Multimedia Graph-Based Processing

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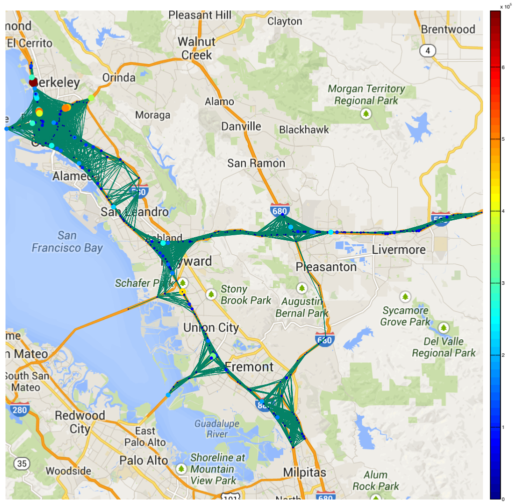
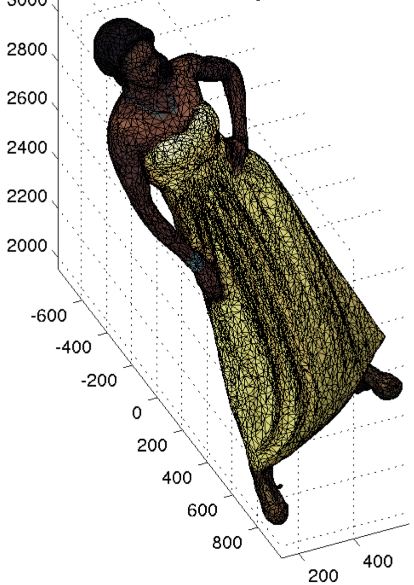
ÉCOLE POLYTECHNIQUE FÉDÉRALE DE LAUSANNE

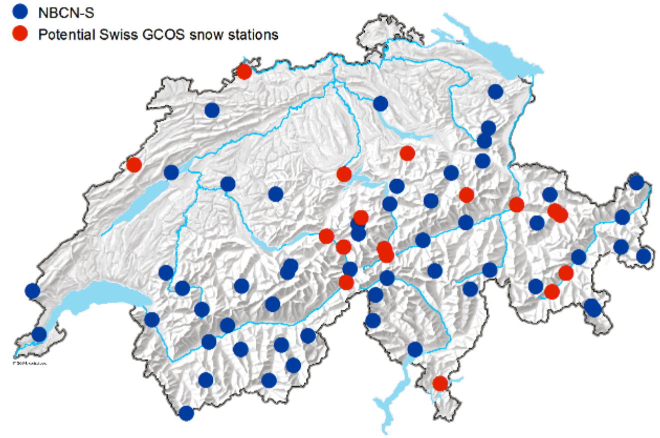
Structured data are everywhere



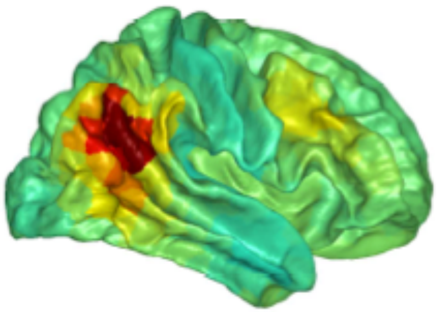
Traffic bottlenecks



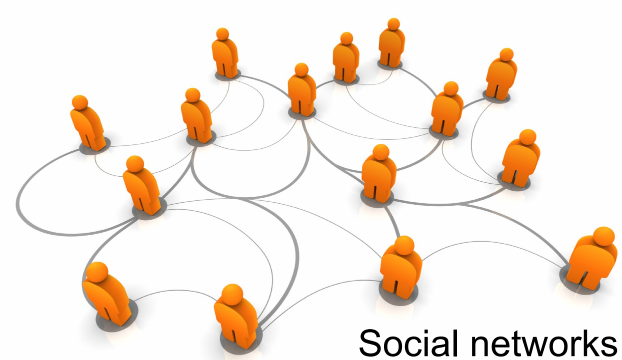
3D point clouds



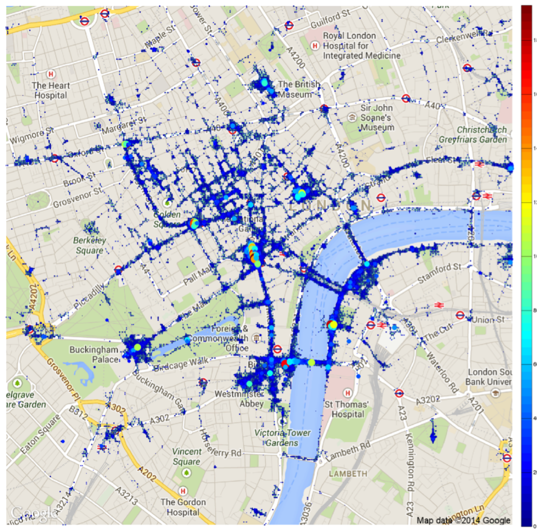
Sensor networks



Brain signals



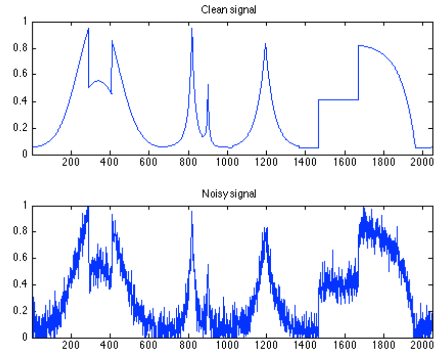
Social networks

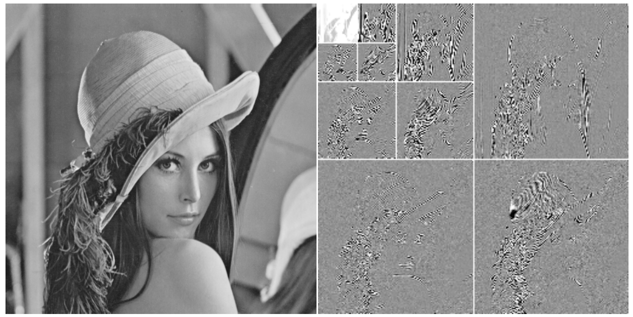


Geo-tagged pictures

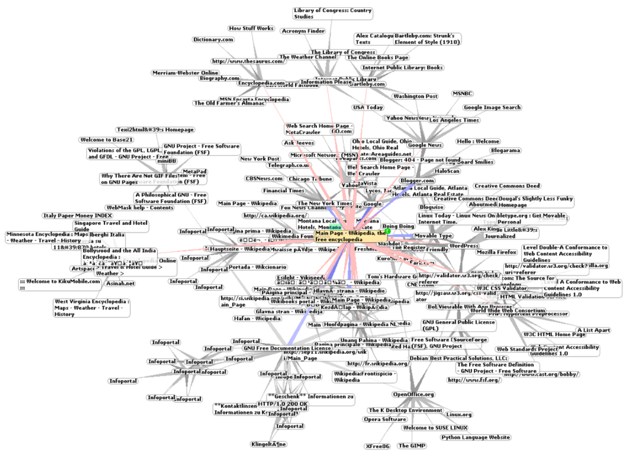
Irregular data structure

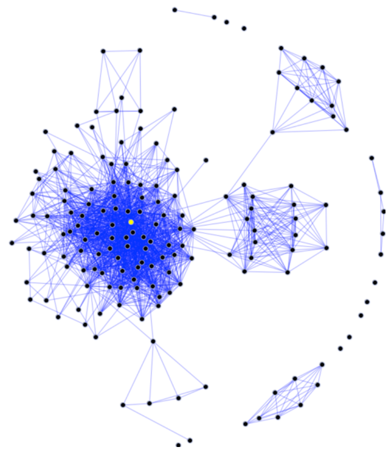
- Traditional signal processing in Euclidean space





- Irregular (graph) structures: new challenges for multimedia signal processing?





Talk outline

- Overview of the Graph Signal Processing framework
  - Tools to process signals that live on irregular structures
  - Emerging field that combines spectral graph theory and harmonic analysis, as well as application expertise

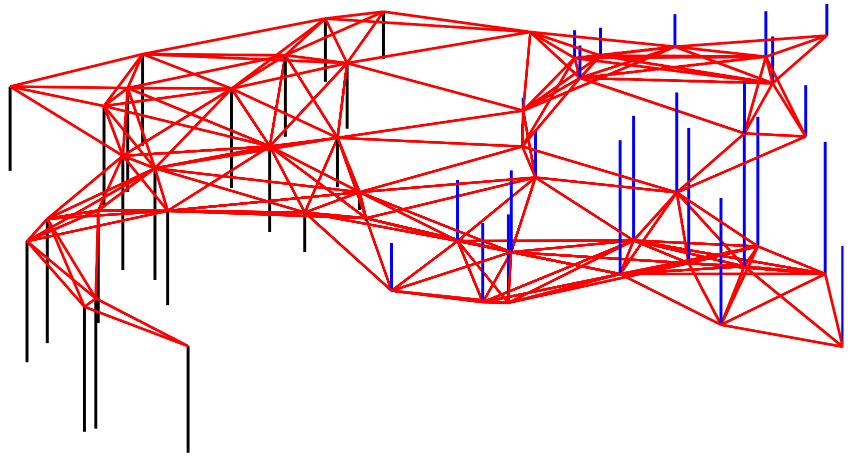
- Illustrative graph-based multimedia applications
  - Data compression
  - Image filtering and enhancement
  - Multimedia data analysis

- Focus on the interplay between data and structure

Signals on Graphs

- Connected, undirected, weighted graph $\mathcal{G} = (V, E, W)$
  where $W_{i,j}$ is the weight of the edge $e = (i, j)$

- Graph signal: a function $f : \mathcal{V} \rightarrow \mathbb{R}$ that assigns real values to each vertex of the graph



- Graph description:
  - Weight matrix $\mathbf{W}$
  - Degree matrix $\mathbf{D}$: diagonal matrix with sum of weights of incident edges
  - Laplacian matrix $\mathcal{L}$: difference operator

(Unormalized) Laplacian

- Laplacian is a difference operator $\mathcal{L} := \mathbf{D} - \mathbf{W}$

$$(\mathcal{L}f)(i) = \sum_{j \in \mathcal{N}_i} W_{i,j}[f(i) - f(j)]$$

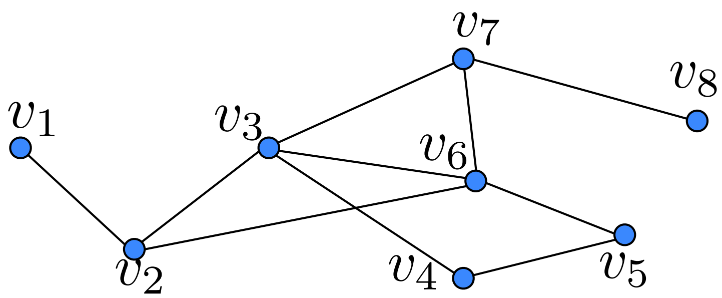
- It is a real symmetric matrix
- It has a complete set of eigenvectors $\{\mathbf{u}_\ell\}_{\ell=0,1,\ldots,N-1}$
- The eigenvectors are associated with real, nonnegative eigenvalues $\{\lambda_\ell\}_{\ell=0,1,\ldots,N-1}$

$$\mathcal{L}\mathbf{u}_\ell = \lambda_\ell \mathbf{u}_\ell, \ \forall \ell = 0, 1, \ldots, N-1$$

- Its spectrum is defined as $\sigma(\mathcal{L}) := \{\lambda_0, \lambda_1, \ldots, \lambda_{N-1}\}$

$$0 = \lambda_0 < \lambda_1 \leq \lambda_2 \ldots \leq \lambda_{N-1} := \lambda_{\max}$$

Laplacian example



$G = \{V, E\}$ with unitary weights

$$D = diag(degree(v_1) \quad \dots \quad degree(v_n))$$

$$\mathcal{L} := \mathbf{D} - \mathbf{W}$$

$$\begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix} \Rightarrow \begin{pmatrix} 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 3 & -1 & 0 & 0 & -1 & 0 & 0 \\ 0 & -1 & 4 & -1 & 0 & -1 & -1 & 0 \\ 0 & 0 & -1 & 2 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 2 & -1 & 0 & 0 \\ 0 & -1 & -1 & 0 & -1 & 4 & -1 & 0 \\ 0 & 0 & -1 & 0 & 0 & -1 & 3 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 \end{pmatrix}$$

$\mathbf{W}$ $\qquad$ $\mathcal{L}$

- Symmetric
- Off-diagonal entries non-positive
- Rows sum up to zero
- Has a complete set of orthonormal eigenvectors: $L = \chi \Lambda \chi^T$

  $0 = \lambda_0 < \lambda_1 \leq \ldots \leq \lambda_{n-1}$

Normalized Laplacian

- Each weight $W_{i,j}$ is normalised by $\dfrac{1}{\sqrt{d_i d_j}}$

$$\tilde{\mathcal{L}} := \mathbf{D}^{-\frac{1}{2}} \mathcal{L} \mathbf{D}^{-\frac{1}{2}}$$

$$(\tilde{\mathcal{L}} f)(i) = \frac{1}{\sqrt{d_i}} \sum_{j \in \mathcal{N}_i} W_{i,j} \left[\frac{f(i)}{\sqrt{d_i}} - \frac{f(j)}{\sqrt{d_j}} \right]$$

- The set of eigenvalues is $0 = \tilde{\lambda}_0 < \tilde{\lambda}_1 \leq \ldots \leq \tilde{\lambda}_{\max} \leq 2$
- The normalized Laplacian has often stability benefits

