What is network science?

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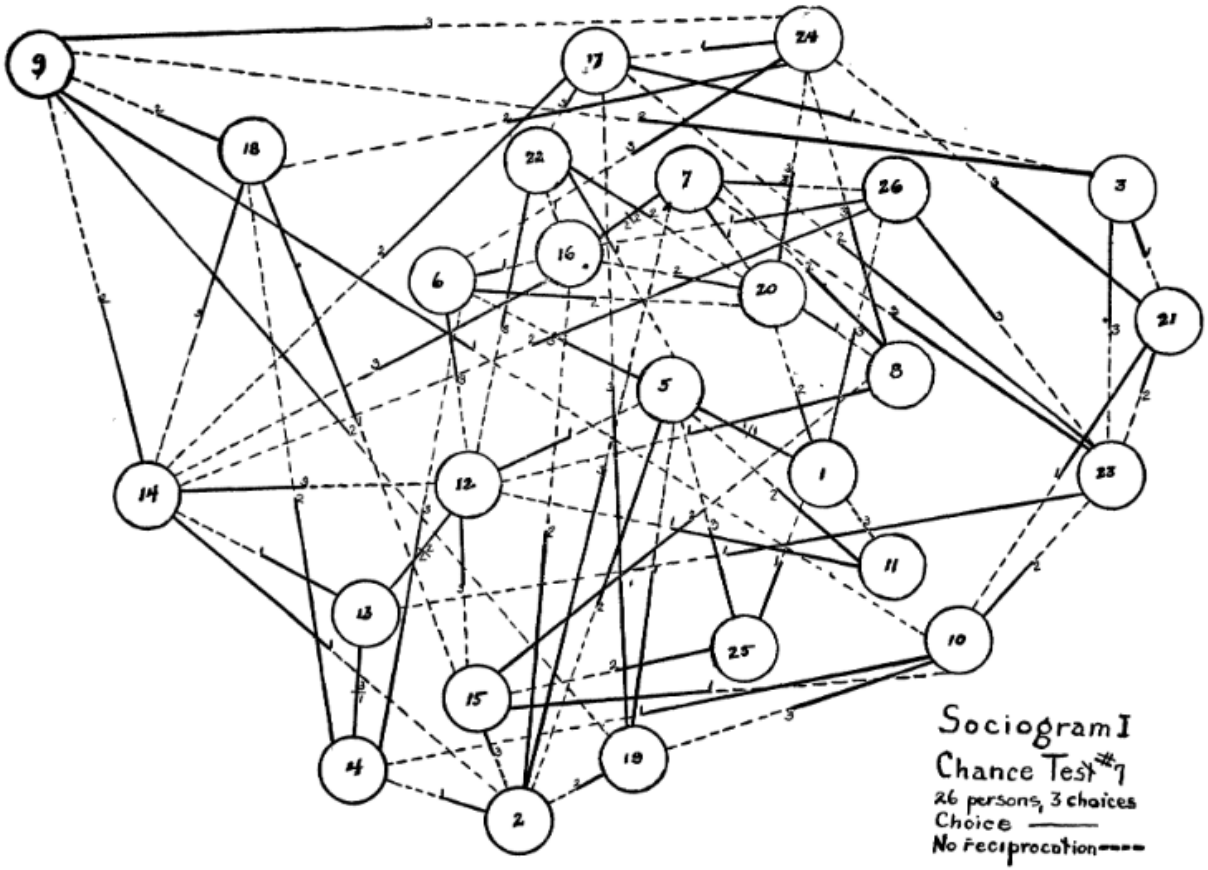
Outline

Multidisciplinary history

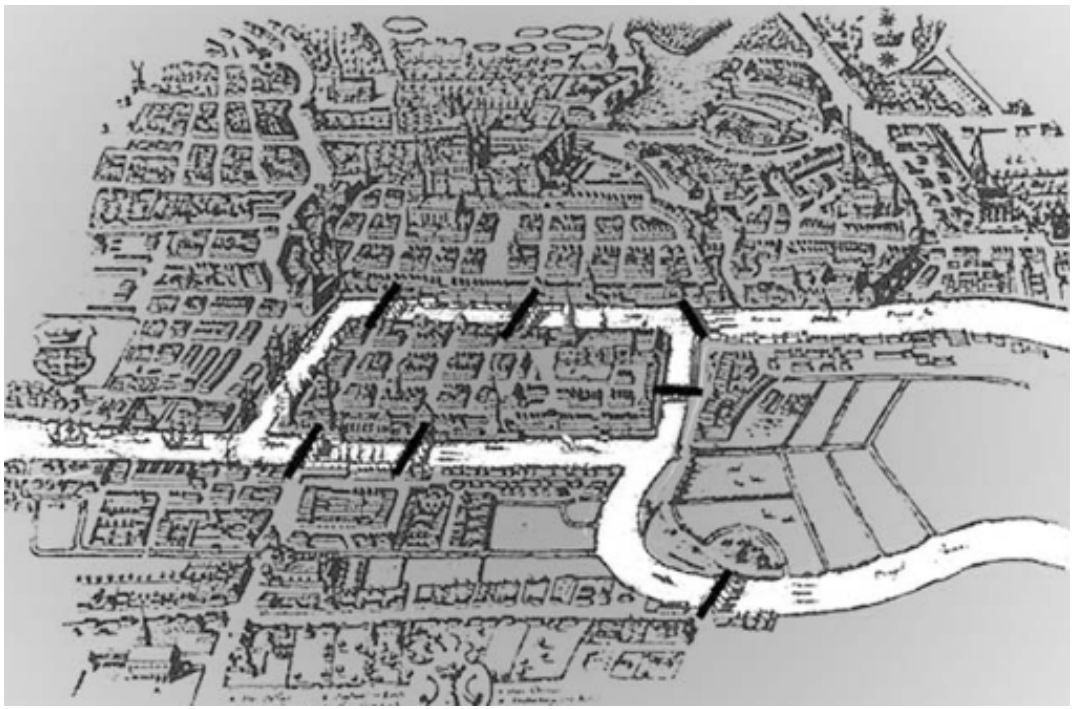
Social network analysis (Sociology)

