合内第 m (0<=m<K) 个点, 则 st[i]=1<<m
4  * endSt=i<<K
5  * dp[tree[i][state] 表示以 i 为根, 连通状态为 state 的生成树值
6  */
7 inline void update(int &x, int y) { if (x == -1) x = y; else if (x > y) x = y; }
8 inline void spfa(int state)
9 {
10     while (!team.empty())
11     {
12         int now = team.front(); team.pop();
13         for (int i = side[now]; i <= n; i++)
14         {
15             int v = to[i];
16             if (f[v][st[v]|state] == -1 || f[v][st[v]|state] > f[now][state]+len[i])
17             {
18                 f[v][st[v]|state] = f[now][state]+len[i];
19                 if ((st[v]|state) != state || vis[v][state]) continue;
20                 vis[v][state] = true; team.push(v);
21             }
22         }
23         vis[now][state] = false;
24     }
25 }
26 inline int work()
27 {
28     endSt = 1<<(K<<1);
29     memset(f, -1, sizeof(f)); memset(st, 0, sizeof(st)); memset(dp, -1, sizeof(dp));
30     memset(vis, false, sizeof(vis)); memset(side, 0, sizeof(side));
31     for (int i = 1; i <= K; ++i) st[i] = 1<<(i-1);
32     for (int i = 1; i <= K; ++i) st[N-K+i] = 1<<(i+K-1);
33     for (int i = 1; i <= N; ++i) f[i][st[i]] = 0;
34     for (int j = 1; j < endSt; ++j)
35     {
36         for (int i = 1; i <= N; ++i)
37         {
38             if (!st[i] || (st[i]&j))
39             for (int sub = (j-1)&j; sub; sub = (sub-1)&j)
40             {
41                 int x = sub|st[i], y = (j-sub)|st[i];
42                 if (f[i][x] != -1 && f[i][y] != -1)
43                     update(f[i][j], f[i][x]+f[i][y]);
44             }
45             if (f[i][j] != -1) team.push(i), vis[i][j] = true;
46         }
47     }
48     spfa(j);
49 }

```

Strongly Connected Component

```

1 int dfn[maxn], low[maxn], timestamp;
2 stack <int> stk; vector <int> scc[maxn];
3 void tarjan(int now)
4 {
5     dfn[now] = low[now] = ++timestamp;
6     stk.push(now);
7     for (int i = side[now]; i <= n; i++)
8     {
9         if (!dfn[to[i]])
10             tarjan(to[i]), low[now] = min(low[now], low[to[i]]);
11         else if (!bel[to[i]]) low[now] = min(low[now], dfn[to[i]]);
12     }
13     if (dfn[now] == low[now])
14     {
15         ++tot;
16         while (stk.top() != now)
17         {
18             scc[tot].push_back(stk.top());
19             bel[stk.top()] = tot; stk.pop();
20         }
21         scc[tot].push_back(stk.top());
22         bel[stk.top()] = tot; stk.pop();
23     }
24 }

```

Suffix Array

```

1 // 记得最后填一个字符集中没有的字符
2 inline void build(char *buf, int *Sa, int *Rank, int *Height, int n, int now, int m)
3 {
4     int i, j, k, *x = t1, *y = t2;
5     memset(c, 0, 4*m);
6     for (i = 0; i < n; ++i) c[x[i] - 'A']++;
7     for (i = 1; i < m; ++i) c[i] += c[i-1];
8     for (i = n-1; i >= 0; --i) Sa[--c[x[i]]] = i;
9     for (k = 1; k < n; k <= m)
10     {
11         int p = 0;
12         for (i = n-k; i < n; ++i) y[p++] = i;
13         for (i = 0; i < n; ++i) if (Sa[i] >= k) y[p++] = Sa[i] - k;
14         memset(c, 0, 4*m);
15         for (i = 0; i < n; ++i) c[x[y[i]]]++;
16         for (i = 1; i < m; ++i) c[i] += c[i-1];
17         for (i = n-1; i >= 0; --i) Sa[--c[x[y[i]]]] = y[i];
18         swap(x, y); p = 1; x[sa[0]] = 0;
19         for (i = 1; i < n; ++i)
20             x[sa[i]] = y[sa[i-1]] == y[sa[i]] && y[sa[i-1]+k] == y[sa[i]+k] ? p-1 : p++;
21         if (p >= n) break; m = p;
22     }
23     for (i = 0; i < n; ++i) Rank[sa[i]] = i;
24     for (i = k = 0; i < n; ++i)
25     {
26         if (k) --k; if (!Rank[i]) continue;
27         j = Sa[Rank[i]-1];
28         while (i+k < n && j+k < n && buf[i+k] == buf[j+k]) ++k;
29         Height[Rank[i]] = k;
30     }
31 }

```

Zhu-Liu Algorithm

```

1 struct Directed_MT
2 {
3     struct Edge
4     {
5         int u, v, w;
6         inline Edge() = default;
7         inline Edge(int _u, int _v, int _w): u(_u), v(_v), w(_w) {}
8     };
9     int n, m, vis[maxn], pre[maxn], id[maxn], in[maxn]; Edge edges[maxn];
10
11     inline void init(int _n) { n = _n; m = 0; }
12     inline void AddEdge(int u, int v, int w) { edges[m++] = Edge(u, v, w); }
13     inline int work(int root)
14     {
15         int ret = 0;
16         while (true)
17         {
18             // 初始化
19             for (int i = 0; i < n; ++i) in[i] = inf+1;
20             for (int i = 0; i < m; ++i)
21             {
22                 int u = edges[i].u, v = edges[i].v;
23                 // 找最小入边, 删除自环
24                 if (edges[i].u < in[v] && u != v)
25                     in[v] = edges[i].w, pre[v] = u;
26             }
27             // 如果没有最小入边, 表示该点不连通, 则最小树形图形成失败
28             for (int i = 0; i < n; ++i)
29             {
30                 if (i == root) continue;
31                 if (in[i] == inf+1) return inf;
32             }
33             int cnt = 0; // 记录缩点
34             memset(id, -1, sizeof(id)); memset(vis, -1, sizeof(vis));
35             in[root] = 0;
36             for (int i = 0; i < n; ++i)
37             {
38                 ret += in[i]; int v = i;
39                 // 找寻自环
40                 while (vis[v] != i && id[v] == -1 && v != root)
41                     vis[v] = i, v = pre[v];
42                 if (v != root && id[v] == -1)
43                 {
44                     // 这里不能从 i 开始找, 因为 i 有可能不在自环内
45                     for (int u = pre[v]; u != v; u = pre[u]) id[u] = cnt;
46                     id[v] = cnt++;
47                 }
48             }
49             // 如果没有自环了, 表示最小树形图成功了
50             if (!cnt) break;
51             // 找到那些不是自环的, 重新给那些点进行标记
52             for (int i = 0; i < n; ++i)
53                 if (id[i] == -1) id[i] = cnt++;
54             for (int i = 0; i < m; ++i)
55             {

```



```

53 Point n = (u.a-u.b)/(v.a-v.b);
54 return fabs((u.a-v.a)*n)/n.norm();
55 }
56 // 两直线夹角 cos
57 inline double angle_cos(const Line &u,const Line &v) { return ((u.a-u.b)*(v.a-v.b))/(u.a-u.b).norm()/(v.a-v.b).norm(); }
58
59 // -----Plane-----
60 struct Plane
61 {
62     Point a,b,c;
63     inline Plane() = default;
64     inline Plane(const Point &a,const Point &b,const Point &c):a(_a),b(_b),c(_c) {}
65     inline Point pvec() const { return (a-b)/(b-c); } // normal vector
66 };
67 // 四点共面
68 inline bool dots_onplane(const Point &a,const Point &b,const Point &c,const Point &d) { return dcmp(Plane(a,b,c).pvec()*(d-a)) == 0; }
69 // 点在三角形内
70 inline bool dot_inplane_in(const Point &p,const Plane &s) { return
    ↪ dcmp(((s.a-s.b)/(s.a-s.c)).norm()-((p-s.a)/(p-s.b)).norm()-((p-s.b)/(p-s.c)).norm()-((p-s.c)/(p-s.a)).norm()) }
71 inline bool dot_inplane_ex(const Point &p,const Plane &s)
72 {
73     if (!dot_inplane_in(p,s.a,s.b,s.c)) return false;
74     int s1 = dcmp((p-s.a)/(p-s.b)),s2 = dcmp((p-s.b)/(p-s.c)),s3 = dcmp((p-s.c)/(p-s.a));
75     return ((s1 > 0&&s2 > 0&&s3 > 0)|| (s1 < 0&&s2 < 0&&s3 < 0));
76 }
77 inline bool same_side(const Point &p1,const Point &p2,const Plane &s) // 同侧
78 {
79     Point v = s.pvec();
80     return dcmp((v/(p1-s.a))*(v/(p2-s.a))) > 0;
81 }
82 inline bool opposite_side(const Point &p1,const Point &p2,const Plane &s) // 异侧
83 {
84     Point v = s.pvec();
85     return dcmp((v/(p1-s.a))*(v/(p2-s.a))) < 0;
86 }
87 // 直线与平面是否有交
88 inline bool intersect_in(const Line &l,const Plane &s)
89 {
90     return !same_side(l.a,l.b,s)&&
91         !same_side(s.a,s.b,Plane(l.a,l.b,s.c))&&
92         !same_side(s.b,s.c,Plane(l.a,l.b,s.a))&&
93         !same_side(s.c,s.a,Plane(l.a,l.b,s.b));
94 }
95 inline bool intersect_ex(const Line &l,const Plane &s)
96 {
97     return opposite_side(l.a,l.b,s)&&
98         opposite_side(s.a,s.b,Plane(l.a,l.b,s.c))&&
99         opposite_side(s.b,s.c,Plane(l.a,l.b,s.a))&&
100         opposite_side(s.c,s.a,Plane(l.a,l.b,s.b));
101 }
102 // 线面交点
103 inline Point intersection(const Line &l,const Plane &s)
104 {
105     Point n = s.pvec();
106     double t = ((n*(s.a-l.a)))/(n*(l.b-l.a));
107     return l.a+(l.b-l.a)*t;
108 }
109 // 求两平面交线
110 inline Line intersection(const Plane &p1,const Plane &p2)
111 {
112     Line ret;
113     ret.a = parallel(Line(p12.a,p12.b),p1)?intersection(Line(p12.b,p12.c),p1):intersection(Line(p12.a,p12.b),p1);
114     ret.b = ret.a+(p1.pvec()/p12.pvec());
115     return ret;
116 }
117 // 点到直线距离
118 inline double ptoline(const Point &p,const Plane &s) { double n = s.pvec(); return fabs(n*(p-s.a))/n.norm(); }
119 inline double angle_cos(const Plane &u,const Plane &v)
120 {
121     Point n1 = u.pvec(),n2 = v.pvec();
122     return (n1*n2)/n1.norm()/n2.norm();
123 }
124 inline double angle_sin(const Line &l,const Plane &s)
125 {
126     Point n = s.pvec();
127     return ((l.a-l.b)*n)/(l.a-l.b).norm()/n.norm();
128 }

```

Convex Hull 2D

```

1 struct Point
2 {
3     double x,y;
4     friend inline bool operator <(const Point &a,const Point &b)
5     {
6         if (a.x != b.x) return a.x < b.x;
7         else return a.y < b.y;
8     }
9 }P[maxn],convex[maxn];
10

```

```

11 inline void ConvexHull()
12 {
13     sort(P+1,P+N+1); //x 第一关键字, y 第二关键字从小到大排序
14     for (int i = 1;i <= N;++i)
15     {
16         while (m > 1&&(convex[m]-convex[m-1])/(P[i]-convex[m-1]) <= 0) --m;
17         convex[++m] = P[i];
18     }
19     int k = m;
20     for (int i = N-1;i-->1)
21     {
22         while (m > k&&(convex[m]-convex[m-1])/(P[i]-convex[m-1]) <= 0) --m;
23         convex[++m] = P[i];
24     }
25     if (N > 1) m--;
26 }

```

Convex Hull 3D

