$$A(x)=\sum_{i=0}^{n} a_i x^i = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$$

$$B(x)=\sum_{i=0}^{n} b_i x^i = b_n x^n + b_{n-1}x^{n-1} + \cdots + b_1 x + b_0$$

$$A(\omega_n^k)=\sum_{i=0}^{n-1} a_i \omega^{ki}, k=0,1,\cdots,n-1$$

$$\begin{aligned} A(\omega_n^k) &= \sum_{i=0}^{n-1} a_i \omega_n^{ki} \\ &= \sum_{i=0}^{\frac{n}{2}-1} a_{2i}\omega_n^{2ki} + \omega_n^k \sum_{i=0}^{\frac{n}{2}-1} a_{2i+1}\omega_n^{2ki} \end{aligned}$$

$$\omega_n^2 = \left(e^{\frac{2\pi i}{n}}\right)^2 = e^{\frac{2\pi i}{n/2}} = \omega_{\frac{n}{2}}$$

设 $k < \frac{n}{2}$:

$$A(\omega_n^k) = \sum_{i=0}^{\frac{n}{2}-1} a_{2i}\omega_{\frac{n}{2}}^{ki} + \omega_n^k \sum_{i=0}^{\frac{n}{2}-1} a_{2i+1}\omega_{\frac{n}{2}}^{ki}$$

$$\begin{aligned} A(\omega_n^{k+\frac{n}{2}}) &= \sum_{i=0}^{\frac{n}{2}-1} a_{2i}\omega_{\frac{n}{2}}^{ki} + \omega_n^{k+\frac{n}{2}} \sum_{i=0}^{\frac{n}{2}-1} a_{2i+1}\omega_{\frac{n}{2}}^{ki} \\ &= \sum_{i=0}^{\frac{n}{2}-1} a_{2i}\omega_{\frac{n}{2}}^{ki} - \omega_n^k \sum_{i=0}^{\frac{n}{2}-1} a_{2i+1}\omega_{\frac{n}{2}}^{ki} \end{aligned}$$

$$T(n) = 2T(\frac{n}{2}) + \mathcal{O}(n) = \mathcal{O}(n\log n)$$

$$\begin{bmatrix} (\omega_n^0)^0 & (\omega_n^0)^1 & \cdots & (\omega_n^0)^{n-1} \\ (\omega_n^1)^0 & (\omega_n^1)^1 & \cdots & (\omega_n^1)^{n-1} \\ \vdots & \vdots & \ddots & \vdots \\ (\omega_n^{n-1})^0 & (\omega_n^{n-1})^1 & \cdots & (\omega_n^{n-1})^{n-1} \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ \vdots \\ a_{n-1} \end{bmatrix} = \begin{bmatrix} A(\omega_n^0) \\ A(\omega_n^1) \\ \vdots \\ A(\omega_n^{n-1}) \end{bmatrix} \tag{1}$$

系数矩阵的逆矩阵:

$$\mathbf{D} = \begin{bmatrix} (\omega_n^{-0})^0 & (\omega_n^{-0})^1 & \cdots & (\omega_n^{-0})^{n-1} \\ (\omega_n^{-1})^0 & (\omega_n^{-1})^1 & \cdots & (\omega_n^{-1})^{n-1} \\ \vdots & \vdots & \ddots & \vdots \\ (\omega_n^{-(n-1)})^0 & (\omega_n^{-(n-1)})^1 & \cdots & (\omega_n^{-(n-1)})^{n-1} \end{bmatrix}$$