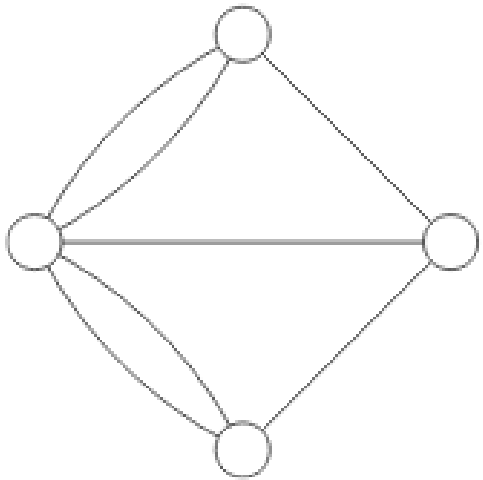


Sociogram — psychiatrist Moreno and social psychologist Jennings.

Moreno, J. L., & Jennings, H. H. (1938). Statistics of social configurations. *Sociometry*, 342-374.

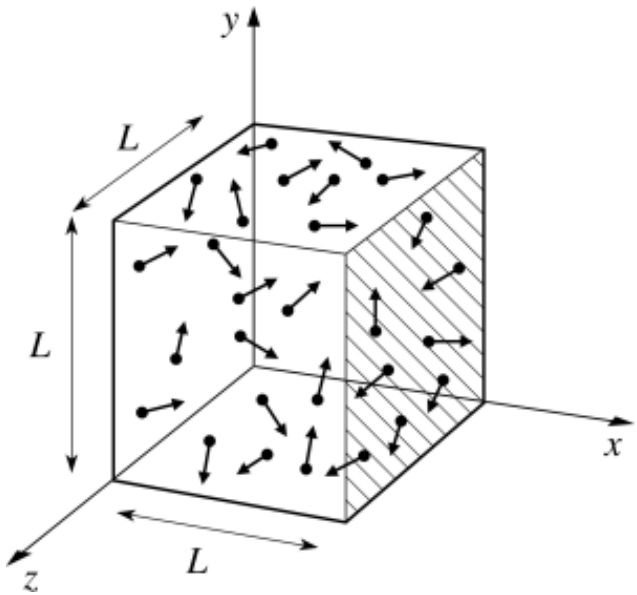
Graph theory (Mathematics)





Königsberg (Kaliningrad) bridge problem — mathematician Leonard Euler (1736).

Statistical mechanics (Physics)



Local (micro) properties explain global (macro) properties — physicist Ludwig Boltzmann (1870s).

Network science

- Modeling real-world networks.

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- Networks as evolving structures.