Graph Fourier Transform

- The eigenvectors of the graph Laplacian are used for defining the Graph Fourier Transform

GFT

$$\hat{f}(\lambda_\ell) := \langle \mathbf{f}, \mathbf{u}_\ell \rangle = \sum_{i=1}^{N} f(i) u_\ell^*(i)$$

IGFT

$$f(i) = \sum_{\ell=0}^{N-1} \hat{f}(\lambda_\ell) u_\ell(i)$$

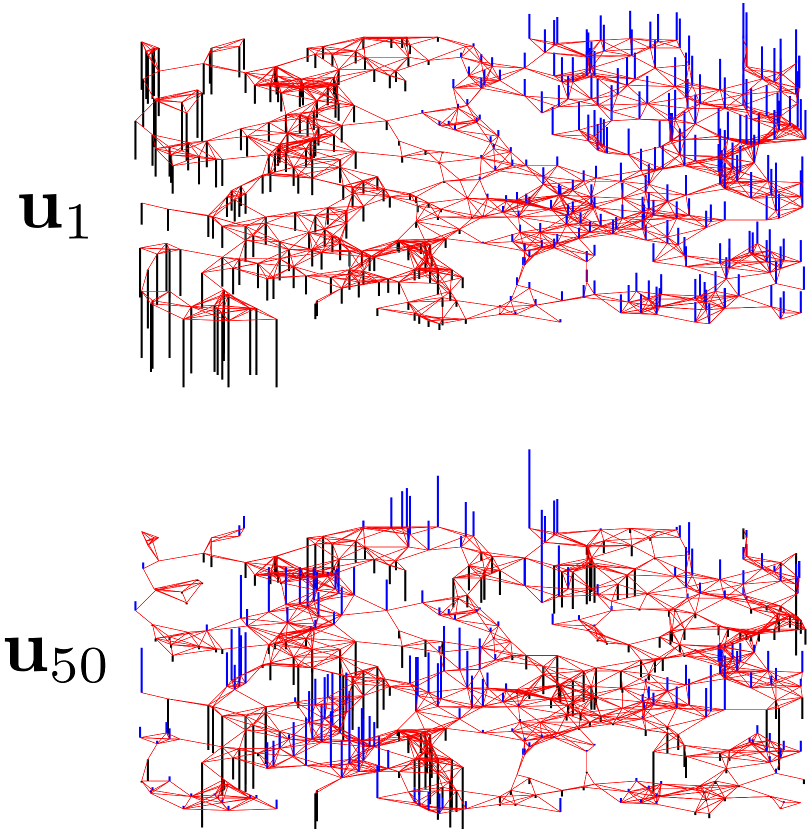
- This is analogous to the classical Fourier Transform built on eigenfunctions of the 1-D Laplace operator

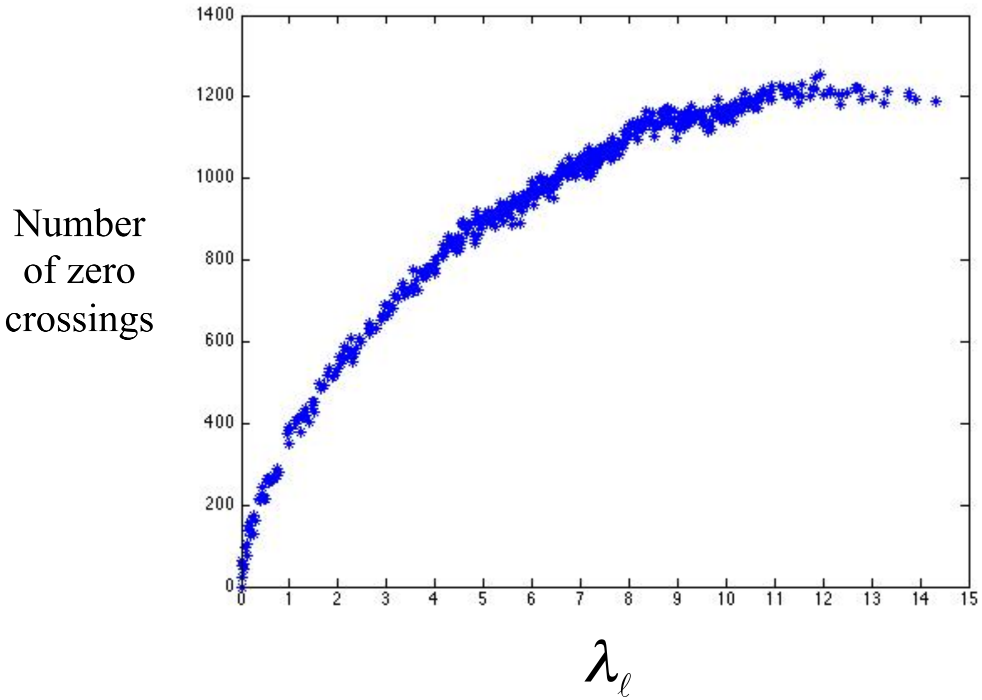
$$\hat{f}(\xi) := \langle f, e^{2\pi i \xi t} \rangle = \int_{\mathbb{R}} f(t) e^{-2\pi i \xi t} dt$$

$$-\Delta(e^{2\pi i \xi t}) = -\frac{\partial^2}{\partial t^2} e^{2\pi i \xi t} = (2\pi\xi)^2 e^{2\pi i \xi t}$$

Notion of 'frequency'

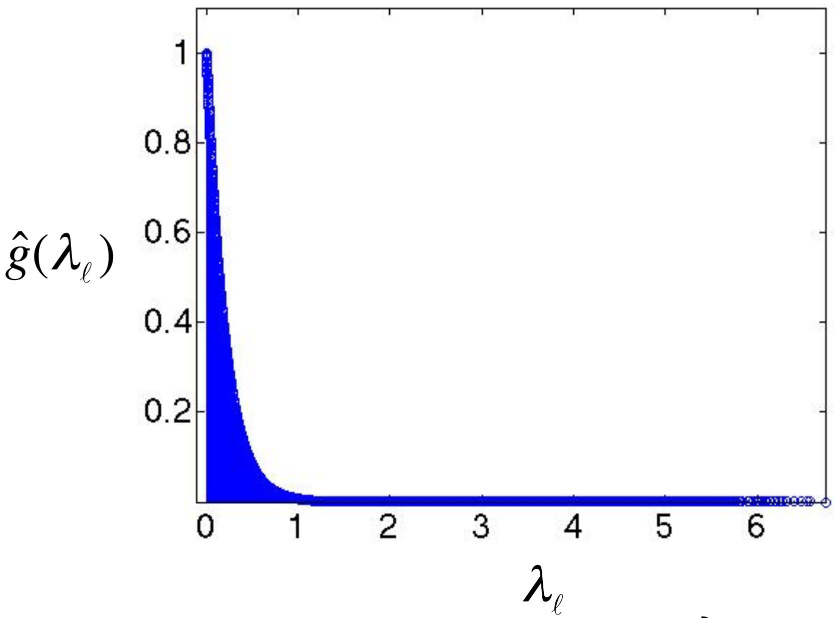
- The graph Laplacian eigenvalues and eigenvectors carry a notion of frequency



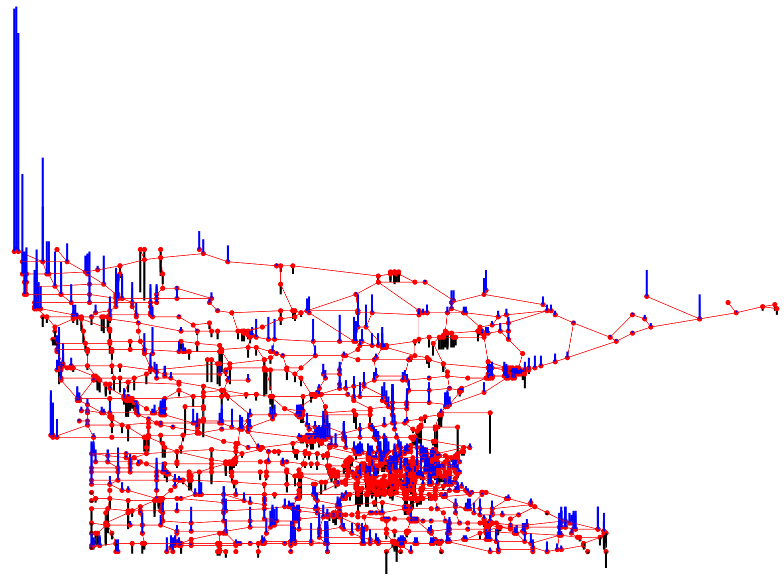


Dual representations

- Graph signals represented in either the vertex or the spectral domains (*kernels, or graph Fourier multipliers*)
  - example: (smooth) heat kernel



$$\hat{g}(\lambda_\ell) = e^{-5\lambda_\ell}$$



$$g(n) \xleftarrow{IGFT} \hat{g}(\lambda_\ell)$$

Local Smoothness

- **Assumption**: strong interplay between signal and graph
  - Signal analysis driven by data structure
- Local smoothness at vertex i

$$\|\nabla_i \mathbf{f}\|_2 := \left[\sum_{j \in \mathcal{N}_i} W_{i,j} \left[f(j) - f(i)\right]^2\right]^{\frac{1}{2}}$$

  - with the gradient $\nabla_i \mathbf{f} := \left[\left\{\sqrt{W_{i,j}}\left[f(j) - f(i)\right]\right\}_{\text{for } j \in \mathcal{V} \text{ s.t. } e=(i,j) \in \mathcal{E}}\right]$

Global Smoothness

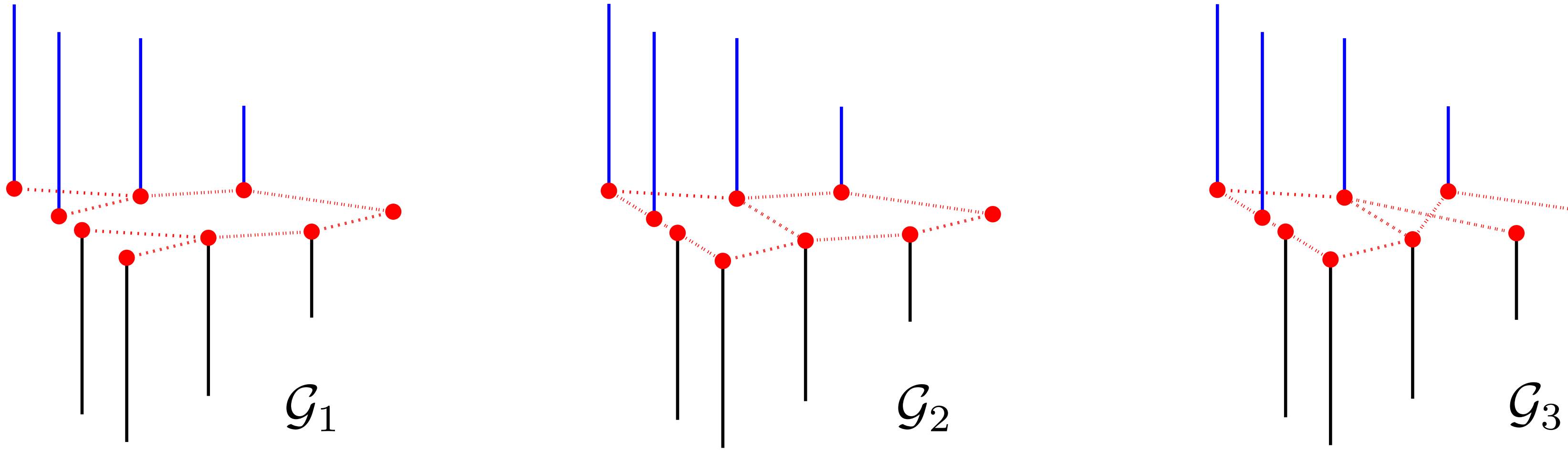
- **Assumption**: strong interplay between signal and graph
  - Signal analysis driven by data structure
- Global smoothness

$$S_p(\mathbf{f}) := \frac{1}{p}\sum_{i\in V}\|\nabla_i \mathbf{f}\|_2^p = \frac{1}{p}\sum_{i\in V}\left[\sum_{j\in\mathcal{N}_i} W_{i,j}\left[f(j)-f(i)\right]^2\right]^{\frac{p}{2}}$$

  - with $p=1$: total variation of the signal wrt the graph
  - with $p=2$: graph Laplacian quadratic form

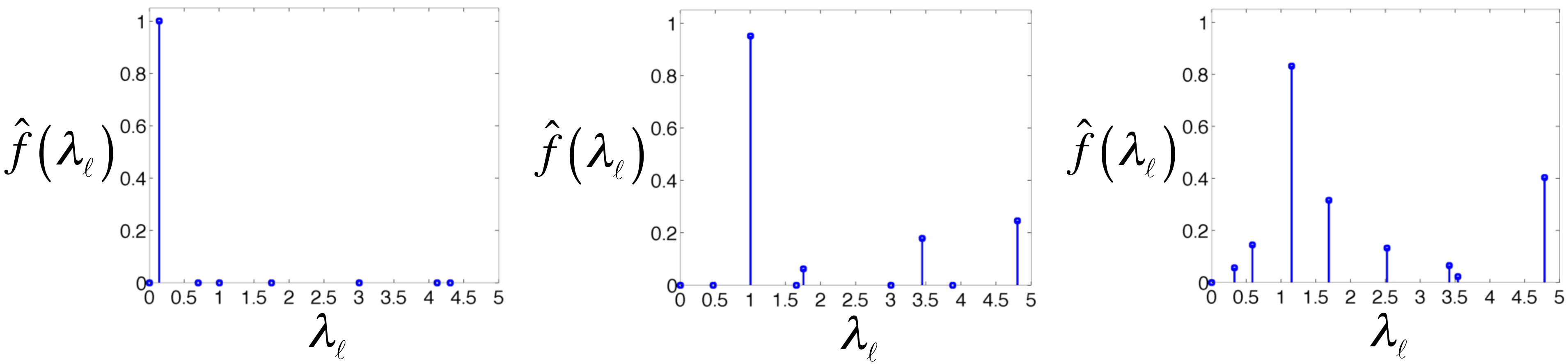
$$S_2(\mathbf{f}) = \frac{1}{2}\sum_{i\in V}\sum_{j\in\mathcal{N}_i} W_{i,j}\left[f(j)-f(i)\right]^2 = \sum_{(i,j)\in\mathcal{E}} W_{i,j}\left[f(j)-f(i)\right]^2 = \mathbf{f}^{\mathrm{T}}\mathcal{L}\mathbf{f}$$

