Mathematical Modeling Exercises

(1) [Griva, Nash, Sofer, 2009] Suppose that a manufacturer of kitchen cabinets is trying to maximize the weekly revenue of a factory. Various orders have come in that the company could accept. They include bookcases with open shelves, cabinets with doors, cabinets with drawers, and custom-designed cabinets. Table 1 indicates the quantities of materials and labor required to assemble the four types of cabinets, as well as the revenue earned. Suppose that 5000 units of wood and 1500 units of labor are available.

Table 1 Quantity of material and labour required to assmeble cabinets

Cabinet	Wood	Labor	Revenue (₹/unit)
Bookshelf	10	2	100
Cabinets with Doors	12	4	150
Cabinets with Drawers	25	8	200
Custom	20	12	400

(a) Model this problem as an optimization problem.
(b) Generalize your model to any number of products n, and from two resources (wood and labor) to any number of resources m. Denoting the unit profit from product j by c_j , the amount available of resource i by b_i , and the amount of resource i used by a unit of product j by $a_{ij.}$
(c) Write your model in a more compact way by introducing matrix-vector notation.

(2) A company has three factories (Supply points) and four warehouses (Demand points). The objective is to determine how many units should be shipped from each factory to each warehouse to minimize transportation cost, while meeting supply and demand constraints. Table 2 shows the relevant data.

Table 2 Cost of transportation, demand and capacities

Factory / Warehouse	Supply	W1	W2	W3	W4
F1	100	2	3	11	7
F2	150	1	0	6	1
F3	200	5	8	15	9
Demand →		80	120	150	100

(a) Model this problem as an optimization problem. (b) Generalize your model to any number of supply points, demand points, etc. Use appropriate notation. (c) Write your model in a more compact way by introducing matrix-vector notation.

(3) A manager wants to assign 4 workers (W1 to W4) to 4 tasks (T1 to T4). The time (in hours) each worker takes to complete each task is shown below. Each worker must be assigned exactly one task, and each task must be assigned to exactly one worker. The objective is to minimize the total time. Assign each worker to one task such that the total time is minimized.

	T1	T2	T3	T4
W1	9	2	7	8
W2	6	4	3	7
W3	5	8	1	8
W4	7	6	9	4

Answer (a), (b), (c) as in the above problems.

(4) A company produces a metal alloy by blending three raw materials: A, B, and C. Each material has a different cost and different percentages of two key components: Nickel and Chromium. The final alloy must have (i) total amount: 100 tons (ii) at least 12% Nickel (iii) at least 15% Chromium.. Table 4 shows the corresponding compositions and the costs. The objective is to minimize the total costs.

Table 4 Costs and compositions of different alloys

Material	**Cost (₹/ton)**	**% Nickel**	**% Chromium**
A	200	10%	20%
B	250	20%	10%
C	180	5%	15%

Answer (a), (b), (c) as in the above problems.

(5) [Bertsimas and Tsitsiklis, 1997] A hospital wants to make a weekly (continuing) night shift (00:00 – 08:00) schedule for its nurses. The demand for nurses on a day j for the night shift is d_j. Every nurse works for 5 consecutive days in a week and then takes a 2-day rest. Find the minimum number of nurses the hospital needs to hire.