Homework

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Exercise 1-6

5

(1)

$\tau\sigma\tau^{-1} = (1)$

(2)

$\tau\sigma\tau^{-1} = (1\ 3\ 4\ 2)$

12

There are totally 6 subgroups.

- $\{(1)\}$.
- $\{(1\ 2), (1)\}$.
- $\{(1\ 3), (1)\}$.
- $\{(2\ 3), (1)\}$.
- $\{(1\ 2\ 3), (1\ 3\ 2), (1)\}$.
- $\{(1\ 2\ 3), (1\ 3\ 2), (1\ 2), (1\ 3), (2\ 3), (1)\}$.

24

Let $G = S_n$. Beacuse odd permutation exists in G, we have $n \geq 2$. Then we get $(1\ 2) \in G$. We define:

$$A = \{\tau \in G \mid \ \tau \text{ is a even permutation}\}$$
$$B = \{\tau \in G \mid \ \tau \text{ is an odd permutation}\}$$

If $\tau \in A$, by the definition of even/odd permutation, we have $(1\ 2) \in B$.
We define a mapping:$f : A \to B$ and $f(\tau) = (1\ 2)\tau$ for all $\tau \in A$.

If $f(\tau_1) = f(\tau_1)$, we have $(1\ 2)\tau_1 = (1\ 2)\tau_2$. We can get $\tau_1 = \tau_2$ by the elimination law of group. So f is injective.

For all $\sigma \in B$, we have $\tau = (2\ 1)\sigma \in A$ such that $f(\tau) = (1\ 2)(2\ 1)\sigma = \sigma$. So f is surjective.

Beacuse f is a bijection between A and B, we have $\mid A \mid = \mid B \mid$.

25

Beacuse $(1) \in H$, H is not empty.

For all $\tau, \sigma \in H$, we have:

$$\tau = (a_1 a_2)(a_3 a_4) \cdots (a_{r-1} a_r)$$
$$\sigma = (b_1 b_2)(b_3 b_4) \cdots (b_{s-1} b_s)$$

(r and s are even numbers,maybe zero.)

Then we have $\tau^{-1}\sigma = (a_r a_{r-1}) \cdots (a_4 a_3)(a_2 a_1)(b_1 b_2)(b_3 b_4) \cdots (b_{s-1} b_s)$ and $r+s$ is a even number. By the definition of even permutation, we have $\tau^{-1}\sigma \in H$.

Above all, $H \subseteq G$.