system_identification

November 11, 2025

1 Checking recorded data

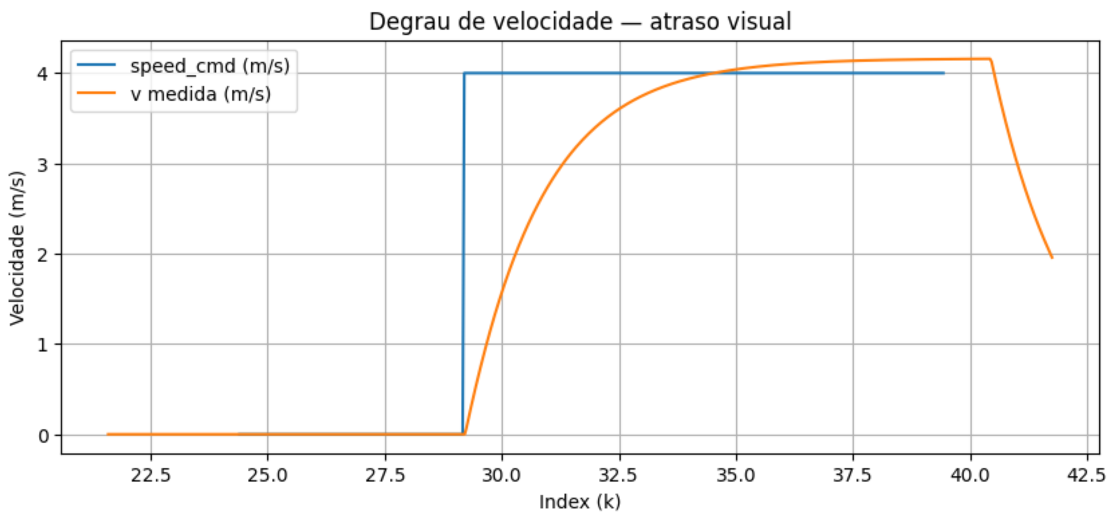
[ ]:
```python
import pandas as pd
import matplotlib.pyplot as plt
import numpy as np

# pd.set_option('display.max_rows', None)

HZ         = 30.0
Ts         = 1.0 / HZ


# === Data files path ===
cmd_speed = "data/speed_only_30hz/speed_step_30hz_cmd_raw.csv"
odom_speed = "data/speed_only_30hz/speed_step_30hz_odom_raw.csv"
cmd_steer = "data/final/final_modelling_cmd_raw.csv"
odom_steer = "data/final/final_modelling_odom_raw.csv"


cmd_s = pd.read_csv(cmd_speed)
odom_s = pd.read_csv(odom_speed)
cmd_d = pd.read_csv(cmd_steer)
odom_d = pd.read_csv(odom_steer)
odom_d.yaw = np.unwrap(odom_d.yaw)
print("CMD ORIGINAL SPEED ONLY")
display(cmd_s)
print("OUTPUT ORIGINAL SPEED ONLY")
display(odom_s)
print("CMD ORIGINAL SPEED STEER")
display(cmd_s)
print("OUTPUT ORIGINAL SPEED STEER")
display(odom_s)

# === SPEED: entrada vs saída ===
plt.figure(figsize=(10,4))
plt.plot(cmd_s.t, cmd_s.speed, label="speed_cmd (m/s)")
plt.plot(odom_s.t, odom_s.v, label="v medida (m/s)")
plt.title("Degrau de velocidade - atraso visual")
plt.xlabel("Index (k)")
plt.ylabel("Velocidade (m/s)")
plt.legend()
plt.grid(True)
plt.show()

# === STEERING: entrada vs yaw ===
plt.figure(figsize=(10,4))
plt.plot(cmd_d.t, cmd_d.steering_angle, label="steering_cmd (rad)")
plt.plot(odom_d.t, odom_d.yaw, label="yaw unwrap (rad)")
plt.title("Degrau de direção - atraso visual")
plt.xlabel("Index (k)")
plt.ylabel("Ângulo (rad)")
plt.legend()
plt.grid(True)
plt.show()

# === STEERING: entrada vs yaw_rate ===
plt.figure(figsize=(10,4))
plt.plot(cmd_d.t, cmd_d.steering_angle, label="steering_cmd (rad)")
plt.plot(odom_d.t, odom_d.yaw_rate, label="yaw_rate (rad/s)")
plt.title("Direction step - yaw_rate")
plt.xlabel("Index (k)")
plt.ylabel("Delta / rate")
plt.legend()
plt.grid(True)
plt.show()
```

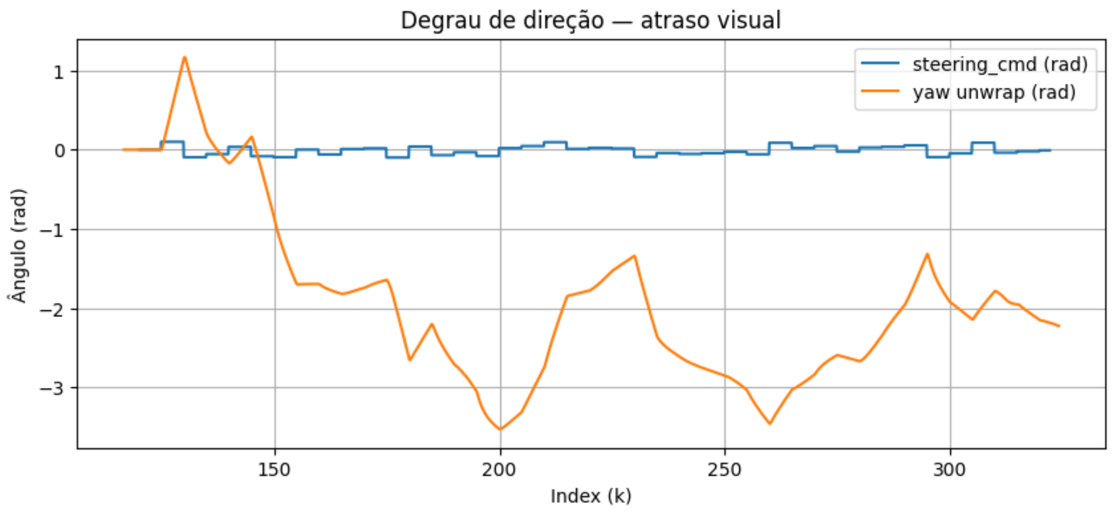
```
CMD ORIGINAL SPEED ONLY

          t  steering_angle  speed
0    24.398             0.0    0.0
1    24.431             0.0    0.0
2    24.464             0.0    0.0
3    24.498             0.0    0.0
4    24.531             0.0    0.0
..      …             …    …
447  39.298             0.0    4.0
448  39.331             0.0    4.0
449  39.364             0.0    4.0
450  39.398             0.0    4.0
451  39.431             0.0    4.0

[452 rows x 3 columns]

OUTPUT ORIGINAL SPEED ONLY

          t          x         y       yaw          v         vy       yaw_rate
```

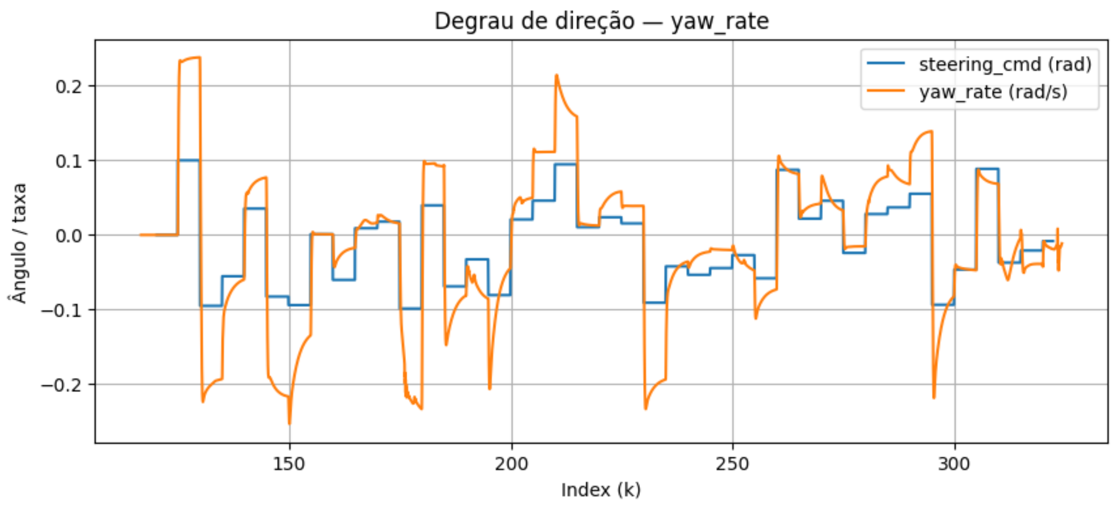
```
0    21.592  -0.016144  0.000007 -0.000004  0.000223 -0.000013  7.216536e-07
1    21.626  -0.016170  0.000007 -0.000004  0.000223 -0.000013  7.255404e-07
2    21.660  -0.016197  0.000007 -0.000004  0.000224 -0.000013  7.256423e-07
3    21.694  -0.016224  0.000007 -0.000004  0.000224 -0.000013  7.216195e-07
4    21.728  -0.016251  0.000007 -0.000004  0.000224 -0.000013  7.217170e-07
..      …         …         …         …         …         …            …
589  41.618  43.416241 -0.000542 -0.000012  2.116369 -0.000039  7.564961e-07
590  41.652  43.487470 -0.000543 -0.000012  2.075799 -0.000038  7.535799e-07
591  41.686  43.557332 -0.000544 -0.000012  2.035918 -0.000038  7.540939e-07
592  41.720  43.625850 -0.000545 -0.000012  1.996716 -0.000037  7.498295e-07
593  41.754  43.693046 -0.000546 -0.000012  1.958181 -0.000037  7.517875e-07

[594 rows x 7 columns]
```

```
CMD ORIGINAL SPEED STEER

          t  steering_angle  speed
0    24.398             0.0    0.0
1    24.431             0.0    0.0
2    24.464             0.0    0.0
3    24.498             0.0    0.0
4    24.531             0.0    0.0
..      …             …    …
447  39.298             0.0    4.0
448  39.331             0.0    4.0
449  39.364             0.0    4.0
450  39.398             0.0    4.0
451  39.431             0.0    4.0

[452 rows x 3 columns]
```

```
OUTPUT ORIGINAL SPEED STEER

          t          x         y       yaw         v        vy      yaw_rate
0    21.592  -0.016144  0.000007 -0.000004  0.000223 -0.000013  7.216536e-07
1    21.626  -0.016170  0.000007 -0.000004  0.000223 -0.000013  7.255404e-07
2    21.660  -0.016197  0.000007 -0.000004  0.000224 -0.000013  7.256423e-07
3    21.694  -0.016224  0.000007 -0.000004  0.000224 -0.000013  7.216195e-07
4    21.728  -0.016251  0.000007 -0.000004  0.000224 -0.000013  7.217170e-07
..      …         …         …         …         …         …            …
589  41.618  43.416241 -0.000542 -0.000012  2.116369 -0.000039  7.564961e-07
590  41.652  43.487470 -0.000543 -0.000012  2.075799 -0.000038  7.535799e-07
591  41.686  43.557332 -0.000544 -0.000012  2.035918 -0.000038  7.540939e-07
592  41.720  43.625850 -0.000545 -0.000012  1.996716 -0.000037  7.498295e-07
593  41.754  43.693046 -0.000546 -0.000012  1.958181 -0.000037  7.517875e-07

[594 rows x 7 columns]
```


Degrau de velocidade — atraso visual
speed_cmd (m/s)
v medida (m/s)
Velocidade (m/s)
Index (k)
0
1
2
3
4
22.5
25.0
27.5
30.0
32.5
35.0
37.5
40.0
42.5


Degrau de direção — atraso visual
steering_cmd (rad)
yaw unwrap (rad)
Ângulo (rad)
Index (k)
1
0
−1
−2
−3
150
200
250
300



```
[ ]: # ===================== Utils ==========================
     def common_interval(t1, t2):
         return max(np.min(t1), np.min(t2)), min(np.max(t1), np.max(t2))

     def resample_to_grid(t_src, data_dict, t_grid, unwrap_keys=None):
         unwrap_keys = set(unwrap_keys or [])
         t_src = np.asarray(t_src)
         order = np.argsort(t_src)
         t_sorted = t_src[order]

         out = {}
         for k, v in data_dict.items():
             v = np.asarray(v)[order]
             if k in unwrap_keys:
                 v = np.unwrap(v)
             out[k] = np.interp(t_grid, t_sorted, v)

         # adiciona número da amostra e tempo como colunas explícitas
         out_df = pd.DataFrame(out)
         out_df.insert(0, "t", t_grid)
         out_df.insert(0, "k", np.arange(len(t_grid), dtype=int))
         return out_df

     def finite_mask(*arrays):
         M = np.column_stack([a if a.ndim>1 else a.reshape(-1,1) for a in arrays])
         return np.isfinite(M).all(axis=1)

     # ================== SPEED-ONLY (v <- speed_cmd) ==================
     t0_s, tf_s = common_interval(cmd_s['t'].values, odom_s['t'].values)
     tgrid_s = np.arange(t0_s, tf_s, Ts)
```

```
cmd_s_rs = resample_to_grid(
    cmd_s['t'].values,
    {
        'speed':    cmd_s['speed'].values,
        'steering_angle': cmd_s['steering_angle'].values,  # deve ser␣
 ↪~constante neste teste
    },
    tgrid_s
)

odom_s_rs = resample_to_grid(
    odom_s['t'].values,
    {
        'v':        odom_s['v'].values,
        'yaw':      odom_s['yaw'].values,
        'yaw_rate': odom_s['yaw_rate'].values,
        'x':        odom_s['x'].values,
        'y':        odom_s['y'].values,
    },
    tgrid_s
)

# speed_df_rs = pd.DataFrame({'t': tgrid_s, **cmd_s_rs, **odom_s_rs})
# Aplica atraso nk na ENTRADA (speed_cmd)
# speed_df_rs['speed_cmd_lag'] = speed_df_rs['speed_cmd'].shift(NK_SPEED).
 ↪bfill()



# steering angle corresponds to the yaw of a virtual wheel located at the
# center of the front axle, like on a tricycle.  Positive yaw is to
# the left. (This is *not* the angle of the steering wheel inside the
# passenger compartment.)
# === Junta entradas e saídas ===
# yae eh o angulo do robo em relacao ao mundo
speed_df = pd.DataFrame({
    "t": cmd_s_rs["t"],
    "speed_cmd": cmd_s_rs["speed"],
    "steering_cmd": cmd_s_rs["steering_angle"],
    "v_measured": odom_s_rs["v"],
    "yaw":      odom_s_rs["yaw"],
    "yaw_rate": odom_s_rs["yaw_rate"],
    "x"       : odom_s_rs["x"],
    "y"       : odom_s_rs["y"]
})
print("Speed Only Dataframe")
```

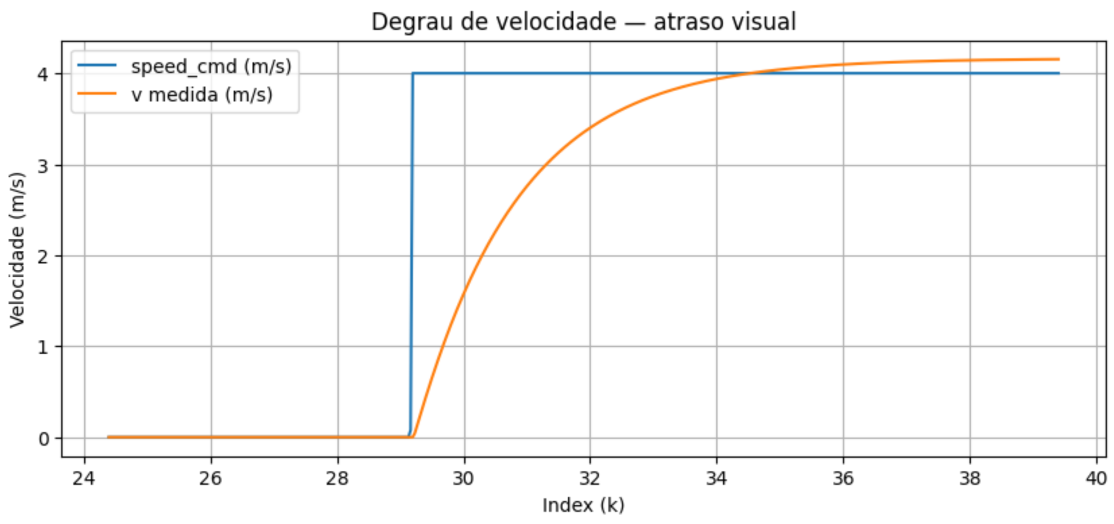
```
display(speed_df)

# ================== SPEED STEER  ==================
t0_d, tf_d = common_interval(cmd_d['t'].values, odom_d['t'].values)
tgrid_d = np.arange(t0_d, tf_d, Ts)

cmd_d_rs = resample_to_grid(
    cmd_d['t'].values,
    {
        'speed':    cmd_d['speed'].values,
        'steering_angle': cmd_d['steering_angle'].values,  # deve ser␣
 ↪~constante neste teste
    },
    tgrid_d
)

odom_d_rs = resample_to_grid(
    odom_d['t'].values,
    {
        'v':        odom_d['v'].values,
        'yaw':      odom_d['yaw'].values,
        'yaw_rate': odom_d['yaw_rate'].values,
        'x':        odom_d['x'].values,
        'y':        odom_d['y'].values,
    },
    tgrid_d
)


steer_df = pd.DataFrame({
    "t": cmd_d_rs["t"],
    "speed_cmd": cmd_d_rs["speed"],
    "steering_cmd": cmd_d_rs["steering_angle"],
    "v_measured": odom_d_rs["v"],
    "yaw": odom_d_rs["yaw"],
    "yaw_rate": odom_d_rs["yaw_rate"],
    "x"       : odom_d_rs["x"],
    "y"       : odom_d_rs["y"]
})

print("Speed Steer Dataframe")
display(steer_df)
```