Importance of the structure



The same signal has different smoothness wrt different graphs

$\mathbf{f}^{\mathrm{T}}\mathcal{L}_1\mathbf{f} = 0.14$ $\mathbf{f}^{\mathrm{T}}\mathcal{L}_2\mathbf{f} = 1.31$ $\mathbf{f}^{\mathrm{T}}\mathcal{L}_3\mathbf{f} = 1.81$



Frequency filtering

- Analogously to classical filtering, one can perform graph spectral filtering with transfer function $\hat{h}(\lambda_\ell)$

$$\hat{f}_{out}(\lambda_\ell) = \hat{f}_{in}(\lambda_\ell)\hat{h}(\lambda_\ell)$$

- Equivalently $$f_{out}(i) = \sum_{\ell=0}^{N-1} \hat{f}_{in}(\lambda_\ell)\hat{h}(\lambda_\ell)u_\ell(i)$$

- In matrix notation: $$\mathbf{f}_{out} = \hat{h}(\mathcal{L})\mathbf{f}_{in}$$

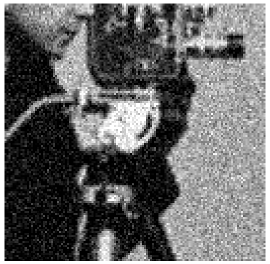
$$\hat{h}(\mathcal{L}) := \mathbf{U} \begin{bmatrix} \hat{h}(\lambda_0) & & \mathbf{0} \\ & \ddots & \\ \mathbf{0} & & \hat{h}(\lambda_{N-1}) \end{bmatrix} \mathbf{U}^{\mathrm{T}}$$

Example: Tikhonov regularization

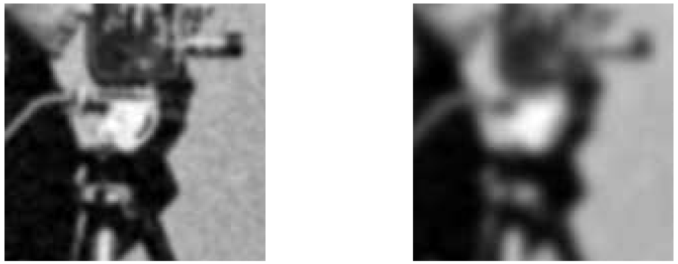
- Consider a classical denoising problem
  - noisy signal $\mathbf{y} = \mathbf{f}_0 + \boldsymbol{\eta}$
  - smooth regularization prior $\mathbf{f}^{\mathrm{T}} \mathcal{L} \mathbf{f}$
  - optimization problem: $\underset{\mathbf{f}}{\operatorname{argmin}} \left\{ \|\mathbf{f} - \mathbf{y}\|_2^2 + \gamma \mathbf{f}^{\mathrm{T}} \mathcal{L} \mathbf{f} \right\}$
  - optimal solution

$$f^*(i) = \sum_{\ell=0}^{N-1} \left[\frac{1}{1 + \gamma \lambda_\ell} \right] \hat{y}(\lambda_\ell) u_\ell(i) \quad \text{or} \quad \mathbf{f} = \hat{h}(\mathcal{L}) \mathbf{y} \;\; \text{with} \;\; \hat{h}(\lambda) := \frac{1}{1 + \gamma \lambda}$$

Original



Noisy



Gaussian filtering

Graph filtering

Filtering in the vertex domain

- Linear combination of values at neighbour vertices

$$f_{out}(i) = b_{i,i} f_{in}(i) + \sum_{j \in \mathcal{N}(i,K)} b_{i,j} f_{in}(j)$$

  - localized linear transform

- Example: polynomial filter as $\hat{h}(\lambda_\ell) = \sum_{k=0}^{K} a_k \lambda_\ell^k$

$$f_{out}(i) = \sum_{\ell=0}^{N-1} \hat{f}_{in}(\lambda_\ell) \hat{h}(\lambda_\ell) u_\ell(i)$$

$$= \sum_{j=1}^{N} f_{in}(j) \sum_{k=0}^{K} a_k \left(\mathcal{L}^k\right)_{i,j} \rightarrow b_{i,j} := \sum_{k=d_{\mathcal{G}}(i,j)}^{K} a_k \left(\mathcal{L}^k\right)_{i,j}$$

Convolution on graphs

- The classical convolution does not generalise to the graph settings
  - $h(t-\tau)$ does not have any equivalent on graphs

$$f_{out}(t) = \int_{\mathbb{R}} f_{in}(\tau)h(t-\tau)d\tau =: (f_{in} * h)(t)$$

- Instead, it can be defined by multiplication in the graph spectral domain

$$(f * h)(i) := \sum_{\ell=0}^{N-1} \hat{f}(\lambda_\ell)\hat{h}(\lambda_\ell)u_\ell(i)$$

Translation on graphs

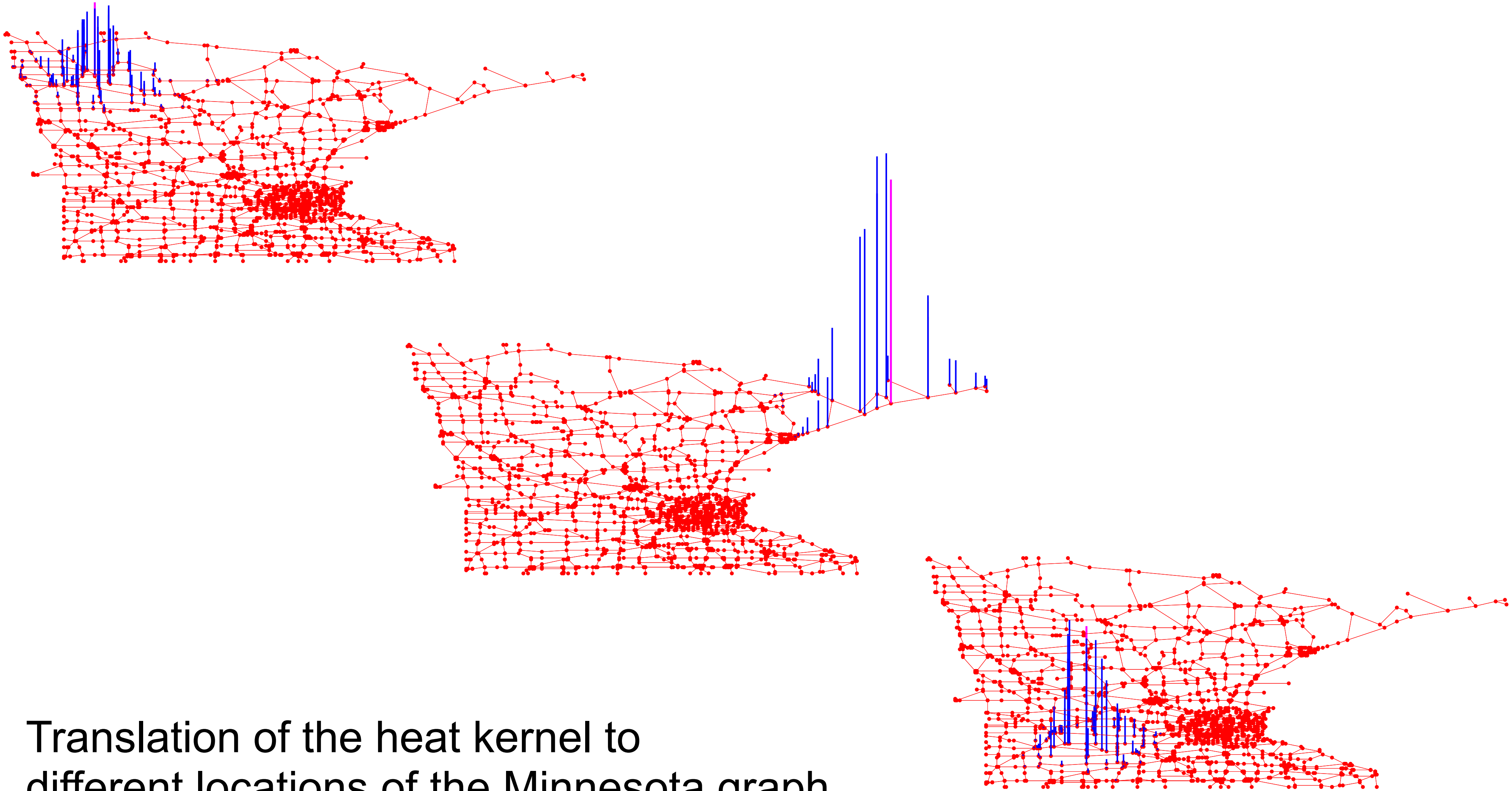
- The classical translation $(T_u f)(t) := f(t-u)$ does not generalise to non-regular graphs
- A generalized translation operator on graphs can still be defined as

$$T_n : \mathbb{R}^N \to \mathbb{R}^N$$

$$\boxed{(T_n g)(i) := \sqrt{N}(g * \delta_n)(i) = \sqrt{N} \sum_{\ell=0}^{N-1} \hat{g}(\lambda_\ell) u_\ell^*(n) u_\ell(i)}$$

$$\delta_n(i) = \begin{cases} 1 & \text{if } i = n \\ 0 & \text{otherwise} \end{cases}$$

Translation example



Translation of the heat kernel to
different locations of the Minnesota graph

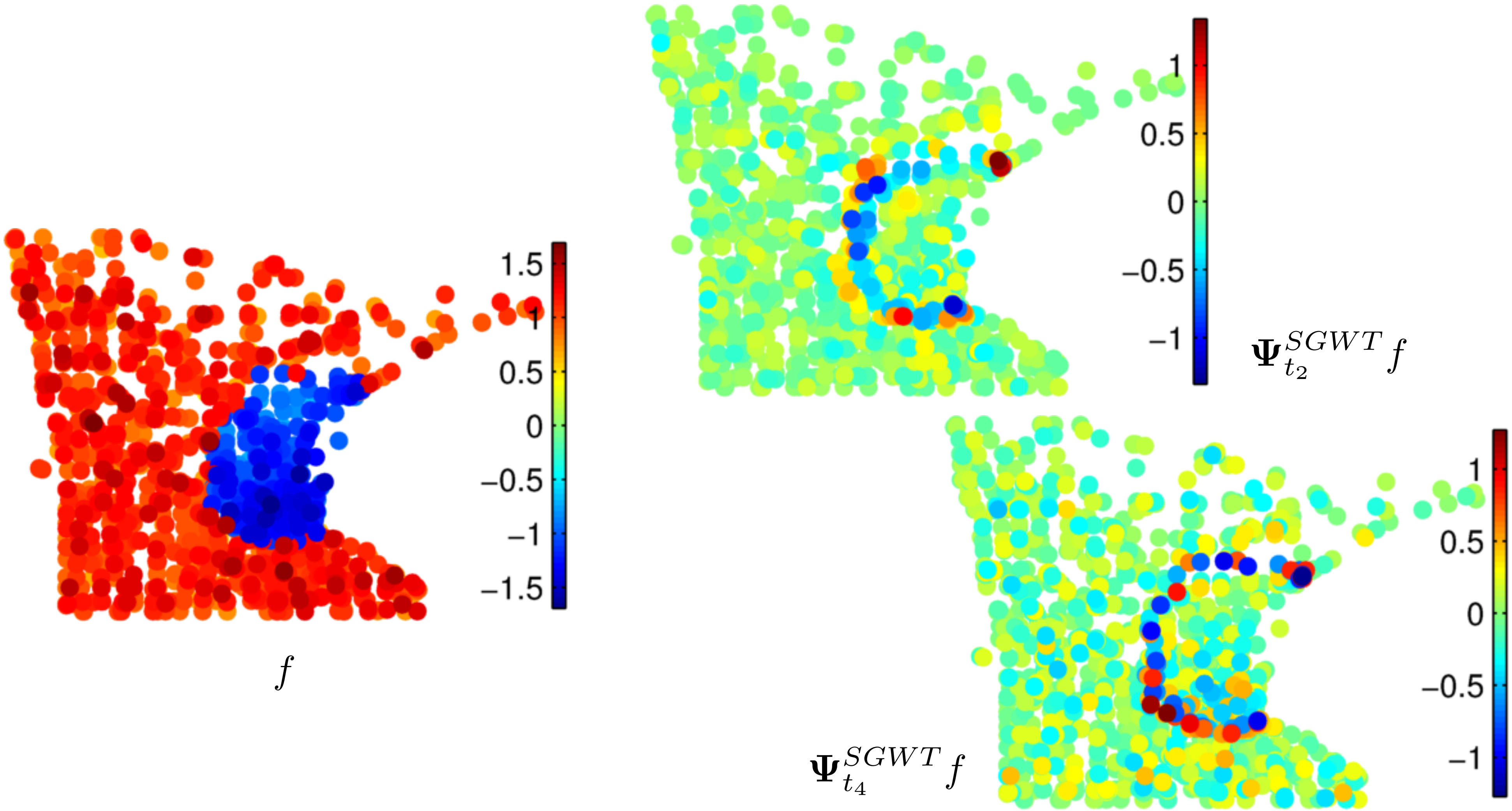
Transforms on graphs

- Localized transforms are ideal to analyse graph signals
  - analysis properties and scalable implementations
- Wavelet transforms are particularly interesting
  - localization in both the vertex and spectral domains
  - different designs in the vertex or the spectral domain [Shuman:2013]
    - Diffusion wavelets [Coifman:2006], Vertex-based graph wavelets [Crovella: 2003], Graph quadrature mirror filterbanks [Narang:2012]
  - *Example*: Spectral Graph Wavelets [Hammond:2011]

$$\mathbf{\Psi}^{SGWT} : \mathbb{R}^N \to \mathbb{R}^{N(K+1)} \qquad \mathbf{\Psi}^{SGWT} = [\mathbf{\Psi}^{SGWT}_{scal}; \mathbf{\Psi}^{SGWT}_{t_1}; \dots ; \mathbf{\Psi}^{SGWT}_{t_K}]$$

