Scaling Analysis of a Hierarchical Parallelization of Large Inverse Multiple-Scattering Solutions

Mert Hidayetoğlu, Carl Pearson, Izzat El Hajj, Weng Cho Chew, Levent Gürel, and Wen-Mei Hwu

Department of Electrical and Computer Engineering

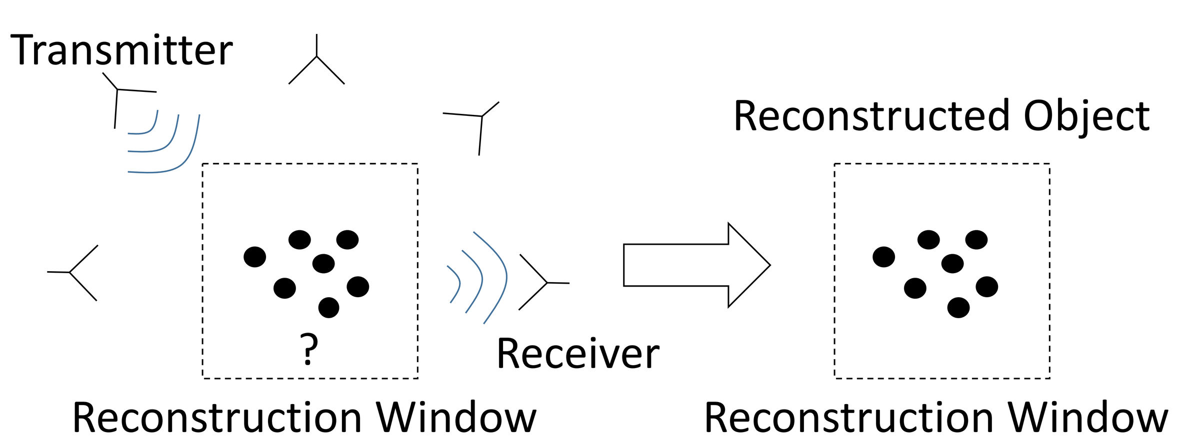
University of Illinois at Urbana-Champaign