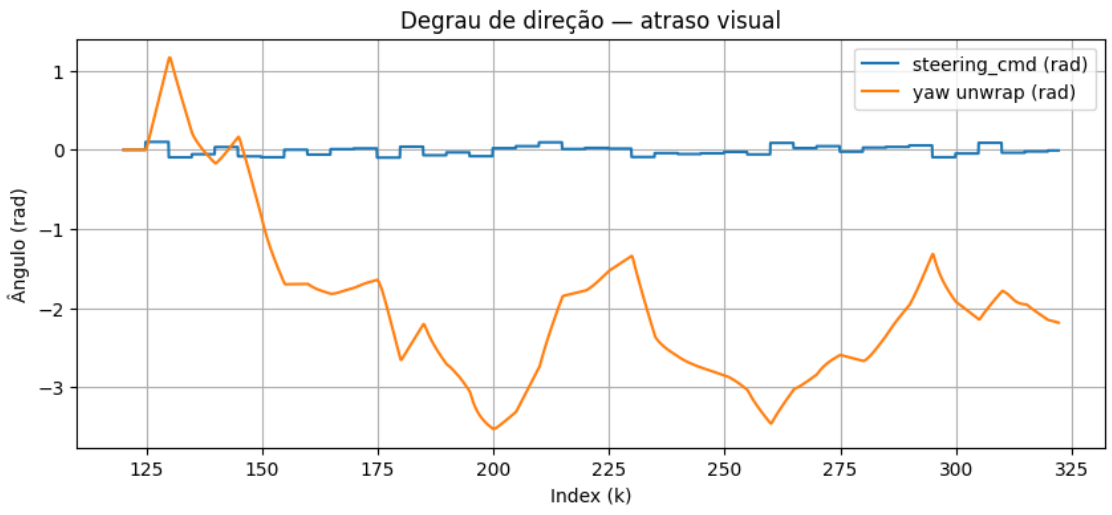
```
Speed Only Dataframe

             t  speed_cmd  steering_cmd  v_measured       yaw      yaw_rate  \
0    24.398000        0.0           0.0    0.000229 -0.000004  7.230685e-07
1    24.431333        0.0           0.0    0.000229 -0.000004  7.243827e-07
```

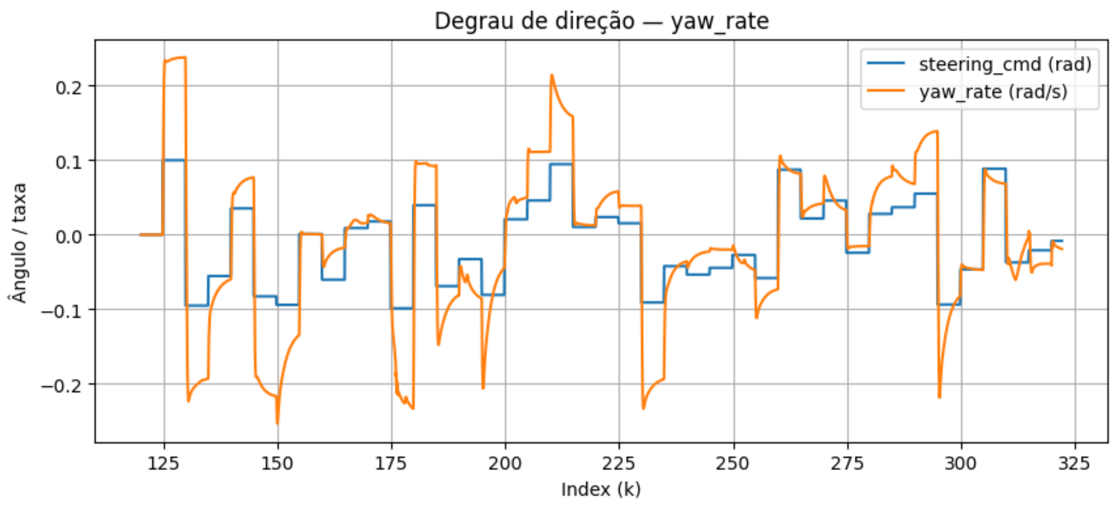
```
2    24.464667         0.0           0.0    0.000229 -0.000004  7.224758e-07
3    24.498000         0.0           0.0    0.000229 -0.000004  7.204708e-07
4    24.531333         0.0           0.0    0.000229 -0.000004  7.222719e-07
..         …        …           …         …        …           …
446  39.264667         4.0           0.0    4.153525 -0.000014  7.354200e-07
447  39.298000         4.0           0.0    4.153706 -0.000014  6.976666e-07
448  39.331333         4.0           0.0    4.153883 -0.000014  7.024131e-07
449  39.364667         4.0           0.0    4.154057 -0.000014  7.433632e-07
450  39.398000         4.0           0.0    4.154227 -0.000014  7.210465e-07

             x         y
0    -0.018350  0.000008
1    -0.018376  0.000008
2    -0.018402  0.000008
3    -0.018428  0.000008
4    -0.018454  0.000008
..         …        …
446  34.970436 -0.000429
447  35.108856 -0.000431
448  35.247282 -0.000433
449  35.385714 -0.000435
450  35.524152 -0.000437

[451 rows x 8 columns]

Speed Steer Dataframe

               t  speed_cmd  steering_cmd  v_measured       yaw      yaw_rate  \
0     120.126000   4.000000       0.00000    0.455235 -0.000021  5.883358e-07
1     120.159333   4.000000       0.00000    0.528374 -0.000021  4.991829e-07
2     120.192667   4.000000       0.00000    0.600271 -0.000021  4.845740e-07
3     120.226000   4.000000       0.00000    0.670947 -0.000021  4.368874e-07
4     120.259333   4.000000       0.00000    0.740421 -0.000021  4.045348e-07
…           …        …           …         …        …           …
6057  322.026000   4.615464      -0.00823   -2.542170 -2.183364 -1.878596e-02
6058  322.059333   4.615464      -0.00823   -2.547595 -2.183970 -1.885668e-02
6059  322.092667   4.615464      -0.00823   -2.552968 -2.184579 -1.893010e-02
6060  322.126000   4.615464      -0.00823   -2.558279 -2.185189 -1.900005e-02
6061  322.159333   4.615464      -0.00823   -2.563569 -2.185803 -1.905187e-02

               x           y
0      -0.046719    0.000040
1      -0.030338    0.000039
2      -0.011540    0.000039
3       0.009633    0.000039
4       0.033140    0.000038
…           …          …
6057 -158.590005 -284.474192
6058 -158.674814 -284.594798
```

```
6059 -158.759804 -284.715505
6060 -158.844971 -284.836308
6061 -158.930315 -284.957204

[6062 rows x 8 columns]
```

[ ]:
```
plt.figure(figsize=(10,4))
plt.plot(cmd_s_rs.t, cmd_s_rs.speed, label="speed_cmd (m/s)")
plt.plot(odom_s_rs.t, odom_s_rs.v, label="v medida (m/s)")
plt.title("Degrau de velocidade - atraso visual")
plt.xlabel("Index (k)")
plt.ylabel("Velocidade (m/s)")
plt.legend()
plt.grid(True)
plt.show()

plt.figure(figsize=(10,4))
plt.plot(cmd_d_rs.t, cmd_d_rs.steering_angle, label="steering_cmd (rad)")
plt.plot(odom_d_rs.t, odom_d_rs.yaw, label="yaw unwrap (rad)")
plt.title("Degrau de direção - atraso visual")
plt.xlabel("Index (k)")
plt.ylabel("Ângulo (rad)")
plt.legend()
plt.grid(True)
plt.show()

plt.figure(figsize=(10,4))
plt.plot(cmd_d_rs.t, cmd_d_rs.steering_angle, label="steering_cmd (rad)")
plt.plot(odom_d_rs.t, odom_d_rs.yaw_rate, label="yaw_rate (rad/s)")
plt.title("Degrau de direção - yaw_rate")
plt.xlabel("Index (k)")
plt.ylabel("Ângulo / taxa")
plt.legend()
plt.grid(True)
plt.show()
```

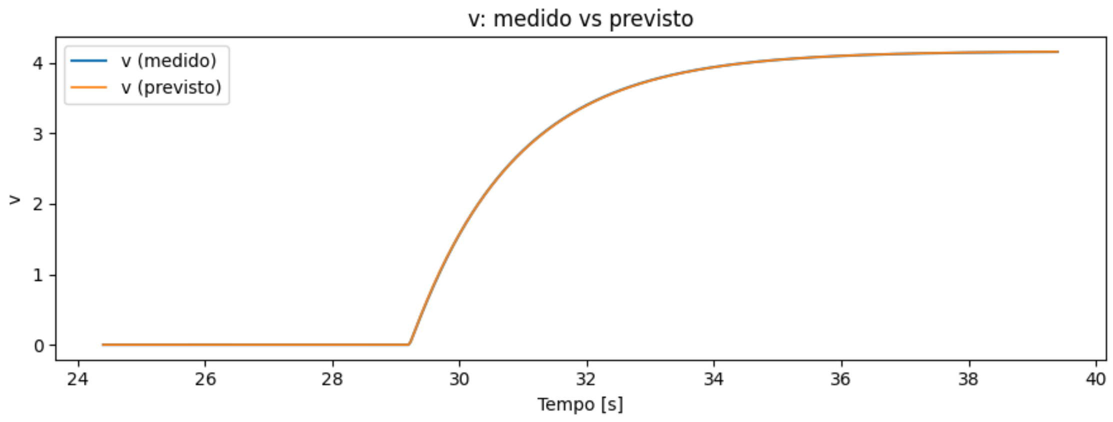

Degrau de velocidade — atraso visual
speed_cmd (m/s)
v medida (m/s)
Velocidade (m/s)
Index (k)
0
1
2
3
4
24
26
28
30
32
34
36
38
40

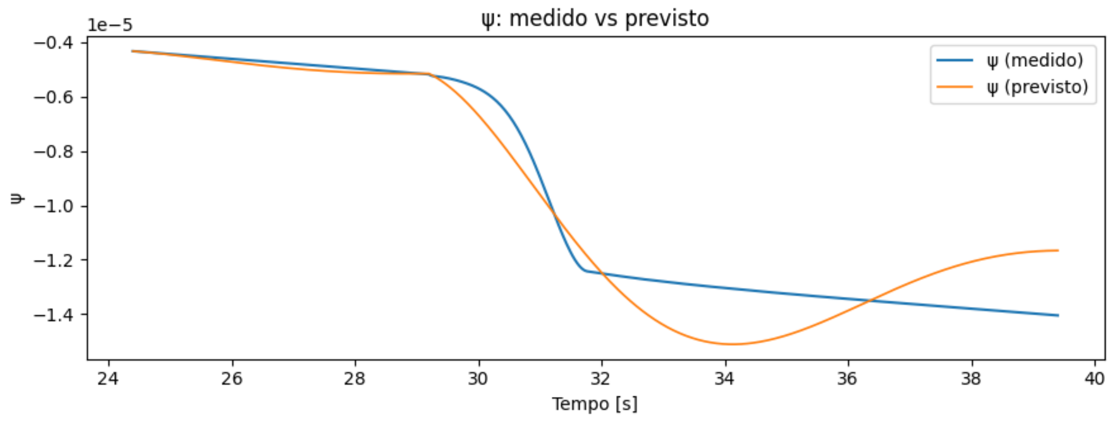

Degrau de direção — atraso visual
steering_cmd (rad)
yaw unwrap (rad)
Ângulo (rad)
Index (k)
1
0
−1
−2
−3
125
150
175
200
225
250
275
300
325

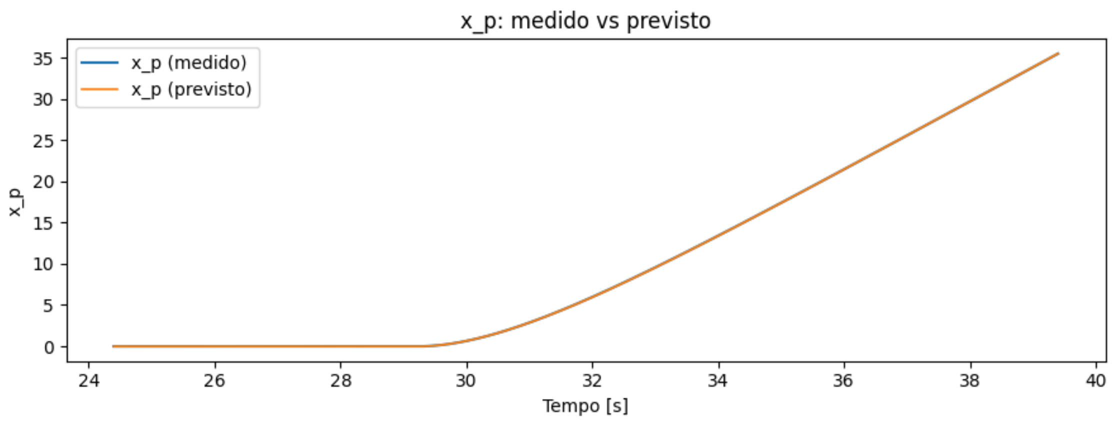


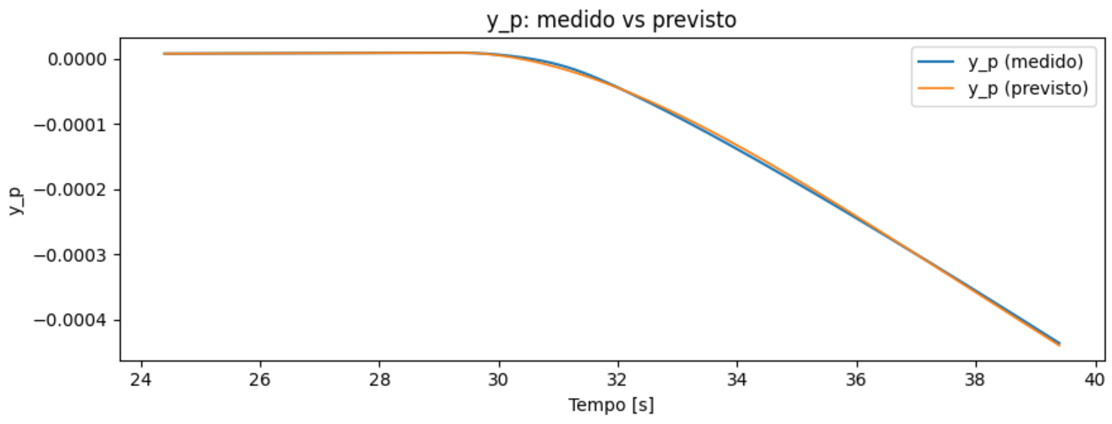
2 ARX Pure Speed Model System Identification

[ ]:
```python
from gekko import GEKKO

# generate time-series model
m = GEKKO(remote=False)

# === SPEED ONLY: ===
t = speed_df['t']
u = speed_df[['speed_cmd', 'steering_cmd']]
y = speed_df[['v_measured', 'yaw', 'x', 'y']]

# system identification
na = 2 # output coefficients
nb = 2 # input coefficients
nk = 2 # pela analise anterior eh igual a 2 para speed e 0 para steering angle
# y predicted, parameters, gain matrix
yp_sp,p_sp,K_sp = m.sysid(t,u,y,na,nb, pred='meas')

names = ['v_measured', 'yaw', 'x', 'y']
titles = {'v_measured':'v', 'yaw':' ', 'x':'x_p', 'y':'y_p'}

for i, name in enumerate(names):
    y_meas = y[name].to_numpy()
    y_pred = yp_sp[:, i]  # coluna i de yp

    plt.figure(figsize=(9,3.5))
    plt.plot(t, y_meas, label=f'{titles[name]} (measured)')
    plt.plot(t, y_pred, label=f'{titles[name]} (predicted)', linewidth=1.2)
    plt.xlabel('MTime [s]')
    plt.ylabel(titles[name])
    plt.title(f'{titles[name]}: measured vs predicted')
    plt.legend()
    plt.tight_layout()
    plt.savefig(f'plot_{name}.png', dpi=200)
    plt.show()
```