```

1 // 0-base
2 const double eps = 1e-9;
3 const int maxn = 1010;
4 int N,cnt,mark[maxn][maxn];
5
6 struct Point
7 {
8     double x,y,z;
9     friend inline bool operator <(const Point &a,const Point &b)
10     {
11         if (dcmp(a.x-b.x)) return a.x < b.x;
12         else if (dcmp(a.y-b.y)) return a.y < b.y;
13         else if (dcmp(a.z-b.z)) return a.z < b.z;
14         else return false;
15     }
16     friend inline bool operator ==(const Point &a,const Point &b) { return !dcmp(a.x-b.x)&&!dcmp(a.y-b.y)&&!dcmp(a.z-b.z); }
17 }info[maxn];
18
19 inline double volume(int a,int b,int c,int d) { return mix(info[b]-info[a],info[c]-info[a],info[d]-info[a]); }
20
21 struct Face
22 {
23     int a,b,c;
24     inline Face() = default;
25     inline Face(int _a,int _b,int _c):a(_a),b(_b),c(_c) {}
26 };
27 vector <Face> face;
28
29 inline void add(int v)
30 {
31     vector <Face> tmp;
32     int a,b,c; cnt++;
33     for (int i = 0;i < (int)face.size();++i)
34     {
35         a = face[i].a; b = face[i].b; c = face[i].c;
36         if (dcmp(volume(v,a,b,c)) < 0)
37             mark[a][b] = mark[b][c] = mark[c][a] = mark[a][c] = cnt;
38         else tmp.push_back(face[i]);
39     }
40     face = tmp;
41     for (int i = 0;i < (int)tmp.size();++i)
42     {
43         a = face[i].a; b = face[i].b; c = face[i].c;
44         if (mark[a][b] == cnt) face.push_back(Face(b,a,v));
45         if (mark[b][c] == cnt) face.push_back(Face(c,b,v));
46         if (mark[c][a] == cnt) face.push_back(Face(a,c,v));
47     }
48 }
49
50 inline bool find()
51 {
52     for (int i = 2;i < N;++i)
53     {
54         Point ndir = (info[0]-info[i])/(info[1]-info[i]);
55         if (!dcmp(ndir.norm())) continue;
56         swap(info[i],info[2]);
57         for (int j = i+1;j < N;++j)
58             if (dcmp(volume(0,1,2,j)) != 0)
59             {
60                 swap(info[j],info[3]);
61                 face.push_back(Face(0,1,2));
62                 face.push_back(Face(0,2,1));
63                 return true;
64             }
65     }
66     return false;
67 }
68
69 inline double ConvexHull()
70 {

```

```

71 sort(info,info+N); N = unique(info,info+N)-info; face.clear();
72 random_shuffle(info,info+N);
73 if (find())
74 {
75     memset(mark,0,sizeof mark); cnt = 0;
76     for (int i = 3;i < N;++i) add(i);
77     double ans = 0;
78     for (int i = 0;i < (int)face.size();++i)
79     {
80         Point p = (info[face[i].a]-info[face[i].b])/(info[face[i].c]-info[face[i].b]);
81         ans += p.norm();
82     }
83     return ans/2;
84 }
85 return -1;
86 }

```

Maximal Weighted Matching in General Graphs

```

1 // 0-base
2 const double eps = 1e-9;
3 const int maxn = 1010;
4 int N,cnt,mark[maxn][maxn];
5
6 struct Point
7 {
8     double x,y,z;
9     friend inline bool operator <(const Point &a,const Point &b)
10     {
11         if (dcmp(a.x-b.x)) return a.x < b.x;
12         else if (dcmp(a.y-b.y)) return a.y < b.y;
13         else if (dcmp(a.z-b.z)) return a.z < b.z;
14         else return false;
15     }
16     friend inline bool operator ==(const Point &a,const Point &b) { return !dcmp(a.x-b.x)&&!dcmp(a.y-b.y)&&!dcmp(a.z-b.z); }
17 }info[maxn];
18
19 inline double volume(int a,int b,int c,int d) { return mix(info[b]-info[a],info[c]-info[a],info[d]-info[a]); }
20
21 struct Face
22 {
23     int a,b,c;
24     inline Face() = default;
25     inline Face(int _a,int _b,int _c):a(_a),b(_b),c(_c) {}
26 };
27 vector<Face> face;
28
29 inline void add(int v)
30 {
31     vector<Face> tmp;
32     int a,b,c; cnt++;
33     for (int i = 0;i < (int)face.size();++i)
34     {
35         a = face[i].a; b = face[i].b; c = face[i].c;
36         if (dcmp(volume(v,a,b,c)) < 0)
37             mark[a][b] = mark[b][a] = mark[b][c] = mark[c][b] = mark[c][a] = mark[a][c] = cnt;
38         else tmp.push_back(face[i]);
39     }
40     face = tmp;
41     for (int i = 0;i < (int)tmp.size();++i)
42     {
43         a = face[i].a; b = face[i].b; c = face[i].c;
44         if (mark[a][b] == cnt) face.push_back(Face(b,a,v));
45         if (mark[b][c] == cnt) face.push_back(Face(c,b,v));
46         if (mark[c][a] == cnt) face.push_back(Face(a,c,v));
47     }
48 }
49
50 inline bool find()
51 {
52     for (int i = 2;i < N;++i)
53     {
54         Point ndir = (info[0]-info[i])/(info[1]-info[i]);
55         if (!dcmp(ndir.norm())) continue;
56         swap(info[i],info[2]);
57         for (int j = i+1;j < N;++j)
58             if (dcmp(volume(0,1,2,j)) != 0)
59             {
60                 swap(info[j],info[3]);
61                 face.push_back(Face(0,1,2));
62                 face.push_back(Face(0,2,1));
63                 return true;
64             }
65     }
66     return false;
67 }
68
69 inline double ConvexHull()
70 {
71     sort(info,info+N); N = unique(info,info+N)-info; face.clear();

```

```

72 random_shuffle(info,info+N);
73 if (find())
74 {
75     memset(mark,0,sizeof mark); cnt = 0;
76     for (int i = 3;i < N;++i) add(i);
77     double ans = 0;
78     for (int i = 0;i < (int)face.size();++i)
79     {
80         Point p = (info[face[i].a]-info[face[i].b])/(info[face[i].c]-info[face[i].b]);
81         ans += p.norm();
82     }
83     return ans/2;
84 }
85 return -1;
86 }

```

Minimal Ball Cover

```

1 #include <iostream>
2 #include<cstdio>
3 #include<algorithm>
4 #include<cstring>
5 #include<cmath>
6 using namespace std;
7 const double eps=1e-7;
8 struct point3D
9 {
10     double x,y,z;
11 } data[35];
12 int n;
13 double dis(point3D a,point3D b)
14 {
15     return sqrt((a.x-b.x)*(a.x-b.x)+(a.y-b.y)*(a.y-b.y)+(a.z-b.z)*(a.z-b.z));
16 }
17 double solve()
18 {
19     double step=100,ans=1e30,mt;
20     point3D z;
21     z.x=z.y=z.z=0;
22     int s=0;
23     while(step>eps)
24     {
25         for(int i=0; i<n; i++)
26             if (dis(z,data[s])<dis(z,data[i])) s=i;
27         mt=dis(z,data[s]);
28         ans=min(ans,mt);
29         z.x+=(data[s].x-z.x)/mt*step;
30         z.y+=(data[s].y-z.y)/mt*step;
31         z.z+=(data[s].z-z.z)/mt*step;
32         step*=0.98;
33     }
34     return ans;
35 }
36 int main()
37 { // freopen("t.txt","r",stdin);
38     double ans;
39     while(~scanf("%d",&n),n)
40     {
41         for(int i=0; i<n; i++)
42             scanf("%lf%lf%lf",&data[i].x,&data[i].y,&data[i].z);
43         ans=solve();
44         printf("%.5f\n",ans);
45     }
46     return 0;
47 }p

```

Minimal Circle Cover

```

1 #include <iostream>
2 #include <cstdio>
3 #include <cstring>
4 #include <algorithm>
5 #include <cmath>
6 #include <vector>
7 using namespace std;
8 typedef long long ll;
9 const int N=2005;
10 const double INF=1e9;
11 const double eps=1e-8;
12 const double pi=acos(-1);
13 inline int read(){
14     char c=getchar();int x=0,f=1;
15     while(c<'0' || c>'9'){if(c=='-')f=-1; c=getchar();}
16     while(c>'0' && c<='9'){x=x*10+c-'0'; c=getchar();}
17     return x*f;
18 }
19
20 inline int sgn(double x){
21     if(abs(x)<eps) return 0;

```

```

22     else return x<0?-1:1;
23 }
24
25 struct Vector{
26     double x,y;
27     Vector(double a=0,double b=0):x(a),y(b){}
28     bool operator <(const Vector &a)const{
29         return sgn(x-a.x)<0|| (sgn(x-a.x)==0&&sgn(y-a.y)<0);
30     }
31     void print(){printf("%lf %lf\n",x,y);}
32 };
33 typedef Vector Point;
34 Vector operator +(Vector a,Vector b){return Vector(a.x+b.x,a.y+b.y);}
35 Vector operator -(Vector a,Vector b){return Vector(a.x-b.x,a.y-b.y);}
36 Vector operator *(Vector a,double b){return Vector(a.x*b,a.y*b);}
37 Vector operator /(Vector a,double b){return Vector(a.x/b,a.y/b);}
38 bool operator ==(Vector a,Vector b){return sgn(a.x-b.x)==0&&sgn(a.y-b.y)==0;}
39 double Dot(Vector a,Vector b){return a.x*b.x+a.y*b.y;}
40 double Cross(Vector a,Vector b){return a.x*b.y-a.y*b.x;}
41 double Len(Vector a){return sqrt(Dot(a,a));}
42 Vector Normal(Vector a){
43     return Vector(-a.y,a.x);//counterclockwise
44 };
45 struct Line{
46     Point s,t;
47     Line(){}
48     Line(Point a,Point b):s(a),t(b){}
49 };
50 bool isLSI(Line l1,Line l2){
51     Vector v=l1.t-l1.s,u=l2.s-l1.s,w=l2.t-l1.s;
52     return sgn(Cross(v,u))!=sgn(Cross(v,w));
53 }
54 Point LI(Line a,Line b){
55     Vector v=a.s-b.s,v1=a.t-a.s,v2=b.t-b.s;
56     double t=Cross(v2,v)/Cross(v1,v2);
57     return a.s+v1*t;
58 }
59 Point Circumcenter(Point a,Point b,Point c){
60     Point p=(a+b)/2,q=(a+c)/2;
61     Vector v=Normal(b-a),u=Normal(c-a);
62     if(sgn(Cross(v,u))==0){
63         if(sgn(Len(a-b)+Len(b-c)-Len(a-c))==0) return (a+c)/2;
64         if(sgn(Len(a-b)+Len(a-c)-Len(b-c))==0) return (b+c)/2;
65         if(sgn(Len(a-c)+Len(b-c)-Len(a-b))==0) return (a+b)/2;
66     }
67     return LI(Line(p,p+v),Line(q,q+u));
68 }
69
70 double minCircleCover(Point p[],int n,Point &c){
71     random_shuffle(p+1,p+1+n);
72     c=p[1];
73     double r=0;
74     for(int i=2;i<=n;i++){
75         if(sgn(Len(c-p[i])-r)>0){
76             c=p[i],r=0;
77             for(int j=1;j<i;j++){
78                 if(sgn(Len(c-p[j])-r)>0){
79                     c=(p[i]+p[j])/2,r=Len(c-p[i]);
80                     for(int k=1;k<j;k++){
81                         if(sgn(Len(c-p[k])-r)>0){
82                             c=Circumcenter(p[i],p[j],p[k]);
83                             r=Len(c-p[i]);
84                         }
85                     }
86                 }
87             }
88         }
89     }
90     return r;
91 }
92
93 int n;
94 Point p[N],c;
95 int main(int argc, const char * argv[]){
96     while(true){
97         n=read();if(n==0) break;
98         for(int i=1;i<=n;i++) scanf("%lf%lf",&p[i].x,&p[i].y);
99         double r=minCircleCover(p,n,c);
100         printf("%.2f %.2f %.2f\n",c.x,c.y,r);
101     }
102 }

```

Stoer Wagner Algorithm