ILLINOIS 150 1867-2017

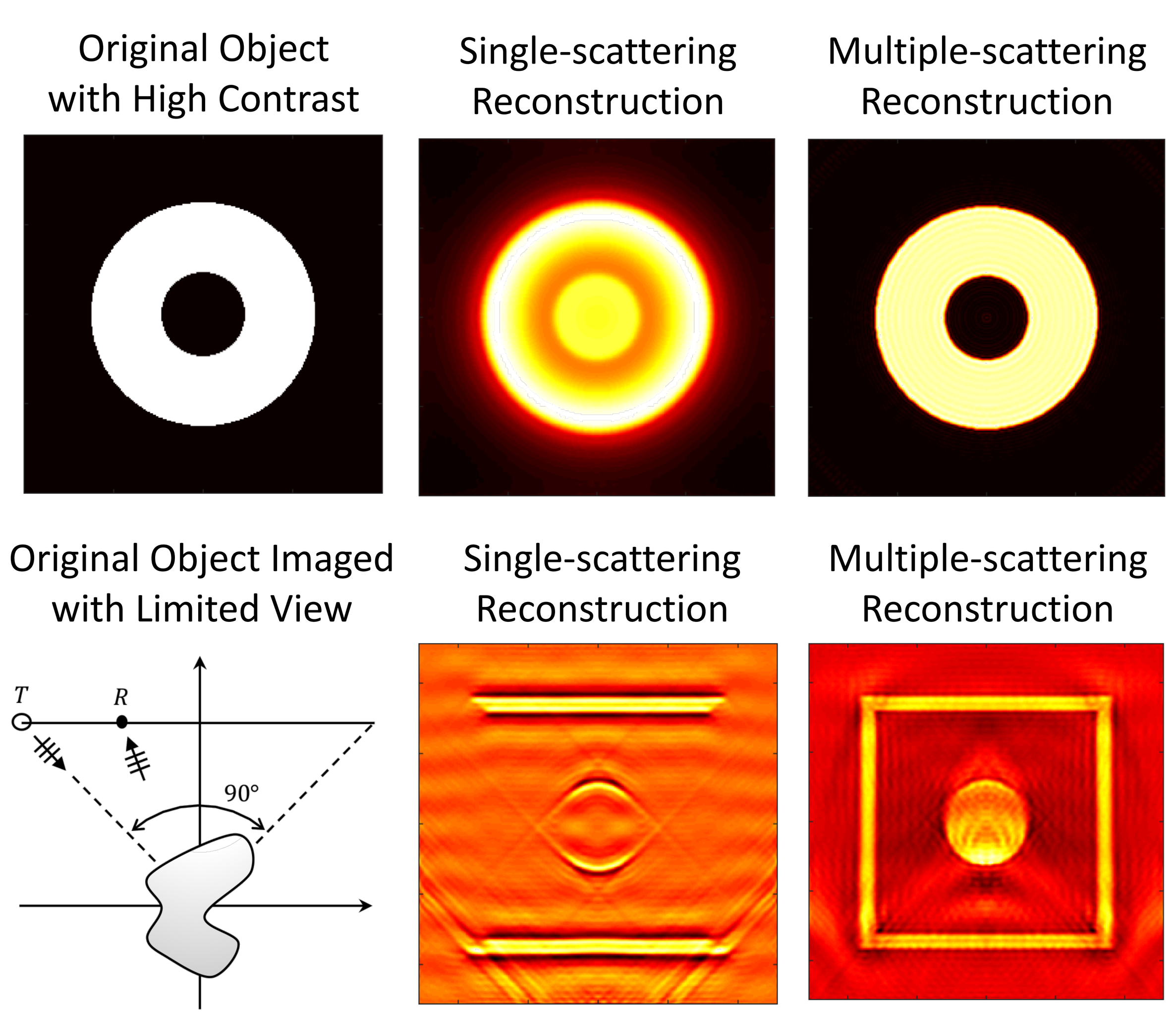
ECE ILLINOIS Department of Electrical and Computer Engineering

BLUE WATERS

Inverse-Scattering Problems



Multiple-scattering reconstructions does not make any fundamental approximation.



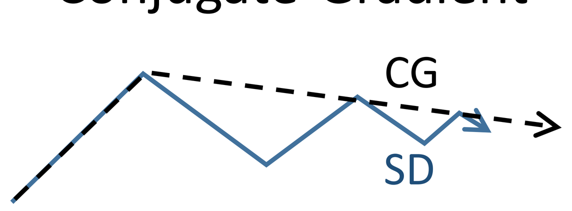
Nonlinear Optimization

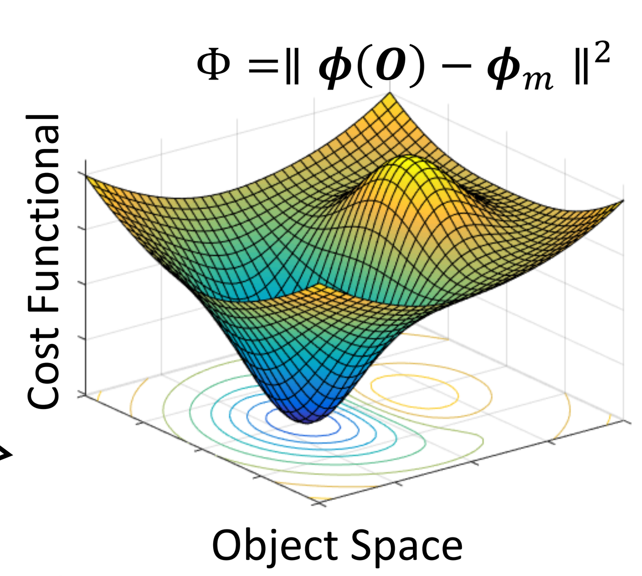
There is not a single way to do this, however, we found out nonlinear conjugate-gradient method is efficient for this algorithm.

