($\rho_m \to \rho_n$ greedy direction)

Lets assume $\rho_m > \rho_n$ then $\Pi_{mn} \rho_n = \alpha_{mn} \rho_m$ is unconditionally accepted

And $\Pi_{nm} \rho_m = \frac{\rho_n}{\rho_m} \alpha_{nm} \rho_m$ → conditionally accept it, where by design ρ_m cancels on the RHS → $\Pi_{nm} \rho_m = \alpha_{nm} \rho_n$

The detail balance conditions means $\sum_n \Pi_{mn} \rho_n = \sum_n \Pi_{nm} \rho_m$ Where the sum is over n → so we can leave out ρ_m, hence

$\sum_n \Pi_{mn} \rho_n = \sum_n \Pi_{nm} \rho_m = \left(\sum_n \Pi_{nm} \right) \rho_m = (1) \rho_m$ which is the normalization condition given that you're in state m

$\Pi \rho = \rho$ meaning $\sum_{n=1}^{N} \Pi_{mn} \rho_n = \rho_m$ using $\Pi_{mn} \rho_n = \Pi_{nm} \rho_m$

for which $\Pi_{mn} = \begin{cases} \alpha_{mn} , & \rho_m \geq \rho_n \quad m \neq n \\ (\rho_m/\rho_n)\alpha_{mn}, & \rho_m < \rho_n \quad m \neq n \\ 1 - \sum_{m \neq n} \Pi_{mn}, & m = n \end{cases}$

then summing over each condition above: (where ρ_m cancels in the 2nd term)

$$\sum_n^N \Pi_{nm} \rho_m = \sum_{\substack{n \neq m \\ \rho_n \geq \rho_m}} \alpha_{nm} \rho_m + \sum_{\substack{n \neq m \\ \rho_n < \rho_m}} \left(\frac{\rho_n}{\rho_m}\right) \alpha_{nm} \cancel{\rho_m} + \sum_{m=n} \left(1 - \sum_{m \neq n} \alpha_{nm}\right) \rho_m$$

simplifies to:

$$= \sum_{\substack{n \neq m \\ \rho_n \geq \rho_m}} \alpha_{nm} \rho_m + \underbrace{\sum_{\substack{n \neq m \\ \rho_n < \rho_m}} \rho_n \alpha_{mn}} + \underbrace{\sum_{m=n} \left(1 - \sum_{m \neq n} \alpha_{nm}\right) \rho_m}_{\rho_m - \sum \alpha_{nm}}$$

Were the 2nd term: $\sum_{\substack{n \neq m \\ \rho_n < \rho_m}} \rho_m \alpha_{nm} \ (n \leftrightarrow m)$

And the third term cancels with first

hence: $= \rho_m$