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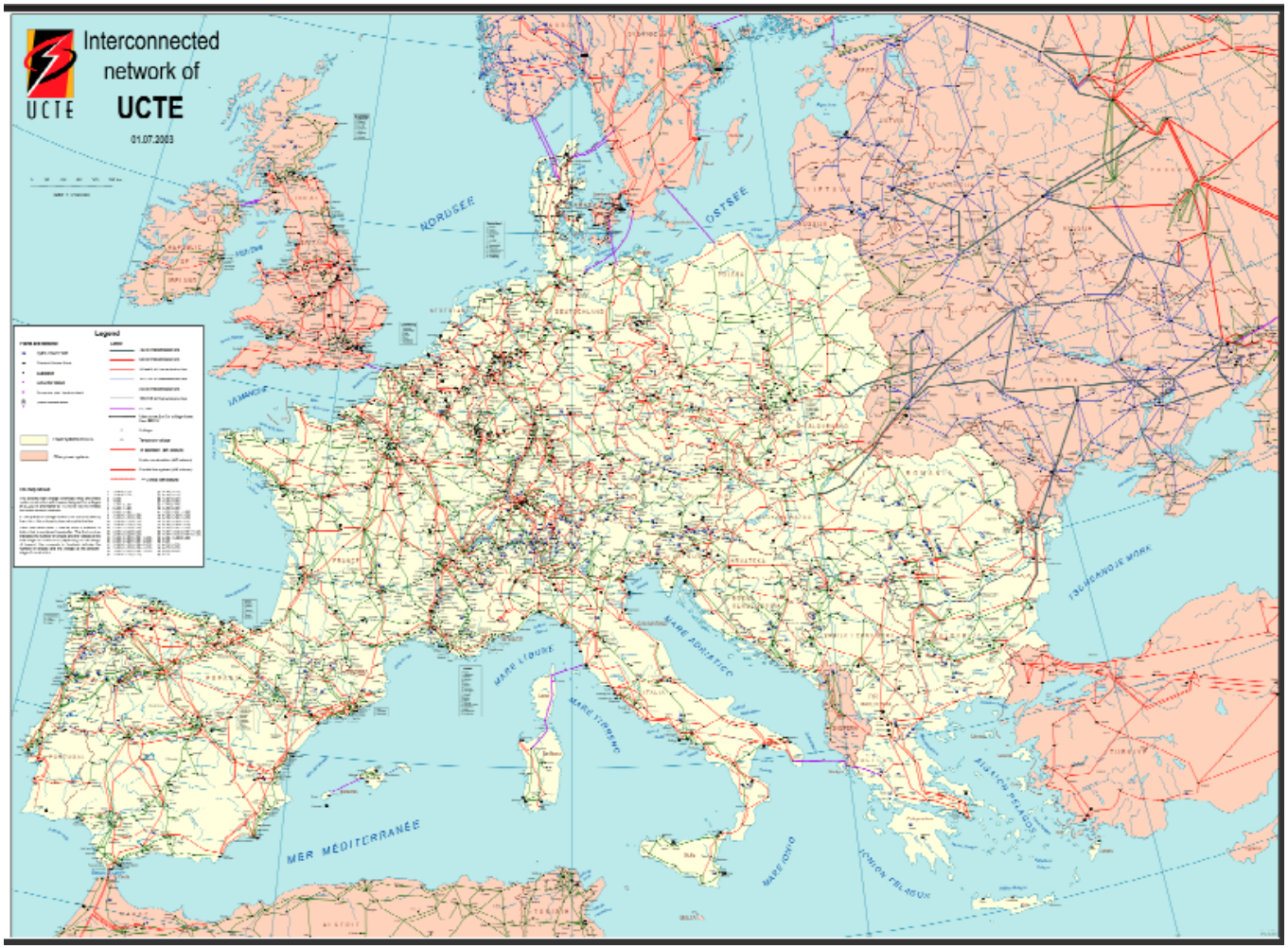
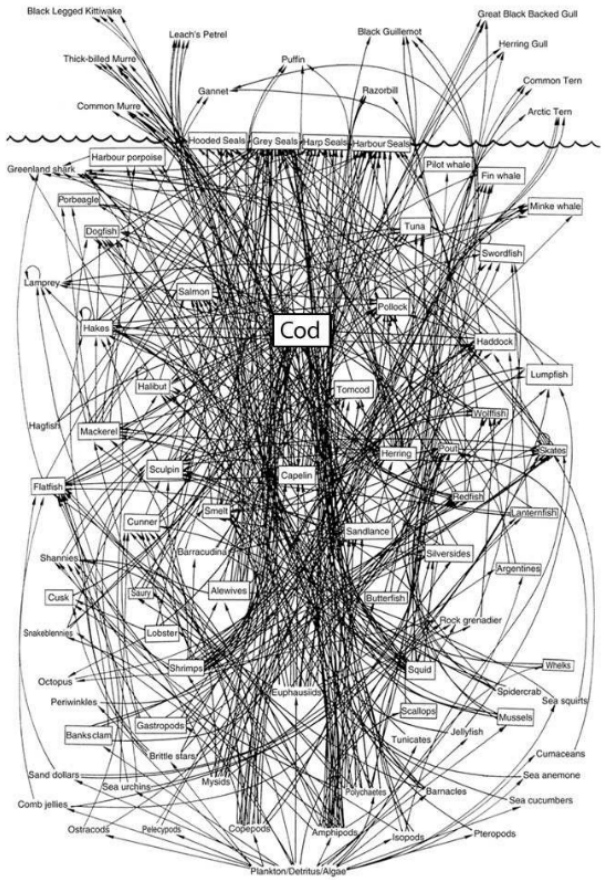
- Networks as evolving structures.
- Networks as dynamical systems.

Engineered systems



* European electricity grid

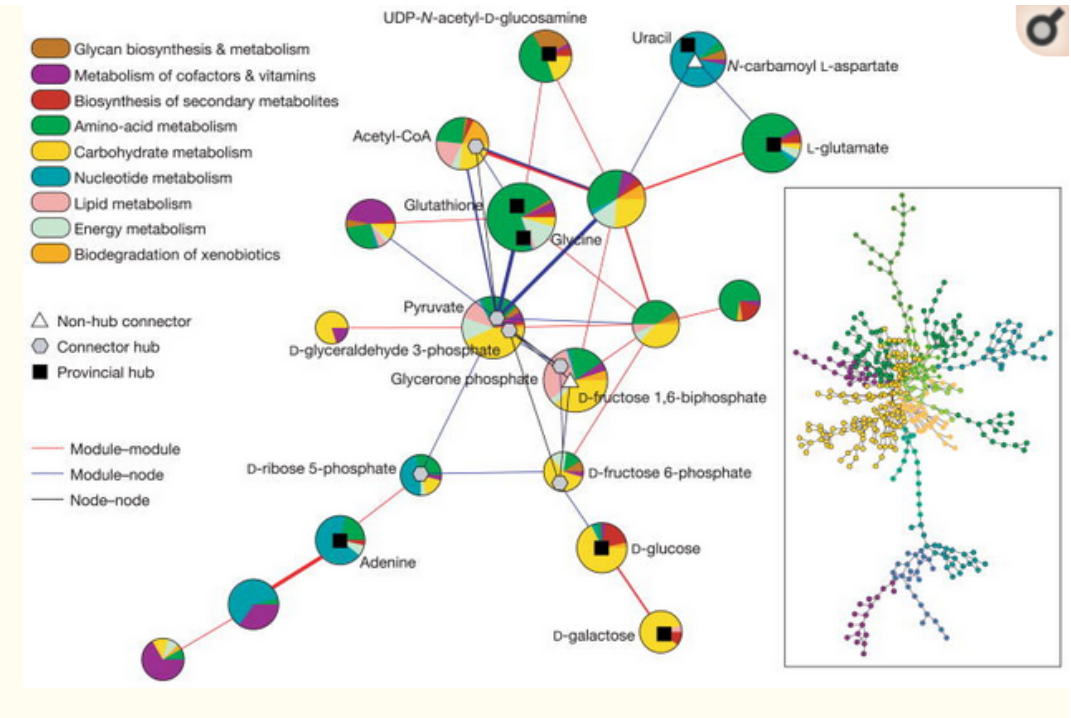
Food-webs



* Northwest Atlantic food-web

Lavigne, D. M. (2003). Marine mammals and fisheries: the role of science in the culling debate. Marine mammals: Fisheries, tourism and management issues, 31-47.

