

Report of fitting

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This is the fitting report of the multi-hawkes point process. In last assignment, the simulation has been implemented. And in this assignment, EM algorithm is used to optimize the likelihood of the model.

EM algorithm

Here is the inductions in Prof. Yan's slides and I just copy them here for simplicity.

Maximum-likelihood estimation

For MLE, we use the data $\{t_i, d_i\}_{i=1}^N$ instead of $\{(t_i^m)_i\}_{m=1}^M$

For a simple multi-dimensional Hawkes processes:

$$\lambda_d = \mu_d + \sum_{i: t_i < t} \alpha_{d d_i} e^{-\beta(t-t_i)}$$

log-likelihood:

$$\begin{aligned} \log L &= \sum_{d=1}^M \left\{ \sum_{(t_i, d_i): d_i=d} \log \lambda_{d_i}(t_i) - \int_0^T \lambda_d(t) dt \right\} \\ &= \sum_{i=1}^n \log \left(\mu_{d_i} + \sum_{t_j < t_i} \alpha_{d_i d_j} e^{-\beta(t_i - t_j)} \right) - T \sum_{d=1}^M \mu_d - \sum_{d=1}^M \sum_{j=1}^n \alpha_{d d_j} G_{d d_j}(T - t_j) \end{aligned}$$

Maximum-likelihood estimation

Jensen equality:

$$\begin{aligned} \log L &= \sum_{i=1}^n \log \left(\mu_{d_i} + \sum_{t_j < t_i} \alpha_{d_i d_j} e^{-\beta(t_i - t_j)} \right) - T \sum_{d=1}^M \mu_d - \sum_{d=1}^M \sum_{j=1}^n \alpha_{d d_j} G_{d d_j}(T - t_j) \\ &\geq \sum_{i=1}^n \left(p_{ii} \log \frac{\mu_{d_i}}{p_{ii}} + \sum_{j=1}^{i-1} p_{ij} \log \frac{\alpha_{d_i d_j} e^{-\beta(t_i - t_j)}}{p_{ij}} \right) - T \sum_{d=1}^M \mu_d - \sum_{d=1}^M \sum_{j=1}^n \alpha_{d d_j} G_{d d_j}(T - t_j) \end{aligned}$$

lower bound

Maximum-likelihood estimation

So for E-step

$$p_{ii}^{(k+1)} = \frac{\mu_{d_i}^{(k)}}{\mu_{d_i}^{(k)} + \sum_{j=1}^{i-1} \alpha_{d_i d_j}^{(k)} e^{-\beta(t_i - t_j)}}$$

$$p_{ij}^{(k+1)} = \frac{\alpha^{(k)} e^{-\beta(t_i - t_j)}}{\mu_{d_i}^{(k)} + \sum_{j=1}^{i-1} \alpha_{d_i d_j}^{(k)} e^{-\beta(t_i - t_j)}}$$

The probability that the event i is triggered by the base intensity μ

The probability that the event i is triggered by the event j

Maximum-likelihood estimation

M-step (do partial differential equation for μ and α)

$$\mu_d^{(k+1)} = \frac{1}{T} \sum_{i=1, d_i=d}^n p_{ii}^{(k+1)}$$

$$\alpha_{uv}^{(k+1)} = \frac{\sum_{i=1, d_i=u}^n \sum_{j=1, d_j=v}^{i-1} p_{ij}^{(k+1)}}{\sum_{j=1, d_j=v}^n G(T - t_j)}$$

For β , if $e^{-\beta(T-t_i)} \approx 0$

$$\beta^{(k+1)} = \frac{\sum_{i>j} p_{ij}^{(k+1)}}{\sum_{i>j} (t_i - t_j) p_{ij}^{(k+1)}}$$

Implementation details

In the implementation, 10 event sequences are simulated out first. Then for each sequence, above algorithm will be used to optimize the parameters. Finally, we will get 10 sets of parameters. And their mean will be used as the result.