- Dilations and translations of a band-pass kernel $\psi^{SGWT}_{t_k,i} := T_i \mathcal{D}_{t_k} \mathbf{g} = \widehat{\mathcal{D}_{t_k} g}(\mathcal{L}) \boldsymbol{\delta}_i$
- Translation of a low-pass kernel $\psi^{SGWT}_{scal,i} := T_i \mathbf{h} = \hat{h}(\mathcal{L}) \boldsymbol{\delta}_i$

SGWT illustration



[Shuman:2013]

Another SGWT illustration

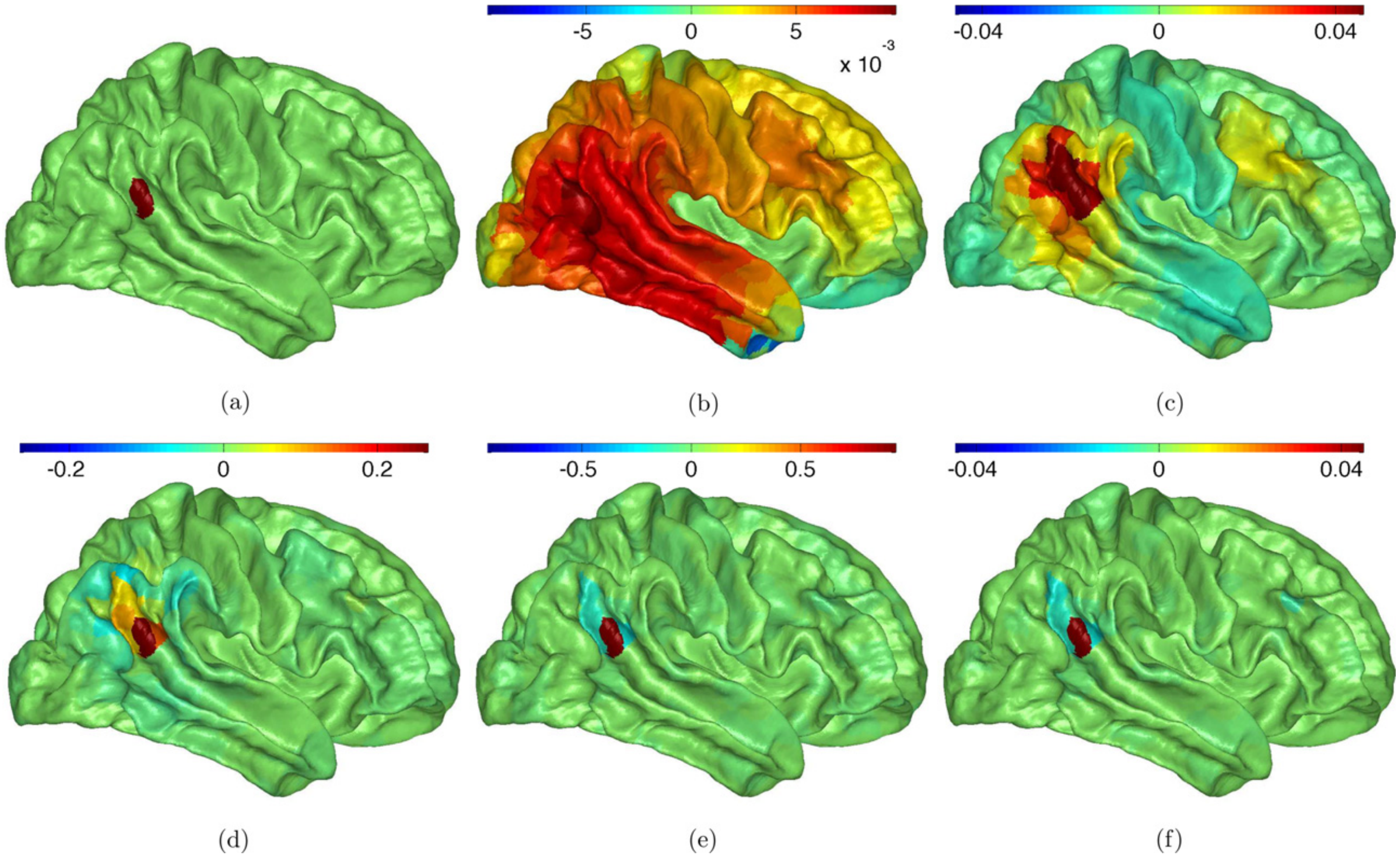
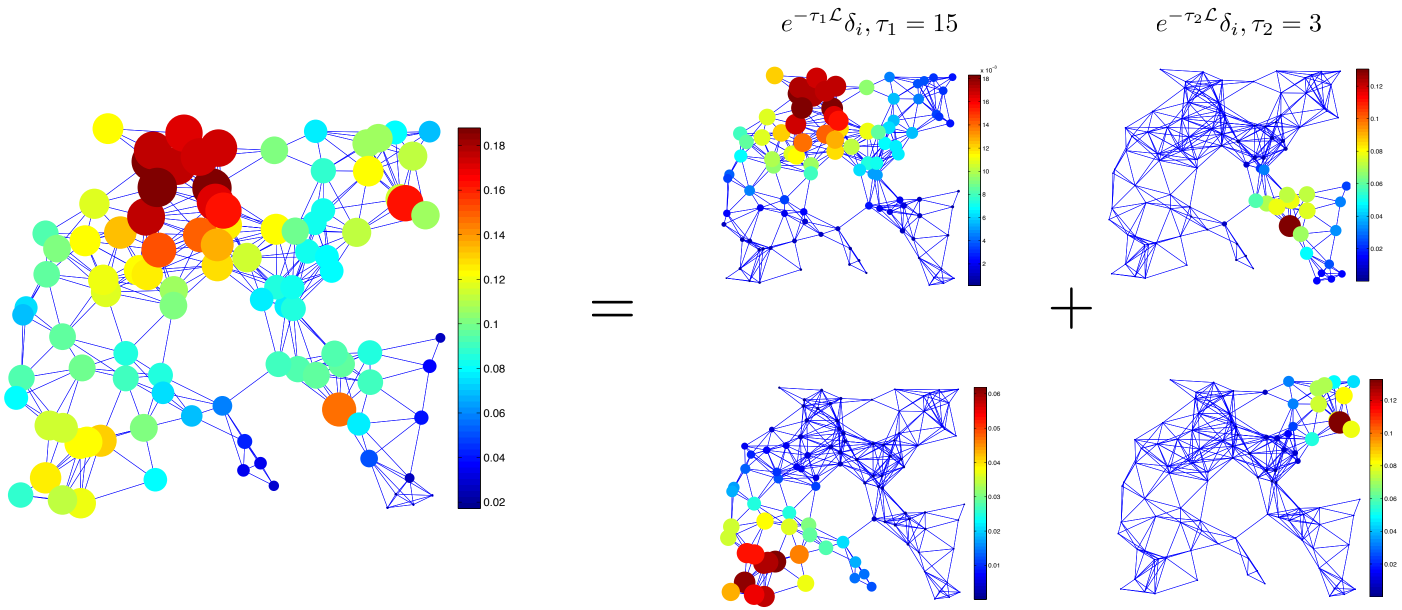


Fig. 5. Spectral graph wavelets on cerebral cortex, with $K = 50$, $J = 4$ scales. (a) ROI at which wavelets are centered, (b) scaling function, (c)–(f) wavelets, scales 1–4.

[Hammond:2011]

Data-adaptive representations

- The representation can be adapted to the data by numerical optimisation
  - Signals = *sparse* linear combinations of *graph atoms*
  - Learning of wavelets [Rustamov:2013] or dictionaries [Thanou:2014]



Graph spectral dictionaries

- A parametric graph dictionary $\mathcal{D} = [\mathcal{D}_1, \mathcal{D}_2, ..., \mathcal{D}_S]$ is a concatenation of S subdictionaries

- Each subdictionary built on a polynomial spectral kernel

$$\mathcal{D}_s = \widehat{g}_s(\mathcal{L}) = \chi \left(\sum_{k=0}^{K} \alpha_{sk} \Lambda^k \right) \chi^T = \sum_{k=0}^{K} \alpha_{sk} \mathcal{L}^k$$

  - Each atom (column of $\mathcal{D}_s$ corresponds to a K-hop localized pattern centered on a node of the graph, i.e.,

$$\frac{1}{\sqrt{N}} T_n g_s$$

[Thanou:2014]

Spectral Dictionary Learning

- Learning consists in computing $\{\alpha_{sk}\}_{s=1,2,\ldots,S;\ k=1,2,\ldots,K}$
- Given a set of training signals $Y = [y_1, y_2, ..., y_M] \in \mathbb{R}^{N \times M}$ on the graph $\mathcal{G}$, solve

$$\underset{\alpha\in\mathbb{R}^{(K+1)S},\ X\in\mathbb{R}^{SN\times M}}{\operatorname{argmin}} \quad \{||Y - \mathcal{D}X||_F^2 + \mu\|\alpha\|_2^2\}$$

$$\text{subject to} \quad \|x_m\|_0 \leq T_0, \quad \forall m \in \{1, ..., M\},$$

$$\mathcal{D}_s = \sum_{k=0}^{K} \alpha_{sk}\mathcal{L}^k, \quad \forall s \in \{1, 2, ..., S\}$$

$$0 \preceq \mathcal{D}_s \preceq c, \quad \forall s \in \{1, 2, ..., S\}$$

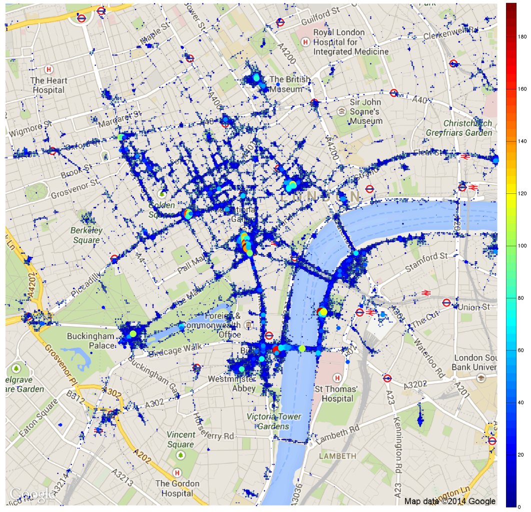
$$(c - \epsilon_1)I \preceq \sum_{s=1}^{S} \mathcal{D}_s \preceq (c + \epsilon_2)I,$$

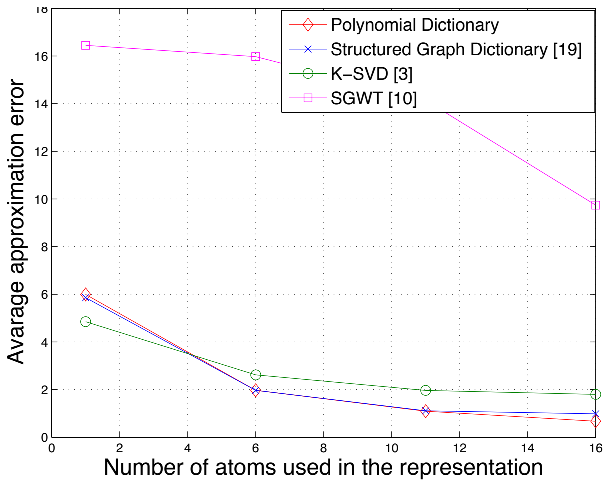
The spectral constraints guarantee that:
1. The learned kernels cover the whole spectrum
2. The dictionary is a frame

[Thanou:2014]

