Math 302, Assignment 1

(1) Let $S = \{1, \{\}, c\}$ be a sample space. List all possible events.
(2) We roll a fair die until the first 1 comes up. What is the probability that the number of tosses is odd?
(3) Assuming a fair poker deal, what is the probability of a
    (a) royal flush
    (b) straight flush
    (c) flush
    (d) straight
    (e) two pair

    See https://en.wikipedia.org/wiki/List_of_poker_hands for the definition of these poker hands.
(4) How many ways are there to deal 52 standard playing cards to four players (so that each player gets 13 cards)? Suppose you are world champion in card dealing, and can deal 52 cards in just one second. Compare the time it would need you to deal all possible combinations with the current age of the universe.
(5) We toss a fair die three times. What is the probability that all tosses produce different outcomes?
(6) **Challenge, not marked.** Prove that the number of unordered sequences of length k with elements from a set X of size n is $\binom{n+k-1}{k}$.
    Hint: For illustration, first consider the example $n = 4, k = 6$. Let the 4 elements of the set X be denoted a, b, c, d. Argue that any unordered sequence of size 6 consisting of elements a, b, c, d can be represented uniquely by a symbol similar to "$\cdot\cdot|\cdot|\cdot\cdot|\cdot$", corresponding to the sequence *aabccd*. Now count the number of choices for the vertical bars.
(7) You own n colours, and want to use them to colour 8 objects. For each object, you randomly choose one of the colours. How large does n have to be so that the probability that two objects sharing the same colour is less than 0.25?