[31]:
```
A_sp, B_sp, C_sp = p_sp['a'], p_sp['b'], p_sp['c']
print("A shape:", A_sp.shape, "B shape:", B_sp.shape, "C shape:", C_sp.shape)
print("A ="); print(A_sp)
print("B ="); print(B_sp)
print("C ="); print(C_sp)
print("K ="); print(K_sp)
```

```
A shape: (2, 4) B shape: (4, 2, 2) C shape: (4,)
A =
[[ 1.19748111  1.98612632  1.9805609   1.99333374]
 [-0.21314866 -0.98655506 -0.98056202 -0.99335265]]
B =
[[[ 1.09299760e-02  0.00000000e+00]
  [ 5.38567662e-03  0.00000000e+00]]

 [[-9.55427341e-09  0.00000000e+00]
  [ 8.74016753e-09  0.00000000e+00]]
 [[ 1.37122319e-04  0.00000000e+00]
  [ 5.44459757e-04  0.00000000e+00]]

 [[ 7.44836097e-09  0.00000000e+00]
  [-1.23493006e-08  0.00000000e+00]]]
C =
[-2.20646085e-06 -2.11664315e-09 -6.00391776e-07  2.39481555e-10]
K =
[[ 1.00276037e+00  0.00000000e+00]
 [-7.59528475e-06  0.00000000e+00]
 [ 6.89738729e+01  0.00000000e+00]
 [-1.03692635e-03  0.00000000e+00]]
```

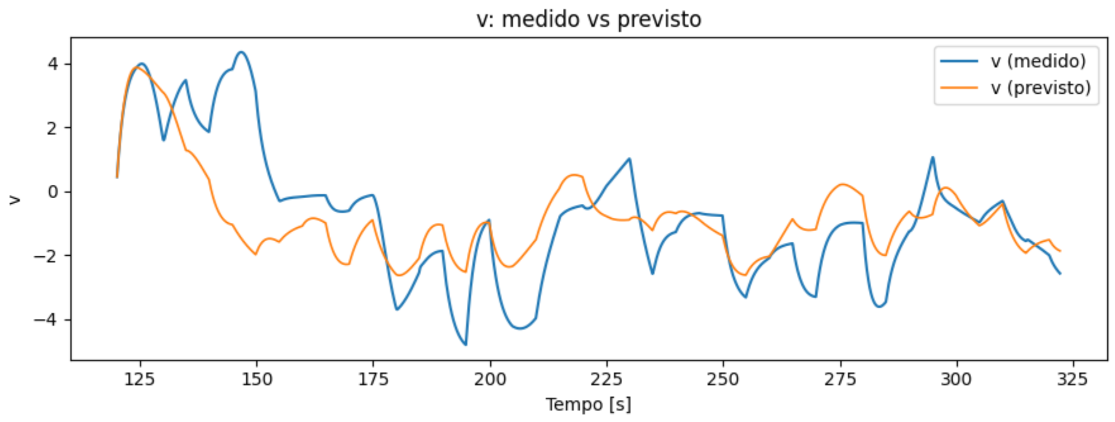
[ ]:
```
na, ny = A_sp.shape
def ar_roots_from_gekko(A):
    roots_per_output = []
    for i in range(ny):
        a = A[:, i]                     # [a1, a2, ..., ana] da saída i
        coeffs = np.r_[1.0, -a]         # z^na - a1 z^{na-1} - ... - a_na
        r = np.roots(coeffs)
        roots_per_output.append(r)
    return roots_per_output

roots_sp = ar_roots_from_gekko(A_sp)

for i, r in enumerate(roots_sp):
    mags = np.abs(r)
    print(f"Output {i}: roots = {r}, | | = {mags}, Stable? {np.all(mags < 1.
  ↪0)}")
```

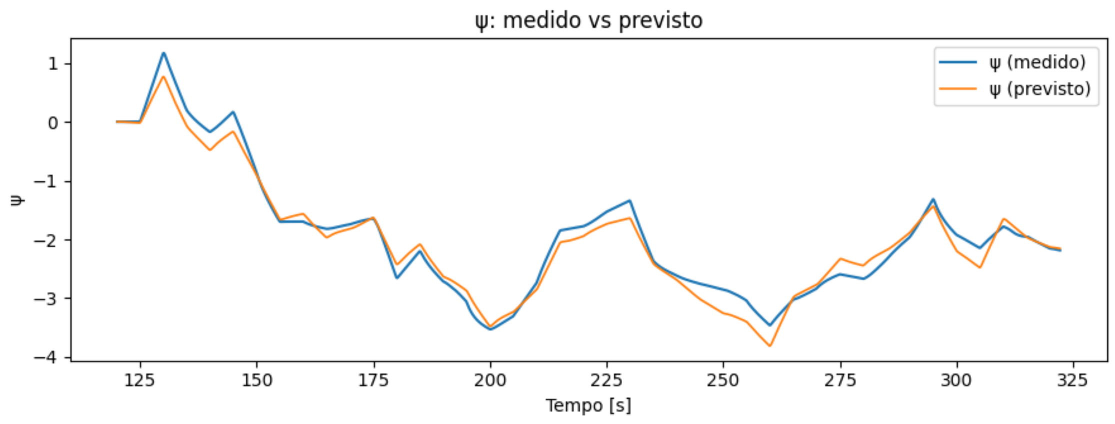
```
Saída 0: raízes = [0.97997748 0.21750363], | | = [0.97997748 0.21750363],
estável? True
Saída 1: raízes = [0.99306316+0.01950956j 0.99306316-0.01950956j], | | =
[0.99325478 0.99325478], estável? True
Saída 2: raízes = [0.99994263 0.98061828], | | = [0.99994263 0.98061828],
estável? True
Saída 3: raízes = [0.99666687+0.00279211j 0.99666687-0.00279211j], | | =
[0.99667078 0.99667078], estável? True
```

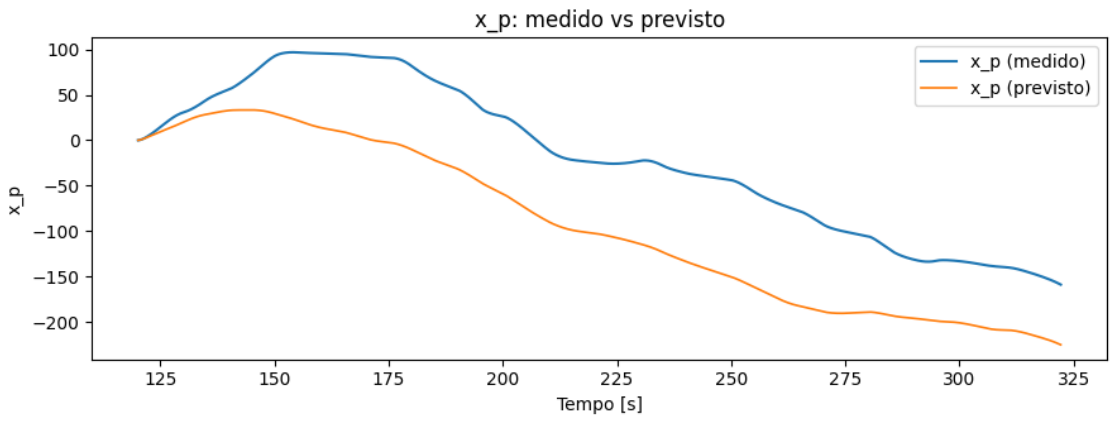
3 ARX Speed Steer model SI

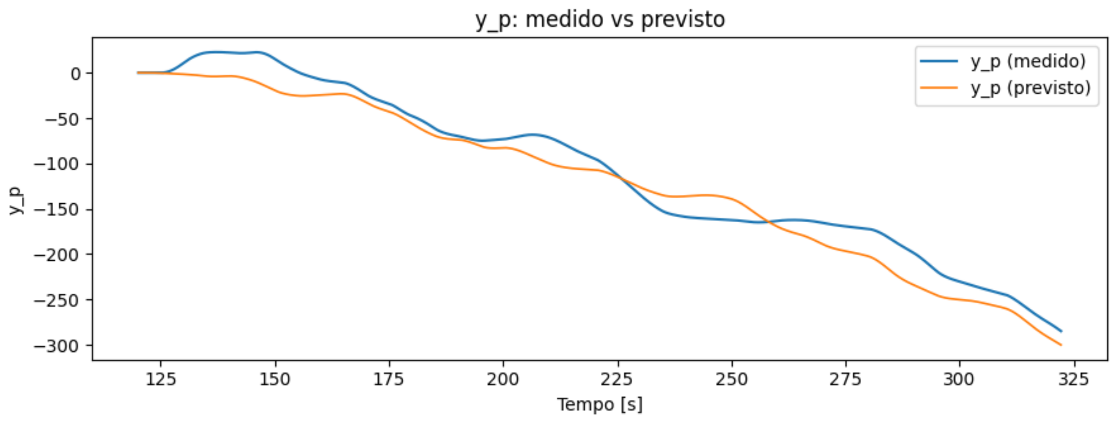
[ ]:
```
# === SPEED STEER: ===
t_st = steer_df['t']
u_st = steer_df[['speed_cmd', 'steering_cmd']]
y_st = steer_df[['v_measured', 'yaw', 'x', 'y']]
```

```
# system identification
na = 3 # output coefficients
nb = 2 # input coefficients
nk = 2 # looking at the data it should be two
# y predicted, parameters, gain matrix
yp_st,p_st,K_st = m.sysid(t_st,u_st,y_st,na,nb,nk=nk,pred='meas')

names = ['v_measured', 'yaw', 'x', 'y']
titles = {'v_measured':'v', 'yaw':' ', 'x':'x_p', 'y':'y_p'}

for i, name in enumerate(names):
    y_meas = y_st[name].to_numpy()
    y_pred = yp_st[:, i]

    plt.figure(figsize=(9,3.5))
    plt.plot(t_st, y_meas, label=f'{titles[name]} (medido)')
    plt.plot(t_st, y_pred, label=f'{titles[name]} (previsto)', linewidth=1.2)
    plt.xlabel('Tempo [s]')
    plt.ylabel(titles[name])
    plt.title(f'{titles[name]}: medido vs previsto')
    plt.legend()
    plt.tight_layout()
    plt.savefig(f'plot_{name}.png', dpi=200)
    plt.show()
```









[34]:
```
A_st, B_st, C_st = p_st['a'], p_st['b'], p_st['c']
print("A shape:", A_st.shape, "B shape:", B_st.shape, "C shape:", C_st.shape)
print("A ="); print(A_st)
print("B ="); print(B_st)
print("C ="); print(C_st)
print("K ="); print(K_st)
```

```
A shape: (3, 4) B shape: (4, 4, 2) C shape: (4,)
A =
[[ 2.31217167  2.84526402  2.9653192   2.99175188]
 [-1.63434081 -2.69376322 -2.93067853 -2.98354702]
 [ 0.32213804  0.84849891  0.96535934  0.99179515]]
B =
[[[-7.89985040e-03 -3.21030900e-03]
  [ 9.40154363e-03  1.55027260e-02]
  [-9.13216500e-04 -9.38073370e-03]
  [-5.89288393e-04 -2.17655105e-03]]

 [[-6.83977254e-06  1.35585826e-03]
  [ 3.78860055e-05  3.94729565e-03]
  [-3.64481297e-05 -3.30290616e-03]
  [ 4.62413091e-06 -1.82573980e-03]]

 [[-1.52612523e-04  4.96497236e-05]
  [-5.25047885e-06  2.40652734e-04]
  [ 1.41421061e-04  2.54055321e-05]
  [ 1.39323401e-05 -2.18151938e-04]]

 [[-1.39582086e-04  3.92627331e-04]
  [-3.17776231e-05  6.04933314e-05]
  [ 1.53109150e-04 -3.96788671e-04]
  [ 1.87571981e-05 -5.45105694e-05]]]
C =
[-2.72973974e-05  2.37925592e-06  6.31961931e-06 -3.09854331e-06]
K =
[[-1.23510785e-02  2.58143774e+00]
 [-2.54057437e+00  1.31542210e+02]
 [ 8.52264314e+00 -7.64523466e+01]
 [-1.48220939e+00 -1.22966955e+00]]
```

[ ]:
```
na, ny = A_st.shape
def ar_roots_from_gekko(A):
    roots_per_output = []
    for i in range(ny):
        a = A[:, i]                    # [a1, a2, ..., ana] da saída i
        coeffs = np.r_[1.0, -a]        # z^na - a1 z^{na-1} - ... - a_na
        r = np.roots(coeffs)
        roots_per_output.append(r)
    return roots_per_output
```

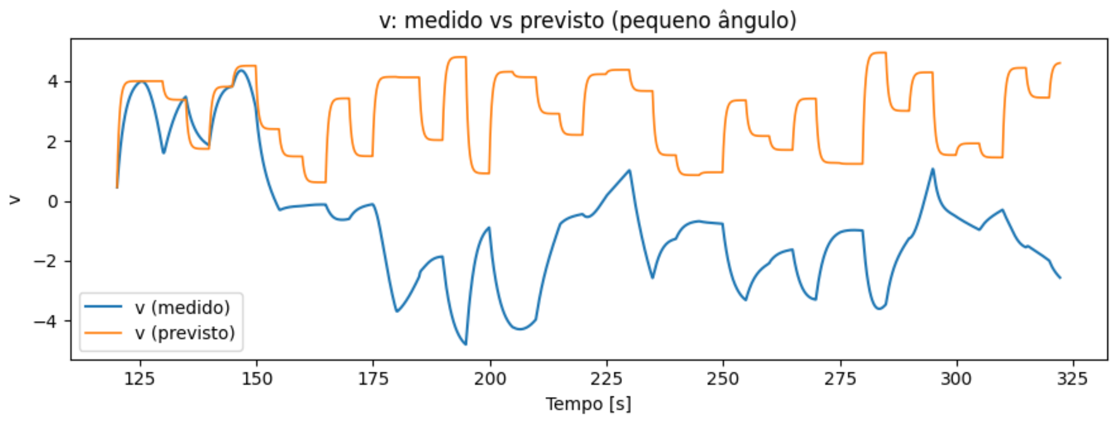
```
roots_st = ar_roots_from_gekko(A_st)

for i, r in enumerate(roots_st):
    mags = np.abs(r)
    print(f"Output {i}: roots = {r}, | | = {mags}, Stable? {np.all(mags < 1.
 ↪0)}")
```

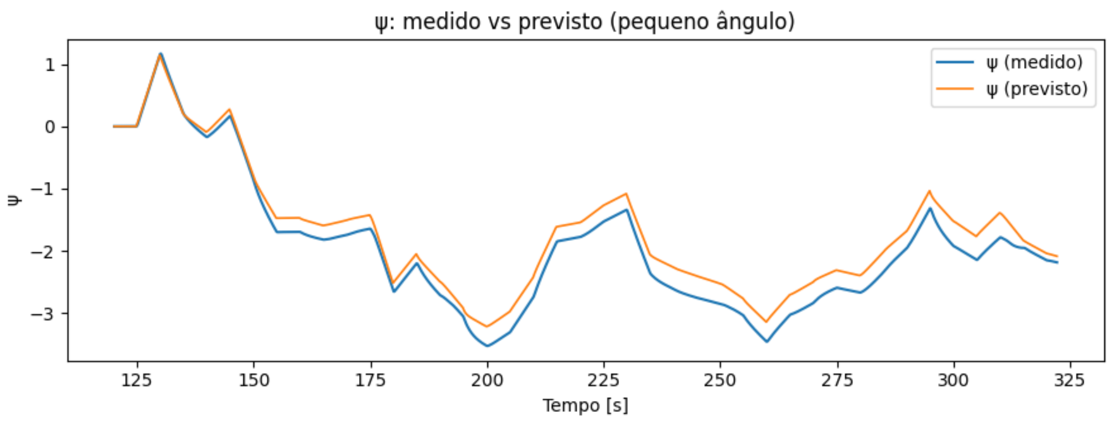
```
Saída 0: raízes = [0.99551243 0.98970209 0.32695715], | | = [0.99551243
0.98970209 0.32695715], estável? True
Saída 1: raízes = [0.99991232 0.97518694 0.87016476], | | = [0.99991232
0.97518694 0.87016476], estável? True
Saída 2: raízes = [1.00011318 0.99868329 0.96652273], | | = [1.00011318
0.99868329 0.96652273], estável? False
Saída 3: raízes = [0.99582147+0.00516895j 0.99582147-0.00516895j 1.00010894+0.j
], | | = [0.99583488 0.99583488 1.00010894], estável? False
```