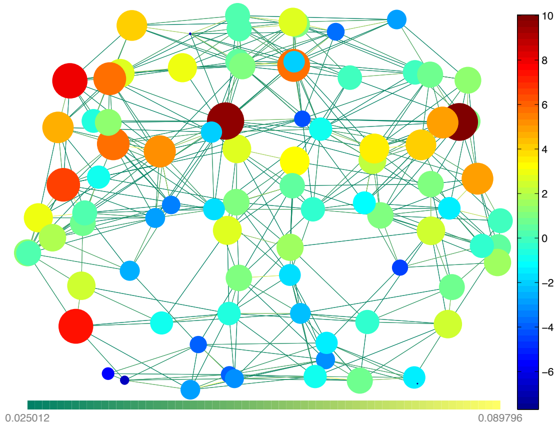
Graph-based transforms examples

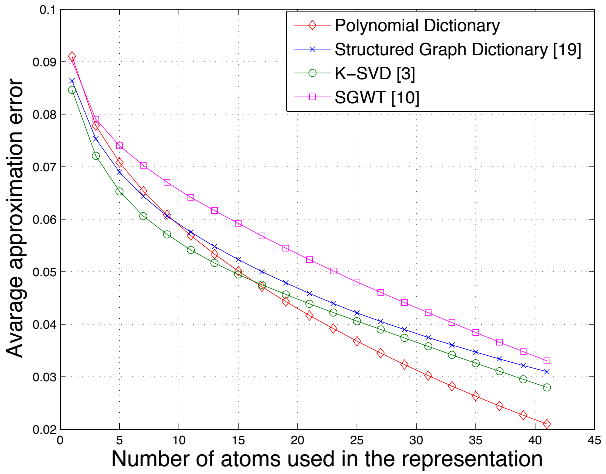
FlickR



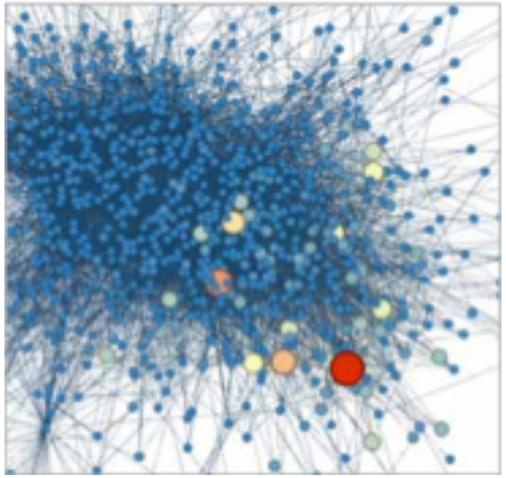


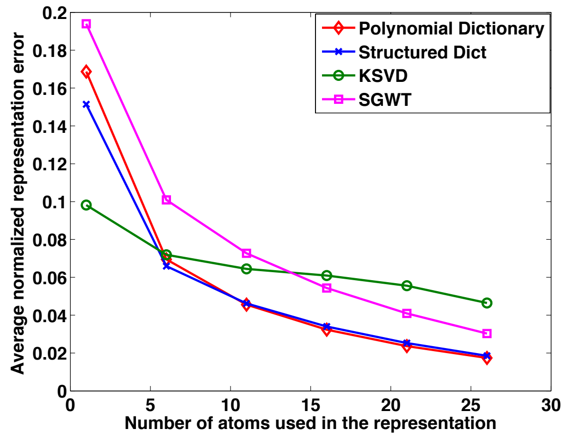
Brain fMRI





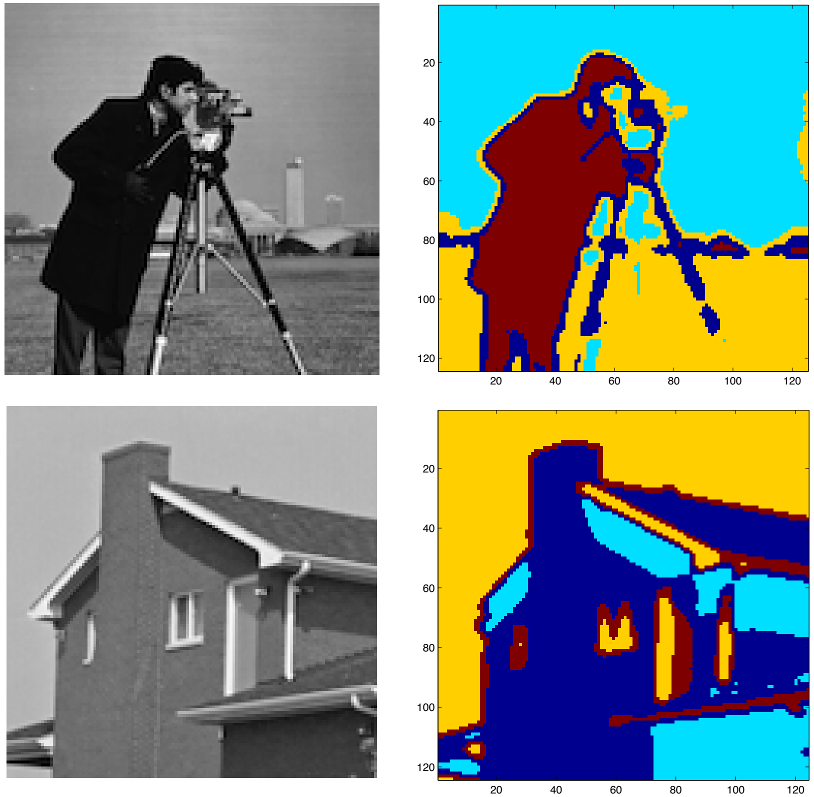
Twitter





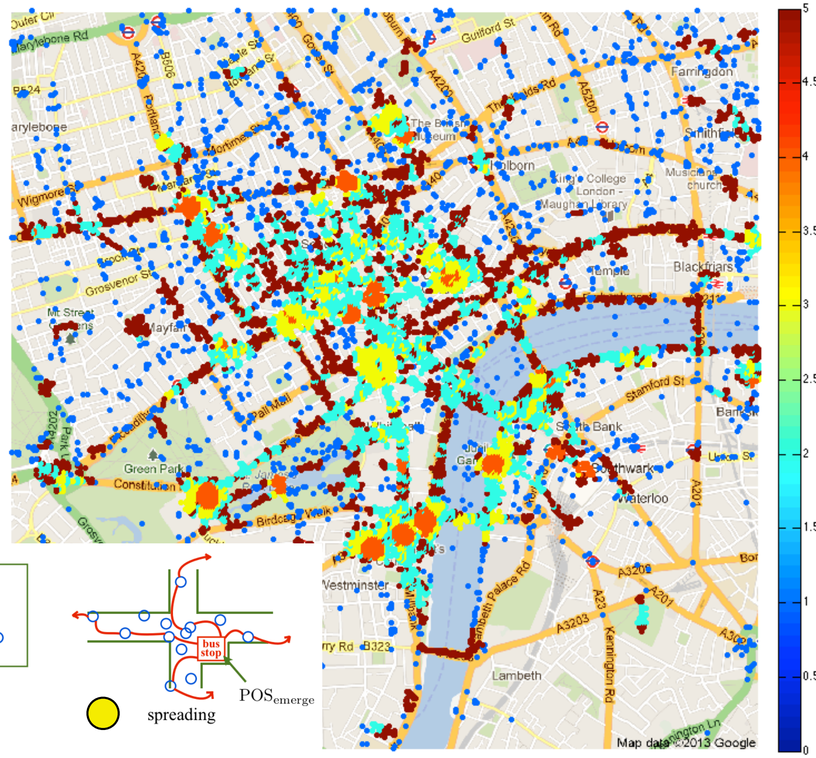
Graph signal analysis examples

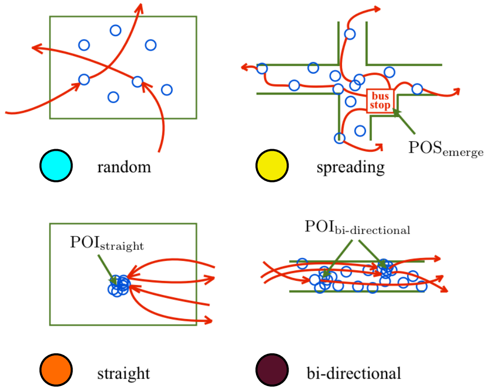
Dictionary-based image segmentation



[Thanou:2014]

SGWT-based mobility inference from FlickR

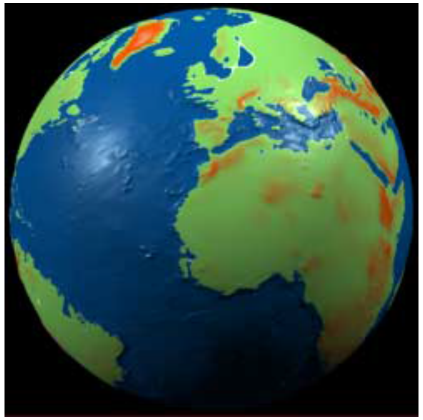




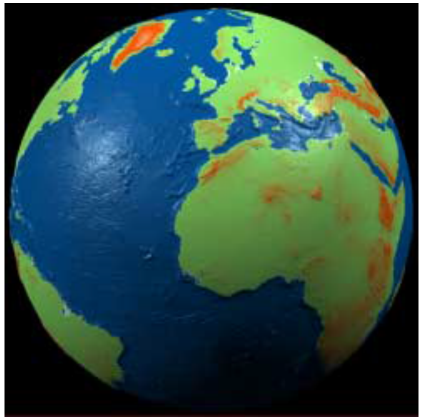
[Dong:2013]

Related frameworks

- Other GSP frameworks
  - GFT based on the adjacency matrix [Sandryhaila:2013]
  - Probabilistic framework [Zhang:2015]
- Discrete calculus [Elmoataz:2008] [Grady:2010] [Lozes:2015]
  - discretized PDE framework for regularization on graphs
- Spherical wavelets
  - efficient mesh-based representation [Schroeder:1995]



15000 wavelets coefficients



190000 wavelets coefficients

[Schroeder:1995]

GSP: what are the benefits?

- Graphs are very generic data representations

- GSP offers flexible processing tools (transforms, shift)
  - data on irregular structures or networks
  - data on regular structures but with irregular boundaries

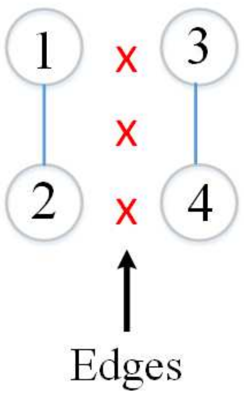
- Intuitions from classical framework extend to graphs
  - spectral representation used for filtering or feature construction

- Multimedia applications are emerging
  - compression, quality enhancement, analysis… many more to come!

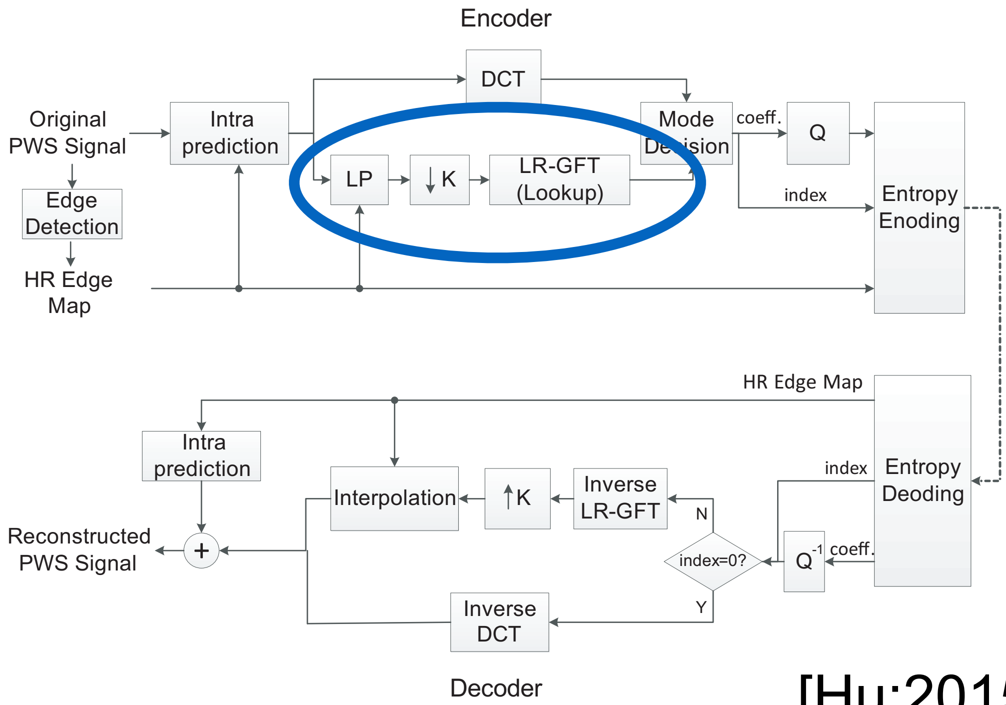
Compression by adaptive transform

- Compression of piecewise-smooth images (depth maps or animation images)
  - Adapt graphs for maximally smooth signals
  - Optimize graph transform for better compression

$$\mathbf{x}^T \mathcal{L} \mathbf{x} = \frac{1}{2} \sum_{i=1}^{N} \sum_{j=1}^{N} W_{i,j} (x_i - x_j)^2$$

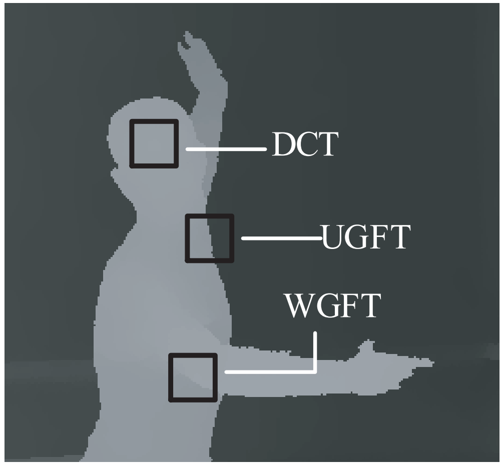


$$\min_{\mathbf{W}} \quad R_\alpha(\mathbf{x}, \mathbf{W}) + R_T(\mathbf{W})$$
$$\text{s.t.} \quad W_{i,j} \in \mathcal{C} \quad \forall\, i, j \in \mathcal{V}$$

