Mathematical Logic Homework 7

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Solution 7.1. By Completeness Theorem, we can derive $\Theta \models \varphi$ from $\Theta \vdash \varphi$. Then for any S-interpretation $\mathfrak{I}$

$$\mathfrak{I} \models \Theta \text{ implies } \mathfrak{I} \models \varphi$$

We can construct a S_0-interpretation $\mathfrak{I}'$ by retaining the symbols occurring in Θ and φ and keep their interpretation unchanged. By Coincidence Lemma,

$$\mathfrak{I}' \models \Theta \text{ implies } \mathfrak{I}' \models \varphi$$

It's obvious that any S_0-interpretation can be expanded to a S-interpretation without changing the interpretation of symbols in S_0. So for any S_0-interpretation $\mathfrak{I}'$,

$$\mathfrak{I}' \models \Theta \text{ implies } \mathfrak{I}' \models \varphi$$

which means $\Theta \models \varphi$ (Now φ is a S_0-formula and so does the formulas in Θ). By Completeness Theorem, $\Theta \vdash \varphi$ and every formula occurs in the proof is a S_0-formula.

□

Solution 7.2. □