Suggested due date: 2018/03/07

Combinatorics: Homework 1

Problem 1. How many odd numbers are there in the 2018-th row of the Pascal triangle?
NOTE: The 2018-th row has the binomial numbers $\binom{2018}{r}$.

Problem 2. The sets are defined as

$$A_0 = \{0\}, \quad A_1 = \{1\}, \quad A_{i+2} = (A_1 + A_{i+1})\Delta A_i \ (i \geq 0),$$

where for two sets of numbers A and B, $A + B = \{a + b : a \in A, b \in B\}$.
(a) Enumerate A_i for $i = 0, 1, \ldots, 15$.
(b) Prove that $|A_n| = 1$ for infinitely many n.

Problem 3. n points form a circle, on one point there are n chips. In each step, one can pick a point where there are at least two chips, and send one to each of its neighbours. The game stops when no point has more than 1 chip.
(a) Prove that when n is even, the game never stops.
(b) Prove that when n is odd, the game always stops.

Problem 4. With cost α (α can be any positive real number), you can buy a strip of land between any two parallel lines (including the lines) in the plane of your choice, as long as the distance between the two lines is at most α. It is easy to see that you can cover all the points in the unit circle with total cost 2. Prove that you cannot do it with less cost.