3. The Regula Falsi Method

Background. The Regula Falsi method is one of the bracketing methods for finding roots of equations.
Implementation. Given a function f(x) and an interval which might contain a root, perform a predetermined number of iterations using the Regula Falsi method.

Theorem (Regula Falsi Theorem). Assume that $f \in C[a, b]$ and that there exists a number $r \in [a, b]$ such that $f(r) = 0$.
If $f(a)$ and $f(b)$ have opposite signs, and

$$c_n = \frac{a_n f(b_n) - b_n f(a_n)}{f(b_n) - f(a_n)}$$

represents the sequence of points generated by the Regula Falsi process, then the sequence $\{c_n\}$ converges to the zero $x = r$.

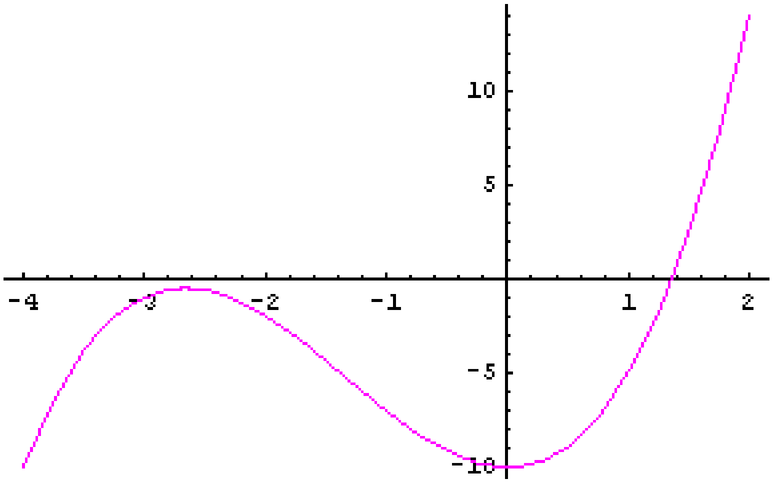
That is, $\lim_{k \to \infty} c_n = r$.

Example 1. Find all the real solutions to the cubic equation $x^3 + 4x^2 - 10 = 0$.
Solution 1.

Remember. The Regula Falsi method can only be used to find a real root in an interval [a,b] in which f[x] changes sign.

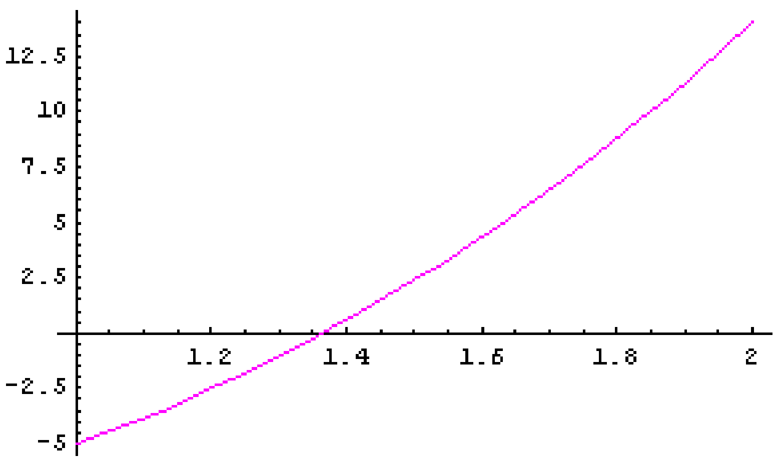
Example 2. **Convergence** Find the solution to the cubic equation $x^3 + 4x^2 - 10 = 0$. Use the starting interval $[a, b] = [-1, 2]$.
Solution 2.

Example 1. Find all the real solutions to the cubic equation $x^3 + 4x^2 - 10 = 0$.
Solution 1.
Plot the function.



$y = f[x] = -10 + 4x^2 + x^3$

There appears to be only one real root which lies in the interval [1,2].



$y = f[x] = -10 + 4x^2 + x^3$

Call the Regula Falsi subroutine on the interval [1,2] using 10 iterations

k	a_k	c_k	b_k	$f[c_k]$
0	1.	1.263157894736842	2.	-1.602274384020995
1	1.263157894736842	1.338827838827839	2.	-0.4303647480045276
2	1.338827838827839	1.358546341824779	2.	-0.1100087884743455

3	1.358546341824779	1.36354744004209	2.	-0.02776209100106009
4	1.36354744004209	1.36480703182678	2.	-0.006983415401172977
5	1.36480703182678	1.365123717884378	2.	-0.001755209032341387
6	1.365123717884378	1.3652033036626	2.	-0.000441063010153453
7	1.3652033036626	1.365223301985543	2.	-0.0001108281334247785
8	1.365223301985543	1.365228327025519	2.	-0.0000278479845592372
9	1.365228327025519	1.365229589673847	2.	$-6.99739040177505\times10^{-6}$
10	1.365229589673847	1.365229906940572	2.	$-1.758239715154986\times10^{-6}$

$$c = 1.365229906940572$$
$$f[c] = -1.758239715154986\times10^{-6}$$

After 10 iterations, the interval has been reduced to [a,b] where

$$a = 1.365229589673847$$
$$b = 2.$$

The root lies somewhere in the interval [a,b] width of which is

$$0.6347704103261533$$

The reported root is alleged to be

$$1.365229906940572$$

The estimate of "how things are going" is the distance between c and the nearest endpoint to the interval.

$$3.172667253359407\times10^{-7}$$

Is this the desired accuracy you want ? If not, more iterations are required.

Example 2. Convergence Find the solution to the cubic equation $x^3 + 4x^2 - 10 = 0$. Use the starting interval $[a, b] = [-1, 2]$.

Solution 2.

k	a_k	c_k	b_k	$f[c_k]$
0	-1.	0.	2.	-10.
1	0.	0.833333333333333	2.	-6.64351851851852
2	0.833333333333333	1.208791208791209	2.	-2.389038325519425
3	1.208791208791209	1.32412611057995	2.	-0.6651566794894461
4	1.32412611057995	1.354781223366369	2.	-0.1716623158084789
5	1.354781223366369	1.362596802578731	2.	-0.04342714574355178
6	1.362596802578731	1.364567874259778	2.	-0.01093061901004688
7	1.364567874259778	1.365063606246616	2.	-0.002747723789759959
8	1.365063606246616	1.365188198210173	2.	-0.0006904969945025208
9	1.365188198210173	1.365219506354675	2.	-0.0001735063715968543
10	1.365219506354675	1.36522737329005	2.	-0.00004359736556747151
11	1.36522737329005	1.36522935002777	2.	-0.00001095475959944636
12	1.36522935002777	1.365229846724515	2.	$-2.752611369505331 \times 10^{-6}$
13	1.365229846724515	1.365229971529886	2.	$-6.916506798404498 \times 10^{-7}$
14	1.365229971529886	1.365230002889821	2.	$-1.737915589217209 \times 10^{-7}$
15	1.365230002889821	1.365230010769655	2.	$-4.366872907723973 \times 10^{-8}$

$$c = 1.365230010769655$$
$$f[c] = -4.366872907723973 \times 10^{-8}$$