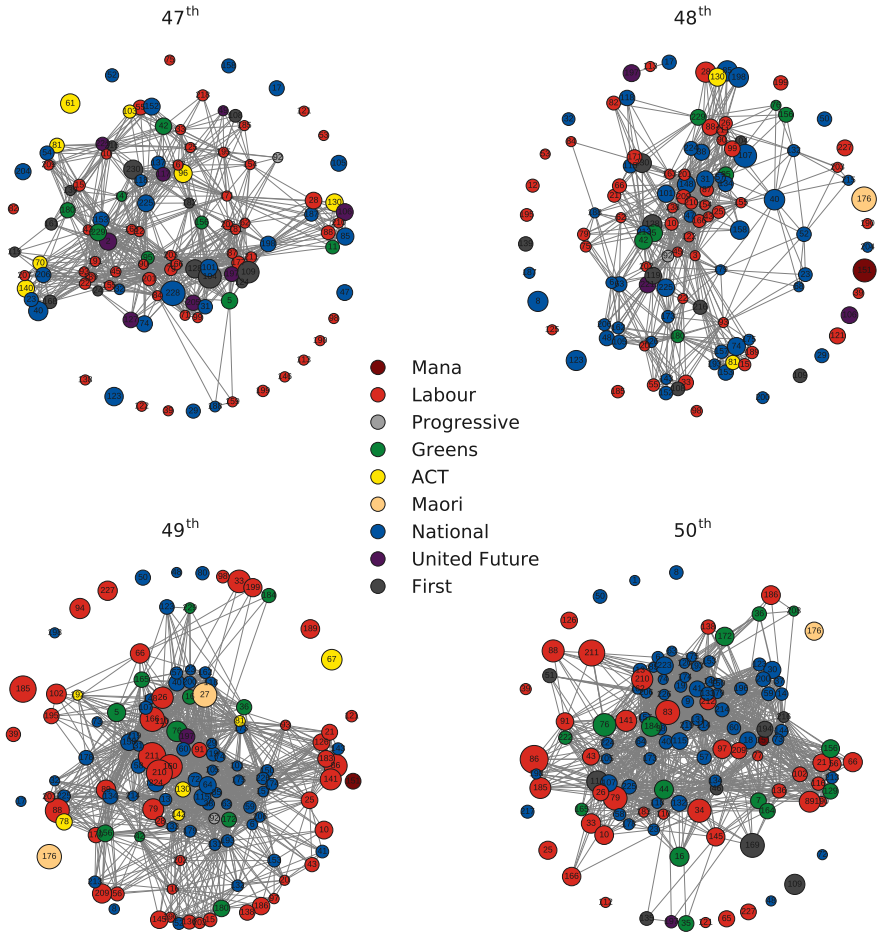
Biomolecular systems



* *E. coli* metabolic network

Guimera, R., & Amaral, L. A. N. (2005). Functional cartography of complex metabolic networks. *Nature*, 433(7028), 895.

Social systems



* Political networks in New Zealand

Curran, B., Higham, K., Ortiz, E., & Vasques Filho, D. (2018). Look who's talking: Two-mode networks as representations of a topic model of New Zealand parliamentary speeches. *PloS One*, 13(6), e0199072.

Defining networks

What are networks?

Mathematical models of interactions/relationships between (many) things.

The representation of the “skeleton” of a system with complex interactions between its parts.

Mathematical definition

A network is a graph $G = \{U, L\}$, where:

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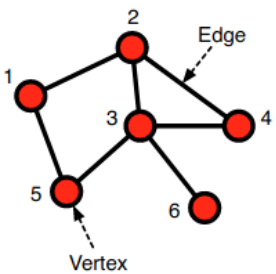
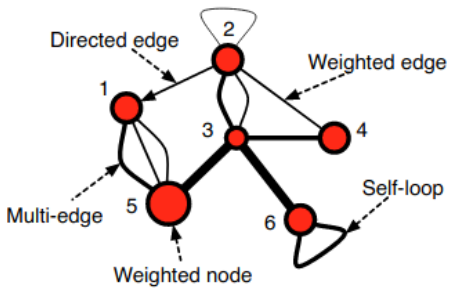
- $L = \{(u, u') : u, u' \in U\}$, $e_{uu'} = (u, u')$

Types of networks

Defined by types of links:

- simple graphs (friendship),
- multigraphs (Wikipedia),
- directed graphs (correspondence),
- weighted graphs (financial transactions),
- signed graphs (alliances and enemies in war)...