Result and discovery

| | | | | | | | | | | |
|----|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| ~U | 0.0026 | 0.0006 | 0.0972 | 0.0058 | 0.0061 | 0.0035 | 0.0035 | 0.0055 | 0.0074 | 0.0058 |
| U | 0.0010 | 0.0000 | 0.1000 | 0.0050 | 0.0070 | 0.0025 | 0.0030 | 0.0069 | 0.0081 | 0.0043 |
| ~A | 0.0717 | 0.0262 | 0.0717 | 0.0211 | 0.0346 | 0.0260 | 0.0465 | 0.0107 | 0.0336 | 0.0138 |
| | 0.0217 | 0.0297 | 0.0077 | 0.0025 | 0.1007 | 0.0016 | 0.0173 | 0.0244 | 0.0307 | 0.0037 |
| | 0.0587 | 0.0086 | 0.0140 | 0.0274 | 0.0086 | 0.0071 | 0.0541 | 0.0092 | 0.0086 | 0.0811 |
| | 0.0178 | 0.0024 | 0.0271 | 0.0676 | 0.0125 | 0.0049 | 0.0172 | 0.0326 | 0.0061 | 0.0047 |
| | 0.0262 | 0.0703 | 0.0065 | 0.0261 | 0.0204 | 0.0059 | 0.0380 | 0.0143 | 0.0236 | 0.0079 |
| | 0.0155 | 0.0032 | 0.0053 | 0.0028 | 0.0043 | 0.0616 | 0.0385 | 0.0373 | 0.0130 | 0.0026 |
| | 0.0729 | 0.0158 | 0.0584 | 0.0276 | 0.0398 | 0.0407 | 0.0202 | 0.0192 | 0.0340 | 0.0312 |
| | 0.0075 | 0.0226 | 0.0091 | 0.0144 | 0.0101 | 0.0443 | 0.0146 | 0.1280 | 0.0204 | 0.0068 |
| | 0.0557 | 0.0610 | 0.0055 | 0.0082 | 0.0223 | 0.0249 | 0.0207 | 0.0317 | 0.0727 | 0.0185 |
| | 0.0155 | 0.0069 | 0.0867 | 0.0067 | 0.0040 | 0.0048 | 0.0489 | 0.0064 | 0.0319 | 0.0917 |
| A | 0.1000 | 0.0720 | 0.0044 | 0.0000 | 0.0023 | 0.0000 | 0.0900 | 0.0000 | 0.0700 | 0.0250 |
| | 0.0000 | 0.0500 | 0.0680 | 0.0000 | 0.0270 | 0.0650 | 0.0000 | 0.0000 | 0.0970 | 0.0000 |
| | 0.0930 | 0.0000 | 0.0062 | 0.0450 | 0.0000 | 0.0000 | 0.0530 | 0.0095 | 0.0000 | 0.0830 |
| | 0.0190 | 0.0033 | 0.0000 | 0.0730 | 0.0580 | 0.0000 | 0.0560 | 0.0000 | 0.0000 | 0.0000 |
| | 0.0450 | 0.0910 | 0.0000 | 0.0000 | 0.0660 | 0.0000 | 0.0000 | 0.0330 | 0.0058 | 0.0000 |
| | 0.0670 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0550 | 0.0630 | 0.0780 | 0.0850 | 0.0095 |
| | 0.0000 | 0.0220 | 0.0013 | 0.0000 | 0.0570 | 0.0910 | 0.0088 | 0.0650 | 0.0000 | 0.0730 |
| | 0.0000 | 0.0900 | 0.0000 | 0.0880 | 0.0000 | 0.0780 | 0.0000 | 0.0900 | 0.0680 | 0.0000 |
| | 0.0000 | 0.0000 | 0.0930 | 0.0000 | 0.0330 | 0.0000 | 0.0690 | 0.0000 | 0.0820 | 0.0330 |
| | 0.0010 | 0.0000 | 0.0890 | 0.0000 | 0.0080 | 0.0000 | 0.0069 | 0.0000 | 0.0000 | 0.0720 |

The mean relative error is: 1.15

I find that the variance of mean relative error is very big among different sequences. (I think it is unreasonable to define the error of this assignment, which mix up the relative error and absolute error)