```

1 int G[maxn][maxn],node[maxn],dis[maxn]; bool visit[maxn];
2
3 inline int solve(int n)
4 {
5     if (n == 1) return inf;
6     int answer = inf;
7     for (int i = 0; i < n; ++i) node[i] = i;
8     while (n > 1)
9     {

```

```

10     int mx = 1;
11     for (int i = 0; i < n; ++i)
12     {
13         dis[node[i]] = G[node[0]][node[i]];
14         if (dis[node[i]] > dis[node[mx]]) mx = i;
15     }
16     int prev = 0;
17     memset(visit,false,sizeof visit);
18     visit[node[0]] = true;
19     for (int i = 1; i < n; ++i)
20     {
21         if (i == n-1)
22         {
23             answer = min(answer,dis[node[mx]]);
24             for (int k = 0; k < n; ++k)
25                 G[node[k]][node[prev]] = (G[node[prev]][node[k]] += G[node[k]][node[mx]]);
26             node[mx] = node[--n];
27         }
28         visit[node[mx]] = true; prev = mx; mx = -1;
29         for (int j = 1; j < n; ++j)
30             if (!visit[node[j]])
31             {
32                 dis[node[j]] += G[node[prev]][node[j]];
33                 if (mx == -1||dis[node[mx]] < dis[node[j]]) mx = j;
34             }
35     }
36 }
37 return answer;
38 }

```

Virtual Tree

```

1 int N,cnt,timestamp,dfn[maxn],f[maxn][26],side[maxn],H[maxn];
2 int dep[maxn],toit[maxn],nxt[maxn],last[maxn],cost[maxn],stk[maxn];
3 ll best[maxn],g[maxn];
4
5 inline void add(int a,int b,int c) { nxt[++cnt] = side[a]; side[a] = cnt; toitt[cnt] = b; cost[cnt] = c; }
6 inline void ins(int a,int b,int c) { add(a,b,c); add(b,a,c); }
7
8 inline void nadd(int a,int b,int idc)
9 {
10     if (a == b) return;
11     if (last[a] != idc) side[a] = 0,last[a] = idc;
12     if (last[b] != idc) side[b] = 0,last[b] = idc;
13     nxt[++cnt] = side[a]; side[a] = cnt; toitt[cnt] = b;
14 }
15
16 inline bool cmp(int a,int b) { return dfn[a] < dfn[b]; }
17
18 inline void dfs(int now)
19 {
20     dfn[now] = ++timestamp;
21     for (int i = 1; i <= 26; ++i)
22         f[now][i] = f[f[now][i-1]][i-1];
23     for (int i = side[now]; i; i = nxt[i])
24         if (toitt[i] != f[now][0])
25         {
26             best[toitt[i]] = min(best[now],(ll)cost[i]);
27             dep[toitt[i]] = dep[now]+1;
28             f[toitt[i]][0] = now; dfs(toitt[i]);
29         }
30 }
31
32 inline int jump(int a,int step) { for (int i = 0; step; step >= 1, ++i) if (step&1) a = f[a][i]; return a; }
33 inline int lca(int a,int b)
34 {
35     if (dep[a] < dep[b]) swap(a,b);
36     a = jump(a,dep[a]-dep[b]);
37     if (a == b) return a;
38     for (int i = 0; i >= 0; )
39     {
40         if (f[a][i] != f[b][i])
41             a = f[a][i], b = f[b][i], ++i;
42         else --i;
43     }
44     return f[a][0];
45 }
46
47 inline void work(int idc)
48 {
49     cnt = 0; int K = gi(),tot,top;
50     for (int i = 1; i <= K; ++i) H[i] = gi();
51     sort(H+1,H+K+1,cmp); H[tot] = 1 = H[1];
52     for (int i = 2; i <= K; ++i) if (lca(H[tot],H[i]) != H[tot]) H[++tot] = H[i];
53     stk[top = 1] = 1;
54     for (int i = 1; i <= tot; ++i)
55     {
56         int ans = lca(H[i],stk[top]);
57         while (true)

```

```
58 {
59     if (dep[ans] >= dep[stk[top-1]]) { nadd(ans,stk[top--],idc); break; }
60     nadd(stk[top-1],stk[top],idc); --top;
61 }
62 if (stk[top] != ans) stk[++top] = ans;
63 if (stk[top] != H[1]) stk[++top] = H[1];
64 }
65 while (--top) nadd(stk[top],stk[top+1],idc);
66 // dp(1); printf("%lld\n",g[1]);
67 }
```

Graph Isomorphism

哈希函数:

$$F_t(i) = \left(F_{t-1}(i) \times A + \sum_{i \rightarrow j} F_{t-1}(j) \times B + \sum_{j \rightarrow i} F_{t-1}(j) \times C + D \times (i == a) \right) \bmod P$$

其中 K, A, B, C, D, P 为自选参数。两个图同构当且仅当 $F_K(i)$ 值一样。

Competition

```
1 ;; Default Font: Courier 10 Pitch Bold      Size: 15
2 ;; Remember to set CUA-mode and save your options.
3 (global-set-key (kbd "M-n") [fforward-paragraph)
4 (global-set-key (kbd "M-p") [fbackward-paragraph)
5 (global-set-key (kbd "RET") [newline-and-indent)
6 (global-set-key (kbd "<f8>") [gud-gdb)
7 (defun compile-cpp ()
8   (interactive)
9   (compile (format "g++ -o %s %s -g -lm -Wall -std=c++11" (file-name-sans-extension (buffer-name)) (buffer-name))))
10 (global-set-key (kbd "<f9>") [compile-cpp)
11 (global-linum-mode t)
12 (setq default-tab-width 4)
13 (setq c-basic-offset 4)
```

孙司宇

FFT

```
1 #include<iostream>
2 #include<cstdio>
3 #include<cmath>
4 using namespace std;
5 const double eps=1e-8;
6 const double PI=acos(-1.0);
7 struct Complex
8 {
9     double real,image;
10     Complex(double _real,double _image)
11     {
12         real=_real;
13         image=_image;
14     }
15     Complex(){real=0;image=0;}
16 };
17
18 Complex operator + (const Complex &c1, const Complex &c2)
19 {
20     return Complex(c1.real + c2.real, c1.image + c2.image);
21 }
22
23 Complex operator - (const Complex &c1, const Complex &c2)
24 {
25     return Complex(c1.real - c2.real, c1.image - c2.image);
26 }
27
28 Complex operator * (const Complex &c1, const Complex &c2)
29 {
30     return Complex(c1.real*c2.real - c1.image*c2.image, c1.real*c2.image + c1.image*c2.real);
31 }
32
33 int rev(int id,int len)
34 {
35     int ret=0;
36     for(int i=0;(1<i)<len;i++)
37     {
38         ret<<=1;
39         if(idk(1<i))
40             ret|=i;
41     }
42     return ret;
43 }
44 Complex* IterativeFFT(Complex* a,int len,int DFT)
45 {
46     Complex* A=new Complex[len];
```

```
47     for(int i=0;i<len;i++)
48         A[rev(i,len)]=a[i];
49     for(int s=1;(1<s)<=len;s++)
50     {
51         int m=(1<<s);
52         Complex wm=Complex(cos(DFT*2*PI/m),sin(DFT*2*PI/m));
53         for(int k=0;k<len;k+=m)
54         {
55             Complex w=Complex(1,0);
56             for(int j=0;j<(m>>1);j++)
57             {
58                 Complex t=w*A[k+j+(m>>1)];
59                 Complex u=A[k+j];
60                 A[k+j]=u+t;
61                 A[k+j+(m>>1)]=u-t;
62                 w=w*wm;
63             }
64         }
65     }
66     if(DFT==1)
67     for(int i=0;i<len;i++)
68     {
69         A[i].real/=len;
70         A[i].image/=len;
71     }
72     return A;
73 }
74 char s[101010],t[101010];
75 Complex a[202020],b[202020],c[202020];
76 int pr[202020];
77 int main()
78 {
79     int len;
80     scanf("%d",&len);
81     scanf("%s",s);
82     scanf("%s",t);
83     for(int i=0;i<len;i++)
84         a[i]=Complex(s[len-i-1]-'0',0);
85     for(int i=0;i<len;i++)
86         b[i]=Complex(t[len-i-1]-'0',0);
87     int tmp=1;
88     while(tmp<=len)
89         tmp*=2;
90     len=tmp/2;
91     Complex* aa=IterativeFFT(a,len,1);
92     Complex* bb=IterativeFFT(b,len,1);
93     for(int i=0;i<len;i++)
94         c[i]=aa[i]*bb[i];
95     Complex* ans=IterativeFFT(c,len,-1);
96     for(int i=0;i<len;i++)
97         pr[i]=round(ans[i].real);
98     for(int i=0;i<len;i++)
99     {
100         pr[i+1]+=pr[i]/10;
101         pr[i]=10;
102     }
103     bool flag=0;
104     for(int i=len-1;i>=0;i--)
105     {
106         if(pr[i]>0)
107             flag=1;
108         if(flag)
109             printf("%d",pr[i]);
110     }
111     printf("\n");
112     return 0;
113 }
```

NTT

```
1 #include <iostream>
2 #include <cstdio>
3 #include <cstring>
4 #include <algorithm>
5 #include <cmath>
6 using namespace std;
7 const int M=(1<<18)+6, INF=1e9;
8 const double PI=acos(-1);
9 long long P=1004535809;
10 long long Pow(long long a, long long b,long long P)
11 {
12     long long ans=1;
13     for(; b; b>>=1, a=a*a%P)
14         if(b&1) ans=ans*a%P;
15     return ans;
16 }
17 struct NumberTheoreticTransform {
18     int n, rev[N];
19     long long g;
20     void ini(int lim) {
```



```

21     g=3;
22     n=1; int k=0;
23     while(n<lim) n<=1, k++;
24     for(int i=0; i<n; i++) rev[i] = (rev[i>>1]>>1) | ((i&1)<<(k-1));
25 }
26 void dft(long long *a, int flag) {
27     for(int i=0; i<n; i++) if(i<rev[i]) swap(a[i], a[rev[i]]);
28     for(int l=2; l<=n; l<=1) {
29         int m=l>>1;
30         long long wn = Pow(g, flag==1 ? (P-1)/l : P-1-(P-1)/l, P);
31         for(long long *p=a; p!=a+n; p+=l) {
32             long long w=1;
33             for(int k=0; k<m; k++) {
34                 long long t = w * p[k+m]%P;
35                 p[k+m]=(p[k]-t+P)%P;
36                 p[k]=(p[k]+t)%P;
37                 w=w*wn%P;
38             }
39         }
40     }
41     if(flag==1) {
42         long long inv=Pow(n, P-2, P);
43         for(int i=0; i<n; i++) a[i]=a[i]*inv%P;
44     }
45 }
46 void mul(long long *a, long long *b, int m) {
47     ini(m);
48     dft(a, 1); dft(b, 1);
49     for(int i=0; i<n; i++) a[i]=a[i]*b[i];
50     dft(a, -1);
51 }
52 }
53 if(
54     int n1, n2, m, c[N];
55     long long a[N], b[N];
56     char s1[N], s2[N];
57     int main()
58     {
59         int n;
60         scanf("%d",&n);
61         scanf("%s%s",s1,s2);
62         n1=strlen(s1); n2=strlen(s2);
63         for(int i=0;i<n1;i++)
64             a[i]=s1[n1-i-1]-'0';
65         for(int i=0;i<n2;i++)
66             b[i]=s2[n2-i-1]-'0';
67         m=n1+n2-1;
68         f.mul(a,b,m);
69         for(int i=0;i<m;i++) c[i]=a[i];
70         for(int i=0;i<m;i++) c[i+1]+=c[i]/10, c[i]%=10;
71         if(c[m])
72             m++;
73         for(int i=m-1; i>=0; i--)
74             printf("%d",c[i]);
75         return 0;
76     }

```

SAM