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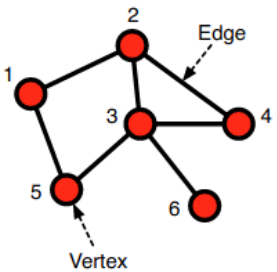
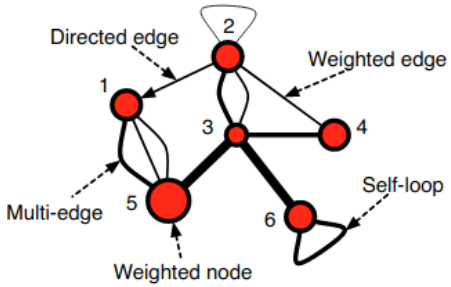
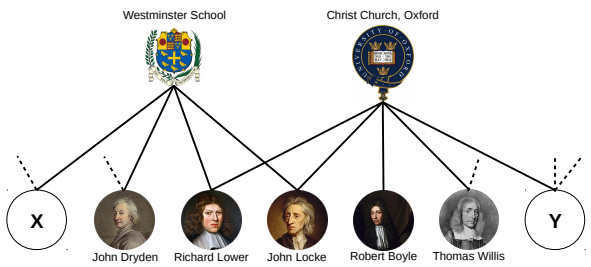


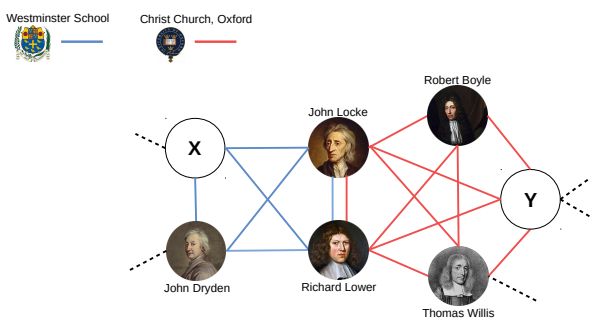


Defined by (number of) types of nodes:

- bipartite networks (co-authorship)

Actors attending, belonging, being affiliated to something (an event, a social unit, a university, and so on.)





Universal (real-world) network properties

Degree distribution and preferential linking

- q_u is the degree of node u
- $2|L| = \sum_u q_u$ (undirected network)
- $\langle q \rangle = \frac{\sum_u q_u}{|U|} = \frac{2|L|}{|U|}$

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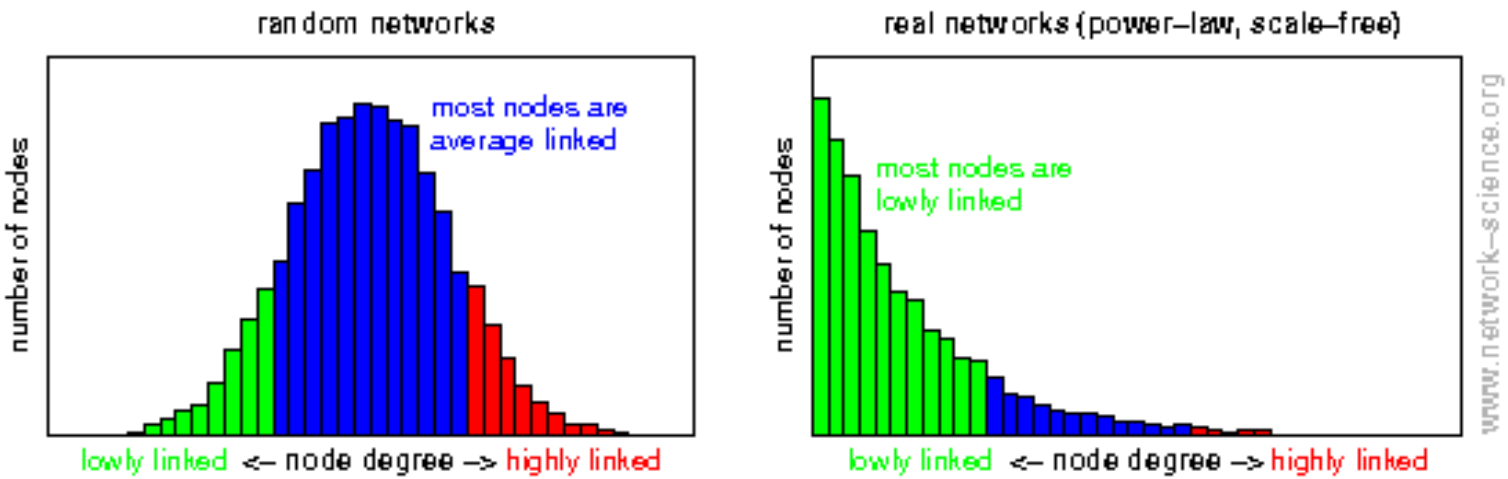

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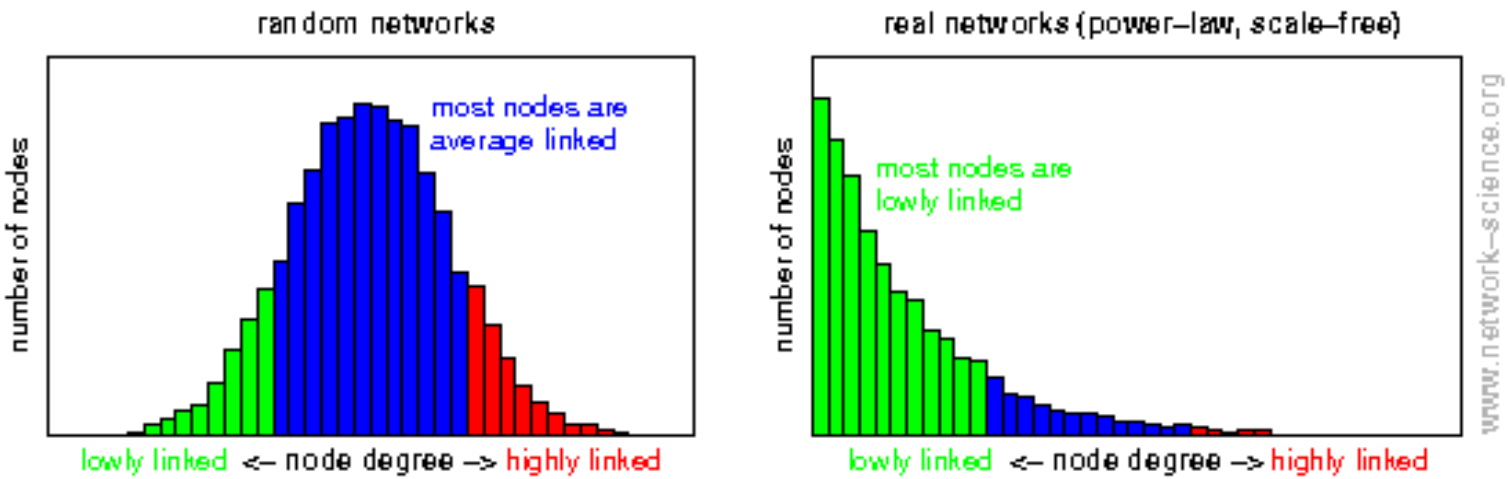

