

# Mathematical Logic Homework 8

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*Solution 8.1.* (1) Wrong

If it's correct, by (2), we can get the equivalence of decidability and enumerability. But it's wrong as discussed in class.

(2) Correct

$\Rightarrow$ : We can enumerate all the elements of  $A^*$  by increasing order of length, and decide whether the element belongs to  $W$ . If it belongs to  $W$ , we print it, otherwise not.

$\Leftarrow$ : We can strictly enumerate the elements in  $W$  and when the element we want to decide occurs, halts and when the length exceeds the length of element we want to decide, prints anything and halts.  $\square$

*Solution 8.2.* Prove by contradiction.

Assume it's R-decidable and the program  $\mathcal{P}_0$  can decide the set.

For all program  $\mathcal{P}$ , construct a mapping:  $\mathcal{P} \rightarrow \mathcal{P}^+$  such as

```
if r0 not empty then
  loop forever
else
  call P
fi
```

(I write the program in pseudo code and the same program of register machine version must exist)

Then we can construct a program  $\mathcal{P}_1$  that can decide  $\Pi_{halt}$  such as

```
get w_P+ from w_P and store it to r0
call P0
```

Then the above program  $\mathcal{P}_1$  can decide  $\Pi_{halt}$ , which is a contradiction.

In conclusion, the set in the problem statement is not decidable.  $\square$