```

1  #include<iostream>
2  #include<cstring>
3  using namespace std;
4  const int MaxPoint=1010101;
5  struct Suffix_AutoMachine{
6      int son[MaxPoint][27],pre[MaxPoint],step[MaxPoint],right[MaxPoint],last,root,num;
7      int NewNode(int stp)
8      {
9          num++;
10         memset(son[num],0,sizeof(son[num]));
11         pre[num]=0;
12         step[num]=stp;
13         return num;
14     }
15     Suffix_AutoMachine()
16     {
17         num=0;
18         root=last=NewNode(0);
19     }
20     void push_back(int ch)
21     {
22         int np=NewNode(step[last]+1);
23         right[np]=i;
24         step[np]=step[last]+1;
25         int p=last;
26         while(p&&!son[p][ch])
27         {
28             son[p][ch]=np;
29             p=pre[p];
30         }
31         if(!p)

```

```

32         pre[np]=root;
33     else
34     {
35         int q=son[p][ch];
36         if(step[q]==step[p]+1)
37             pre[np]=q;
38     else
39     {
40         int nq=NewNode(step[p]+1);
41         memcpy(son[nq],son[q],sizeof(son[q]));
42         step[nq]=step[p]+1;
43         pre[nq]=pre[q];
44         pre[q]=pre[np]=nq;
45         while(p&&son[p][ch]==q)
46         {
47             son[p][ch]=nq;
48             p=pre[p];
49         }
50     }
51     }
52     last=np;
53 }
54 };
55 /*
56
57 int arr[1010101];
58 bool Step_Cmp(int x,int y)
59 {
60     return S.step[x]<S.step[y];
61 }
62 void Get_Right()
63 {
64     for(int i=1;i<=S.num;i++)
65         arr[i]=i;
66     sort(arr+1,arr+S.num+1,Step_Cmp);
67     for(int i=S.num;i>=2;i--)
68         S.right[S.pre[arr[i]]]+=S.right[arr[i]];
69 }
70 */
71 int main()
72 {
73     return 0;
74 }
75

```

manacher

```

1  #include<iostream>
2  #include<cstring>
3  using namespace std;
4  char Mana[202020];
5  int cher[202020];
6  int Manacher(char *S)
7  {
8      int len=strlen(S),id=0,mx=0,ret=0;
9      Mana[0]='$';
10     Mana[1]='#';
11     for(int i=0;i<len;i++)
12     {
13         Mana[2*i+2]=S[i];
14         Mana[2*i+3]='#';
15     }
16     Mana[2*len+2]=0;
17     for(int i=1;i<=2*len+1;i++)
18     {
19         if(i<mx)
20             cher[i]=min(cher[2*id-i],mx-i);
21         else
22             cher[i]=0;
23         while(Mana[i+cher[i]+1]==Mana[i-cher[i]-1])
24             cher[i]++;
25         if(cher[i]+i>mx)
26         {
27             mx=cher[i]+i;
28             id=i;
29         }
30         ret=max(ret,cher[i]);
31     }
32     return ret;
33 }
34 char S[101010];
35 int main()
36 {
37     ios::sync_with_stdio(false);
38     cin.tie(0);
39     cout.tie(0);
40     cin>>S;
41     cout<<Manacher(S)<<endl;
42     return 0;
43 }

```

中国剩余定理

```

1 // 51nod 1079
2 #include<iostream>
3 using namespace std;
4 int gcd(int x,int y)
5 {
6     if(x==0)
7         return y;
8     if(y==0)
9         return x;
10    return gcd(y,x/y);
11 }
12 long long exgcd(long long a,long long b,long long &x,long long &y)
13 {
14     if(b==0)
15     {
16         x=1;
17         y=0;
18         return a;
19     }
20     long long ans=exgcd(b,a/b,x,y);
21     long long temp=x;
22     x=y;
23     y=temp-a/b*y;
24     return ans;
25 }
26 void fix(long long &x,long long &y)
27 {
28     x%=y;
29     if(x<0)
30         x+=y;
31 }
32 bool solve(int n, std::pair<long long, long long> input[],std::pair<long long, long long> &output)
33 {
34     output = std::make_pair(1, 1);
35     for(int i = 0; i < n; ++i)
36     {
37         long long number, useless;
38         exgcd(output.second, input[i].second, number, useless);
39         long long divisor = gcd(output.second, input[i].second);
40         if((input[i].first - output.first) % divisor)
41         {
42             return false;
43         }
44         number *= (input[i].first - output.first) / divisor;
45         fix(number,input[i].second);
46         output.first += output.second * number;
47         output.second *= input[i].second / divisor;
48         fix(output.first, output.second);
49     }
50     return true;
51 }
52 pair<long long,long long> input[101010],output;
53 int main()
54 {
55     int n;
56     cin>>n;
57     for(int i=0;i<n;i++)
58         cin>>input[i].second>>input[i].first;
59     solve(n,input,output);
60     cout<<output.first<<endl;
61     return 0;
62 }

```

回文自动机

```

1 //Tsinsen A1280 最长双回文串
2 #include<iostream>
3 #include<cstring>
4 using namespace std;
5
6 const int maxn = 100005;// n(空间复杂度 o(n*ALP)), 实际开 n 即可
7 const int ALP = 26;
8
9 struct PAM{ // 每个节点代表一个回文串
10     int next[maxn][ALP]; // next 指针, 参照 Trie 树
11     int fail[maxn]; // fail 失配后缀链接
12     int cnt[maxn]; // 此回文串出现个数
13     int num[maxn];
14     int len[maxn]; // 回文串长度
15     int s[maxn]; // 存放添加的字符
16     int last; // 指向上一个字符所在的节点, 方便下一次 add
17     int n; // 已添加字符个数
18     int p; // 节点个数
19
20     int newnode(int w)
21     {
22         // 初始化节点, w= 长度
23         for(int i=0;i<ALP;i++)
24             next[p][i] = 0;
25     }
26 }

```

```

24     cnt[p] = 0;
25     num[p] = 0;
26     len[p] = w;
27     return p++;
28 }
29 void init()
30 {
31     p = 0;
32     newnode(0);
33     newnode(-1);
34     last = 0;
35     n = 0;
36     s[n] = -1; // 开头放一个字符集中没有的字符, 减少特判
37     fail[0] = 1;
38 }
39 int get_fail(int x)
40 { // 和 KMP 一样, 失配后找一个尽量最长的
41     while(s[n-len[x]-1] != s[s[n]]) x = fail[x];
42     return x;
43 }
44 int add(int c)
45 {
46     c -= 'a';
47     s[++n] = c;
48     int cur = get_fail(last);
49     if(!next[cur][c])
50     {
51         int now = newnode(len[cur]+2);
52         fail[now] = next[get_fail(fail[cur])][c];
53         next[cur][c] = now;
54         num[now] = num[fail[now]] + 1;
55     }
56     last = next[cur][c];
57     cnt[last]++;
58     return len[last];
59 }
60 void count()
61 {
62     // 最后统计一遍每个节点出现个数
63     // 父亲累加儿子的 cnt, 类似 SAM 中 parent 树
64     // 满足 parent 拓扑关系
65     for(int i=p-1;i>=0;i--)
66         cnt[fail[i]] += cnt[i];
67 }
68 }pam;
69 char S[101010];
70 int l[101010],r[101010];
71 int main()
72 {
73     cin>>S;
74     int len=strlen(S);
75     pam.init();
76     for(int i=0;i<len;i++)
77         l[i]=pam.add(S[i]);
78     pam.init();
79     for(int i=len-1;i>=0;i--)
80         r[i]=pam.add(S[i]);
81     pam.init();
82     int ans=0;
83     for(int i=0;i<len-1;i++)
84         ans=max(ans,l[i]+r[i+1]);
85     cout<<ans<<endl;
86     return 0;
87 }

```

多项式开方

```

1 // 2
2 //Nlog^2N
3 #include <cstdio>
4 #include <algorithm>
5 #define FDR(i,j,k) for(i=j;i<=k;++i)
6 #define rep(i,j,k) for(i=j;i<=k;++i)
7 #define gmod(x) (((x)%mod+mod)%mod)
8 const int N = 262144, mod = 998244353, inv2 = 499122177;
9 using namespace std;
10 typedef long long ll;
11 ll qpow(ll x, int y) {
12     ll z = 1;
13     for (; y; x = x % mod, y /= 2)
14         if (y & 1) z = z * x % mod;
15     return z;
16 }
17 namespace NTT {
18     int n, rev[N], inv_n, m = -1;
19     void init(int c) {
20         int k = -1, i;
21         if (m == c) return; else m = c;
22         for (n = 1; n <= m; n <= 1) ++k;
23     }
24 }

```

```

23     inv_n = qpow(n, mod - 2);
24     rep(i,0,n) rev[i] = (rev[i >> 1] >> 1) | ((i & 1) << k);
25 }
26 void ntt(int *a, int f) {
27     int h, i, j;
28     rep(i,0,n) if (i < rev[i]) swap(a[i], a[rev[i]]);
29     for (h = 2; h <= n; h *= 2) {
30         int wn = qpow(3, (mod - 1) / h);
31         for (i = 0; i < n; i += h) {
32             int w = 1;
33             rep(j,0,h/2) {
34                 int u = a[i + j], t = 111 * a[i + j + h / 2] * w % mod;
35                 a[i + j + h / 2] = (u - t + mod) % mod;
36                 a[i + j] = (u + t) % mod;
37                 w = 111 * w * wn % mod;
38             }
39         }
40     }
41     if (f) {
42         rep(i,1,n/2) swap(a[i], a[n - i]);
43         rep(i,0,n) a[i] = 111 * a[i] * inv_n % mod;
44     }
45 }
46 void inv(int *a, int *b, int n) {
47     static int t[N];
48     int i;
49     if (n == 1) { b[0] = qpow(a[0], mod - 2); return; }
50     inv(a, b, n / 2);
51     rep(i,0,n) t[i] = a[i]; rep(i,n,2*n) t[i] = 0;
52     NTT::init(n);
53     NTT::ntt(t, 0); NTT::ntt(b, 0);
54     rep(i,0,NTT::n) t[i] = (11) b[i] * gmod(211 - (11) t[i] * b[i] % mod) % mod;
55     NTT::ntt(t, 1);
56     rep(i,0,n) b[i] = t[i]; rep(i,n,2*n) b[i] = 0;
57 }
58 void sqrt(int *a, int *b, int n) {
59     static int t[N], b1[N];
60     if (n == 1) { b[0] = 1; return; }
61     int i;
62     sqrt(a, b, n / 2);
63     rep(i,0,n) b1[i] = 0;
64     inv(b, b1, n);
65     rep(i,0,n) t[i] = a[i]; rep(i,n,2*n) t[i] = 0;
66     NTT::init(n);
67     NTT::ntt(t, 0), NTT::ntt(b, 0), NTT::ntt(b1, 0);
68     rep(i,0,NTT::n) t[i] = inv2 * ((b[i] + (11) b1[i] * t[i] % mod) % mod) % mod;
69     NTT::ntt(t, 1);
70     rep(i,0,n) b[i] = t[i]; rep(i,n,2*n) b[i] = 0;
71 }
72 int main() {
73     static int c[N], sc[N], ic[N];
74     int i, x, n, m, l;
75     scanf("%d%d", &n, &m);
76     FOR(i,1,n) scanf("%d", &x), ++c[x];
77     c[0] = gmod(1 - c[0]);
78     FOR(i,1,m) c[i] = gmod(-4 * c[i]);
79     for (l = 1; l <= m; l <= 1);
80     sqrt(c, sc, 1);
81     (++sc[0]) %= mod;
82     inv(sc, ic, 1);
83     FOR(i,0,m) ic[i] = 211 * ic[i] % mod;
84     FOR(i,1,m) printf("%d\n", ic[i]);
85     return 0;
86 }

```

多项式求逆