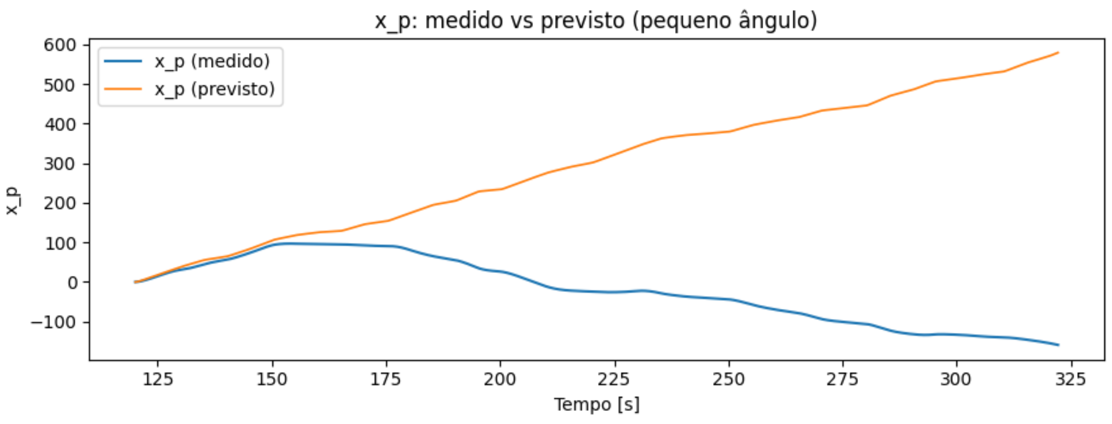
4 Linearized Bycicle Model

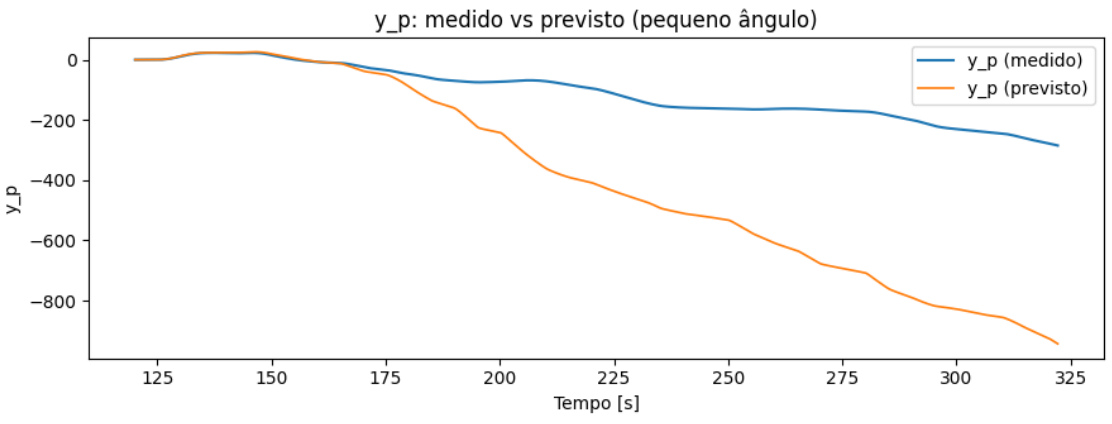
[ ]:
```
L = 1.75        # from the manual we get 1.75
tau_v = 0.5

# --- amostragem ---
Ts = np.median(np.diff(t_st))

# --- sinais no seu padrão ---
v_meas = y_st['v_measured'].to_numpy()
yaw    = y_st['yaw'].to_numpy()
x_m    = y_st['x'].to_numpy()
y_m    = y_st['y'].to_numpy()

delta  = u_st['steering_cmd'].to_numpy()    # [rad]
spd    = u_st['speed_cmd'].to_numpy()       # [m/s]

X_meas = np.stack([v_meas, yaw, x_m, y_m], axis=1)  # [v, psi, x, y]
U_meas = np.stack([spd, delta], axis=1)             # [speed_cmd, steering_cmd]

def f_small_angle(x, u):
    v, psi, xg, yg = x
    spd_u, delt_u  = u
    v_dot   = (spd_u - v)/tau_v
    psi_dot = (v/L)*delt_u
    x_dot   = v
    y_dot   = v*psi
    return np.array([v_dot, psi_dot, x_dot, y_dot], dtype=float)
```

```
Xp = np.zeros_like(X_meas)
Xp[0] = X_meas[0]

for k in range(len(t_st)-1):
    Xp[k+1] = Xp[k] + Ts * f_small_angle(Xp[k], U_meas[k])

yp_st = np.zeros_like(X_meas)
yp_st[:,0] = Xp[:,0]   # v
yp_st[:,1] = Xp[:,1]   # yaw
yp_st[:,2] = Xp[:,2]   # x
yp_st[:,3] = Xp[:,3]   # y

names  = ['v_measured', 'yaw', 'x', 'y']
titles = {'v_measured':'v', 'yaw':' ', 'x':'x_p', 'y':'y_p'}

for i, name in enumerate(names):
    y_meas = y_st[name].to_numpy()
    y_pred = yp_st[:, i]

    plt.figure(figsize=(9,3.5))
    plt.plot(t_st, y_meas, label=f'{titles[name]} (Measured)')
    plt.plot(t_st, y_pred, label=f'{titles[name]} (Predicted)', linewidth=1.2)
    plt.xlabel('Time [s]')
    plt.ylabel(titles[name])
    plt.title(f'{titles[name]}: measured vs predicted (small angles)')
    plt.legend()
    plt.tight_layout()
    plt.savefig(f'plot_{name}_lin_small.png', dpi=200)
    plt.show()
```





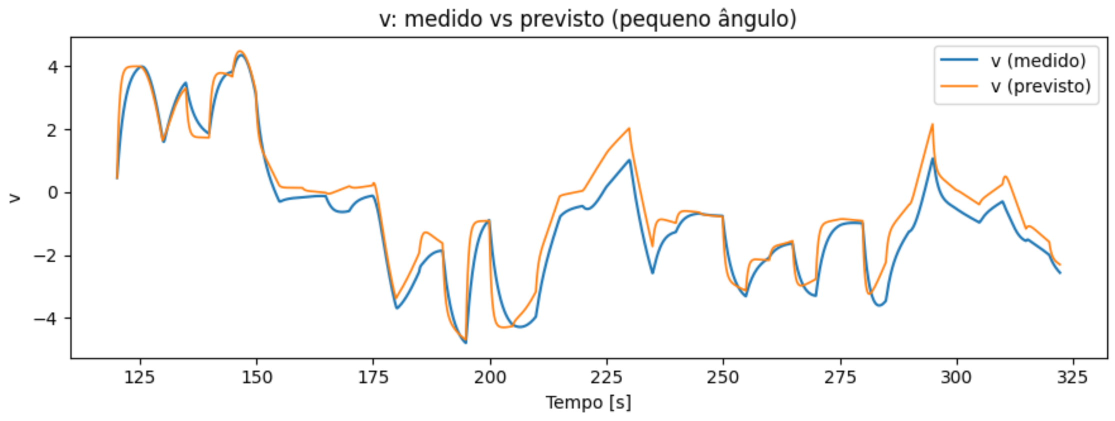


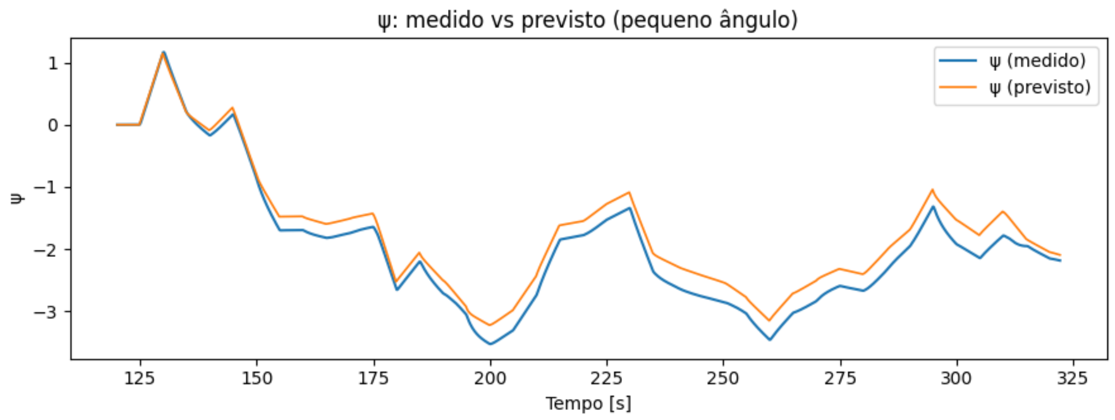


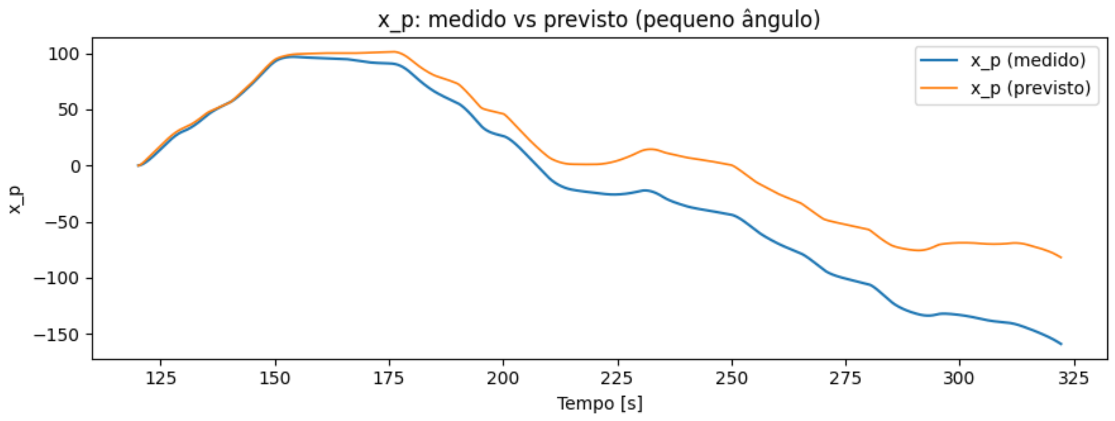
The robot has the speed not on its body frame, but instead on the world one, so we can't just apply the bicycle model, but need to apply a rotation matrix to it first

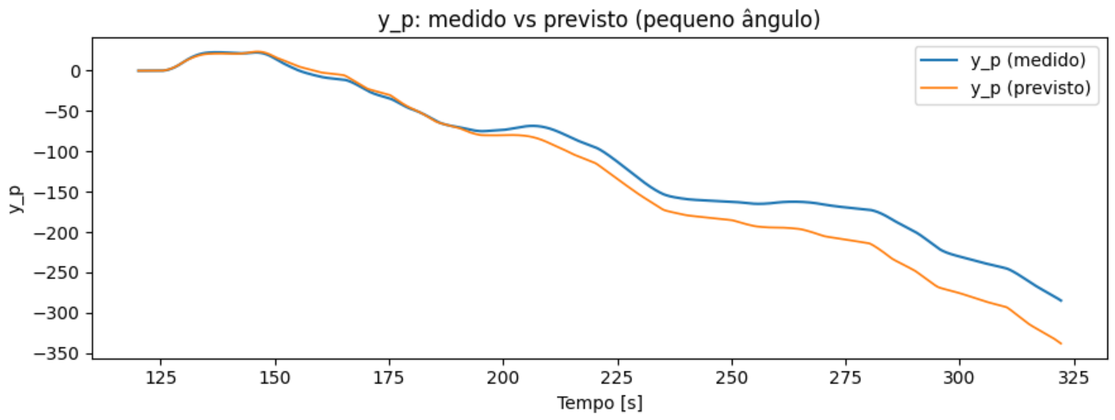
```
[ ]: L = 1.75
     tau_v = 0.5

     Ts = np.median(np.diff(t_st))


     v_meas = y_st['v_measured'].to_numpy()

     yaw    = y_st['yaw'].to_numpy()
     x_m    = y_st['x'].to_numpy()
     y_m    = y_st['y'].to_numpy()

     delta  = u_st['steering_cmd'].to_numpy()   # [rad]
     spd    = u_st['speed_cmd'].to_numpy()      # [m/s]


     X_meas = np.stack([v_meas, yaw, x_m, y_m], axis=1)  # [v, psi, x, y]
     U_meas = np.stack([spd, delta], axis=1)             # [speed_cmd, steering_cmd]


     def f_small_angle(x, u):
         v, psi, xg, yg = x
         spd_u, delt_u  = u

         # speeds ast the robot frame (with no lateral speed): [v, 0]
         vx_b = v
         vy_b = 0.0

         # rotation body->woorld
         c, s = np.cos(psi), np.sin(psi)
         vx_w =  c*vx_b - s*vy_b      # = v*cos(psi)
         vy_w =  s*vx_b + c*vy_b      # = v*sin(psi)

         # dinâmicas
         v_dot   = (spd_u - v)/tau_v
         psi_dot = (v/L)*np.tan(delt_u)
         x_dot   = vx_w
         y_dot   = vy_w

         return np.array([v_dot, psi_dot, x_dot, y_dot], dtype=float)

     Xp = np.zeros_like(X_meas)
     Xp[0] = X_meas[0]

     for k in range(len(t_st)-1):
         Xp[k+1] = Xp[k] + Ts * f_small_angle(Xp[k], U_meas[k])
```

```
yp_st = np.zeros_like(X_meas)
v_long_pred = Xp[:,0]
psi_pred    = Xp[:,1]


v_pred_plot = v_long_pred * np.cos(psi_pred)

yp_st[:,0] = v_pred_plot
yp_st[:,1] = Xp[:,1]  # yaw
yp_st[:,2] = Xp[:,2]  # x
yp_st[:,3] = Xp[:,3]  # y

names  = ['v_measured', 'yaw', 'x', 'y']
titles = {'v_measured':'v', 'yaw':' ', 'x':'x_p', 'y':'y_p'}

for i, name in enumerate(names):
    y_meas = y_st[name].to_numpy()
    y_pred = yp_st[:, i]

    plt.figure(figsize=(9,3.5))
    plt.plot(t_st, y_meas, label=f'{titles[name]} (medido)')
    plt.plot(t_st, y_pred, label=f'{titles[name]} (previsto)', linewidth=1.2)
    plt.xlabel('Tempo [s]')
    plt.ylabel(titles[name])
    plt.title(f'{titles[name]}: medido vs previsto (pequeno ângulo)')
    plt.legend()
    plt.tight_layout()
    plt.savefig(f'plot_{name}_lin_small.png', dpi=200)
    plt.show()

vx = np.gradient(y_st['x'].to_numpy(), t_st)
vy = np.gradient(y_st['y'].to_numpy(), t_st)
psi = y_st['yaw'].to_numpy()
```




ψ: medido vs previsto (pequeno ângulo)
ψ (medido)
ψ (previsto)
ψ
Tempo [s]
1
0
−1
−2
−3
125
150
175
200
225
250
275
300
325


x_p: medido vs previsto (pequeno ângulo)
x_p (medido)
x_p (previsto)
x_p
Tempo [s]
100
50
0
−50
−100
−150
125
150
175
200
225
250
275
300
325


y_p: medido vs previsto (pequeno ângulo)
y_p (medido)
y_p (previsto)
y_p
Tempo [s]
0
−50
−100
−150
−200
−250
−300
−350
125
150
175
200
225
250
275
300
325

5 Identifying parameters using nonlinear greybox

[40]:
```python
import numpy as np
from gekko import GEKKO

def identify_bicycle_greybox(t, speed_cmd, steer_cmd,
                             v_meas, yaw_meas, x_meas, y_meas,
                             L_init=1.75, tau_v_init=0.5):
    t = np.asarray(t); t = t - t[0]

    m = GEKKO(remote=False)
    m.time = list(t)

    # Inputs (known)
    u_v = m.Param(value=list(speed_cmd))
    u_d = m.Param(value=list(steer_cmd))

    # Measured outputs as Params
    v_p   = m.Param(value=list(v_meas))
    psi_p = m.Param(value=list(yaw_meas))
    x_p   = m.Param(value=list(x_meas))
    y_p   = m.Param(value=list(y_meas))

    # Unknown constants to estimate (FVs)
    tau_v = m.FV(value=tau_v_init, lb=0.05, ub=5.0);  tau_v.STATUS  = 1
    L_eff = m.FV(value=L_init,     lb=1.0,  ub=4.0);  L_eff.STATUS  = 1
    k_del = m.FV(value=1.0,        lb=0.5,  ub=2.0);  k_del.STATUS  = 1  #␣
↪steering gain / ratio

    # States
    v   = m.Var(value=float(v_meas[0]))
    psi = m.Var(value=float(yaw_meas[0]))
    xg  = m.Var(value=float(x_meas[0]))
    yg  = m.Var(value=float(y_meas[0]))

    # Nonlinear dynamics (keep smooth)
    m.Equation(v.dt()   == (u_v - v)/tau_v)
    m.Equation(psi.dt() == (v/L_eff) * m.tan(k_del*u_d))
    m.Equation(xg.dt()  == v*m.cos(psi))
    m.Equation(yg.dt()  == v*m.sin(psi))

    # Fit objective (weights are tunable)
    wv, wpsi, wx, wy = 1.0, 5.0, 1.0, 1.0
    m.Minimize(wv  * (v   - v_p)**2)
    m.Minimize(wpsi* (psi - psi_p)**2)
    m.Minimize(wx  * (xg  - x_p)**2 + wy*(yg - y_p)**2)
    # tiny regularization to keep params near initials
    m.Minimize(1e-6 * ((tau_v-tau_v_init)**2 + (L_eff-L_init)**2 + (k_del-1.
↪0)**2))

    m.options.IMODE   = 6        # dynamic estimation over full dataset
    m.options.NODES   = 3
    m.options.SOLVER  = 3        # IPOPT
    m.options.SCALING = 1
    m.options.MAX_ITER = 300

    m.solve(disp=False)

    return {
        'tau_v' : float(tau_v.VALUE[0]),
        'L_eff' : float(L_eff.VALUE[0]),
        'k_del' : float(k_del.VALUE[0])
    }
```

[41]:
```
pars = identify_bicycle_greybox(t_st, spd, delta, v_meas, yaw, x_m, y_m)
```

display(pars)

