Mathematical Logic Homework 4

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Solution 4.1.

1	Γ	$\neg\varphi$	ψ	premise
2	Γ	$\neg\varphi$	$\neg\psi$	premise
	Γ		φ	second contradiction by 1, 2(regard $\Gamma\neg\varphi$ as Γ')

□

Solution 4.2.

(a)

1	Γ		φ	premise
2	Γ	$\neg\varphi$	φ	antecedent by 1
3	Γ	$\neg\varphi$	$\neg\neg\varphi$	contraposition by 2
4	Γ	$\neg\neg\varphi$	$\neg\neg\varphi$	assumption
	Γ		$\neg\neg\varphi$	

(b)

1	Γ		$\neg\neg\varphi$	premise
2	Γ	$\neg\varphi$	$\neg\neg\varphi$	antecedent by 1
3	Γ	$\neg\varphi$	$\neg\varphi$	assumption
	Γ		φ	contradiction by 2, 3

□

Solution 4.3.

It's not derivable.

Prove by contradiction.

Assume that it's derivable. Let Γ be empty.

Let $\mathfrak{A}$ be the S-structure of natural number. Let φ be $x \equiv 1$.

Then $\mathfrak{A} \models \exists x\varphi$, by our assumption, $\mathfrak{A} \models \forall x\varphi$. But it's obvious that $\mathfrak{A} \not\models \forall x\varphi$. It's a contradiction.

So it's not derivable.

□