Scale-free networks have degree distributions that closely follow a power law:

$$P(q) \propto q^{-\gamma}$$

Degree distribution and preferential linking

Easy way of visualizing in log scale:
$\ln P(q) \propto \ln q^{-\gamma}$
$\ln P(q) \propto -\gamma \ln + \text{constant}$

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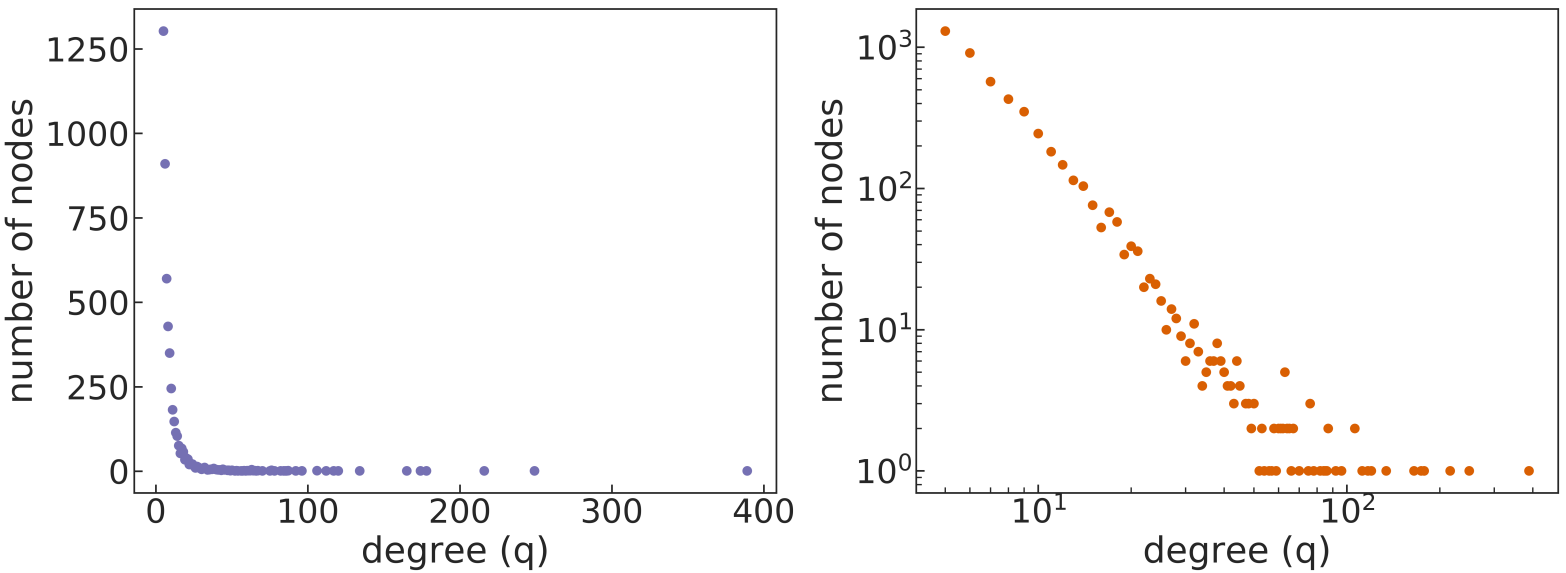
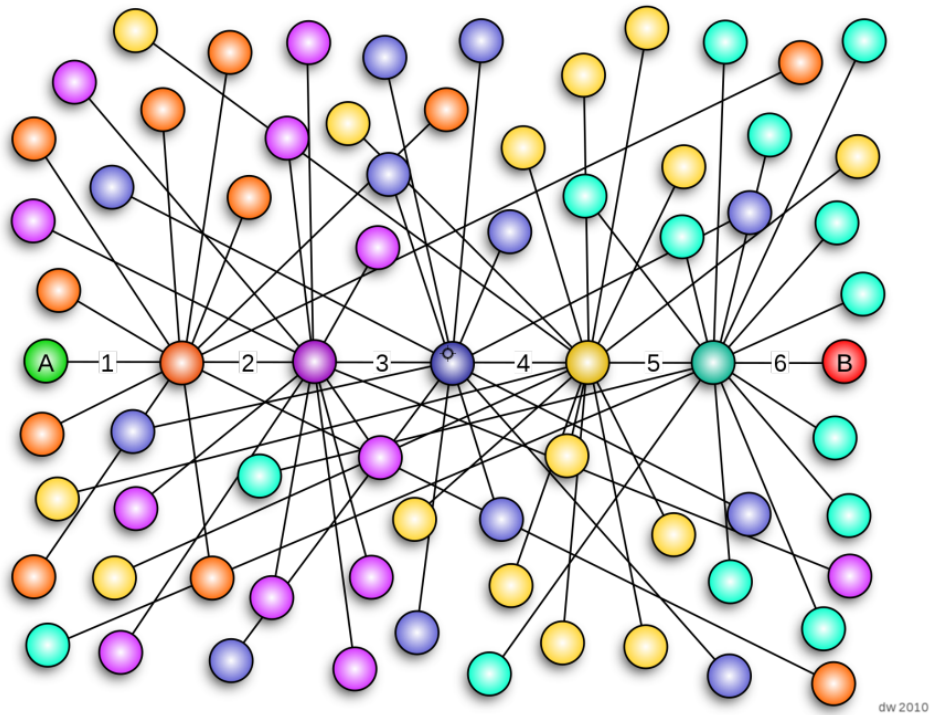