6 Non linear graybox System Identification

[ ]:
```
def identify_bicycle_greybox_plus(t, speed_cmd, steer_cmd,
                                  v_meas, yaw_meas, x_meas, y_meas,
                                  L0=1.75, tau_v0=0.5):
    """Estimativa de parâmetros com atuadores + slip  ( ) e viés de  ."""
    t = np.asarray(t, float); t = t - t[0]
    m = GEKKO(remote=False); m.time = list(t)

    u_v = m.Param(value=list(speed_cmd))
    u_d = m.Param(value=list(steer_cmd))


    v_p   = m.Param(value=list(v_meas))
    psi_p = m.Param(value=list(yaw_meas))
    x_p   = m.Param(value=list(x_meas))
    y_p   = m.Param(value=list(y_meas))

    tau_acc  = m.FV(value=0.20, lb=0.02, ub=3.0);  tau_acc.STATUS  = 1
    tau_str  = m.FV(value=0.15, lb=0.02, ub=3.0);  tau_str.STATUS  = 1
    tau_v    = m.FV(value=tau_v0, lb=0.05, ub=5.0);tau_v.STATUS    = 1
    L_eff    = m.FV(value=L0,    lb=1.0,  ub=4.0); L_eff.STATUS    = 1
    k_del    = m.FV(value=1.0,   lb=0.5,  ub=2.0); k_del.STATUS    = 1
    k_b1     = m.FV(value=0.0,   lb=-0.6, ub=0.6); k_b1.STATUS     = 1
    k_b3     = m.FV(value=0.0,   lb=-1.0, ub=1.0); k_b3.STATUS     = 1
    psi_bias = m.FV(value=0.0,   lb=-0.3, ub=0.3); psi_bias.STATUS = 1

    v   = m.Var(value=float(v_meas[0]), lb=-10, ub=10)
    psi = m.Var(value=float(yaw_meas[0]))
    xg  = m.Var(value=float(x_meas[0]))
    yg  = m.Var(value=float(y_meas[0]))
    uva = m.Var(value=float(v_meas[0]), lb=-10, ub=10)
    da  = m.Var(value=0.0,               lb=-0.8, ub=0.8)


    m.Equation(uva.dt() == (u_v - uva)/tau_acc)
    m.Equation(da.dt()  == (u_d - da)/tau_str)


    m.Equation(v.dt()   == (uva - v)/tau_v)
    m.Equation(psi.dt() == (v/L_eff) * m.tan(k_del*da))
    beta = m.Intermediate(k_b1*da + k_b3*da**3)
    m.Equation(xg.dt()  == v*m.cos(psi + psi_bias + beta))
    m.Equation(yg.dt()  == v*m.sin(psi + psi_bias + beta))


    wv, wpsi, wxy = 5.0, 10.0, 0.2
    # m.Minimize(wv  * (v   - v_p)**2)
    vx_hat_m = m.Intermediate( v*m.cos(psi + psi_bias + beta) )
    m.Minimize(wv * (vx_hat_m - v_p)**2)
    m.Minimize(wpsi* (psi - psi_p)**2)
    m.Minimize(wxy * ((xg - x_p)**2 + (yg - y_p)**2))
    m.Minimize(1e-6*((tau_acc-0.2)**2 + (tau_str-0.15)**2 + (tau_v-tau_v0)**2 +
                     (L_eff-L0)**2 + (k_del-1.0)**2 + k_b1**2 + k_b3**2 +␣
↪psi_bias**2))

    # Solver
    m.options.IMODE=6; m.options.NODES=3; m.options.SOLVER=3
    m.options.SCALING=1; m.options.MAX_ITER=300
    m.solver_options = ['print_level 0','max_iter 300','tol␣
↪1e-6','acceptable_tol 1e-4']

    m.solve(disp=False)

    pars = {
        'tau_acc': float(tau_acc.VALUE[0]),
        'tau_str': float(tau_str.VALUE[0]),
        'tau_v'  : float(tau_v.VALUE[0]),
        'L_eff'  : float(L_eff.VALUE[0]),
        'k_del'  : float(k_del.VALUE[0]),
        'k_b1'   : float(k_b1.VALUE[0]),
        'k_b3'   : float(k_b3.VALUE[0]),
        'psi_bias':float(psi_bias.VALUE[0]),
    }

    v_hat   = np.array(list(v.VALUE),   dtype=float)
    psi_hat = np.array(list(psi.VALUE), dtype=float)
    x_hat   = np.array(list(xg.VALUE),  dtype=float)
    y_hat   = np.array(list(yg.VALUE),  dtype=float)
    da_hat  = np.fromiter(da.VALUE,   dtype=float)

    beta_hat = pars['k_b1']*da_hat + pars['k_b3']*(da_hat**3)
    vx_hat   = v_hat * np.cos(psi_hat + pars['psi_bias'] + beta_hat)

    return pars, (v_hat, psi_hat, x_hat, y_hat, vx_hat)
```

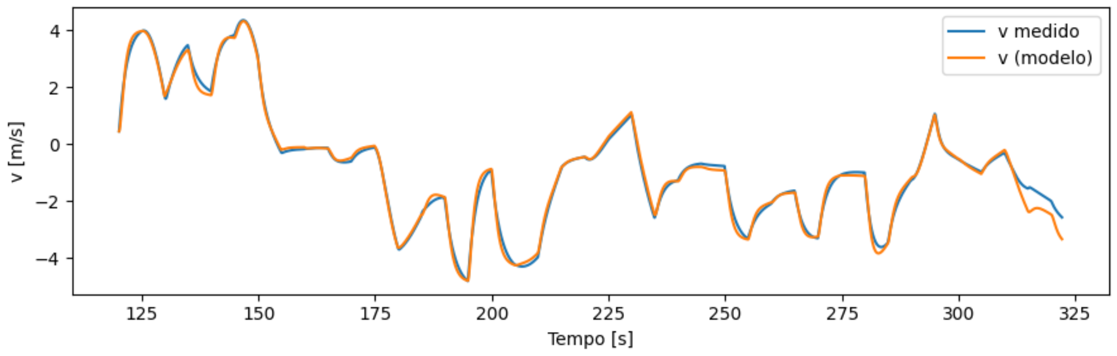
[43]:
```
t         = np.asarray(t_st)
speed     = u_st['speed_cmd'].to_numpy().astype(float)
steer     = u_st['steering_cmd'].to_numpy().astype(float)
v_meas    = y_st['v_measured'].to_numpy().astype(float)
vx_meas   = y_st['v_measured'].to_numpy().astype(float)      # isto é v_x no␣
 ↪MUNDO
yaw_meas = np.unwrap(y_st['yaw'].to_numpy().astype(float))
x_meas    = y_st['x'].to_numpy().astype(float)
y_meas    = y_st['y'].to_numpy().astype(float)

pars, (v_hat, psi_hat, x_hat, y_hat, vx_hat) = identify_bicycle_greybox_plus(
    t, speed, steer, v_meas, yaw_meas, x_meas, y_meas,
    L0=1.75, tau_v0=0.5
)
print(pars)
```

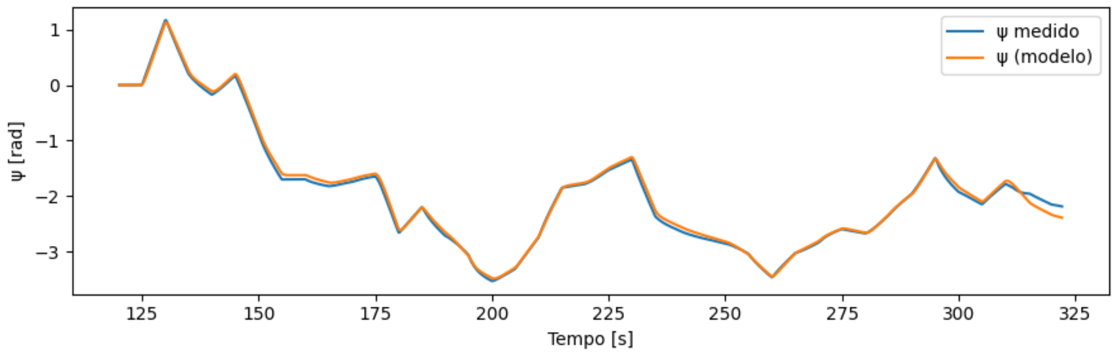
```
{'tau_acc': 0.85716492375, 'tau_str': 0.4119392892, 'tau_v': 0.43809159234,
'L_eff': 2.8168931892, 'k_del': 1.666545649, 'k_b1': 0.6, 'k_b3': 1.0,
'psi_bias': -0.020810914222}
```

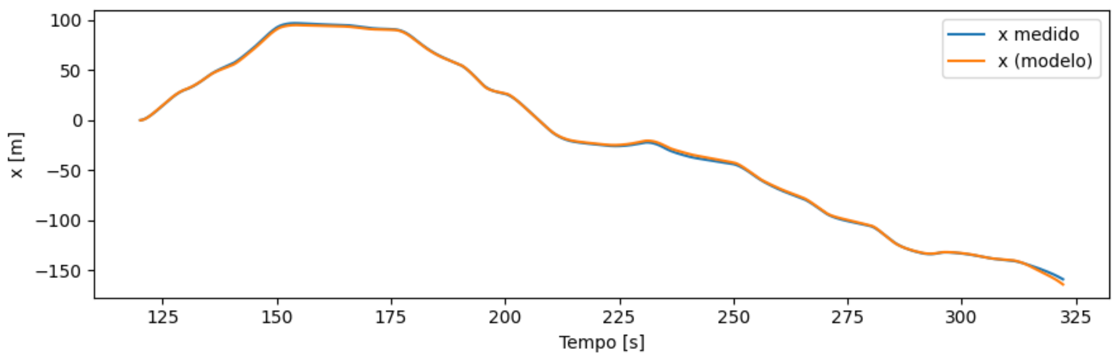
[ ]:
```
# --- 1) Preparar dados (mesmo tamanho; yaw des-enrolado) ---
t         = np.asarray(t_st)
speed     = u_st['speed_cmd'].to_numpy().astype(float)
steer     = u_st['steering_cmd'].to_numpy().astype(float)
v_meas    = y_st['v_measured'].to_numpy().astype(float)
yaw_meas = np.unwrap(y_st['yaw'].to_numpy().astype(float))
x_meas    = y_st['x'].to_numpy().astype(float)
y_meas    = y_st['y'].to_numpy().astype(float)

# (opcional) máscara para remover NaN / inf
mask = np.isfinite(t*0+1) & np.isfinite(speed) & np.isfinite(steer) \
       & np.isfinite(v_meas) & np.isfinite(yaw_meas) \
       & np.isfinite(x_meas) & np.isfinite(y_meas)
```

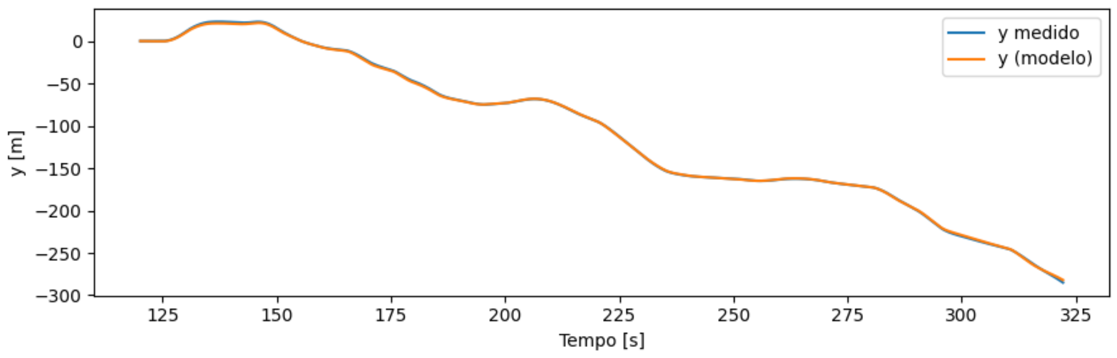
```
t, speed, steer, v_meas, yaw_meas, x_meas, y_meas = \
    t[mask], speed[mask], steer[mask], v_meas[mask], yaw_meas[mask],␣
  ↪x_meas[mask], y_meas[mask]

# (opcional) usar só um trecho para testar
idx = slice(0, len(t))  # ex.: slice(100, 2000)

# --- 2) Identificar e obter predições ---
# pars, (v_hat, psi_hat, x_hat, y_hat, vx_hat) = identify_bicycle_greybox_plus(
#     t[idx], speed[idx], steer[idx], v_meas[idx], yaw_meas[idx], x_meas[idx],␣
  ↪y_meas[idx],
#     L0=1.75, tau_v0=0.5
# )
print('Parâmetros:', pars)

# --- 3) Métricas simples (RMSE) ---
def rmse(a,b):
    a,b = np.asarray(a), np.asarray(b)
    return float(np.sqrt(np.mean((a-b)**2)))
print(f'RMSE v:   {rmse(v_meas[idx], v_hat):.3f} m/s')
print(f'RMSE  :   {rmse(yaw_meas[idx], psi_hat):.3f} rad')
print(f'RMSE x:   {rmse(x_meas[idx], x_hat):.3f} m')
print(f'RMSE y:   {rmse(y_meas[idx], y_hat):.3f} m')

# --- 4) Plots de validação ---
tt = t[idx]

plt.figure(figsize=(9,3))
plt.plot(tt, v_meas[idx], label='v measured')
plt.plot(tt, vx_hat,       label='v (model)')
plt.xlabel('Time [s]'); plt.ylabel('v [m/s]'); plt.legend(); plt.tight_layout();
  ↪ plt.show()

plt.figure(figsize=(9,3))
plt.plot(tt, yaw_meas[idx], label=' measured')
plt.plot(tt, psi_hat,       label=' (model)')
plt.xlabel('Time [s]'); plt.ylabel(' [rad]'); plt.legend(); plt.tight_layout();
  ↪ plt.show()

plt.figure(figsize=(9,3))
plt.plot(tt, x_meas[idx], label='x measured')
plt.plot(tt, x_hat,       label='x (model)')
plt.xlabel('Time [s]'); plt.ylabel('x [m]'); plt.legend(); plt.tight_layout();␣
  ↪plt.show()

plt.figure(figsize=(9,3))
plt.plot(tt, y_meas[idx], label='y measured')
plt.plot(tt, y_hat,        label='y (model)')
plt.xlabel('Time [s]'); plt.ylabel('y [m]'); plt.legend(); plt.tight_layout();␣
↪plt.show()
```

```
Parâmetros: {'tau_acc': 0.85716492375, 'tau_str': 0.4119392892, 'tau_v':
0.43809159234, 'L_eff': 2.8168931892, 'k_del': 1.666545649, 'k_b1': 0.6, 'k_b3':
1.0, 'psi_bias': -0.020810914222}
RMSE v:   4.367 m/s
RMSE  :   0.060 rad
RMSE x:   1.219 m
RMSE y:   0.799 m
```