```

1  /*  ㊦  bzoj3456
2  #include<iostream>
3  #include<cstdio>
4  #include<algorithm>
5  #include<string>
6  #include<cmath>
7  #define N 5000003
8  #define LL long long
9  #define p 1004535809
10 using namespace std;
11 int n,m;
12 int a[N],b[N],c[N],jc[N],inv_j[N],wn[N];
13 LL quickpow(LL num,LL x)
14 {
15     LL base=num%p; LL ans=1;
16     while (x) {
17         if (x&1) ans=ans*base%p;
18         x>>=1;
19         base=base*base%p;
20     }
21     return ans;
22 }

```

```

23 void init()
24 {
25     jc[0]=1; inv_j[0]=quickpow(jc[0],p-2);
26     for (int i=1;i<=n;i++)
27         jc[i]=(LL)jc[i-1]*i%p,inv_j[i]=quickpow(jc[i],p-2);
28     for (int i=1;i<=n*8;i<=1)
29         wn[i]=quickpow(3,(p-1)/(i<<1));
30 }
31 void NTT(int n,int *a,int opt)
32 {
33     for (int i=0,j=0;i<=n;i++) {
34         if (i>j) swap(a[i],a[j]);
35         for (int l=n>>1;j*=2;l>>=1);
36     }
37     for (int i=1;i<=n;i<=1) {
38         LL wn1=wn[i];
39         for (int pl=i<<1,j=0;j<=n;j+=pl) {
40             LL w=1;
41             for (int k=0;k<=i;k++,w=(LL)w*wn1%p) {
42                 int x=a[j+k]; int y=(LL)a[j+k+i]*w%p;
43                 a[j+k]=(x+y)%p; a[j+k+i]=(x-y+p)%p;
44             }
45         }
46     }
47     if (opt==1) reverse(a+1,a+n);
48 }
49 void inverse(int n,int *a,int *b,int *c)
50 {
51     if (n==1) b[0]=quickpow(a[0],p-2);
52     else {
53         inverse((n+1)>>1,a,b,c);
54         int k=0;
55         for (k=i;k<=n;i<=1;k<=1);
56         for (int i=0;i<=n;i++) c[i]=a[i];
57         for (int i=n;i<=k;i++) c[i]=0;
58         NTT(k,c,1);
59         NTT(k,b,1);
60         for (int i=0;i<=k;i++) {
61             b[i]=(LL)(2-(LL)c[i]*b[i]%p)+b[i]%p;
62             if (b[i]<0) b[i]+=p;
63         }
64         NTT(k,b,-1);
65         int inv=quickpow(k,p-2);
66         for (int i=0;i<=k;i++) b[i]=(LL)b[i]*inv%p;
67         for (int i=n;i<=k;i++) b[i]=0;
68     }
69 }
70 int main()
71 {
72     scanf("%d",&n); init();
73     int n1=0;
74     for (n1=1;n1<=n*2;n1<=1);
75     a[0]=1;
76     for (int i=1;i<=n;i++) a[i]=(LL)quickpow(2,(LL)i*(i-1)/2)*inv_j[i]%p;
77     inverse(n1,a,b,c);
78     memset(c,0,sizeof(c));
79     for (int i=1;i<=n;i++) c[i]=(LL)quickpow(2,(LL)i*(i-1)/2)*inv_j[i-1]%p;
80     NTT(n1,b,1); NTT(n1,c,1);
81     for (int i=0;i<=n1;i++) b[i]=(LL)b[i]*c[i]%p;
82     NTT(n1,b,-1);
83     LL inv=quickpow(n1,p-2);
84     for (int i=0;i<=n1;i++) b[i]=(LL)b[i]*inv%p;
85     printf("%d\n", (LL)b[n]*jc[n-1]%p);
86     return 0;
87 }

```

广义 SAM

```

1  #include<iostream>
2  #include<string>
3  using namespace std;
4  const int MaxPoint=1010101;
5  struct Suffix_AutoMachine{
6      int son[MaxPoint][27],pre[MaxPoint],step[MaxPoint],right[MaxPoint],root,num;
7      int NewNode(int stp)
8      {
9          num++;
10         memset(son[num],0,sizeof(son[num]));
11         pre[num]=0;
12         step[num]=stp;
13         return num;
14     }
15     Suffix_AutoMachine()
16     {
17         num=0;
18         root=NewNode(0);
19     }
20     int push_back(int ch,int p)

```

```

21 {
22     int np=NewNode(step[p]+1);
23     right[np]=1;
24     step[np]=step[p]+1;
25     while(p&&!son[p][ch])
26     {
27         son[p][ch]=np;
28         p=pre[p];
29     }
30     if(!p)
31         pre[np]=root;
32     else
33     {
34         int q=son[p][ch];
35         if(step[q]==step[p]+1)
36             pre[np]=q;
37         else
38         {
39             int nq=NewNode(step[p]+1);
40             memcpy(son[nq],son[q],sizeof(son[q]));
41             step[nq]=step[p]+1;
42             pre[nq]=pre[q];
43             pre[q]=pre[np]=nq;
44             while(p&&son[p][ch]==q)
45             {
46                 son[p][ch]=nq;
47                 p=pre[p];
48             }
49         }
50     }
51     return np;
52 }
53 };
54 int main()
55 {
56     return 0;
57 }
58

```

循环串最小表示

```

1 int getmin( char s[] )
2 {
3     int i , j , k , m , t ;
4     m = strlen( s ) ;
5     i = 0 ; j = 1 ; k = 0 ;
6     while( i < m && j < m && k < m )
7     {
8         t = s[( i + k ) % m] - s[( j + k ) % m] ;
9         if( !t )
10             ++ k ;
11         else
12         {
13             if( t > 0 )
14                 i += k + 1 ;
15             else
16                 j += k + 1 ;
17             if( i == j )
18                 j ++ ;
19             k = 0 ;
20         }
21     }
22     return min(i,j);
23 }
24

```

最大团搜索

```

1 #include<iostream>
2 using namespace std;
3 int ans;
4 int num[1010];
5 int path[1010];
6 int a[1010][1010],n;
7 bool dfs(int *adj,int total,int cnt)
8 {
9     int i,j,k;
10    int t[1010];
11    if(total==0)
12    {
13        if(ans<cnt)
14        {
15            ans=cnt;
16            return 1;
17        }
18        return 0;
19    }
20    for(i=0;i<total;i++)
21    {
22        if(cnt+(total-i)<=ans)

```

```

23        return 0;
24        if(cnt+num[adj[i]]<=ans)
25            return 0;
26        for(k=0,j=i+1;j<total;j++)
27            if(a[adj[i]][adj[j]])
28                t[k++]=adj[j];
29        if(dfs(t,k,cnt+1))
30            return 1;
31    }
32    return 0;
33 }
34 int MaxClique()
35 {
36     int i,j,k;
37     int adj[1010];
38     if(n<=0)
39         return 0;
40     ans=1;
41     for(i=n-1;i>=0;i--)
42     {
43         for(k=0,j=i+1;j<n;j++)
44             if(a[i][j])
45                 adj[k++]=j;
46         dfs(adj,k,1);
47         num[i]=ans;
48     }
49     return ans;
50 }
51 int main()
52 {
53     ios::sync_with_stdio(0);
54     cin.tie(0);
55     cout.tie(0);
56     while(cin>>n)
57     {
58         if(n==0)
59             break;
60         for(int i=0;i<n;i++)
61             for(int j=0;j<n;j++)
62                 cin>>a[i][j];
63         cout<<MaxClique()<<endl;
64     }
65     return 0;
66 }

```

求原根

```

1 //51Nod - 1135
2 #include <iostream>
3 #include <string.h>
4 #include <algorithm>
5 #include <stdio.h>
6 #include <math.h>
7 #include <bitset>
8
9 using namespace std;
10 typedef long long LL;
11
12 const int N = 1000010;
13
14 bitset<N> prime;
15 int p[N],pri[N];
16 int k,cnt;
17
18 void isprime()
19 {
20     prime.set();
21     for(int i=2; i<N; i++)
22     {
23         if(prime[i])
24         {
25             p[k++] = i;
26             for(int j=i+i; j<N; j+=i)
27                 prime[j] = false;
28         }
29     }
30 }
31
32 void Divide(int n)
33 {
34     cnt = 0;
35     int t = (int)sqrt(1.0*n);
36     for(int i=0; p[i]<=t; i++)
37     {
38         if(n%p[i]==0)
39         {
40             pri[cnt++] = p[i];
41             while(n%p[i]==0) n /= p[i];
42         }

```

```

43     }
44     if(n > 1)
45         pri[cnt++] = n;
46 }
47
48 LL quick_mod(LL a,LL b,LL m)
49 {
50     LL ans = 1;
51     a %= m;
52     while(b)
53     {
54         if(b&1)
55         {
56             ans = ans * a % m;
57             b--;
58         }
59         b >>= 1;
60         a = a * a % m;
61     }
62     return ans;
63 }
64
65 int main()
66 {
67     int P;
68     isprime();
69     while(cin>>P)
70     {
71         Divide(P-1);
72         for(int g=2; g<P; g++)
73         {
74             bool flag = true;
75             for(int i=0; i<cnt; i++)
76             {
77                 int t = (P - 1) / pri[i];
78                 if(quick_mod(g,t,P) == 1)
79                 {
80                     flag = false;
81                     break;
82                 }
83             }
84             if(flag)
85             {
86                 int root = g;
87                 cout<<root<<endl;
88                 break;
89             }
90         }
91     }
92     return 0;
93 }

```

线性递推多项式

```

1 //µs
2 void linear_recurrence(long long n, int m, int a[], int c[], int p)
3 {
4     long long v[M] = {1 % p}, u[M << 1], msk = !n;
5     for(long long i(n); i > 1; i >>= 1)
6     {
7         msk <<= 1;
8     }
9     for(long long x(0); msk; msk >>= 1, x <<= 1)
10    {
11        fill_n(u, m << 1, 0);
12        int b(!!(n & msk));
13        x |= b;
14        if(x < m)
15        {
16            u[x] = 1 % p;
17        }
18        else
19        {
20            for(int i(0); i < m; i++)
21            {
22                for(int j(0), t(i + b); j < m; j++, t++)
23                {
24                    u[t] = (u[t] + v[i] * v[j]) % p;
25                }
26            }
27            for(int i((m << 1) - 1); i >= m; i--)
28            {
29                for(int j(0), t(i - m); j < m; j++, t++)
30                {
31                    u[t] = (u[t] + c[j] * u[i]) % p;
32                }
33            }
34        }
35        copy(u, u + m, v);
36    }

```

```

37 for(int i(m); i < 2 * m; i++)
38 {
39     a[i] = 0;
40     for(int j(0); j < m; j++)
41     {
42         a[i] = (a[i] + (long long)c[j] * a[i + j - m]) % p;
43     }
44 }
45 for(int j(0); j < m; j++)
46 {
47     b[j] = 0;
48     for(int i(0); i < m; i++)
49     {
50         b[j] = (b[j] + v[i] * a[i + j]) % p;
51     }
52 }
53 for(int j(0); j < m; j++)
54 {
55     a[j] = b[j];
56 }
57 }

```

经纬度球面距离

```

1 double sphereDis(double lon1, double lat1, double lon2, double lat2, double R) {
2     return R*acos(cos(lat1)*cos(lat2)+cos(lon1-lon2)*sin(lat1)*sin(lat2));
3 }

```

日期公式

```

1 int zeller(int y, int m, int d) { // y 年 m 月 d 日是星期几
2     if (m <= 2) y--, m += 12; int c = y / 100; y %= 100;
3     int w = ((c >> 2) - (c << 1) + y + (y >> 2) + (13 * (m + 1) / 5) + d - 1) % 7;
4     if (w < 0) w += 7; return w;
5 }
6 int getId(int y, int m, int d) { // y 年 m 月 d 日的日期编号
7     if (m < 3) {y--; m += 12;}
8     return 365 * y + y / 4 - y / 100 + y / 400 + (153 * m + 2) / 5 + d;
9 }

```

Manacher

注意事项：1-based 算法，请注意下标。