- Gradient-Descent

$$-\nabla\Phi = -\boldsymbol{F}^H\boldsymbol{b}$$

- Conjugate-Gradient



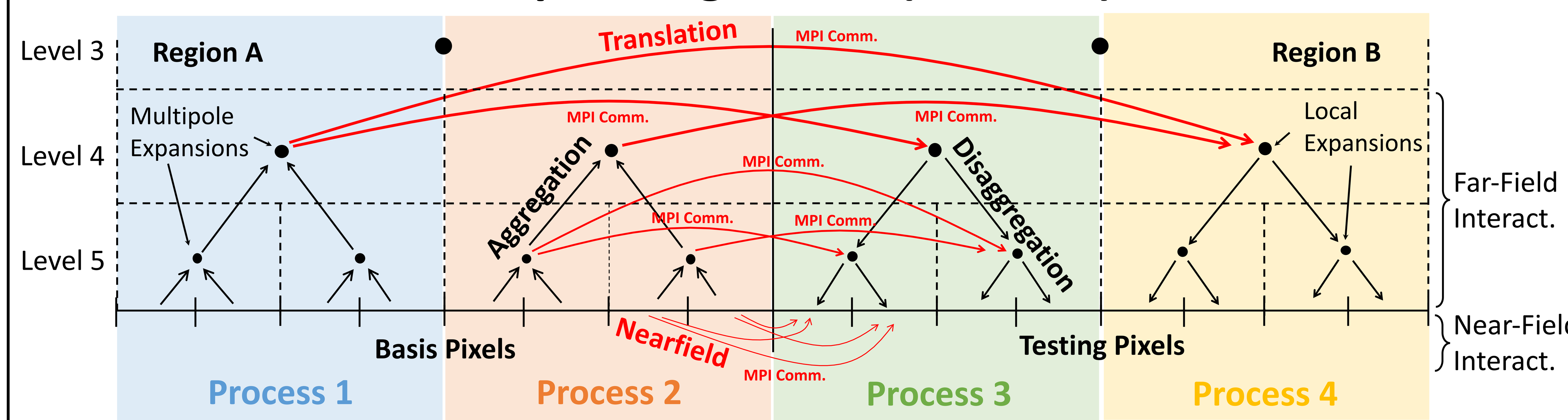


- Newton-Type Methods

$$[\mu^2\boldsymbol{I} + \boldsymbol{F}^H\boldsymbol{F}]\delta\boldsymbol{O} = \nabla\Phi$$