γ is the slope of the line. Scale-free!

Path length and the small world phenomenon

- $l_{uu'}$ is the distance between nodes u and u'
- $\langle l \rangle = \frac{1}{|U|(|U|-1)} \sum_{u \neq u'} l_{uu'}$

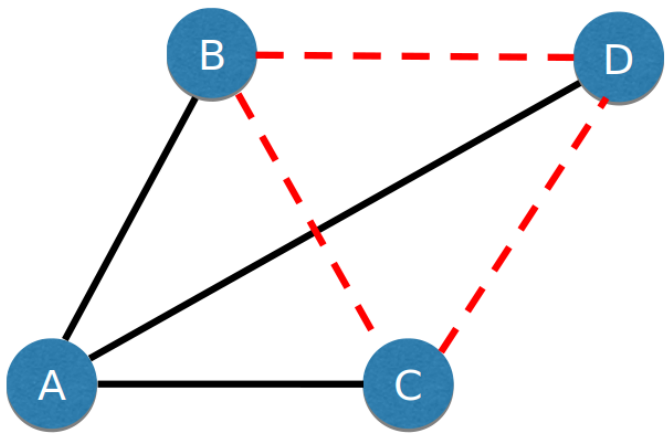


- $\langle l \rangle \propto \log |U|$

*https://en.wikipedia.org/wiki/Small-world_experiment

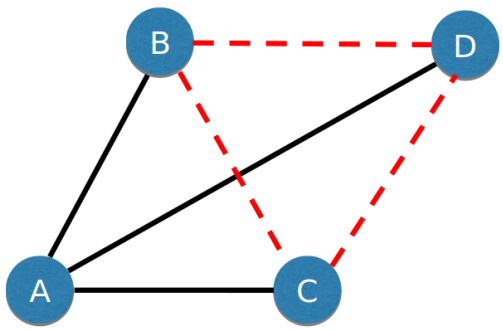
Clustering coefficient and transitivity

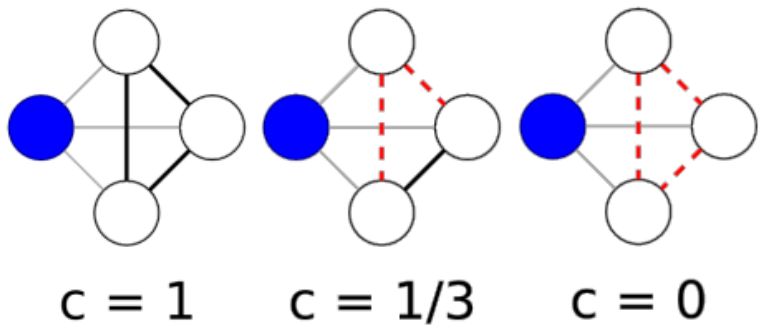
- Transitivity is the formation of triangles (friend of my friend is also my friend).



Clustering coefficient and transitivity

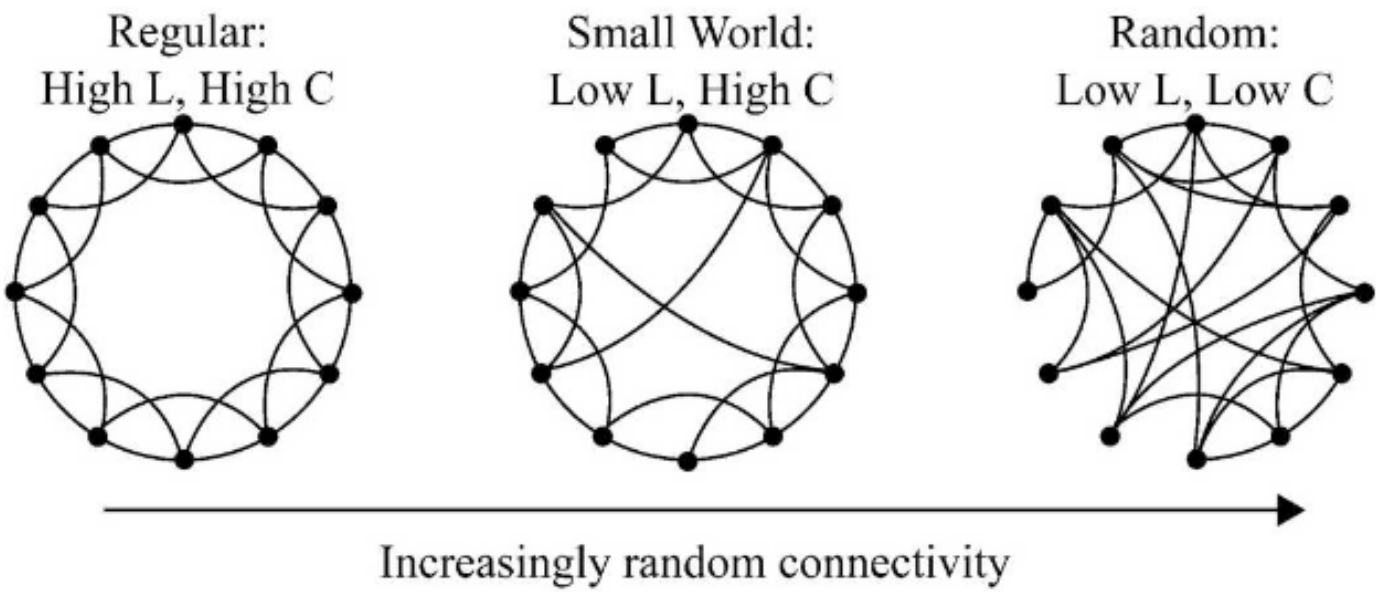
- Transitivity is the formation of triangles (friend of my friend is also my friend).
- How well the neighbors of a node are connected is the clustering coefficient of such node.
- $cc_u = \frac{2|L_u|}{q_u(q_u-1)}$.