```

1 int manacher(char *text, int length, int *palindrome) {
2     static char buffer[MXN];
3     for (int i = 1; i <= length; i++) {
4         buffer[2 * i - 1] = text[i];
5         if (i != 0) buffer[2 * i] = '#';
6     }
7     palindrome[1] = 1;
8     for (int i = 2, j = 0; i <= 2 * length - 1; ++i) {
9         if (j + palindrome[j] <= i) palindrome[i] = 0;
10        else palindrome[i] = std::min(palindrome[(j << 1) - i], j + palindrome[j] - i);
11        while (i - palindrome[i] >= 1 && i + palindrome[i] <= 2 * length - 1 && buffer[i - palindrome[i]] == buffer[i + palindrome[i]]) {
12            palindrome[i]++;
13        }
14        if (i + palindrome[i] > j + palindrome[j]) j = i;
15    }
16    int answer = 0;
17    for (int i = 1; i < 2 * length; i++) {
18        if (i & 1) answer = std::max(answer, 2 * (palindrome[i] - 1 >> 1) + 1);
19        else answer = std::max(answer, 2 * (palindrome[i] >> 1));
20    }
21    return answer;
22 }

```

丁尧尧

kth.shortest.path

```

1 #include <stdio>
2 #include <cstring>
3 #include <queue>
4 using namespace std;
5
6 const int N = 1010;
7 const int M = 100010;
8 const int oo = 0x3f3f3f3f;
9
10 struct Elist {
11     int head[N], _dest[M], _dist[M], _last[M], etot;
12     inline void adde( int u, int v, int d ) {
13         etot++;
14         _dest[etot] = v;
15         _dist[etot] = d;
16         _last[etot] = _head[u];
17         _head[u] = etot;
18     }

```

```

19     inline int head( int u ) { return _head[u]; }
20     inline int dest( int t ) { return _dest[t]; }
21     inline int dist( int t ) { return _dist[t]; }
22     inline int last( int t ) { return _last[t]; }
23 };
24 struct Stat {
25     int u, d;
26     Stat(){}
27     Stat( int u, int d ):u(u),d(d){}
28 };
29
30 int n, m, K;
31 Elist e, re;
32 int src, dst;
33 int rdis[N];
34 bool done[N];
35
36 bool operator<( const Stat &r, const Stat &s ) {
37     return r.d + rdis[r.u] > s.d + rdis[s.u];
38 }
39
40 void dijkstra() {
41     memset( done, false, sizeof(done) );
42     memset( rdis, 0x3f, sizeof(rdis) );
43     priority_queue<pair<int,int>> Q;
44     Q.push( make_pair(0,dst) );
45     rdis[dst] = 0;
46     while( !Q.empty() ) {
47         int u = Q.top().second;
48         Q.pop();
49         if( done[u] ) continue;
50         done[u] = true;
51         for( int t = re.head(u); t; t = re.last(t) ) {
52             int v = re.dest(t), d = re.dist(t);
53             if( done[v] ) continue;
54             if( rdis[v] > rdis[u] + d ) {
55                 rdis[v] = rdis[u] + d;
56                 Q.push( make_pair( -rdis[v], v ) );
57             }
58         }
59     }
60
61     int astar() {
62         int pcnt = 0;
63         priority_queue<Stat> Q;
64
65         if( rdis[src] == oo ) return -1;
66         if( src == dst ) K++;
67         Q.push( Stat( src, 0 ) );
68         while( !Q.empty() ) {
69             Stat s = Q.top();
70             Q.pop();
71             if( s.u == dst ) {
72                 pcnt++;
73                 if( pcnt == K )
74                     return s.d;
75             }
76             for( int t = e.head(s.u); t; t = e.last(t) ) {
77                 int v = e.dest(t), d = e.dist(t);
78                 if( rdis[v] == oo ) continue;
79                 Q.push( Stat( v, s.d + d ) );
80             }
81         }
82         return -1;
83     }
84
85     int main() {
86         scanf( "%d%d", &n, &m );
87         for( int i = 1; i <= m; i++ ) {
88             int u, v, d;
89             scanf( "%d%d%d", &u, &v, &d );
90             e.adde( u, v, d );
91             re.adde( v, u, d );
92         }
93         scanf( "%d%d%d", &src, &dst, &K );
94         dijkstra();
95         printf( "%d\n", astar() );
96     }
97 }

```

弦图

一些定义:

弦图是一种特殊图: 它的所有极大环都只有 3 个顶点。

单纯点: 该顶点与其邻接点在原图中的导出子图是一个完全图。

图 G 的完美消去序列: 一个顶点序列 $a_1 a_2 a_3 \dots a_n$, 使得对于每个元素 a_i , a_i 在 $a_i, a_i+1, a_i+2 \dots a_n$ 的导出子图中是一个单纯点。

弦图有一个性质: 任何一个弦图都至少存在一个单纯点 (该点及其邻接点组成一个完全图)

弦图另一个性质: 一个图是弦图当且仅当存在完美消去序列。(归纳证明)

最大势算法 (msc): 若原图是弦图, 则该算法计算出的序列是完美消去序列。

算法大致思想: 从后往前计算序列, 每次选择点 v 作为序列中的元素, v 是还未选的点中与已经选了的点连边最多的点。

然后检查该序列是否是完美消去序列。

```

12 #include <stdio>
13 #include <cstring>
14 #define N 1010
15 #define M N*N*2
16 int n, m;
17 bool c[N][N];
18 int qu[N], inq[N], dgr[N];
19 int stk[M], top;
20 void msc() {
21     dgr[0] = -1;
22     for( int i=n; i>=1; i-- ) {
23         int s = 0;
24         for( int u=1; u<=n; u++ )
25             if( !inq[u] && dgr[u]>dgr[s] ) s=u;
26         qu[i] = s;
27         inq[s] = true;
28         for( int u=1; u<=n; u++ )
29             if( !inq[u] && c[s][u] ) dgr[u]++;
30     }
31 }
32
33 bool check() {
34     for( int i=n; i>=1; i-- ) {
35         int s=qu[i];
36         top = 0;
37         for( int j=i+1; j<=n; j++ )
38             if( c[s][qu[j]] ) stk[++top] = qu[j];
39         if( top==0 ) continue;
40         for( int j=2; j<=top; j++ )
41             if( !c[stk[j]][stk[j]] ) return false;
42     }
43     return true;
44 }
45
46 int main() {
47     scanf( "%d%d", &n, &m );
48     for( int i=1,u,v; i<=m; i++ ) {
49         scanf( "%d%d", &u, &v );
50         c[u][v] = c[v][u] = 1;
51     }
52     msc();
53     printf( "%s\n", check() ? "Perfect" : "Imperfect" );
54 }
55
56 /*
57 给定一个弦图, 问最少染色数。
58 对于弦图的一个完美消去序列, 从后往前染色, 每次染可以染的最小编号的颜色,
59 由完美消去序列的定义, 序列任一后缀的点的导出子图中, 由该后缀第一个元素
60 及其邻接点导出的子图一定是完全图, 所以, 序列中 某一元素染的颜色编号是该
61 完全图的大小。所以最小染色数小于等于最大团的点数, 而显然前者又大于等于后者,
62 故弦图的最小染色数等于最大团的大小。
63 */
64 #include <stdio>
65 #include <vector>
66 #define maxn 10010
67 using namespace std;
68 int n, m;
69 vector<int> g[maxn];
70 bool done[maxn];
71 int label[maxn], pos[maxn];
72 int msc() {
73     int rt = 0;
74     for( int i=n; i>=1; i-- ) {
75         int mu = 0;
76         for( int u=1; u<=n; u++ ) {
77             if( !done[u] ) {
78                 if( !mu || label[u]>label[mu] )
79                     mu = u;
80             }
81         }
82         done[mu] = true;
83         pos[mu] = i;
84         int cnt = 0;
85         for( int t=0; t<g[mu].size(); t++ ) {
86             int v = g[mu][t];
87             if( done[v] ) {
88                 cnt++;
89             } else {
90                 label[v]++;
91             }
92         }
93         rt = max( rt, cnt+1 );
94     }
95     return rt;
96 }
97
98 int main() {
99     scanf( "%d%d", &n, &m );
100     for( int i=1,u,v; i<=m; i++ ) {
101         scanf( "%d%d", &u, &v );
102         g[u].push_back(v);
103     }
104 }

```

```
100         g[v].push_back(u);
101     }
102     printf( "%d\n", msc() );
103 }
```

集合幂级数

```
1  #include <cstdio>
2  const int N = 10;
3  int n, U;
4  int a[1<N], b[1<N], c[1<N];
5  void trans( int a[], int flag ) {
6      for( int b=0; b<n; b++ ) {
7          int u = U ^ (1<b);
8          for( int s=u, t=1<(n-1); t; s=(s-1)&u, t-- ) {
9              int la[s], r=a[s|(1<b)];
10             /* NOT AND
11             if( flag==1 ) { a[s] = l+r; a[s|(1<b)] = r;
12             } else { a[s] = r; a[s|(1<b)] = l-r; } */
13             /* NOT XOR
14             if( flag==1 ) { a[s] = l+r; a[s|(1<b)] = l-r;
15             } else { a[s] = (l-r)/2; a[s|(1<b)] = (l+r)/2; } */
16             /* NOT OR
17             if( flag==1 ) { a[s] = l; a[s|(1<b)] = l+r;
18             } else { a[s] = r-l; a[s|(1<b)] = l; } */
19             /* OR
20             if( flag==1 ) { a[s] = l; a[s|(1<b)] = l+r;
21             } else { a[s] = l; a[s|(1<b)] = r-l; } */
22             /* AND
23             if( flag==1 ) { a[s] = l+r; a[s|(1<b)] = r;
24             } else { a[s] = l-r; a[s|(1<b)] = r; } */
25             /* XOR
26             if( flag==1 ) { a[s] = l+r; a[s|(1<b)] = l-r;
27             } else { a[s] = (l+r)/2; a[s|(1<b)] = (l-r)/2; } */
28         }
29     }
30 }
31 int main() {
32     scanf( "%d", &n );
33     U = (1<n)-1;
34     for( int i=0; i<U; i++ ) scanf( "%d", &a[i] );
35     for( int i=0; i<U; i++ ) scanf( "%d", &b[i] );
36     trans(a,1); trans(b,1);
37     for( int s=0; s<U; s++ ) c[s] = a[s]*b[s];
38     trans(c,-1);
39     for( int s=0; s<U; s++ ) printf( "%d ", c[s] );
40     printf( "\n" );
41 }
```

其他

Java Hints

```
1  import java.util.*;
2  import java.math.*;
3  import java.io.*;
4  public class Main{
5      static class Task{
6          void solve(int testId, InputReader cin, PrintWriter cout) {
7              // Write down the code you want
8          }
9      };
10     public static void main(String args[]) {
11         InputStream inputStream = System.in;
12         OutputStream outputStream = System.out;
13         InputReader in = new InputReader(inputStream);
14         PrintWriter out = new PrintWriter(outputStream);
15         TaskA solver = new TaskA();
16         solver.solve(1, in, out);
17         out.close();
18     }
19     static class InputReader {
20         public BufferedReader reader;
21         public StringTokenizer tokenizer;
22         public InputReader(InputStream stream) {
23             reader = new BufferedReader(new InputStreamReader(stream), 32768);
24             tokenizer = null;
25         }
26         public String next() {
27             while (tokenizer == null || !tokenizer.hasMoreTokens()) {
28                 try {
29                     tokenizer = new StringTokenizer(reader.readLine());
30                 } catch (IOException e) {
31                     throw new RuntimeException(e);
32                 }
33             }
34             return tokenizer.nextToken();
35         }
36     }
37 }
```