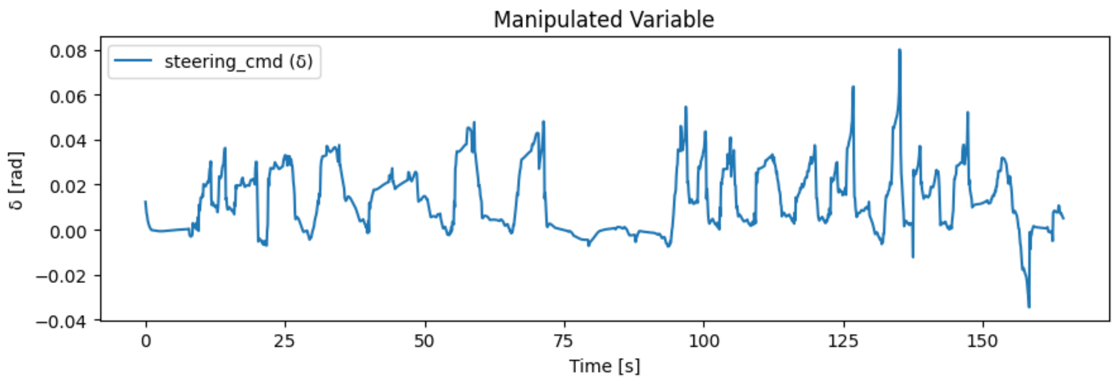


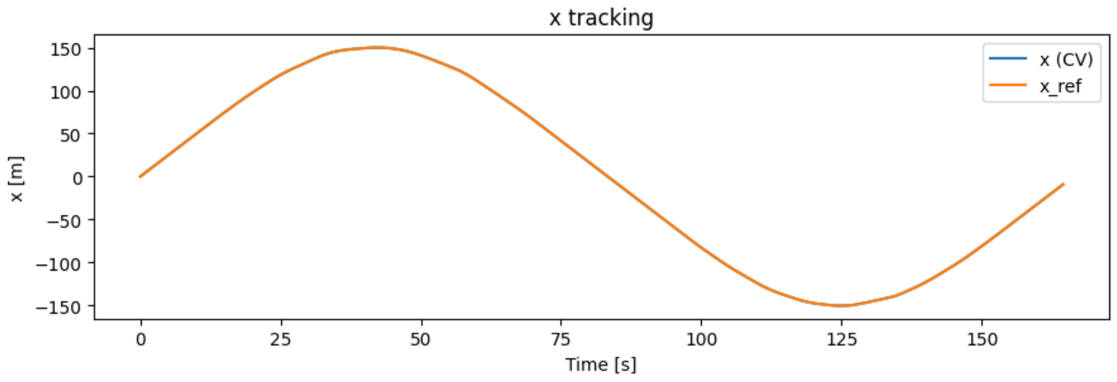


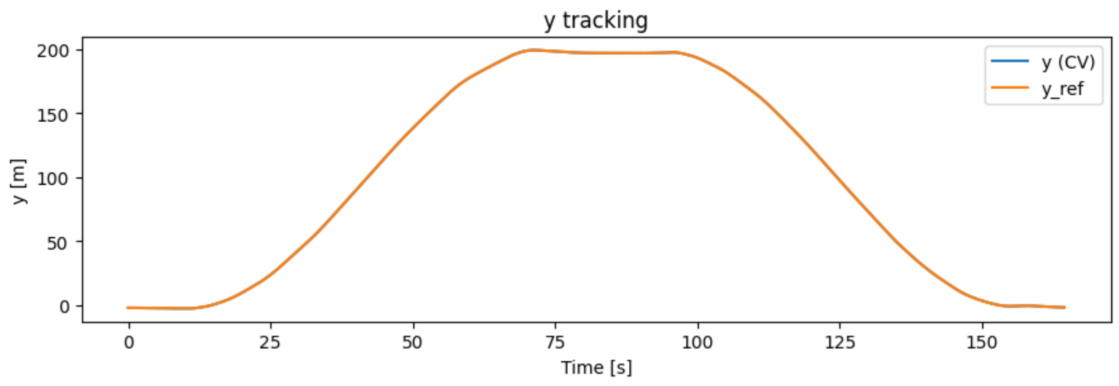
7 MPC Controller

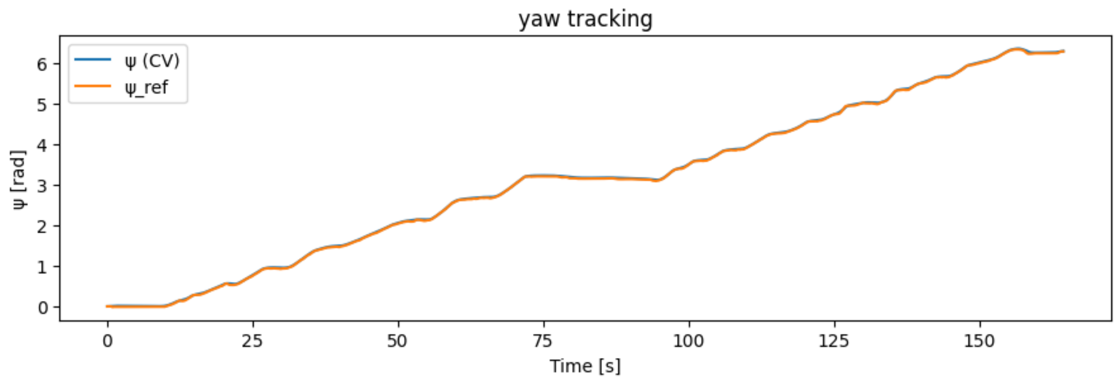
```
[ ]: # --------------- 1) Load CSV and build time-aligned references ---------------
def load_wp_csv_make_refs(csv_path, v_fixed=5.0, Ts=0.1, use_csv_yaw=True):
    """
    csv columns: [x_world, y_world, yaw, v_world, vref_body]
    We discard the speeds and create time from arc-length with constant v_fixed.
    Returns t, x_ref, y_ref, psi_ref, v_cmd (constant)
    """
    df = pd.read_csv(csv_path, header=None)
    df.columns = ['x','y','psi','v_world','vref_body']

    x = df['x'].to_numpy(dtype=float)
    y = df['y'].to_numpy(dtype=float)

    if use_csv_yaw and 'psi' in df:
        psi = np.unwrap(df['psi'].to_numpy(dtype=float))
    else:
        dx = np.gradient(x)
        dy = np.gradient(y)
        psi = np.unwrap(np.arctan2(dy, dx))

    # arc-length
    ds = np.hypot(np.diff(x), np.diff(y))
    s  = np.concatenate(([0.0], np.cumsum(ds)))

    # time from constant speed
    v_fixed = float(v_fixed)
    t_wp = s / max(v_fixed, 1e-6)

    # resample to uniform Ts
    if Ts is None:
        Ts = float(np.median(np.diff(t_wp[t_wp>0]))) if len(t_wp)>1 else 0.1
    t = np.arange(0.0, t_wp[-1] + 0.5*Ts, Ts)


    x_ref   = np.interp(t, t_wp, x)
    y_ref   = np.interp(t, t_wp, y)
    psi_ref = np.interp(t, t_wp, psi)

    v_cmd = np.full_like(t, v_fixed, dtype=float)
    return t, x_ref, y_ref, psi_ref, v_cmd, Ts

# --------------- 2) Steering-only MPC builder (tracks x,y,psi) ---------------
def mpc_xypsi_steer_only(t, pars, x_ref, y_ref, psi_ref, v_cmd,
                         Ts=None, Np=20,
                         steer_bounds=(-0.6,0.6), dmax_steer=0.20,
                         w_pos=1.0, w_yaw=0.2, w_u=1e-6):
    """
    Steering-only MPC that tracks (x, y, psi). Speed is a known input v_cmd(t).
    'pars' comes from grey-box identification; if None, defaults are used.
    """
    if Ts is None:
        Ts = float(np.median(np.diff(t)))
    H = np.linspace(0.0, Np*Ts, Np+1)

    # defaults if pars not given
    if pars is None:
        pars = dict(tau_acc=0.20, tau_str=0.15, tau_v=0.5, L_eff=1.75,
                    k_del=1.0, k_b1=0.0, k_b3=0.0, psi_bias=0.0)

    m = GEKKO(remote=False)
    m.time = H
    m.options.IMODE   = 6
    m.options.NODES   = 2
    m.options.SOLVER  = 3
    m.options.SCALING = 1
    m.options.MAX_ITER = 120
    m.solver_options  = ['print_level 0','max_iter 120','tol␣
↪1e-6','acceptable_tol 1e-4','linear_solver ma27']

    # fixed params
    tau_acc = m.Param(value=pars['tau_acc'])
    tau_str = m.Param(value=pars['tau_str'])
    tau_v   = m.Param(value=pars['tau_v'])
    L_eff   = m.Param(value=pars['L_eff'])
    k_del   = m.Param(value=pars['k_del'])
    k_b1    = m.Param(value=pars['k_b1'])
    k_b3    = m.Param(value=pars['k_b3'])
    psi_b   = m.Param(value=pars['psi_bias'])

    # horizon references and known speed command
    x_sp   = m.Param(value=x_ref[:Np+1].copy())
    y_sp   = m.Param(value=y_ref[:Np+1].copy())
    psi_sp = m.Param(value=psi_ref[:Np+1].copy())
    u_vp   = m.Param(value=v_cmd[:Np+1].copy())

    # states
    v   = m.Var(value=0.0, lb=-10, ub=10)
    psi = m.Var(value=float(psi_ref[0]))
    xg  = m.Var(value=float(x_ref[0]))
    yg  = m.Var(value=float(y_ref[0]))
    uva = m.Var(value=v_cmd[0], lb=-10, ub=10)     # speed actuator
    da  = m.Var(value=0.0,     lb=steer_bounds[0], ub=steer_bounds[1])

    # MV: steering command
    delt_u = m.MV(value=0.0, lb=steer_bounds[0], ub=steer_bounds[1])
    delt_u.STATUS = 1
    delt_u.DMAX   = dmax_steer
    delt_u.DCOST  = 1.0

    # dynamics
    m.Equation(uva.dt() == (u_vp - uva)/tau_acc)
    m.Equation(da.dt()  == (delt_u - da)/tau_str)

    m.Equation(v.dt()   == (uva - v)/tau_v)
    m.Equation(psi.dt() == (v/L_eff) * m.tan(k_del*da))
    beta = m.Intermediate(k_b1*da + k_b3*da**3)
    m.Equation(xg.dt()  == v*m.cos(psi + psi_b + beta))
    m.Equation(yg.dt()  == v*m.sin(psi + psi_b + beta))

    # objective
    ex = m.Intermediate(xg - x_sp)
    ey = m.Intermediate(yg - y_sp)
    dpsi = m.Intermediate(psi - psi_sp)
    m.Minimize(w_pos*(ex**2 + ey**2))
    m.Minimize(w_yaw*(1 - m.cos(dpsi)))
    m.Minimize(w_u*(delt_u**2))

    def set_refs_and_speed(xseg, yseg, psiseg, vseg):
        x_sp.value   = xseg[:Np+1]
        y_sp.value   = yseg[:Np+1]
        psi_sp.value = psiseg[:Np+1]
        u_vp.value   = vseg[:Np+1]

    def solve_first_move(v0, psi0, x0, y0, uva0, da0):
        v.value, psi.value, xg.value, yg.value = v0, psi0, x0, y0
        uva.value, da.value = uva0, da0
        head_err = float((psi_sp.value[min(5,Np)] - psi0 + np.pi)%(2*np.pi) -␣
 ↪np.pi)
        d_guess  = np.clip(head_err, steer_bounds[0], steer_bounds[1])
        delt_u.value = [d_guess]*(Np+1)
        m.solve(disp=False)
        return float(delt_u.NEWVAL)

    return set_refs_and_speed, solve_first_move, dict(Ts=Ts, Np=Np)

# --------------- 3) Example: read CSV, run MPC at v_fixed=5 m/s ---------------
csv_path = 'data/wps.csv'
t, x_ref, y_ref, psi_ref, v_cmd, Ts = load_wp_csv_make_refs(csv_path, v_fixed=5.
 ↪0, Ts=0.1)

try:
    pars
except NameError:
    pars = None  # will use defaults inside the builder

# Build controller
set_hor, solve_move, info = mpc_xypsi_steer_only(
    t=t, pars=pars,
    x_ref=x_ref, y_ref=y_ref, psi_ref=psi_ref,
    v_cmd=v_cmd,
    Ts=Ts, Np=25,
    steer_bounds=(-0.6,0.6), dmax_steer=0.20,
    w_pos=2.0, w_yaw=0.5, w_u=1e-6
)
Ts, Np = info['Ts'], info['Np']

# Closed-loop simulation (plant matches controller model)
M = len(t) - Np - 1
tt = t[:M+1]
```

```python
X = np.zeros((M+1,6))  # [v, psi, x, y, uva, da]
Udel = np.zeros(M)

# initial states from first point
X[0] = [v_cmd[0], psi_ref[0], x_ref[0], y_ref[0], v_cmd[0], 0.0]

for k in range(M):
    xseg   = x_ref[k:k+Np+1]
    yseg   = y_ref[k:k+Np+1]
    psiseg = psi_ref[k:k+Np+1]
    vseg   = v_cmd[k:k+Np+1]
    set_hor(xseg, yseg, psiseg, vseg)

    v0, psi0, x0, y0, uva0, da0 = X[k]
    u_del = solve_move(v0, psi0, x0, y0, uva0, da0)
    Udel[k] = u_del

    # simulate one Ts (Euler)
    uva = uva0 + Ts*(vseg[0] - uva0)/max(pars['tau_acc'] if pars else 0.2, 1e-6)
    da  = da0  + Ts*(u_del      - da0 )/max(pars['tau_str'] if pars else 0.15,␣
 ↪1e-6)
    v   = v0   + Ts*(uva - v0)/max(pars['tau_v'] if pars else 0.5, 1e-6)
    L   = pars['L_eff'] if pars else 1.75
    kdel= pars['k_del'] if pars else 1.0
    kb1 = pars['k_b1']  if pars else 0.0
    kb3 = pars['k_b3']  if pars else 0.0
    psb = pars['psi_bias'] if pars else 0.0
    psi = psi0 + Ts*(v0/L)*np.tan(kdel*da)
    beta = kb1*da + kb3*(da**3)
    x   = x0 + Ts*(v0*np.cos(psi0 + psb + beta))
    y   = y0 + Ts*(v0*np.sin(psi0 + psb + beta))
    X[k+1] = [v, psi, x, y, uva, da]

# --------------- 4) Plots ---------------
plt.figure(figsize=(9,3.2))
plt.plot(t[:M], Udel, label='steering_cmd ( )')
plt.xlabel('Time [s]'); plt.ylabel('  [rad]'); plt.title('Manipulated␣
 ↪Variable'); plt.legend(); plt.tight_layout(); plt.show()

plt.figure(figsize=(9,3.2))
plt.plot(tt, X[:M+1,2], label='x (CV)'); plt.plot(tt, x_ref[:M+1],␣
 ↪label='x_ref')
plt.xlabel('Time [s]'); plt.ylabel('x [m]'); plt.title('x tracking'); plt.
 ↪legend(); plt.tight_layout(); plt.show()

plt.figure(figsize=(9,3.2))
plt.plot(tt, X[:M+1,3], label='y (CV)'); plt.plot(tt, y_ref[:M+1],␣
 ↪label='y_ref')
plt.xlabel('Time [s]'); plt.ylabel('y [m]'); plt.title('y tracking'); plt.
 ↪legend(); plt.tight_layout(); plt.show()

plt.figure(figsize=(9,3.2))
psi_track = np.unwrap(X[:M+1,1]); psi_r = np.unwrap(psi_ref[:M+1])
plt.plot(tt, psi_track, label=' (CV)'); plt.plot(tt, psi_r, label=' _ref')
plt.xlabel('Time [s]'); plt.ylabel('  [rad]'); plt.title('yaw tracking'); plt.
 ↪legend(); plt.tight_layout(); plt.show()
```

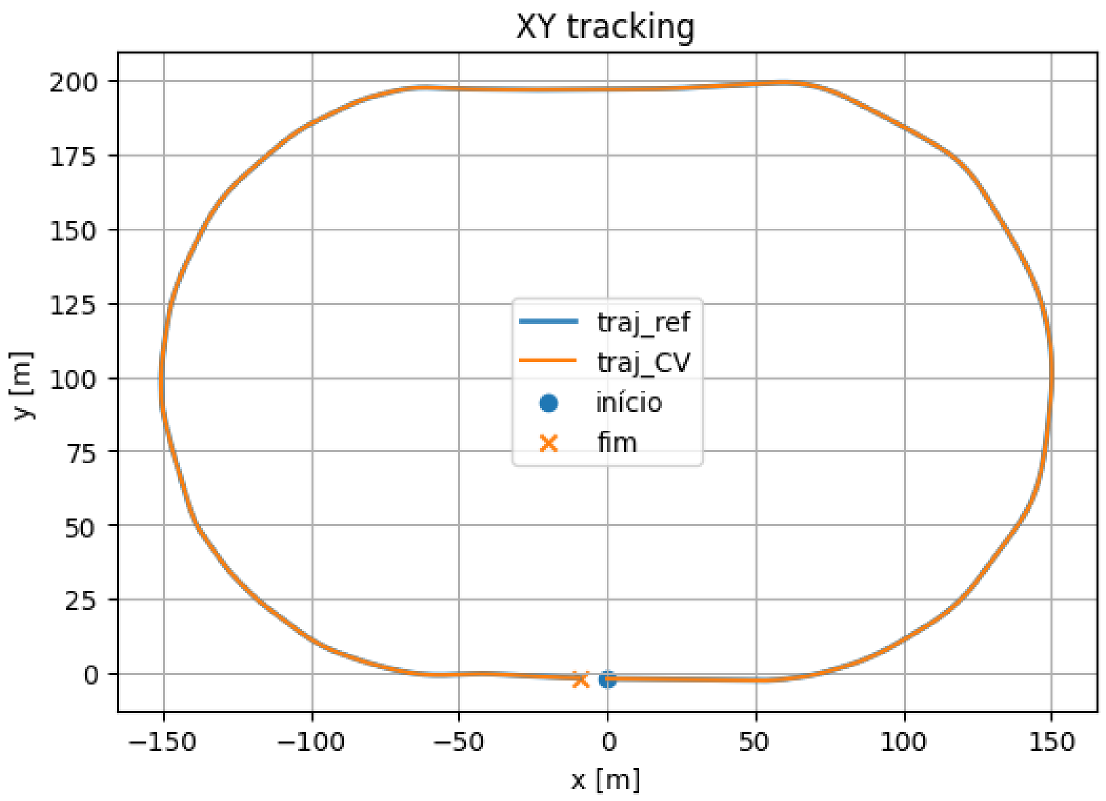








```
[46]: x_cv = X[:M+1, 2]    # x do controlador (CV)
      y_cv = X[:M+1, 3]    # y do controlador (CV)
      x_r  = x_ref[:M+1]   # x de referência
      y_r  = y_ref[:M+1]   # y de referência

      plt.figure(figsize=(6,6))
      plt.plot(x_r,  y_r,  label='traj_ref', linewidth=2, alpha=0.9)
      plt.plot(x_cv, y_cv, label='traj_CV',  linewidth=1.5)
      # marcos de início/fim (opcional)
      plt.scatter([x_r[0]], [y_r[0]], marker='o', label='início')
      plt.scatter([x_r[-1]],[y_r[-1]], marker='x', label='fim')

      plt.gca().set_aspect('equal', adjustable='box')  # escala igual em x e y
      plt.xlabel('x [m]'); plt.ylabel('y [m]')
      plt.title('XY tracking')
      plt.legend(); plt.grid(True); plt.tight_layout(); plt.show()
```