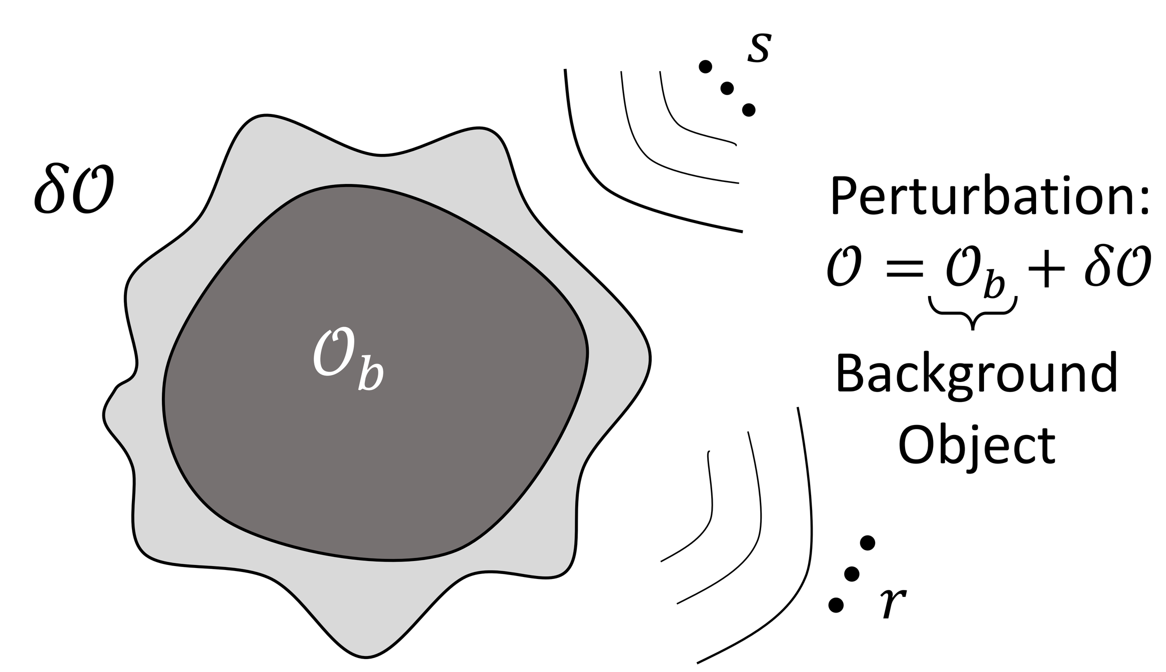
$\boldsymbol{F}$: Functional Derivative Operator

Multilevel Fast Multipole Algorithm (MLFMA) Schematic



MLFMA Provides fast solutions of forward-scattering problems with O(N) computational complexity.

Distorted-Born Approximation



Perturbation: $\mathcal{O} = \underbrace{\mathcal{O}_b}_{\text{Background Object}} + \delta\mathcal{O}$

Scattering Equation: $\phi = \phi_0 + \mathcal{G}_0\mathcal{O}\phi$

Variational Equation: $\delta\phi = \mathcal{G}_b\delta\mathcal{O}\phi_b + \underbrace{\mathcal{G}_b\delta\mathcal{O}\delta\phi}_{\text{Higher-order Variations (neglected)}}$

Distorted-Born approximation provides a semi-analytical way to find functional derivatives.

$$\delta\phi \approx \delta\phi^{(1)} = \mathcal{G}_b\delta\mathcal{O}\phi_b = \mathcal{G}_b\phi_b\delta\mathcal{O}$$

$$\delta\phi^{(1)} = \mathcal{F}\delta\mathcal{O}$$

$$\boldsymbol{F} = \boldsymbol{G}_R\{\boldsymbol{I} + \boldsymbol{O}_b\overbrace{[\boldsymbol{I} - \boldsymbol{G}_0\boldsymbol{O}_b]^{-1}}^{\text{MLFMA}}\boldsymbol{G}_0\}\,\mathrm{diag}\{\underbrace{[\boldsymbol{I} - \boldsymbol{G}_0\boldsymbol{O}_b]^{-1}}_{\text{MLFMA}}\boldsymbol{G}_T\boldsymbol{s}\}$$

Discretization with a subspace projection method.