```
35     }
36     public int nextInt() {
37         return Integer.parseInt(next());
38     }
39 }
40 };
41 // Arrays
42 int a[];
43 .fill(a[,int fromIndex,int toIndex],val);|.sort(a[, int fromIndex, int toIndex])
44 // String
45 String s;
46 .charAt(int i);|compareTo(String)|compareToIgnoreCase ()|contains(String)|
47 length ()|substring(int l, int len)
48 // BigInteger
49 .abs()|.add()|bitLength()|subtract()|divide()|remainder()|divideAndRemainder()|
50 modPow(b, c)|pow(int) | multiply () | compareTo () |
51 gcd() | intValue () | longValue () | isProbablePrime(int c) (1 - 1/2^c) |
52 nextProbablePrime () | shiftLeft(int) | valueOf ()
53 // BigDecimal
54 .ROUND_CEILING | ROUND_DOWN_FLOOR | ROUND_HALF_DOWN | ROUND_HALF_EVEN | ROUND_HALF_UP | ROUND_UP
55 .divide(BigDecimal b, int scale , int round_mode) | doubleValue () | movePointLeft(int) | pow(int) |
56 setScale(int scale , int round_mode) | stripTrailingZeros ()
57 // StringBuilder
58 StringBuilder sb = new StringBuilder ();
59 sb.append(elem) | out.println(sb)
60 // TODO Java STL 的使用方法以及上面这些方法的检验
```

常用结论

上下界网络流

$B(u,v)$ 表示边 (u,v) 流量的下界, $C(u,v)$ 表示边 (u,v) 流量的上界, $F(u,v)$ 表示边 (u,v) 的流量。设 $G(u,v) = F(u,v) - B(u,v)$, 显然有: $0 \leq G(u,v) \leq C(u,v) - B(u,v)$

无源汇的上下界可行流

建立超级源点 S^* 和超级汇点 T^* , 对于原图每条边 (u,v) 在新网络中连如下三条边: $S^* \rightarrow v$, 容量为 $B(u,v)$; $u \rightarrow T^*$, 容量为 $B(u,v)$; $u \rightarrow v$, 容量为 $C(u,v) - B(u,v)$ 。最后求新网络的最大流, 判断从超级源点 S^* 出发的边是否都满流即可, 边 (u,v) 的最终解中的实际流量为 $G(u,v) + B(u,v)$ 。

有源汇的上下界可行流

从汇点 T 到源点 S 连一条上界为 ∞ , 下界为 0 的边。按照无源汇的上下界可行流一样做即可, 流量即为 $T \rightarrow S$ 边上的流量。

有源汇的上下界最大流

1. 在有源汇的上下界可行流中, 从汇点 T 到源点 S 的边改为连一条上界为 ∞ , 下届为 x 的边。 x 满足二分性质, 找到最大的 x 使得新网络存在无源汇的上下界可行流即为原图的最大流。
2. 从汇点 T 到源点 S 连一条上界为 ∞ , 下界为 0 的边, 变成无源汇的网络。按照无源汇的上下界可行流的方法, 建立超级源点 S^* 和超级汇点 T^* , 求一遍 $S^* \rightarrow T^*$ 的最大流, 再将从汇点 T 到源点 S 的这条边拆掉, 求一次 $S \rightarrow T$ 的最大流即可。

有源汇的上下界最小流

1. 在有源汇的上下界可行流中, 从汇点 T 到源点 S 的边改为连一条上界为 x , 下界为 0 的边。 x 满足二分性质, 找到最小的 x 使得新网络存在无源汇的上下界可行流即为原图的最小流。
2. 按照无源汇的上下界可行流的方法, 建立超级源点 S^* 与超级汇点 T^* , 求一遍 $S^* \rightarrow T^*$ 的最大流, 但是注意这一次不加上汇点 T 到源点 S 的这条边, 即不使之改为无源汇的网络去求解。求完后, 再加上那条汇点 T 到源点 S 上界 ∞ 的边。因为这条边下界为 0 , 所以 S^*, T^* 无影响, 再直接求一次 $S^* \rightarrow T^*$ 的最大流。若超级源点 S^* 出发的边全部满流, 则 $T \rightarrow S$ 边上的流量即为原图的最小流, 否则无解。

上下界费用流

来源: BZOJ 3876 设汇 t , 源 s , 超级源 S , 超级汇 T , 本质是每条边的下界为 1 , 上界为 MAX , 跑一遍有源汇的上下界最小费用最小流。(因为上界无穷大, 所以只要满足所有下界的最小费用最小流)

1. 对每个点 x : 从 x 到 t 连一条费用为 0 , 流量为 MAX 的边, 表示可以任意停止当前的剧情(接下来的剧情从更优的路径去走, 画个样例就知道了)
2. 对于每一条边权为 z 的边 $x \rightarrow y$:
 - 从 S 到 y 连一条流量为 1 , 费用为 z 的边, 代表这条边至少要被走一次。
 - 从 x 到 y 连一条流量为 MAX , 费用为 z 的边, 代表这条边除了至少走的一次之外还可以随便走。
 - 从 x 到 T 连一条流量为 1 , 费用为 0 的边。(注意是每一条 $x \rightarrow y$ 的边都连, 或者你可以记下 x 的出边数 K_x , 连一次流量为 K_x , 费用为 0 的边)。

建完图后从 S 到 T 跑一遍费用流, 即可。(当前跑出来的就是满足上下界的最小费用最小流了)

弦图相关

1. 团数 \leq 色数, 弦图团数 = 色数
2. 设 $next(v)$ 表示 $N(v)$ 中最前的点, 令 w^* 表示所有满足 $A \in B$ 的 w 中最后的一个点, 判断 $v \cup N(v)$ 是否为极大团, 只需判断是否存在一个 w , 满足 $Next(w) = v$ 且 $|N(v)| + 1 \leq |N(w)|$ 即可.
3. 最小染色: 完美消除序列从后往前依次给每个点染色, 给每个点染上可以染的最小的颜色
4. 最大独立集: 完美消除序列从前往后能选就选
5. 弦图最大独立集数 = 最小团覆盖数, 最小团覆盖: 设最大独立集为 $\{p_1, p_2, \dots, p_t\}$, 则 $\{p_1 \cup N(p_1), \dots, p_t \cup N(p_t)\}$ 为最小团覆盖

Bernoulli 数

1. 初始化: $B_0(n) = 1$
2. 递推公式: $B_m(n) = n^m - \sum_{k=0}^{m-1} \binom{m}{k} \frac{B_k(n)}{m-k+1}$
3. 应用: $\sum_{k=1}^n k^m = \frac{1}{m+1} \sum_{k=0}^m \binom{m+1}{k} n^{m+1-k}$

常见错误

1. 数组或者变量类型开错, 例如将 double 开成 int;
2. 函数忘记返回返回值;
3. 初始化数组没有初始化完全;
4. 对空间限制判断不足导致 MLE

博弈游戏

巴什博弈

1. 只有一堆 n 个物品, 两个人轮流从这堆物品中取物, 规定每次至少取一个, 最多取 m 个。最后取光者得胜。
2. 显然, 如果 $n = m + 1$, 那么由于一次最多只能取 m 个, 所以, 无论先取者拿走多少个, 后取者都能够一次拿走剩余的物品, 后者取胜。因此我们发现了如何取胜的法则: 如果 $n = m + 1 \cdot r + s$, (r 为任意自然数, $s \leq m$), 那么先取者要拿走 s 个物品, 如果后取者拿走 k ($k \leq m$) 个, 那么先取者再拿走 $m + 1 - k$ 个, 结果剩下 $(m + 1)(r - 1)$ 个, 以后保持这样的取法, 那么先取者肯定获胜。总之, 要保持给对手留下 $(m + 1)$ 的倍数, 就能最后获胜。

威佐夫博弈

1. 有两堆各若干个物品, 两个人轮流从某一堆或同时从两堆中取同样多的物品, 规定每次至少取一个, 多者不限, 最后取光者得胜。
2. 判断一个局势 (a, b) 为奇异局势 (必败态) 的方法: $a_k = [k(1 + \sqrt{5})/2]$ $b_k = a_k + k$

阶梯博弈

1. 博弈在一列阶梯上进行, 每个阶梯上放着自然数个点, 两个人进行阶梯博弈, 每一步则是将一个阶梯上的若干个点 (至少一个) 移到前面去, 最后没有点可以移动的人输。
2. 解决方法: 把所有奇数阶梯看成 N 堆石子, 做 NIM. (把石子从奇数堆移动到偶数堆可以理解为拿走石子, 就相当于几个奇数堆的石子在做 Nim)

图上删边游戏

链的删边游戏

1. 游戏规则: 对于一条链, 其中一个端点是根, 两人轮流删边, 脱离根的部分也算被删去, 最后没边可删的人输。
2. 做法: $sg[i] = n - dist(i) - 1$ (其中 n 表示总点数, $dist(i)$ 表示离根的距离)

树的删边游戏

1. 游戏规则: 对于一棵有根树, 两人轮流删边, 脱离根的部分也算被删去, 没边可删的人输。
2. 做法: 叶子结点的 $sg = 0$, 其他节点的 sg 等于儿子结点的 $sg + 1$ 的异或和。

局部连通图的删边游戏

1. 游戏规则: 在一个局部连通图上, 两人轮流删边, 脱离根的部分也算被删去, 没边可删的人输。局部连通图的构图规则是, 在一棵基础树上加边得到, 所有形成的环保证不共用边, 且只与基础树有一个公共点。
2. 做法: 去掉所有的偶环, 将所有的奇环变为长度为 1 的链, 然后做树的删边游戏。

常用数学公式

求和公式

1. $\sum_{k=1}^n (2k-1)^2 = \frac{n(4n^2-1)}{3}$
2. $\sum_{k=1}^n k^3 = [\frac{n(n+1)}{2}]^2$
3. $\sum_{k=1}^n (2k-1)^3 = n^2(2n^2-1)$
4. $\sum_{k=1}^n k^4 = \frac{n(n+1)(2n+1)(3n^2+3n-1)}{30}$
5. $\sum_{k=1}^n k^5 = \frac{n^2(n+1)^2(2n^2+2n-1)}{12}$
6. $\sum_{k=1}^n k(k+1) = \frac{n(n+1)(n+2)}{3}$
7. $\sum_{k=1}^n k(k+1)(k+2) = \frac{n(n+1)(n+2)(n+3)}{4}$
8. $\sum_{k=1}^n k(k+1)(k+2)(k+3) = \frac{n(n+1)(n+2)(n+3)(n+4)}{5}$
9. $\frac{1}{(1-x)^{n+1}} = \sum_{i=0}^n \binom{i+n}{i} x^i$
10. $\frac{1}{\sqrt{1-4x}} = \sum_{i=0}^n \binom{2i}{i} x^i$

斐波那契数列

1. $fib_0 = 0, fib_1 = 1, fib_n = fib_{n-1} + fib_{n-2}$
2. $fib_{n+2} \cdot fib_n - fib_{n+1}^2 = (-1)^{n+1}$
3. $fib_{-n} = (-1)^{n-1} fib_n$
4. $fib_{n+k} = fib_k \cdot fib_{n+1} + fib_{k-1} \cdot fib_n$
5. $gcd(fib_m, fib_n) = fib_{gcd(m,n)}$
6. $fib_m | fib_n^2 \Leftrightarrow n | fib_m$

错排公式

$$D_n = (n-1)(D_{n-2} - D_{n-1}) = n! \cdot (1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \dots + \frac{(-1)^n}{n!})$$

莫比乌斯函数

$$\mu(n) = \begin{cases} 1 & \text{若 } n=1 \\ (-1)^k & \text{若 } n \text{ 无平方数因子, 且 } n=p_1 p_2 \dots p_k \\ 0 & \text{若 } n \text{ 有大于 } 1 \text{ 的平方数因数} \end{cases}$$

$$\sum_{d|n} \mu(d) = \begin{cases} 1 & \text{若 } n=1 \\ 0 & \text{其他情况} \end{cases}$$

$$g(n) = \sum_{d|n} f(d) \Leftrightarrow f(n) = \sum_{d|n} \mu(d) g(\frac{n}{d})$$

$$g(x) = \sum_{n=1}^{[x]} f(\frac{x}{n}) \Leftrightarrow f(x) = \sum_{n=1}^{[x]} \mu(n) g(\frac{x}{n})$$

Burnside 引理

设 G 是一个有限群, 作用在集合 X 上。对每个 g 属于 G , 令 X^g 表示 X 中在 g 作用下的不动元素, 轨道数 (记作 $|X/G|$) 为 $|X/G| = \frac{1}{|G|} \sum_{g \in G} |X^g|$.

五边形数定理

$$\text{设 } p(n) \text{ 是 } n \text{ 的拆分数, 有 } p(n) = \sum_{k \in \mathbb{Z} \setminus \{0\}} (-1)^{k-1} p\left(n - \frac{k(3k-1)}{2}\right)$$