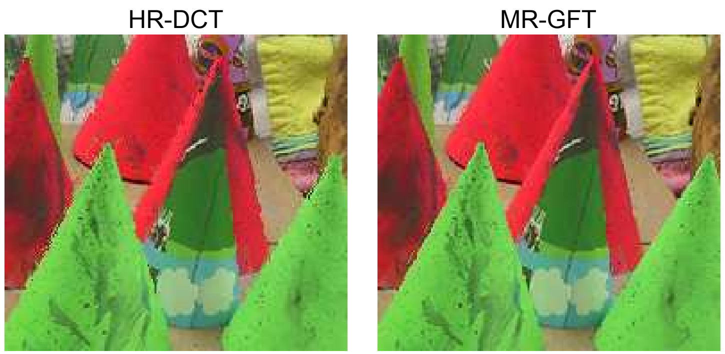


[Hu:2015]

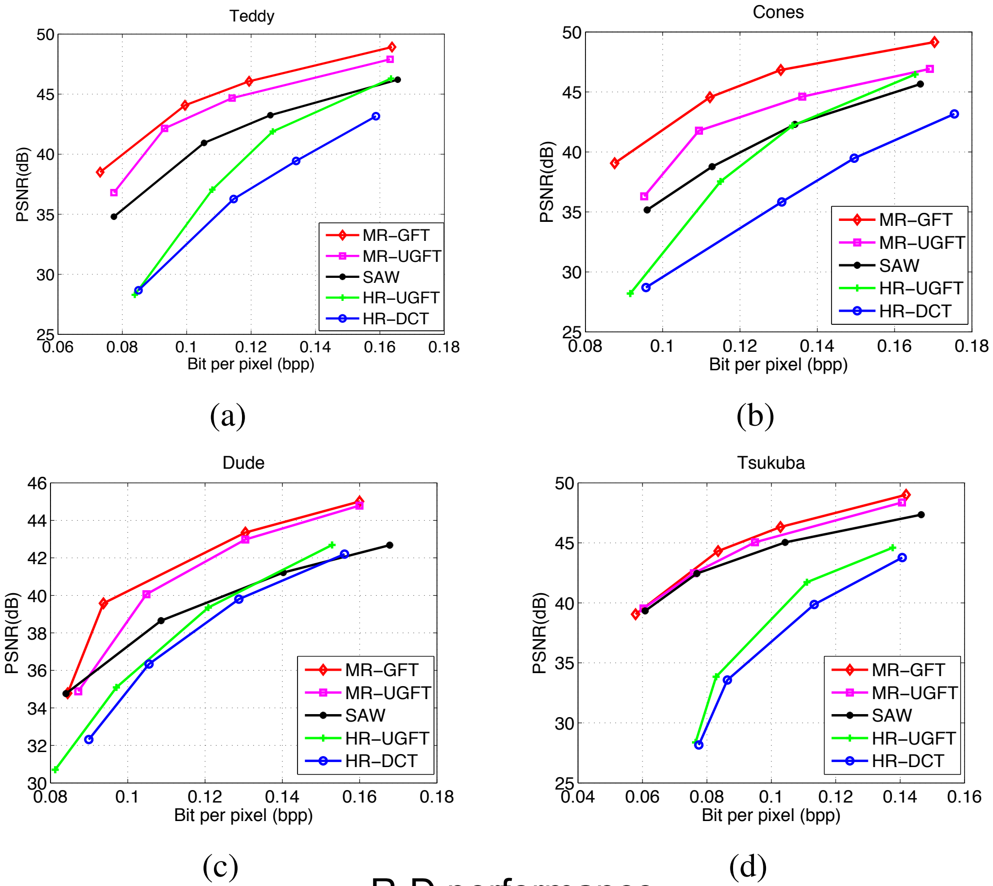
Compression of PWS images



Adaptive Graph Transform



DIBR synthesis

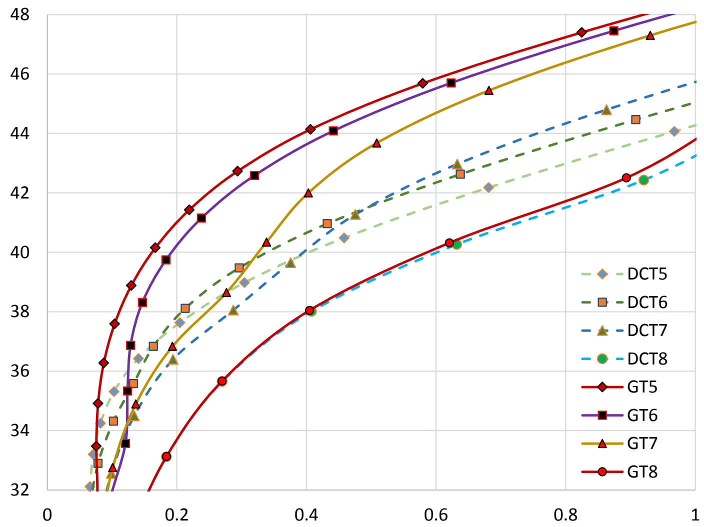


R-D performance

[Hu:2015]

Coding of point cloud attributes

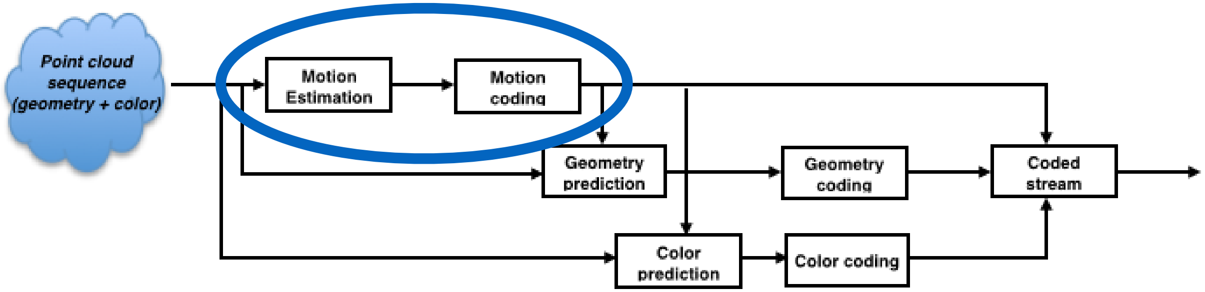
- Compression of colors or normal directions in 3D point clouds [Zhang:2014]
  - Octree representation transformed into a graph
    - weights inversely proportional to the distance between voxels
  - Values of attributes become signals on the graph
- Graph transform, quantization and entropy coding

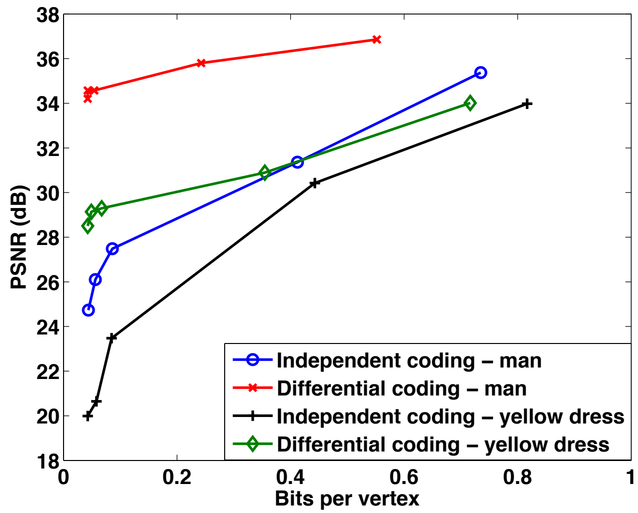


Original Q=4 Q=64

Point cloud sequences coding

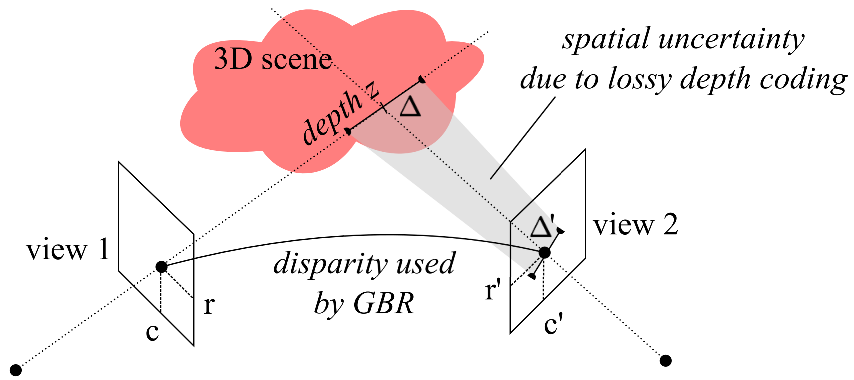
- Graph signals from point clouds
  - Geometry and color signals
- Graph matching for ME
  - Pairing of SGWT features
- Graph-based coding of residuals





[Thanou:2015]

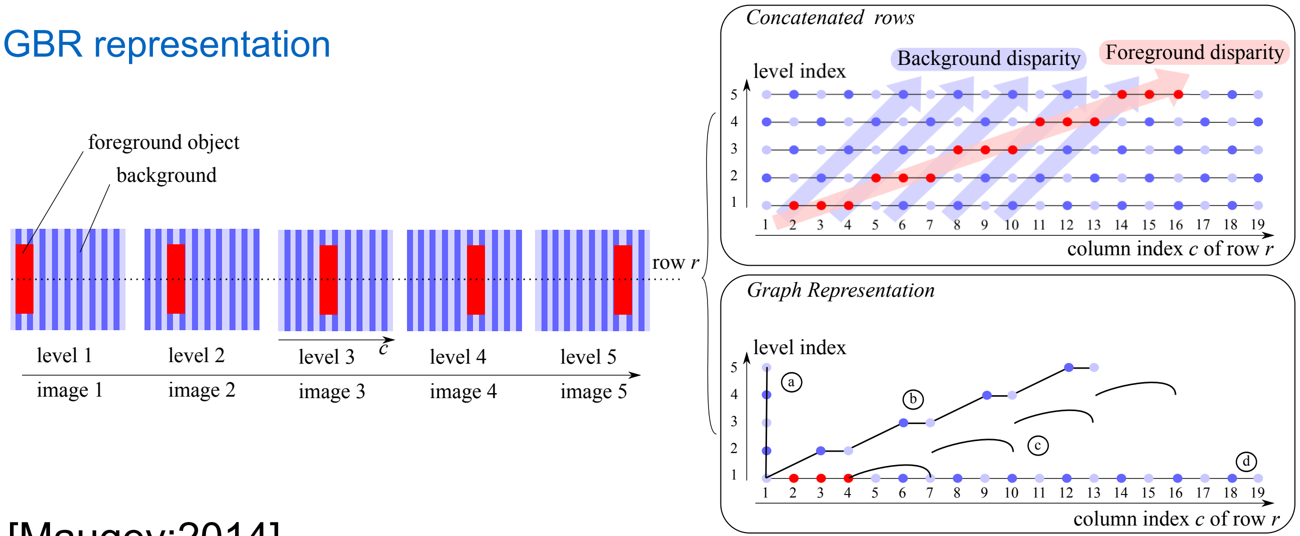
Coding in flexible representations



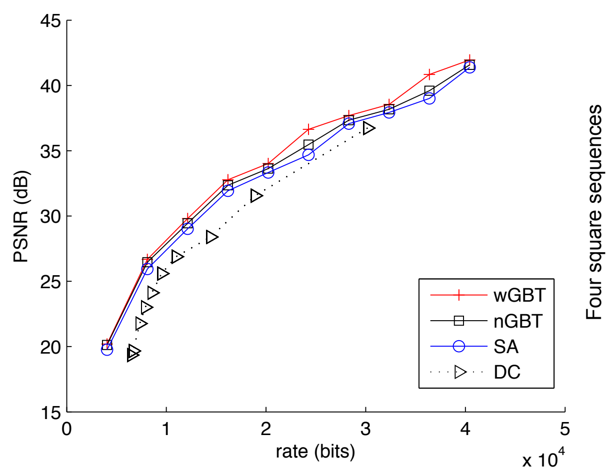
Graph-based representation of multiview images

- graph to connect corresponding pixels in different views
- graph-based transform to code luminance values on GBR

GBR representation

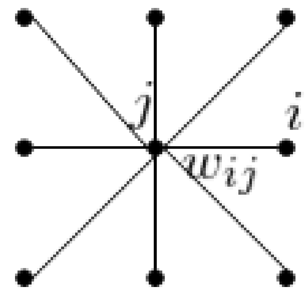


Illustrative performance



[Maugey:2014]

Graph-based bilateral filtering



- Bilateral filtering computes weighted average

$$x_{out}[j] = \sum_i \frac{w_{ij}}{\sum_i w_{ij}} x_{in}[i]$$

  - data-adaptive weights (euclidian and photometric distance)

$$w_{ij} = \exp\left(-\frac{\|p_i - p_j\|^2}{2\sigma_d^2}\right) \exp\left(-\frac{(x_{in}[i] - x_{in}[j])^2}{2\sigma_r^2}\right)$$

- Graph spectral interpretation

$$\mathbf{x}_{out} = \mathbf{D}^{-1}\mathbf{W}\mathbf{x}_{in} \quad \text{with} \quad \mathbf{D}_{jj} = \sum_i w_{ij}$$