$\overline{\boldsymbol{G}}_0$: Dense, $N\times N$

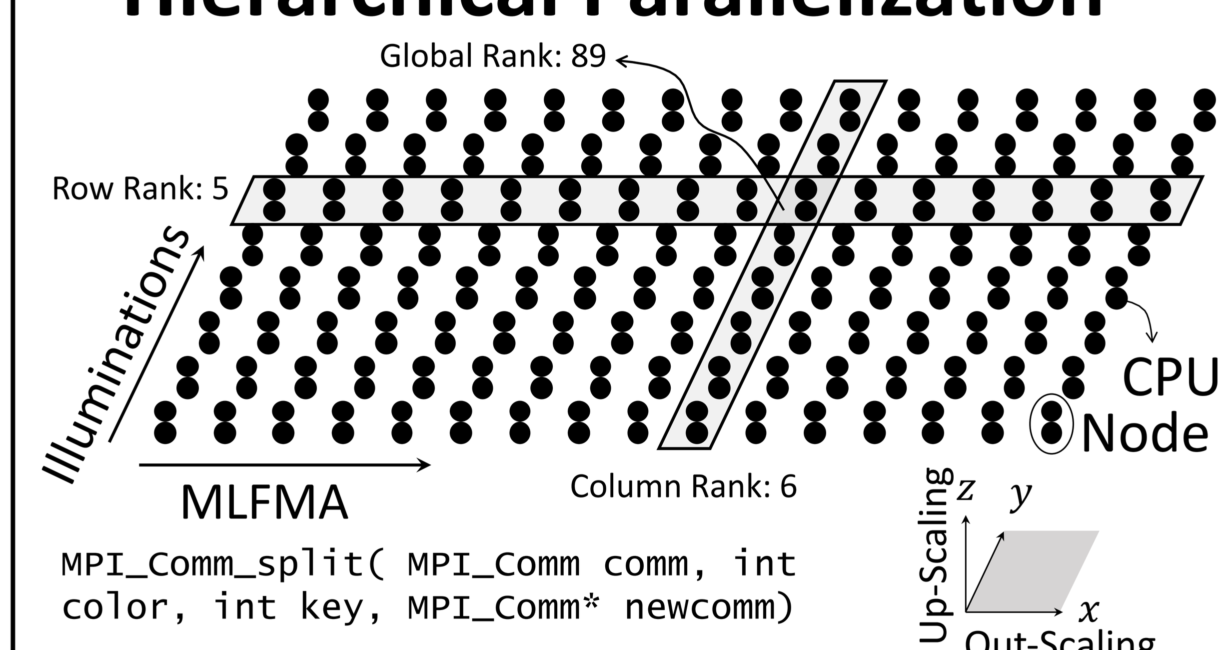
$\overline{\boldsymbol{G}}_T$: Dense, $N\times T$