树的计数

1. 有根树计数: $n+1$ 个结点的有根树的个数为 $a_{n+1} = \frac{\sum_{j=1}^n j \cdot a_j \cdot S_{n,j}}{n}$, 其中, $S_{n,j} = \sum_{i=1}^{n/j} a_{n+1-ij} = S_{n-j,j} + a_{n+1-j}$
2. 无根树计数: 当 n 为奇数时, n 个结点的无根树的个数为 $a_n - \sum_{i=1}^{n/2} a_i a_{n-i}$, 当 n 为偶数时, n 个结点的无根树的个数为 $a_n - \sum_{i=1}^{n/2} a_i a_{n-i} + \frac{1}{2} a_{\frac{n}{2}} (a_{\frac{n}{2}} + 1)$
3. n 个结点的完全图的生成树个数为: n^{n-2}
4. 矩阵 - 树定理: 图 G 由 n 个结点构成, 设 $\mathbf{A}[G]$ 为图 G 的邻接矩阵, $\mathbf{D}[G]$ 为图 G 的度数矩阵, 则图 G 的不同生成树的个数为 $\mathbf{C}[G] = \mathbf{D}[G] - \mathbf{A}[G]$ 的任意一个 $n-1$ 阶主子式的行列式值。

欧拉公式

平面图形的顶点个数、边数和面的个数有如下关系: $V - E + F = C + 1$

其中, V 是顶点的数目, E 是边的数目, F 是面的数目, C 是组成图形的连通部分的数目。当图是单连通图的时候, 公式简化为: $V - E + F = 2$

皮克定理

给定顶点坐标均是整点 (或正方形格点) 的简单多边形, 其面积 A 和内部格点数目 i 、边上格点数目 b 的关系: $A = i + \frac{b}{2} - 1$

牛顿恒等式

设

$$\prod_{i=1}^n (x - x_i) = a_n + a_{n-1}x + \cdots + a_1x^{n-1} + a_0x^n$$

$$p_k = \sum_{i=1}^n x_i^k$$

则

$$a_0 p_k + a_1 p_{k-1} + \cdots + a_{k-1} p_1 + k a_k = 0$$

特别地, 对于

$$|\mathbf{A} - \lambda \mathbf{E}| = (-1)^n (a_n + a_{n-1}\lambda + \cdots + a_1\lambda^{n-1} + a_0\lambda^n)$$

有

$$p_k = \text{Tr}(\mathbf{A}^k)$$

平面几何公式

三角形

1. 面积: $S = \frac{a \cdot H_a}{2} = \frac{ab \cdot \sin C}{2} = \sqrt{p(p-a)(p-b)(p-c)} \left(\frac{a+b+c}{2} \right)$
2. 中线: $M_a = \frac{\sqrt{2(b^2+c^2)-a^2}}{2} = \frac{\sqrt{b^2+c^2+2bc \cdot \cos A}}{2}$
3. 角平分线: $T_a = \frac{\sqrt{bc \cdot [(b+c)^2 - a^2]}}{b+c} = \frac{2bc}{b+c} \cos \frac{A}{2}$
4. 高线: $H_a = b \sin C = c \sin B = \sqrt{b^2 - \left(\frac{a^2 + b^2 - c^2}{2a} \right)^2}$
5. 内切圆半径

$$\begin{aligned} r &= \frac{S}{p} = \frac{\arcsin \frac{B}{2} \cdot \sin \frac{C}{2}}{\sin \frac{B+C}{2}} = 4R \cdot \sin \frac{A}{2} \sin \frac{B}{2} \sin \frac{C}{2} \\ &= \sqrt{\frac{(p-a)(p-b)(p-c)}{p}} = p \cdot \tan \frac{A}{2} \tan \frac{B}{2} \tan \frac{C}{2} \end{aligned}$$

6. 外接圆半径: $R = \frac{abc}{4S} = \frac{a}{2\sin A} = \frac{b}{2\sin B} = \frac{c}{2\sin C}$

四边形

D_1, D_2 为对角线, M 为对角线中点连线, A 为对角线夹角, p 为半周长

1. $a^2 + b^2 + c^2 + d^2 = D_1^2 + D_2^2 + 4M^2$
2. $S = \frac{1}{2} D_1 D_2 \sin A$
3. 对于圆内接四边形: $ac + bd = D_1 D_2$
4. 对于圆内接四边形: $S = \sqrt{(p-a)(p-b)(p-c)(p-d)}$

正 n 边形

R 为外接圆半径, r 为内切圆半径

1. 中心角: $A = \frac{2\pi}{n}$
2. 内角: $C = \frac{n-2}{n} \pi$
3. 边长: $a = 2\sqrt{R^2 - r^2} = 2R \cdot \sin \frac{A}{2} = 2r \cdot \tan \frac{A}{2}$
4. 面积: $S = \frac{nar}{2} = nr^2 \cdot \tan \frac{A}{2} = \frac{nR^2}{2} \cdot \sin A = \frac{na^2}{4 \cdot \tan \frac{A}{2}}$

圆

1. 弧长: $l = rA$
2. 弦长: $a = 2\sqrt{2hr - h^2} = 2r \cdot \sin \frac{A}{2}$
3. 弓形高: $h = r - \sqrt{r^2 - \frac{a^2}{4}} = r(1 - \cos \frac{A}{2}) = \frac{1}{2} \cdot \arctan \frac{A}{4}$
4. 扇形面积: $S_1 = \frac{rl}{2} = \frac{r^2 A}{2}$
5. 弓形面积: $S_2 = \frac{rl - a(r-h)}{2} = \frac{r^2}{2} (A - \sin A)$

棱柱

1. 体积 (A 为底面积, h 为高): $V = Ah$
2. 侧面积 (l 为棱长, p 为直截面周长): $S = lp$
3. 全面积: $T = S + 2A$

棱锥

1. 体积 (A 为底面积, h 为高): $V = Ah$
2. 正棱锥侧面积 (l 为棱长, p 为直截面周长): $S = lp$
3. 正棱锥全面积: $T = S + 2A$

棱台

1. 体积 (A_1, A_2 为上下底面积, h 为高): $V = (A_1 + A_2 + \sqrt{A_1 A_2}) \cdot \frac{h}{3}$
2. 正棱台侧面积 (p_1, p_2 为上下底面周长, l 为斜高): $S = \frac{p_1 + p_2}{2} l$
3. 正棱台全面积: $T = S + A_1 + A_2$

圆柱

1. 侧面积: $S = 2\pi r h$
2. 全面积: $T = 2\pi r(h + r)$
3. 体积: $V = \pi r^2 h$

圆锥

1. 母线: $l = \sqrt{h^2 + r^2}$
2. 侧面积: $S = \pi r l$
3. 全面积: $T = \pi r(l + r)$
4. 体积: $V = \frac{\pi}{3} r^2 h$

圓台

1. 母线: $l = \sqrt{h^2 + (r_1 - r_2)^2}$
2. 側面积: $S = \pi(r_1 + r_2)l$
3. 全面积: $T = \pi r_1(l + r_1) + \pi r_2(l + r_2)$
4. 体积: $V = \frac{\pi}{3}(r_1^2 + r_2^2 + r_1r_2)h$

球

1. 全面积: $T = 4\pi r^2$
2. 体积: $V = \frac{4}{3}\pi r^3$

球台

1. 側面积: $S = 2\pi rh$
2. 全面积: $T = \pi(2rh + r_1^2 + r_2^2)$
3. 体积: $V = \frac{\pi h[3(r_1^2 + r_2^2) + h^2]}{6}$

球扇形

1. 全面积 (h 为球冠高, r_0 为球冠底面半径): $T = \pi r(2h + r_0)$
2. 体积: $V = \frac{2}{3}\pi r^2h$

立体几何公式

球面三角公式

设 a, b, c 是边长, A, B, C 是所对的二面角, 有余弦定理

$$\cos a = \cos b \cdot \cos c + \sin b \cdot \sin c \cdot \cos A$$

正弦定理

$$\frac{\sin A}{\sin a} = \frac{\sin B}{\sin b} = \frac{\sin C}{\sin c}$$

三角形面积是 $A + B + C - \pi$

四面体体积公式

U, V, W, u, v, w 是四面体的 6 条棱, U, V, W 构成三角形, $(U, u), (V, v), (W, w)$ 互为对棱, 则

$$V = \frac{\sqrt{(s - 2a)(s - 2b)(s - 2c)(s - 2d)}}{192uvw}$$

其中

$$\left\{\begin{array}{lcl}a & = & \sqrt{xYZ}, \\b & = & \sqrt{yZX}, \\c & = & \sqrt{zXY}, \\d & = & \sqrt{xyz}, \\s & = & a + b + c + d, \\X & = & (w - U + v)(U + v + w), \\x & = & (U - v + w)(v - w + U), \\Y & = & (u - V + w)(V + w + u), \\y & = & (V - w + u)(w - u + V), \\Z & = & (v - W + u)(W + u + v), \\z & = & (W - u + v)(u - v + W)\end{array}\right.$$

附录

NTT 素数及原根列表

Id	Primes	PRT	Id	Primes	PRT	Id	Primes	PRT
1	7340033	3	38	311427073	7	75	786432001	7
2	13631489	15	39	330301441	22	76	799014913	13
3	23068673	3	40	347078657	3	77	800063489	3
4	26214401	3	41	359661569	3	78	802160641	11
5	28311553	5	42	361758721	29	79	818937857	5
6	69206017	5	43	377487361	7	80	824180737	5
7	70254593	3	44	383778817	5	81	833617921	13
8	81788929	7	45	387973121	6	82	850395137	3
9	101711873	3	46	399507457	5	83	862978049	3
10	104857601	3	47	409993217	3	84	880803841	26
11	111149057	3	48	415236097	5	85	883949569	7
12	113246209	7	49	447741953	3	86	897581057	3
13	120586241	6	50	459276289	11	87	899678209	7
14	132120577	5	51	463470593	3	88	907018241	3
15	136314881	3	52	468713473	5	89	913309697	3
16	138412033	5	53	469762049	3	90	918552577	5
17	141557761	26	54	493879297	10	91	919601153	3
18	147849217	5	55	531628033	5	92	924844033	5
19	155189249	6	56	576716801	6	93	925892609	3
20	158334977	3	57	581959681	11	94	935329793	3
21	163577857	23	58	595591169	3	95	938475521	3
22	167772161	3	59	597688321	11	96	940572673	7
23	169869313	5	60	605028353	3	97	943718401	7
24	185597953	5	61	635437057	11	98	950009857	7
25	186646529	3	62	639631361	6	99	957349889	6
26	199229441	3	63	645922817	3	100	962592769	7
27	204472321	19	64	648019969	17	101	972029953	10
28	211812353	3	65	655360001	3	102	975175681	17
29	221249537	3	66	666894337	5	103	976224257	3
30	230686721	6	67	683671553	3	104	985661441	3
31	246415361	3	68	710934529	17	105	998244353	3
32	249561089	3	69	715128833	3	106	1004535809	3
33	257949697	5	70	718274561	3	107	1007681537	3
34	270532609	22	71	740294657	3	108	1012924417	5
35	274726913	3	72	745537537	5	109	1045430273	3
36	290455553	3	73	754974721	11	110	1051721729	6
37	305135617	5	74	770703361	11	111	1053818881	7