Homework

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Exercise 3-3

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$$I = \{(a - 2b) + (b + 2a)i \mid a, b \in \mathbb{Z}\}$$
$$\mathbb{Z}[i]/I = \{I, i + I, 2i + I, 3i + I, 4i + I\}$$

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Let $z_1, z_2 \in IJ$, it's obvious that $z_1 - z_2 \in IJ$, thus IJ is a subgroup of R on $+$. If $z = \sum x_i y_i \in IJ$ and $t \in R$, then

$$tz = \sum (tx_i) y_i \in IJ \quad \text{and} \quad zt = \sum x_i (y_i t) \in IJ,$$

thus IJ is an ideal of R.

If $z = \sum x_i y_i \in IJ$, then $z = \sum x_i y_i = \sum (x_i)' \in I$ and $z = \sum x_i y_i = \sum (y_i)' \in J$, then $z \in I \cap J$, thus $IJ \subset I \cap J$.

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(1)

$\langle 3 \rangle$.

(2)

$\langle 30 \rangle$.

(3)

$\langle 90 \rangle$.

Exercise 3-4

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If f is the homomorphism from $\mathbb{Z}_4$ to $\mathbb{Z}_{20}$, then we have

$$f(0) = 0 \quad \text{and} \quad f(1) = 1$$

Beacuse $f(a + b) = f(a) + f(b)$, we have $f(n) = n$ for $n = 0, 1, 2, 3$. But $f(3+3) = f(3)+f(3) = 6$ and $f(3+3) = f(2) = 2$, so f is not a homomorphism. Thus the homomorphism does not exist.

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(2)

Similarly with above, there is only one homomorphism:
$f(n) = n \quad (n = 0, 1, \ldots, 9)$.

(3)

There is only one homomorphism:$f(n) = n \quad (n = 0, 1, \ldots, 11)$.