8 Speed-Steer Controller

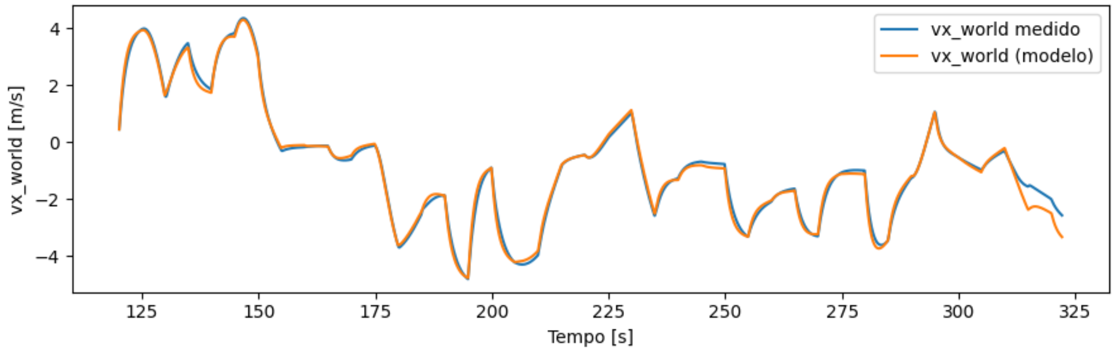
```
[ ]: def identify_bicycle_greybox_plus_speed(t, speed_cmd, steer_cmd,
                                         vx_world_meas, yaw_meas, x_meas, y_meas,
                                         v_body_meas=None,           # <-- NOVO␣
      ↪(opcional)
                                         L0=1.75, tau_v0=0.5,
                                         w_vx=5.0, w_v=2.0, w_psi=10.0, w_xy=0.2):
         import numpy as np
         from gekko import GEKKO

         t = np.asarray(t, float); t = t - t[0]
         m = GEKKO(remote=False); m.time = list(t)

         # Entradas conhecidas
         u_v = m.Param(value=list(speed_cmd))
         u_d = m.Param(value=list(steer_cmd))

         # Medidas (  deve vir unwrapped)
         vx_p  = m.Param(value=list(vx_world_meas))
```

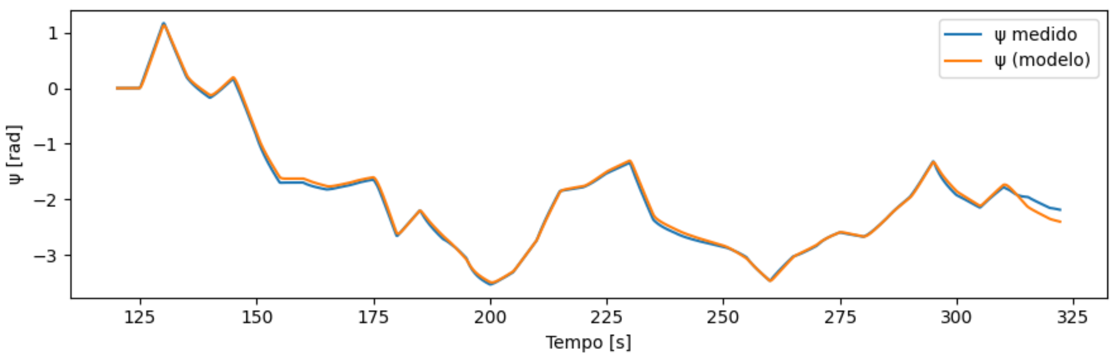
```
    psi_p = m.Param(value=list(yaw_meas))
    x_p   = m.Param(value=list(x_meas))
    y_p   = m.Param(value=list(y_meas))
    v_p   = None
    if v_body_meas is not None:
        v_p = m.Param(value=list(v_body_meas))

    # Parâmetros a estimar (FVs)
    tau_acc  = m.FV(value=0.20, lb=0.02, ub=3.0);  tau_acc.STATUS  = 1
    tau_str  = m.FV(value=0.15, lb=0.02, ub=3.0);  tau_str.STATUS  = 1
    tau_v    = m.FV(value=tau_v0, lb=0.05, ub=5.0);tau_v.STATUS    = 1
    L_eff    = m.FV(value=L0,    lb=1.0,  ub=4.0); L_eff.STATUS    = 1
    k_del    = m.FV(value=1.0,   lb=0.5,  ub=2.0); k_del.STATUS    = 1
    k_b1     = m.FV(value=0.0,   lb=-0.6, ub=0.6); k_b1.STATUS     = 1
    k_b3     = m.FV(value=0.0,   lb=-1.0, ub=1.0); k_b3.STATUS     = 1
    psi_bias = m.FV(value=0.0,   lb=-0.3, ub=0.3); psi_bias.STATUS = 1


    v   = m.Var(value=float(vx_world_meas[0]), lb=-10, ub=10)
    psi = m.Var(value=float(yaw_meas[0]))
    xg  = m.Var(value=float(x_meas[0]))
    yg  = m.Var(value=float(y_meas[0]))
    uva = m.Var(value=float(speed_cmd[0]), lb=-10, ub=10)
    da  = m.Var(value=0.0,                 lb=-0.8, ub=0.8)

    m.Equation(uva.dt() == (u_v - uva)/tau_acc)
    m.Equation(da.dt()  == (u_d - da)/tau_str)

    m.Equation(v.dt()   == (uva - v)/tau_v)
    m.Equation(psi.dt() == (v/L_eff) * m.tan(k_del*da))
    beta = m.Intermediate(k_b1*da + k_b3*da**3)
    m.Equation(xg.dt()  == v*m.cos(psi + psi_bias + beta))
    m.Equation(yg.dt()  == v*m.sin(psi + psi_bias + beta))

    vx_hat_m = m.Intermediate( v*m.cos(psi + psi_bias + beta) )
    m.Minimize(w_vx * (vx_hat_m - vx_p)**2)
    if v_p is not None:
        m.Minimize(w_v * (v - v_p)**2)      # <-- NOVO
    m.Minimize(w_psi* (psi - psi_p)**2)
    m.Minimize(w_xy * ((xg - x_p)**2 + (yg - y_p)**2))

    m.Minimize(1e-6*((tau_acc-0.2)**2 + (tau_str-0.15)**2 + (tau_v-tau_v0)**2 +
                     (L_eff-L0)**2 + (k_del-1.0)**2 + k_b1**2 + k_b3**2 +␣
↪psi_bias**2))

    # Solver
    m.options.IMODE=6; m.options.NODES=3; m.options.SOLVER=3
    m.options.SCALING=1; m.options.MAX_ITER=300
    m.solver_options = ['print_level 0','max_iter 300','tol␣
↪1e-6','acceptable_tol 1e-4']

    m.solve(disp=False)

    pars = {
        'tau_acc': float(tau_acc.VALUE[0]),
        'tau_str': float(tau_str.VALUE[0]),
        'tau_v'  : float(tau_v.VALUE[0]),
        'L_eff'  : float(L_eff.VALUE[0]),
        'k_del'  : float(k_del.VALUE[0]),
        'k_b1'   : float(k_b1.VALUE[0]),
        'k_b3'   : float(k_b3.VALUE[0]),
        'psi_bias':float(psi_bias.VALUE[0]),
    }

    # Trajetórias previstas
    v_hat    = np.fromiter(v.VALUE,   float)
    psi_hat  = np.fromiter(psi.VALUE, float)
    x_hat    = np.fromiter(xg.VALUE,  float)
    y_hat    = np.fromiter(yg.VALUE,  float)
    da_hat   = np.fromiter(da.VALUE,  float)

    # projeção para o mundo (mesma grandeza de vx_world_meas)
    beta_hat = pars['k_b1']*da_hat + pars['k_b3']*(da_hat**3)
    vx_hat   = v_hat * np.cos(psi_hat + pars['psi_bias'] + beta_hat)

    return pars, (v_hat, psi_hat, x_hat, y_hat, vx_hat)
```

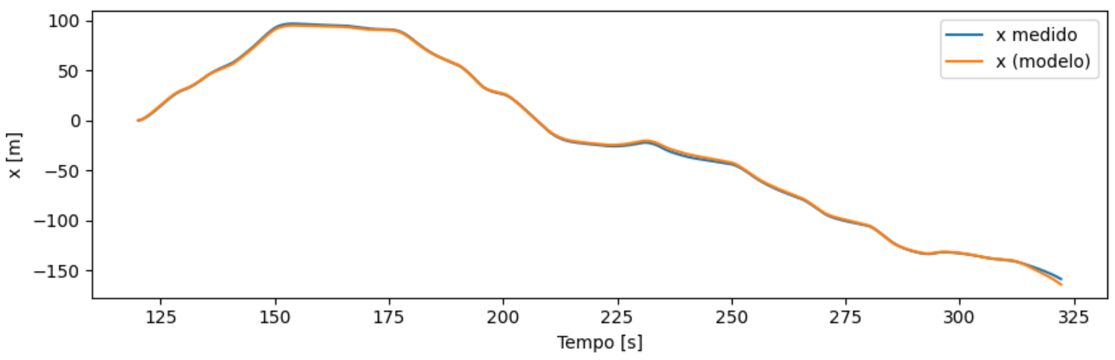
[ ]:
```
t         = np.asarray(t_st, float)
speed     = u_st['speed_cmd'].to_numpy().astype(float)
steer     = u_st['steering_cmd'].to_numpy().astype(float)
yaw_meas = np.unwrap(y_st['yaw'].to_numpy().astype(float))
x_meas    = y_st['x'].to_numpy().astype(float)
y_meas    = y_st['y'].to_numpy().astype(float)

vx_world_meas = y_st['v_measured'].to_numpy().astype(float)


mask = (np.isfinite(t) & np.isfinite(speed) & np.isfinite(steer) &
        np.isfinite(vx_world_meas) & np.isfinite(yaw_meas) &
        np.isfinite(x_meas) & np.isfinite(y_meas))
t, speed, steer, vx_world_meas, yaw_meas, x_meas, y_meas = \
    t[mask], speed[mask], steer[mask], vx_world_meas[mask], \
    yaw_meas[mask], x_meas[mask], y_meas[mask]
```

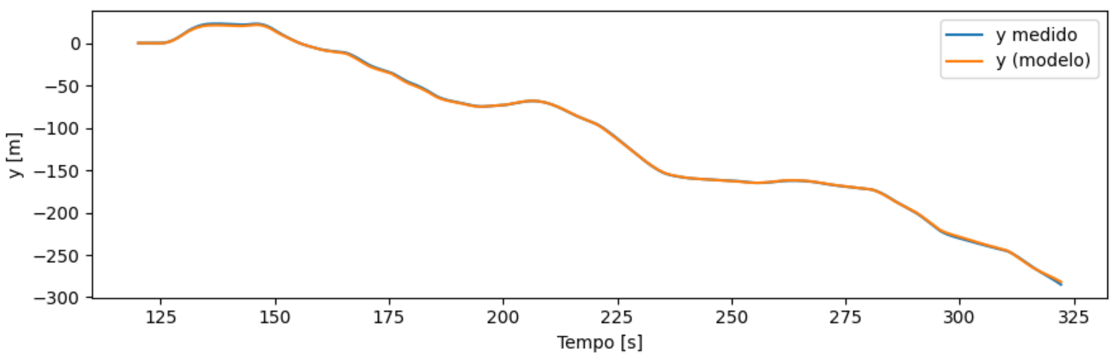
```
idx = slice(0, len(t))


vx_w = np.gradient(x_meas, t)
vy_w = np.gradient(y_meas, t)
v_body_est = vx_w*np.cos(yaw_meas) + vy_w*np.sin(yaw_meas)

pars, (v_hat, psi_hat, x_hat, y_hat, vx_hat) =␣
 ↪identify_bicycle_greybox_plus_speed(
    t=t[idx],
    speed_cmd=speed[idx],
    steer_cmd=steer[idx],
    vx_world_meas=vx_world_meas[idx],
    yaw_meas=yaw_meas[idx],
    x_meas=x_meas[idx],
    y_meas=y_meas[idx],
    v_body_meas=v_body_est[idx],
    L0=1.75, tau_v0=0.5,
    w_vx=5.0, w_v=2.0, w_psi=10.0, w_xy=0.2
)
print('Parameters:', pars)


def rmse(a,b):
    a,b = np.asarray(a), np.asarray(b)
    return float(np.sqrt(np.mean((a-b)**2)))

tt = t[idx]
print(f'RMSE vx(world): {rmse(vx_world_meas[idx], vx_hat):.3f} m/s')
print(f'RMSE  :        {rmse(yaw_meas[idx],       psi_hat):.3f} rad')
print(f'RMSE x:         {rmse(x_meas[idx],         x_hat):.3f} m')
print(f'RMSE y:         {rmse(y_meas[idx],         y_hat):.3f} m')

print(f'RMSE v(body):   {rmse(v_body_est[idx],     v_hat):.3f} m/s')

# --- 4) Plots de validação ---
plt.figure(figsize=(9,3))
plt.plot(tt, vx_world_meas[idx], label='vx_world measured')
plt.plot(tt, vx_hat,              label='vx_world (model)')
plt.xlabel('Time [s]'); plt.ylabel('vx_world [m/s]')
plt.legend(); plt.tight_layout(); plt.show()

plt.figure(figsize=(9,3))
plt.plot(tt, yaw_meas[idx], label='  measured')
plt.plot(tt, psi_hat,       label='  (model)')
plt.xlabel('Time [s]'); plt.ylabel('  [rad]')
plt.legend(); plt.tight_layout(); plt.show()
plt.figure(figsize=(9,3))
plt.plot(tt, x_meas[idx], label='x measured')
plt.plot(tt, x_hat,       label='x (model)')
plt.xlabel('Time [s]'); plt.ylabel('x [m]')
plt.legend(); plt.tight_layout(); plt.show()

plt.figure(figsize=(9,3))
plt.plot(tt, y_meas[idx], label='y measured')
plt.plot(tt, y_hat,       label='y (model)')
plt.xlabel('Time [s]'); plt.ylabel('y [m]')
plt.legend(); plt.tight_layout(); plt.show()

# Opcional: se passou v_body_est, plote também v (corpo)
plt.figure(figsize=(9,3))
plt.plot(tt, v_body_est[idx], label='v_body estimado')
plt.plot(tt, v_hat,           label='v_body (model)')
plt.xlabel('Time [s]'); plt.ylabel('v_body [m/s]')
plt.legend(); plt.tight_layout(); plt.show()
```

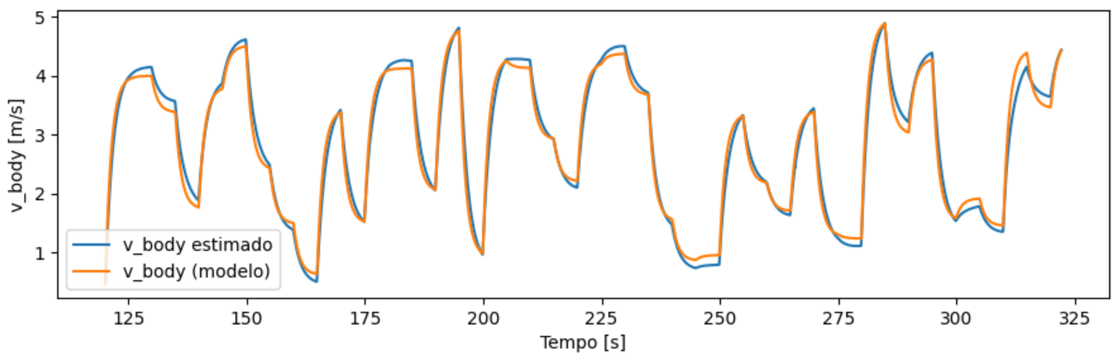
```
Parâmetros: {'tau_acc': 0.053690972223, 'tau_str': 0.36371978163, 'tau_v':
1.2296913679, 'L_eff': 1.0, 'k_del': 0.594227598, 'k_b1': 0.6, 'k_b3': 1.0,
'psi_bias': -0.014286499961}
RMSE vx(world): 0.180 m/s
RMSE  :         0.057 rad
RMSE x:          1.274 m
RMSE y:          0.821 m
RMSE v(body):    0.163 m/s
```




ψ medido
ψ (modelo)
ψ [rad]
Tempo [s]


x medido
x (modelo)
x [m]
Tempo [s]


y medido
y (modelo)
y [m]
Tempo [s]