$\overline{\boldsymbol{G}}_R$: Dense, $R\times N$

$\overline{\boldsymbol{O}}_b$: Diagonal, $N\times N$

$\overline{\boldsymbol{I}}$: Diagonal, $N\times N$

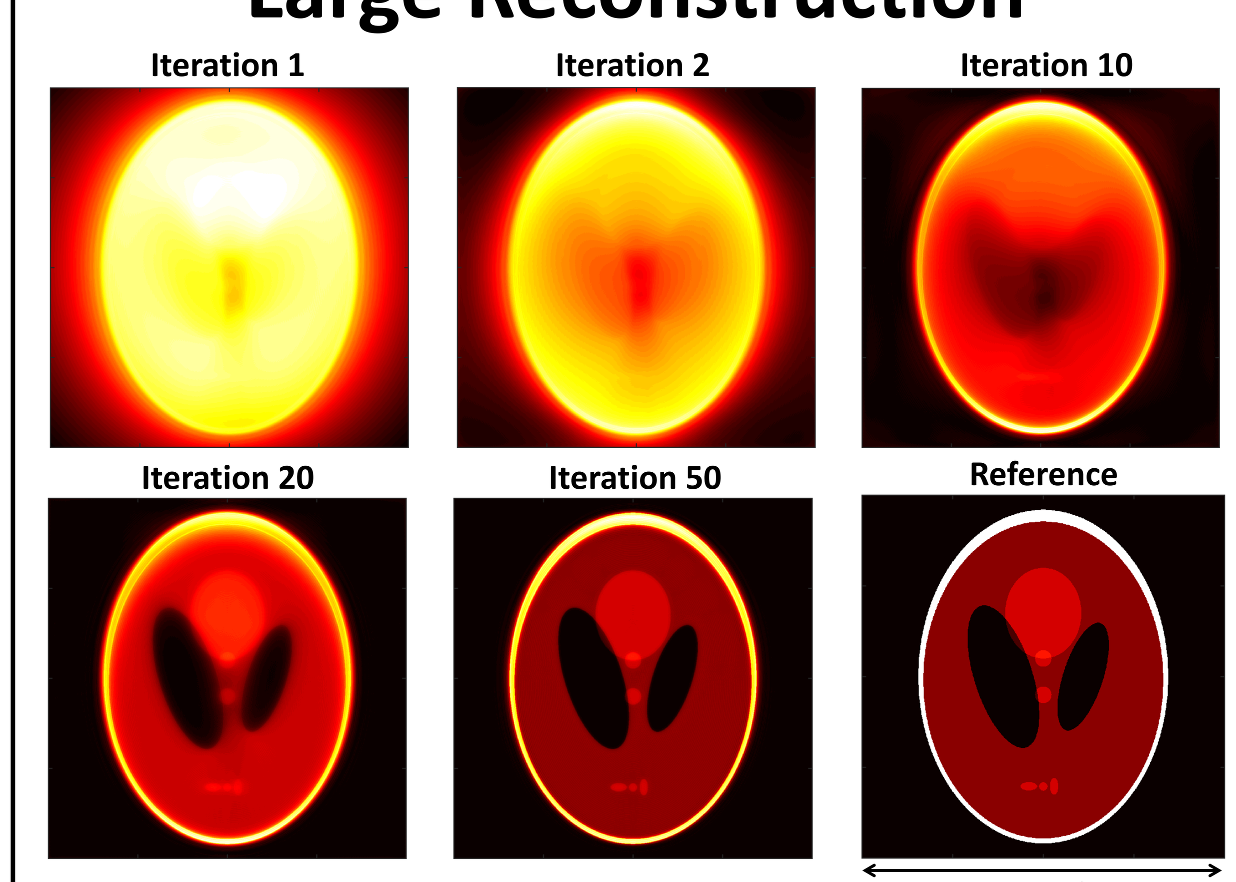
Hierarchical Parallelization



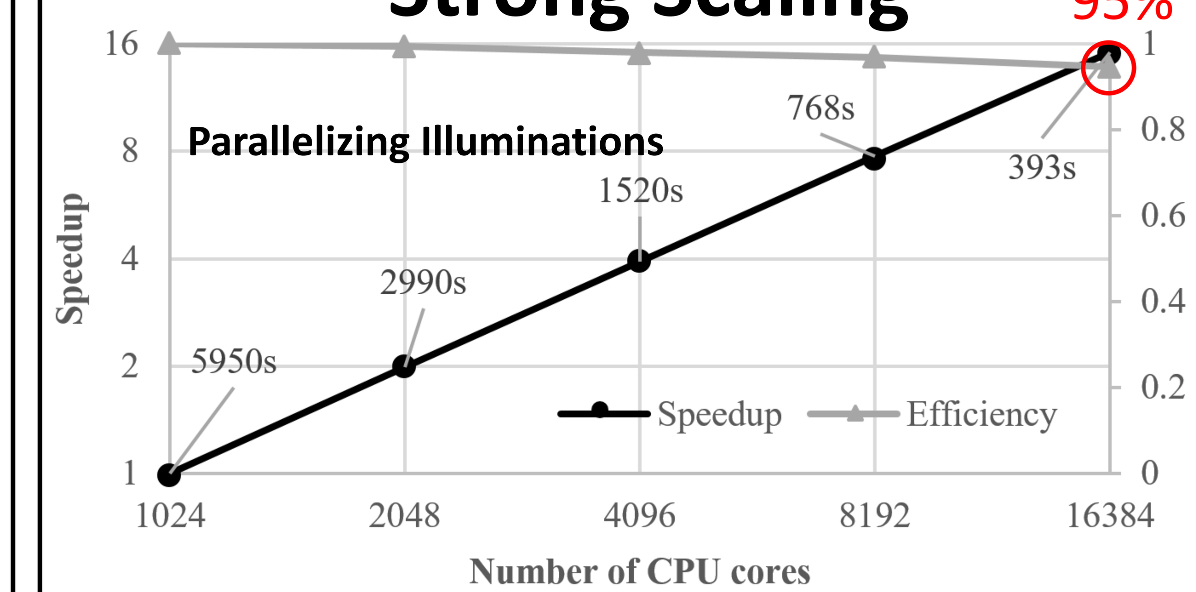

```
MPI_Comm_split( MPI_Comm comm, int
color, int key, MPI_Comm* newcomm)
```

