Homework

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Exercise 1-3

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- Closure: If $a, b \in G$, there are $c, d \in G$ such that $a = c^m, b = d^m$. Then $ab = c^m d^m = (cd)^m \in G$(Beacuse group G is an abelian group).
- Identity: $e = e^m \in G$.
- Inverse: If $a = b^m \in G$, then $a^{-1} = (b^m)^{-1} = (b^{-1})^m \in G$.

Above all, $H \preccurlyeq G$.

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Beacuse H is not empty, gHg^{-1} is not empty.

For all $a, b \in gHg^{-1}$, there are $c, d \in H$ such that $a = gcg^{-1}, b = gdg^{-1}$.

Then $a^{-1}b = (gcg^{-1})^{-1}gdg^{-1} = gc^{-1}g^{-1}gdg^{-1} = gc^{-1}dg^{-1}$.

Beacuse $c^{-1}d \in H$, we have $a^{-1}b \in gHg^{-1}$.

Finally, we get $gHg^{-1} \preccurlyeq G$.

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Beacuse $ea = ae$, $e \in C(a)$. For all $b, c \in C(a)$, we have $ba = ab$ and $ca = ac$.

Then $b^{-1}c = (aba^{-1})^{-1}aca^{-1} = ab^{-1}a^{-1}aca^{-1} = ab^{-1}ca^{-1}$, which imply that $ab^{-1}c = b^{-1}ca$, $b^{-1} \in C(a)$.

Above all, $C(a) \preccurlyeq G$.

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$g \in C(G) \Leftrightarrow \forall a \in G, ga = ag \Leftrightarrow \forall a \in G, g \in C(a) \Leftrightarrow g \in \bigcap_{a \in G} C(a)$, then we have:

$$C(G) = \bigcap_{a \in G} C(a)$$

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- $< m, n > \subseteq < d >$: $a \in < m, n > \Rightarrow \exists k_1, k_2, a = k_1 m + k_2 n = (k_1 \frac{m}{d} + k_2 \frac{n}{d}) d \Rightarrow a \in < d >$.
- $< d > \subseteq < m, n >$: By the Euclid theorem, $\exists k_1, k_2, d = k_1 m + k_2 n$. If $a \in < d >$, we have $a = kd = k(k_1 m + k_2 n) = (kk_1)m + (kk_2)n$, which means $a \in < m, n >$.

Above all, $< m, n > = < d >$.

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It's obvious that $m = \pm n$ imply $< m > = < n >$, we only need to prove $< m > = < n > \Rightarrow m = \pm n$.

If $< m > = < n >$, we have $m \in < m > = < n >$.

There two cases:

- $m = 0$ or $n = 0$: we have $< m > = < n > = 0$, which means $m = n = 0$. So $m = \pm n$.
- $m \neq 0$ and $n \neq 0$: $n \in < n > = < m >$, which means $m \mid n$. We can get $n \mid m$ by the same method. $n \mid m$ and $m \mid n$ means $m \pm n$.