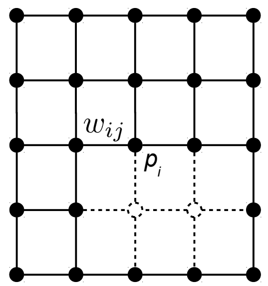
$$\mathbf{x}_{out} = \mathbf{D}^{-\frac{1}{2}}\mathbf{D}^{-\frac{1}{2}}\mathbf{W}\mathbf{D}^{-\frac{1}{2}}\mathbf{D}^{\frac{1}{2}}\mathbf{x}_{in}$$

$$\mathbf{D}^{\frac{1}{2}}\mathbf{x}_{out} = (\mathbf{I} - \mathcal{L})\mathbf{D}^{\frac{1}{2}}\mathbf{x}_{in} \quad \Longleftrightarrow \quad \mathbf{x}_{out} = \underbrace{\mathbf{U}}_{\substack{\text{Inverse} \\ \text{GFT}}} \underbrace{h(\mathbf{\Lambda})}_{\substack{\text{Spectral} \\ \text{response}}} \underbrace{\mathbf{U}^t\mathbf{x}_{in}}_{\text{GFT}} = h(\boldsymbol{\mathcal{L}})\mathbf{x}_{in}$$

[Gadde:2013]

Graph-based 3D image enhancement

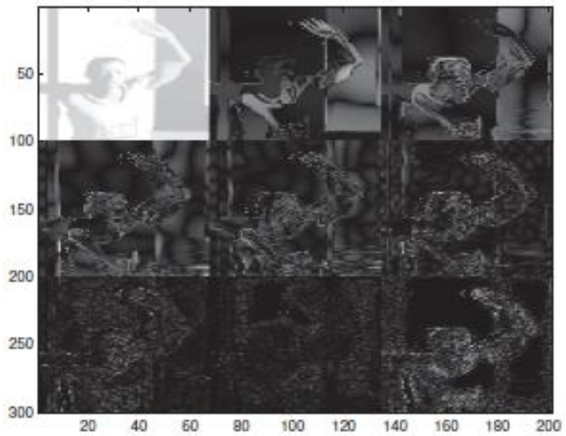
- Multiview images of asymmetric quality
  - Enhance low quality image with help of high quality one
  - Construct graph using warped high quality image as guiding image



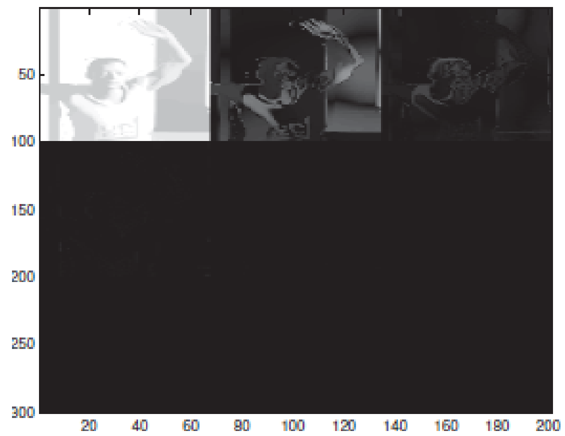
$$w_{ij} = \exp\left(-\frac{\|p_i - p_j\|^2}{2\sigma_d^2}\right) \exp\left(-\frac{(x_{in}[i] - x_{in}[j])^2}{2\sigma_r^2}\right)$$

$$\boxed{\mathbf{x}^* = \arg\min_{\mathbf{x}\in\mathbb{R}^N} \frac{1}{2}\|\mathbf{x} - \mathbf{b}\|^2 + \frac{\rho}{2}\|h_R\mathbf{x}\|^2}$$

Graph spectral decomposition



Filtered decomposition

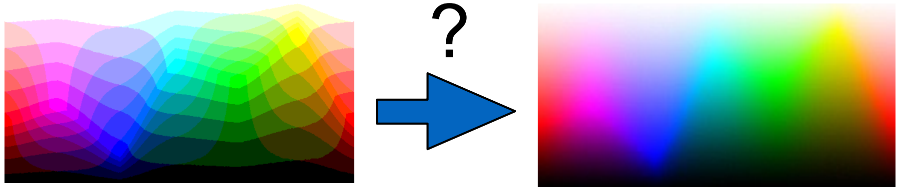


Denoising results

Seqs	Kendo	Poznan_Street
JBF	32.53	32.78
3-POLY	32.28	33.05
3-CG	35.76	34.49
3-CHEB	35.67	34.35

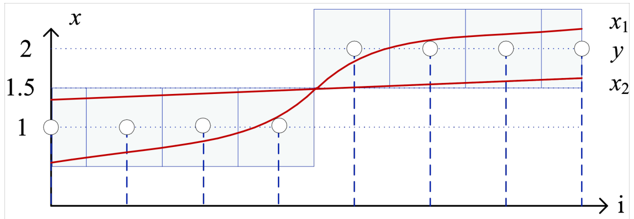
[Tian:2014]

Image Bit-Depth Enhancement



Quantized depth input

Target output



MAP problem: $\hat{x}^* = \underset{\hat{x}}{\operatorname{argmin}} \int \|\hat{x} - x\|_2^2 f(x|y)\,dx$

Posterior: $f(x|y) \propto f(y|x) f(x)$

Likelihood:

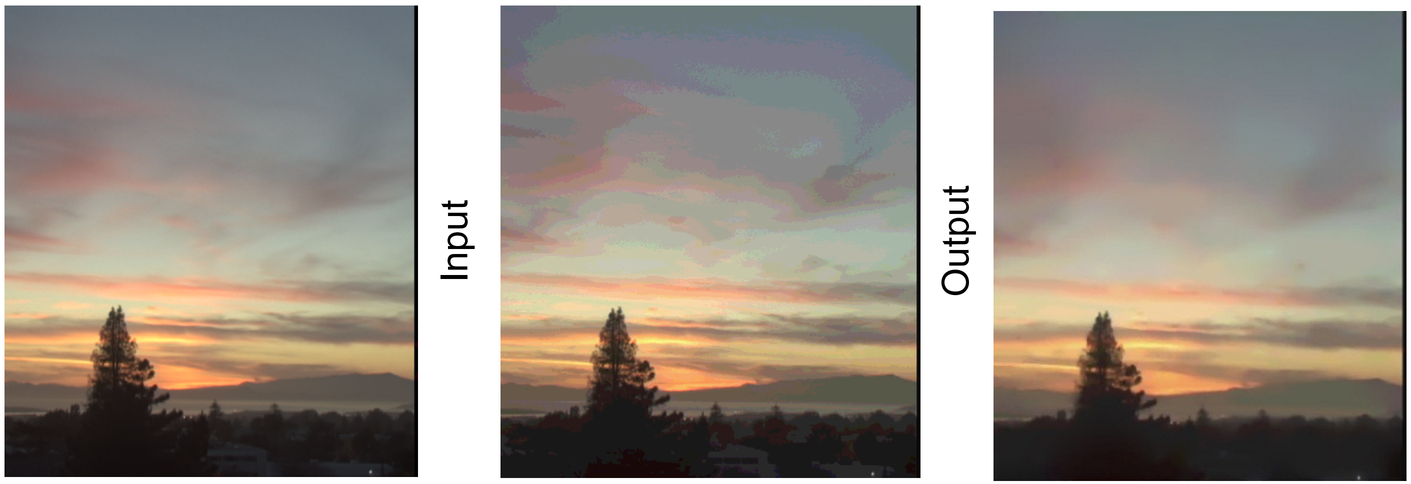
$$f(y|x) = \begin{cases} 1 & \text{if } \mathrm{quant}(x_i) = y_i, \forall i \\ 0 & \text{otherwise} \end{cases}$$

Graph-signal smoothness: $f(x) = \frac{1}{K} \exp(-\sigma x^T L x)$

Original

Input

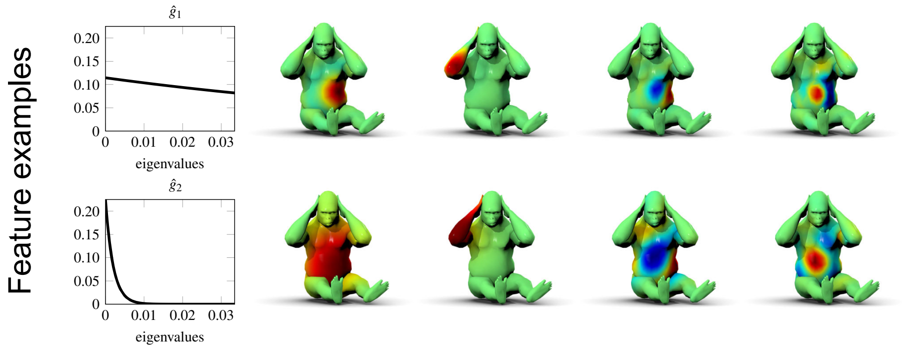
Output

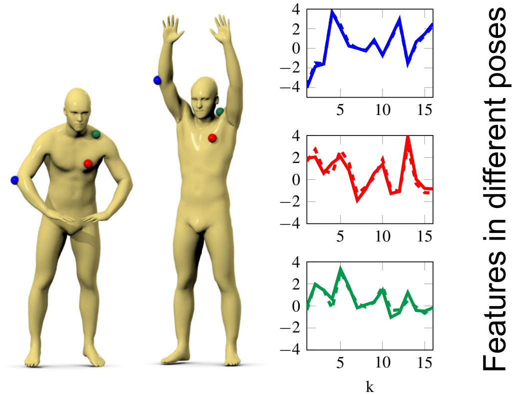


[Wan:2014]

Deformable shape analysis

- Discrete shape manifold approximation with graphs
  - Laplace-Beltrami operator approximated by graph Laplacian
- Localized features with Windowed GFT [Shuman:2015]





  - Features fed into a localised spectral CNN (using graph convolution)

[Boscaini:2015]

Graph-based Active SSL

Active SSL problem

Data points in feature space

Construct similarity graph

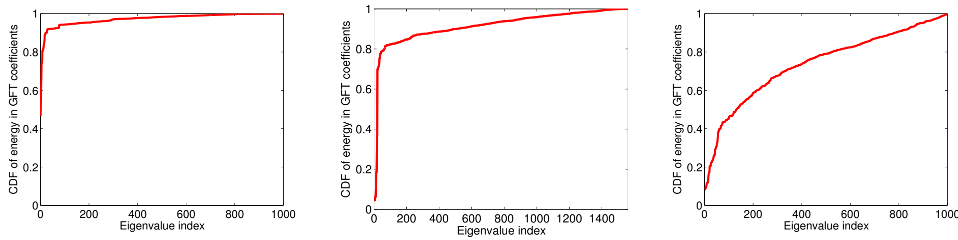
Choose points to label

Predict labels for the rest

- Graph nodes are data points, weights are similarity values
- Graph signals are class membership functions

$$\mathbf{f}^c(j) = \begin{cases} 1, & \text{if node } j \text{ is in class } c \\ 0, & \text{otherwise} \end{cases}$$

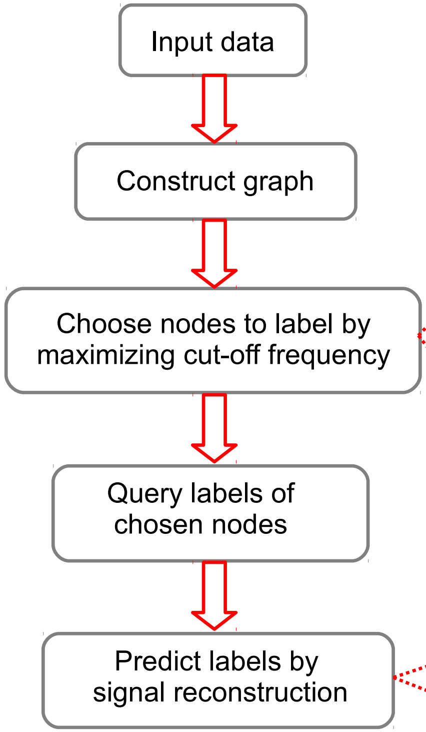
- these functions can be approximated by bandlimited graph signals



(a) USPS (b) Isolet (c) 20 newsgroups

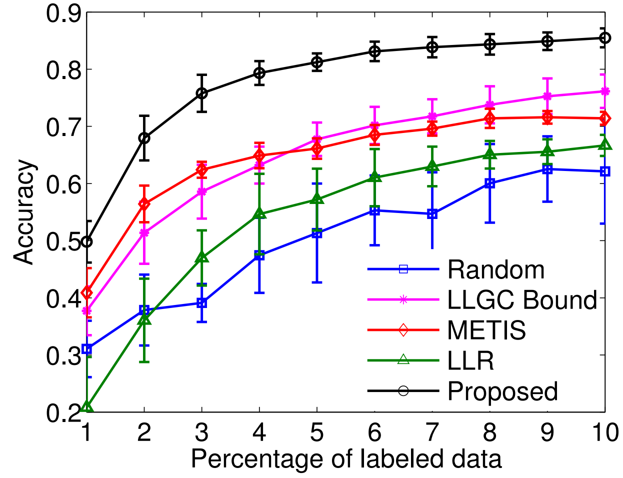
[Gadde:2014]

Graph sampling-based learning

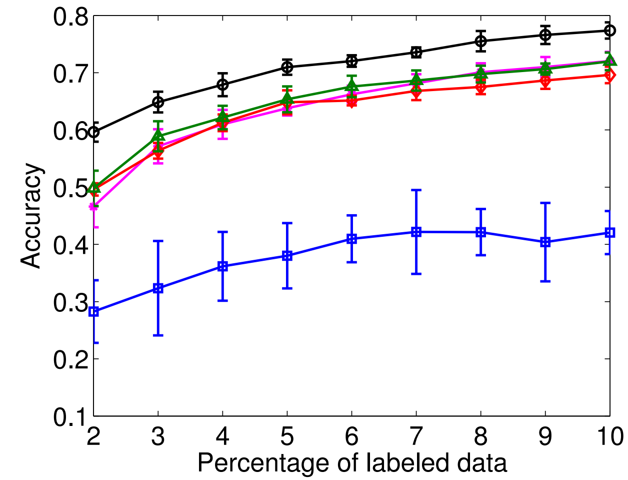


Input: $G = \{\mathcal{V}, E\}, \mathbf{L}$, target size m, parameter $k \in \mathbb{Z}^+$
Initialize: $\mathcal{S} = \{\emptyset\}$
while $|\mathcal{S}| \leq m$
For $\mathcal{S}$, compute the smoothest signal $\phi_k^{\text{opt}} \in L_2(\mathcal{S}^c)$
$v \leftarrow \arg\max_i \left[(\phi_k^{\text{opt}}(i))^2\right]$
$\mathcal{S} \leftarrow \mathcal{S} \cup v$
end while