- **Hierarchical parallelization provides high granularity for out-scaling.**
- **Each MPI process summons a set of shared-memory OpenMP threads to utilize the multi-core CPUs for up-scaling.**

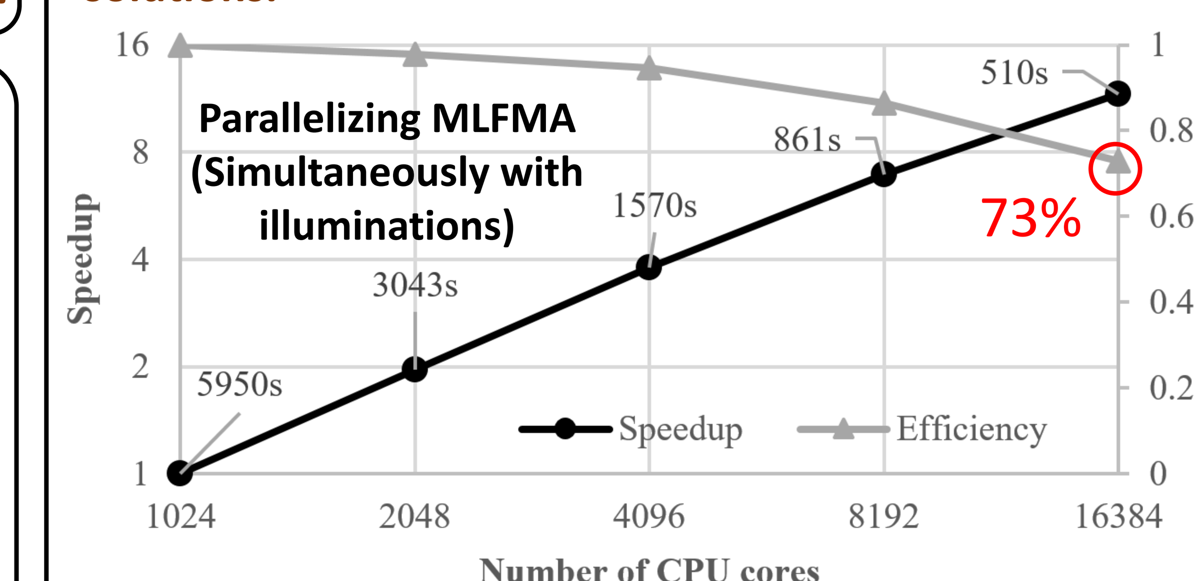
Large Reconstruction



Strong Scaling



Parallelizing illuminations is almost perfectly efficient due to the independent nature of forward solutions.



MLFMA scaling is not perfect due to the MPI communications in each matrix-vector multiplication.

Conclusions

- A hierarchical parallelization strategy to improve the scalability of inverse multiple-scattering solutions is proposed.
- An inverse problem involving a large Shepp-Logan phantom is solved on up to 1,024 CPU nodes of the Blue Waters supercomputer in order to demonstrate the strong-scaling efficiency of the proposed parallelization scheme.

See More Results & Animations:

Future Plans

- **GPU nodes will be employed for massively-parallel solutions (done & submitted: 4k GPUs. Approx. 4x speedup over CPUs).**
- **Real-life problems will be solved with real measurement data. Not trivial because of noise, calibration, etc.**