1. Consider the problem [Vanderbei 5th edition, Exercise 1.1]. Write this as a Linear Programming problem (in the standard inequality form). You must explain your notation and variables.

2. Consider the problem [Vanderbei. 5th edition, Exercise 1.2].

   Write this as a Linear Programming problem (in the standard inequality form). You must explain your notation and variables.

3. For a nonempty set $S \subset \mathbf{R}^n$ and a positive real number $r \in \mathbf{R}$ (and $r > 0$), define the set rS as follows:

$$rS := \{z \in \mathbf{R}^n \mid z = rx,\, x \in S\}.$$

   Here rx is the multiplication of the vector $x \in \mathbf{R}^n$ by the scalar $r \in \mathbf{R}$; in your more familiar notation, $r\vec{x}$. The set rS is the set of all points that are obtained by multiplying r with the vectors $x \in S$.

   For a given nonempty set $S \subset \mathbf{R}^n$ and a given positive number $r > 0$, prove that if S is a convex set then rS is a convex set as well.

4. For given two nonempty sets $S_1, S_2 \subset \mathbf{R}^n$, define the operation $S_1 + S_2$ as follows:

$$S_1 + S_2 := \{z \in \mathbf{R}^n \mid \text{ there exist some } x \in S_1 \text{ and some } y \in S_2 \text{ such that } z = x + y \quad \}$$

   that is, each point $z \in S_1 + S_2$ is the one that can be expressed as the sum $x + y$ for some $x \in S_1$, and $y \in S_2$; here the sum $x + y$ is the vector sum between the two vectors. One does this for all $x \in S_1$ and $y \in S_2$ and get the set $S_1 + S_2$.

   (a) Consider $S_1 = \{(x_1, x_2) \in \mathbf{R}^2 \mid |x_1 - 1| \leq 1 \,\&\, |x_2 - 2| \leq 1\}$ and $S_2 = \{(x_1, x_2) \in \mathbf{R}^2 \mid |x_1| \leq 2 \,\&\, |x_2| \leq 1\}$. Sketch the set $S_1 + S_2$. You do not need to explain your solution for this question. But, your sketch should be neat and very clear, indicating all the relevant coordinate values.

   (b) Is it true that $S_1 + S_2$ must be convex for **any** nonempty convex sets S_1 and S_2 in $\mathbf{R}^n$? Justify your answer carefully. [**This problem is independent of part (a). The sets S_1, S_2 are arbitrary convex sets in this question, not the particular example given in part (a). If you do this problem only for the sets of part (a) or a particular example, you will get zero mark.**]