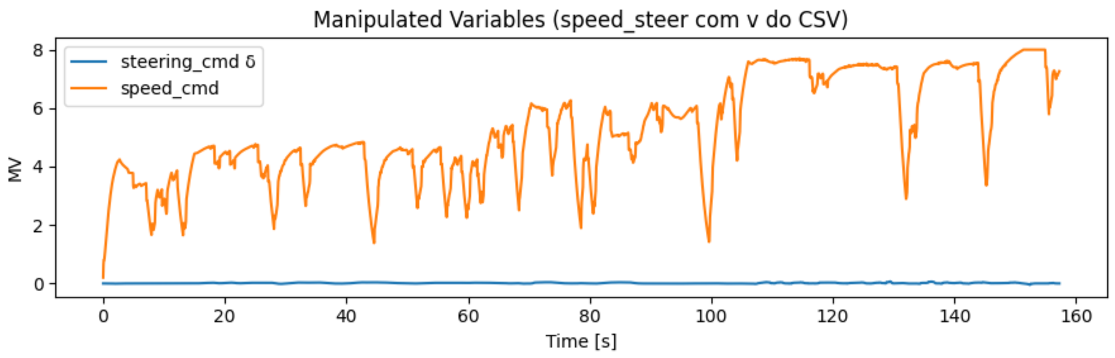
```
[ ]: def mpc_xypsi(t, pars, x_ref, y_ref, psi_ref, v_ref,
                   Ts=None, Np=20, mode='steer_only',
                   steer_bounds=(-0.6,0.6), dmax_steer=0.20,
                   spd_bounds=(0.0,8.0), dmax_spd=0.5,
                   w_pos=1.0, w_yaw=0.3, w_v=0.2,
                   w_u_steer=1e-6, w_u_spd=1e-6):
         import numpy as np
         from gekko import GEKKO
         assert mode in ('steer_only','speed_steer')

         t = np.asarray(t, float)
         if Ts is None:
             Ts = float(np.median(np.diff(t)))
         H = np.linspace(0.0, Np*Ts, Np+1)

         if pars is None:
             pars = dict(tau_acc=0.20, tau_str=0.15, tau_v=0.5, L_eff=1.75,
                         k_del=1.0, k_b1=0.0, k_b3=0.0, psi_bias=0.0)

         m = GEKKO(remote=False)
         m.time = H
         m.options.IMODE=6; m.options.NODES=2; m.options.SOLVER=3
         m.options.SCALING=1; m.options.MAX_ITER=120
         m.solver_options=['print_level 0','max_iter 120','tol 1e-6',
                           'acceptable_tol 1e-4','linear_solver ma27']

         tau_acc = m.Param(value=pars['tau_acc'])
         tau_str = m.Param(value=pars['tau_str'])
         tau_v   = m.Param(value=pars['tau_v'])
         L_eff   = m.Param(value=pars['L_eff'])
         k_del   = m.Param(value=pars['k_del'])
         k_b1    = m.Param(value=pars['k_b1'])
         k_b3    = m.Param(value=pars['k_b3'])
         psi_b   = m.Param(value=pars['psi_bias'])
```

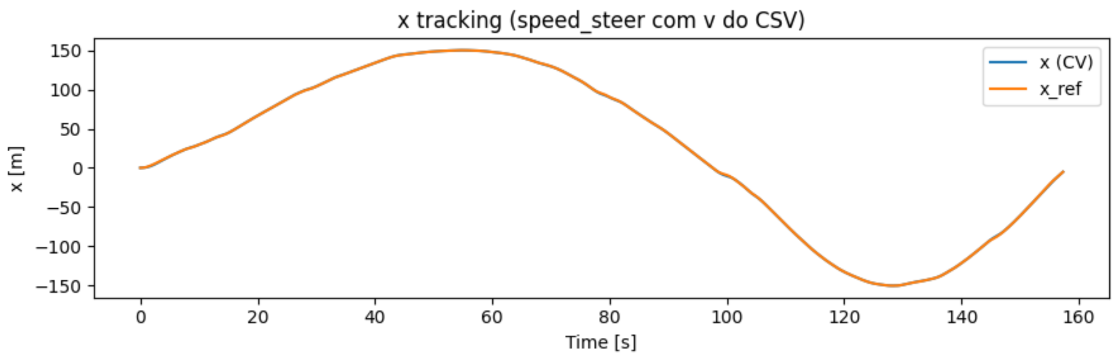
```
    x_sp   = m.Param(value=np.array(x_ref[:Np+1],   float))
    y_sp   = m.Param(value=np.array(y_ref[:Np+1],   float))
    psi_sp = m.Param(value=np.array(psi_ref[:Np+1], float))
    v_sp   = m.Param(value=np.array(v_ref[:Np+1],   float))

    v   = m.Var(value=float(v_ref[0]), lb=-10, ub=10)
    psi = m.Var(value=float(psi_ref[0]))
    xg  = m.Var(value=float(x_ref[0]))
    yg  = m.Var(value=float(y_ref[0]))
    uva = m.Var(value=float(v_ref[0]), lb=-10, ub=10)     # atuador de␣
↪velocidade
    da  = m.Var(value=0.0,             lb=steer_bounds[0], ub=steer_bounds[1])


    delt_u = m.MV(value=0.0, lb=steer_bounds[0], ub=steer_bounds[1])
    delt_u.STATUS = 1; delt_u.DMAX = dmax_steer; delt_u.DCOST = 1.0

    if mode=='speed_steer':
        spd_u = m.MV(value=float(v_ref[0]), lb=spd_bounds[0], ub=spd_bounds[1])
        spd_u.STATUS = 1; spd_u.DMAX = dmax_spd; spd_u.DCOST = 1.0
        spd_src = spd_u
    else:
        spd_param = m.Param(value=np.array(v_ref[:Np+1], float))
        spd_src   = spd_param

    m.Equation(uva.dt() == (spd_src - uva)/tau_acc)
    m.Equation(da.dt()  == (delt_u - da)/tau_str)
    m.Equation(v.dt()   == (uva - v)/tau_v)
    m.Equation(psi.dt() == (v/L_eff) * m.tan(k_del*da))
    beta = m.Intermediate(k_b1*da + k_b3*da**3)
    m.Equation(xg.dt()  == v*m.cos(psi + psi_b + beta))
    m.Equation(yg.dt()  == v*m.sin(psi + psi_b + beta))

    ex = m.Intermediate(xg - x_sp)
    ey = m.Intermediate(yg - y_sp)
    dpsi = m.Intermediate(psi - psi_sp)
    m.Minimize(w_pos*(ex**2 + ey**2))
    m.Minimize(w_yaw*(1 - m.cos(dpsi)))
    m.Minimize(w_u_steer*(delt_u**2))
    if mode=='speed_steer':
        m.Minimize(w_v*(v - v_sp)**2)
        m.Minimize(w_u_spd*(spd_u**2))

    def set_refs_and_speed(xseg, yseg, psiseg, vseg):
        x_sp.value   = np.array(xseg[:Np+1],   float)
        y_sp.value   = np.array(yseg[:Np+1],   float)
        psi_sp.value = np.array(psiseg[:Np+1], float)
        v_sp.value   = np.array(vseg[:Np+1],   float)
        if mode=='steer_only':
            spd_param.value = np.array(vseg[:Np+1], float)

    def solve_first_move(v0, psi0, x0, y0, uva0, da0):
        v.value, psi.value, xg.value, yg.value = float(v0), float(psi0),␣
↪float(x0), float(y0)
        uva.value, da.value = float(uva0), float(da0)
        head_err = float((psi_sp.value[min(5,Np)] - psi0 + np.pi)%(2*np.pi) -␣
↪np.pi)
        d_guess  = float(np.clip(head_err, steer_bounds[0], steer_bounds[1]))
        delt_u.value = [d_guess]*(Np+1)
        if mode=='speed_steer':
            spd_u.value = list(np.clip(v_sp.value, spd_bounds[0],␣
↪spd_bounds[1]))
        m.solve(disp=False)
        if mode=='speed_steer':
            return float(spd_u.NEWVAL), float(delt_u.NEWVAL)
        else:
            return float(v_sp.value[0]), float(delt_u.NEWVAL)

    return set_refs_and_speed, solve_first_move, dict(Ts=Ts, Np=Np, mode=mode)
```

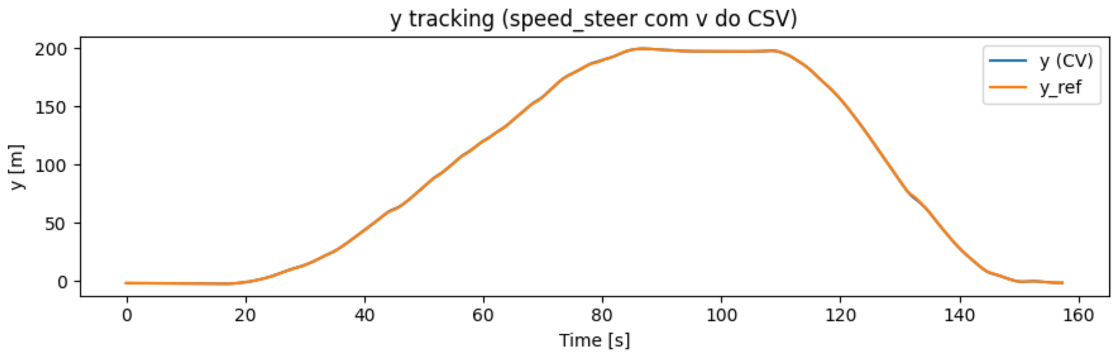
[ ]:
```
def load_wp_csv_with_speed(csv_path, Ts=0.1, use_csv_yaw=True,
                           speed_source='vref_body', v_min=0.2):
    import pandas as pd, numpy as np
    df = pd.read_csv(csv_path, header=None)
    df.columns = ['x','y','psi','v_world','vref_body']

    x   = df['x'].to_numpy(float)
    y   = df['y'].to_numpy(float)
    psi = (np.unwrap(df['psi'].to_numpy(float))
           if use_csv_yaw else np.unwrap(np.arctan2(np.gradient(y), np.
↪gradient(x))))

    v_path = df['vref_body'].to_numpy(float) if speed_source=='vref_body' \
             else df['v_world'].to_numpy(float)
    v_path = np.clip(v_path, v_min, None)

    ds   = np.hypot(np.diff(x), np.diff(y))
    dt   = ds / np.maximum(v_path[:-1], v_min)
    t_wp = np.concatenate(([0.0], np.cumsum(dt)))

    if Ts is None:
        Ts = float(np.median(np.diff(t_wp))) if len(t_wp)>1 else 0.1
    t = np.arange(0.0, t_wp[-1] + 0.5*Ts, Ts)
    x_ref   = np.interp(t, t_wp, x)
    y_ref   = np.interp(t, t_wp, y)
    psi_ref = np.interp(t, t_wp, psi)
    v_ref   = np.interp(t, t_wp, v_path)


    return t, x_ref, y_ref, psi_ref, v_ref, float(Ts)

csv_path = 'data/wps.csv'
t, x_ref, y_ref, psi_ref, v_ref_profile, Ts = load_wp_csv_with_speed(
    csv_path, Ts=0.05, use_csv_yaw=True, speed_source='vref_body'
)
print('v_ref range:', v_ref_profile.min(), '→', v_ref_profile.max())
```

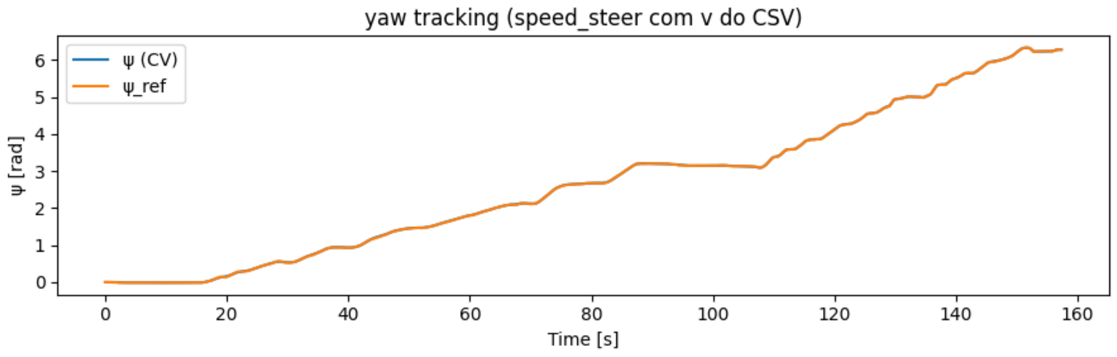
```
v_ref range: 0.2 → 8.225572457042867
```

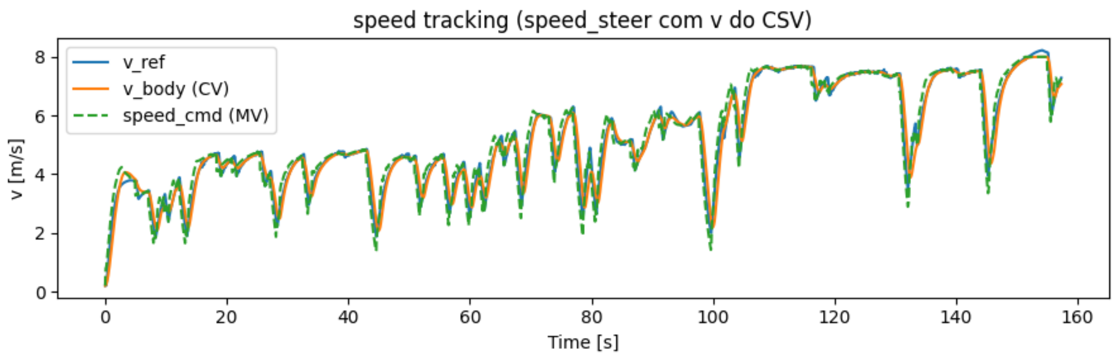
[ ]:
```
pars = None
t, x_ref, y_ref, psi_ref, v_ref_profile, Ts = load_wp_csv_with_speed(
    'data/wps.csv', Ts=0.05, speed_source='vref_body'
)

res = run_and_plot_mpc(
    t=t, x_ref=x_ref, y_ref=y_ref, psi_ref=psi_ref,
    v_ref_profile=v_ref_profile, pars=pars,
    Ts=Ts, Np=25, mode='speed_steer',
    spd_bounds=(0.0, 8.0), dmax_spd=0.6,
    w_pos=2.0, w_yaw=0.5, w_v=1.0,
    title_tag='(speed_steer with v from CSV)'
)
```

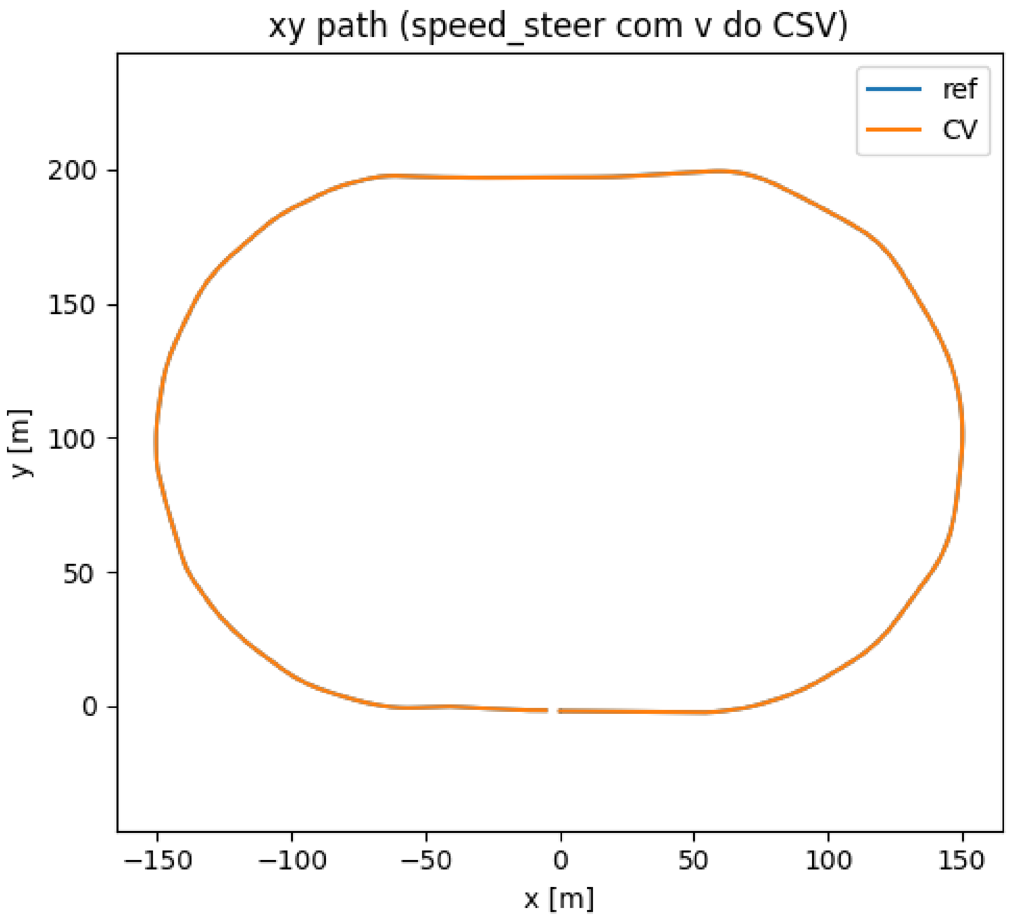



x tracking (speed_steer com v do CSV)
x (CV)
x_ref
x [m]
Time [s]
150
100
50
0
−50
−100
−150
0
20
40
60
80
100
120
140
160


y tracking (speed_steer com v do CSV)
y (CV)
y_ref
y [m]
Time [s]
200
150
100
50
0
0
20
40
60
80
100
120
140
160


yaw tracking (speed_steer com v do CSV)
ψ (CV)
ψ_ref
ψ [rad]
Time [s]
6
5
4
3
2
1
0
0
20
40
60
80
100
120
140
160


speed tracking (speed_steer com v do CSV)
v_ref
v_body (CV)
speed_cmd (MV)
v [m/s]
Time [s]
0
2
4
6
8
0
20
40
60
80
100
120
140
160


xy path (speed_steer com v do CSV)
ref
CV
y [m]
x [m]
200
150
100
50
0
−150
−100
−50
0
50
100
150