*http://tberg.dk/post/clustering-in-complex-networks/

Small-world networks



*Watts, D. J., & Strogatz, S. H. (1998). Collective dynamics of 'small-world' networks. *Nature*, 393(6684), 440.

Other properties

Centrality measures

Geometric:

- closeness centrality (reaching the other nodes),
- betweenness centrality (being the “middle man”)...

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Connectivity:

- eigenvector centrality (influencing),
- Page Rank (being important),
- Katz centrality (more influence)...

Assortativity (homophily)

The tendency that nodes have to connect to similar others with respect to a particular node attribute (property):

- gender, race, age...

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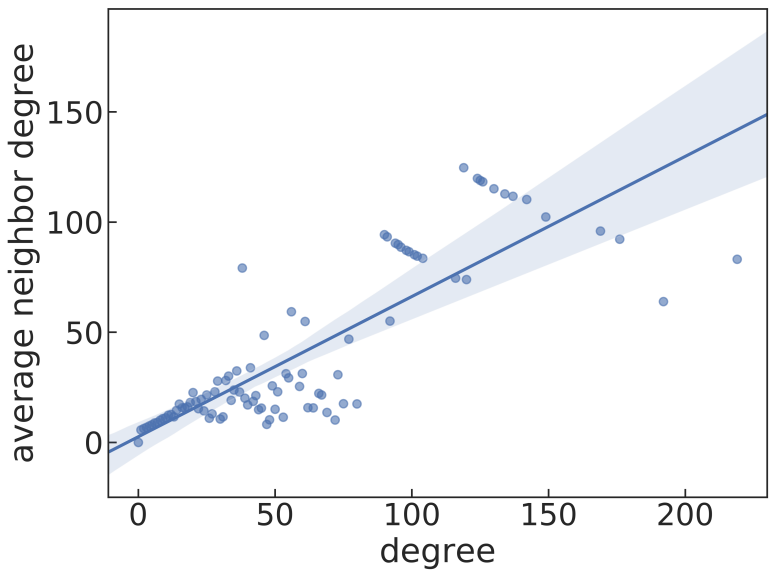
- e.g. degree assortativity (popular connecting to popular)



Community structure

- local-scale, at node level: degree, clustering, centrality, paths...
- global-scale, at network level: degree distribution, transitivity, assortativity...

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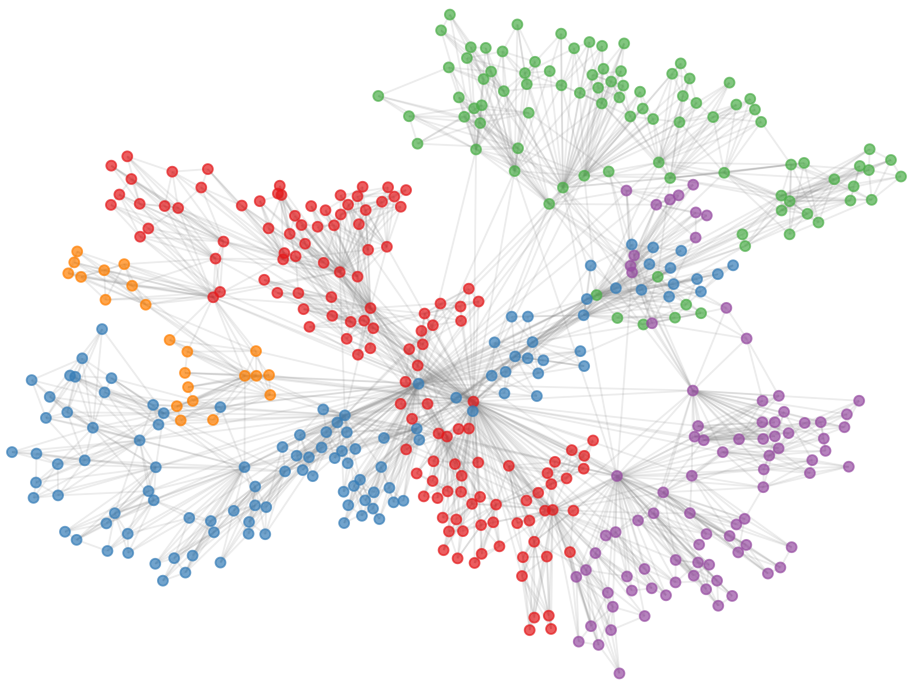
- local-scale, at node level: degree, clustering, centrality, paths...
- global-scale, at network level: degree distribution, transitivity, assortativity...
- meso-scale, between node and network levels: communities.



Danke schön! Fragen?

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