POCS iteration: $\mathbf{f}_{i+1} = \mathbf{P}_{\mathcal{C}_1}\mathbf{P}_{\mathcal{C}_2}\mathbf{f}_i$
Label of node $n = \arg\max_c \mathbf{f}^c(n)$



USPS: handwritten digits

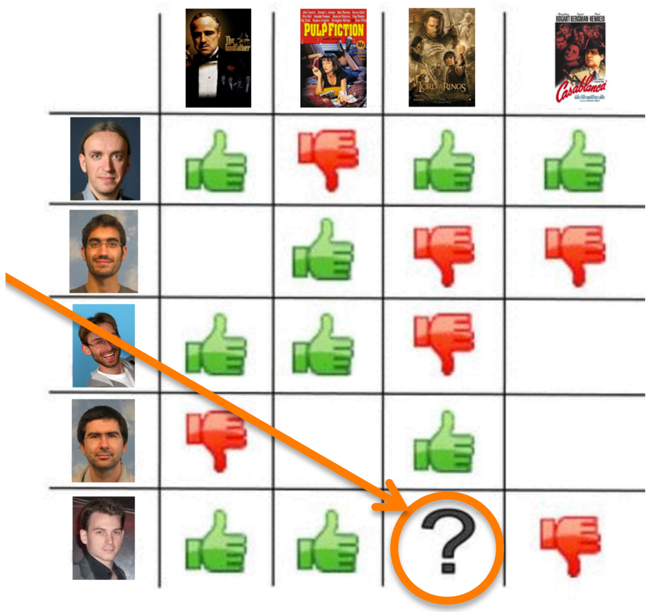


ISOLET: spoken letters

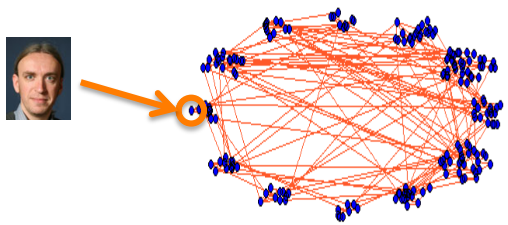
[Gadde:2014]

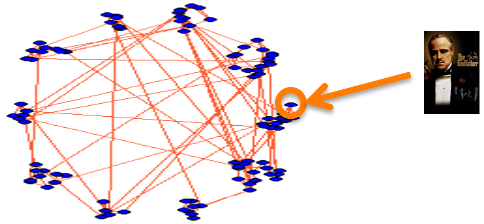
Matrix completion on graphs

- Assumption: users and movies each form a smooth low-dimensional manifold
  - Manifold samples form a graph
  - Graph Laplacian used to 'force' consistency in sample distributions



Graphs: *(1) Row/user graph:* $G^r = (V^r, E^r, W^r)$ *(2) Column/movie graph:* $G^c = (V^c, E^c, W^c)$





[Kalofolias:2014]

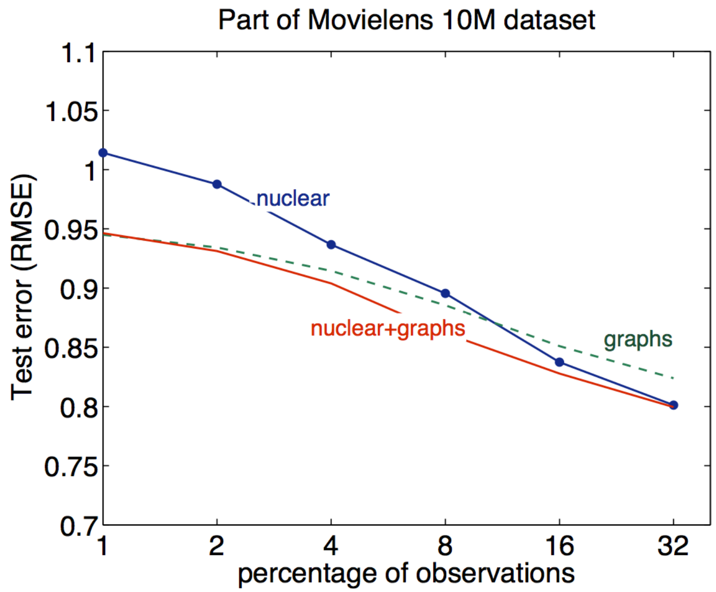
Graph-based low rank solution

$$\min_{X} \ \gamma_n \underbrace{||X||_*}_{\text{Nuclear norm}} + \underbrace{||A_\Omega \circ (X - M)||_{\mathcal{F}}^2}_{\text{Frobenius norm}} + \frac{\gamma_r}{2} \underbrace{\|X\|_{\mathcal{D},r}}_{\substack{\text{Dirichlet norm} \\ \text{w.r.t. rows}}} + \frac{\gamma_c}{2} \underbrace{\|X\|_{\mathcal{D},c}}_{\substack{\text{Dirichlet norm} \\ \text{w.r.t. columns}}}$$

Hadamard product

$$\|X\|_{\mathcal{D},r} = \operatorname{tr}(X L_r X^t) = \sum_{j,j'} w^r_{j,j'} \|X^r_j - X^r_{j'}\|_2^2$$

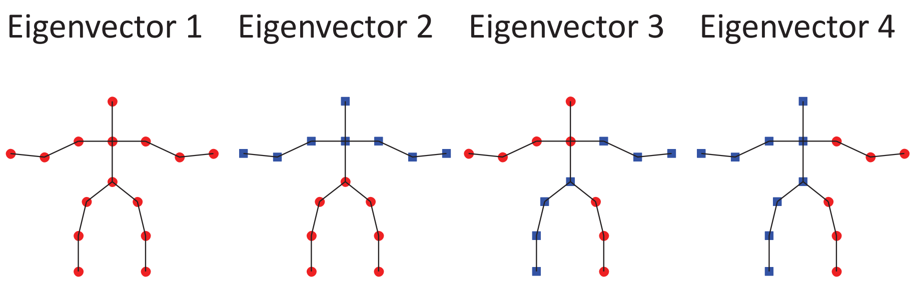
Output: low rank solutions with the rows and columns structured wrt to proximity in the respective graphs



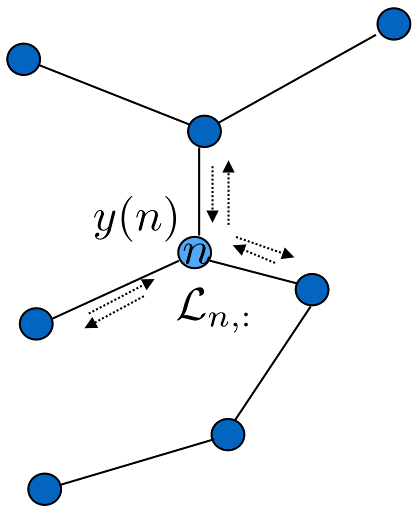
[Kalofolias:2014]

Other applications

- Representation of human motion [Kao:2014]
  - graph-based representation
  - spectral analysis

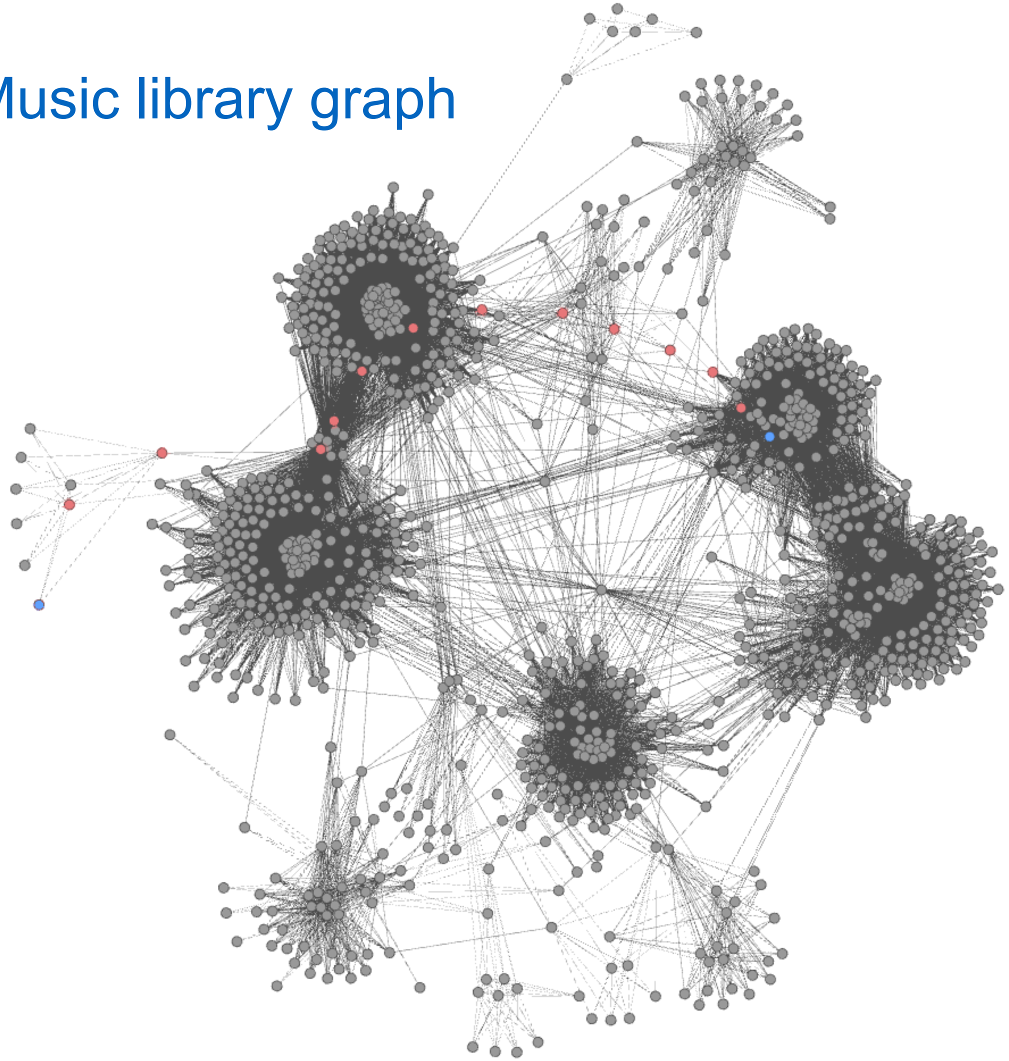


- Speech semantic analysis
  - sentences constructed by GSP from words on a graph [Trifan:2015]
- Compressed Sensing applications
  - Sensor networks [Zhu:2012]
  - Image texture reconstruction [Colonnese:2014]
- Distributed data processing in networks [Shuman:2011]

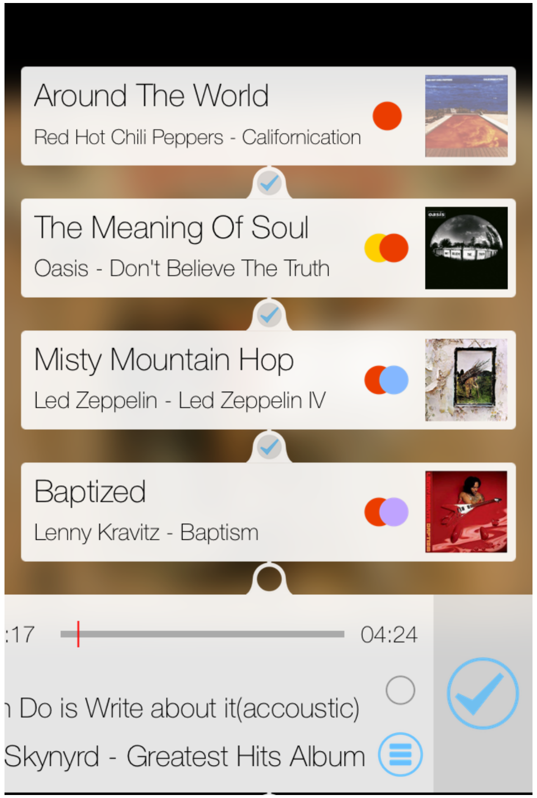


A realworld application: Genezik

Music library graph



Shortest path playlist



http://kirellbenzi.com/genezik/

Summary

- Take-home messages:
  - Emerging framework for representing, compressing and analysing multimedia data living on irregular structures
  - Many of the classical data processing operators extend to graphs
  - Very generic framework with numerous multimedia applications
  - Opportunities for inter-disciplinary work

- Still many open questions:
  - What is a good graph (when not naturally given)?
  - Smoothness priors have been mostly used so far, how about sparsity?
  - How to develop effective implementations at large scale?

Acknowledgments

- Graph-based Signal Processing collaborators
  - Dorina Thanou, Xiaowen Dong, David Shuman, Thomas Maugey, Pierre Vandergheynst, Antonio Ortega, Phil Chou, Gene Cheung
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  - Xavier Bresson, Kirell Benzi, Anthony Vetro, Gene Cheung, Antonio Ortega

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